

Analysis of charm $\rightarrow PP$ based on SU(3) symmetry and final-state interactions

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It is shown that SU(3) symmetry together with final-state interactions goes a long way toward understanding all the decay modes of the kind $(D^+, D^0, D_s^+) \rightarrow PP$. The data are consistent with a small SU(3) breaking. Several predictions of the branching ratios into different decay modes are made.

A great deal of good-statistics data now exist¹⁻³ on the two-body decays of $D^0, D^+,$ and D_s^+ (old F^+) mesons. On the theoretical side considerable inroads adopting different approaches in understanding these data have been made.⁴⁻⁸ In this Brief Report we have investigated all the Cabibbo-angle-favored and the Cabibbo-angle-suppressed decays of $D^0, D^+,$ and D_s^+ with two assumptions: (a) the decay amplitudes are SU(3) symmetric^{9,10} and (b) that the final-state interactions simply rotate the amplitudes.¹¹ One advantage of this approach (as is also the case in the analogous approach of Ref. 7) is that it relates the parameters of the Cabibbo-angle-allowed and the Cabibbo-angle-suppressed decays and allows us to make fairly firm predictions within the scheme. The weakness of this approach is that SU(3) breaking enters only through final-state interactions. Furthermore, final-state interactions do not simply rotate the amplitude but also alter its magnitude through the principal part of the Omnes function¹² (or the absorptive part of the phase). This we disregard since there is no simple way to calculate such effects short of doing an intractable multichannel calculation. The general structure of the $\Delta C=1$ decay amplitude for charm decay into two pseudoscalar mesons is given by¹⁰

$$A(\text{charm} \rightarrow PP) = c(P_a^m P_m^c P^b)H_{[bc]}^a + d(P_a^m P_m^c P^b)H_{\{bc\}}^a + e(P^m P_m^b P_a^c)H_{\{bc\}}^a \quad (1)$$

We follow the notation of Ref. 10. P^a is the $C=1$ triplet (D^0, D^+, D_s^+) ; P_b^a is the pseudoscalar nonet. We ignore η - η' mixing throughout. $H_{[bc]}^a$ represents⁹ the weak spurion belonging to the 6-dimensional representation of SU(3) and $H_{\{bc\}}^a$ to the 15*-dimensional representation of SU(3). For Cabibbo-angle-favored decays $H_{bc}^a \equiv H_{31}^2$ and for Cabibbo-angle-suppressed decays $H_{bc}^a = H_{13}^3 - H_{12}^2$. The coefficients $c, d,$ and e contain the QCD correction factors. We anticipate that $(d, e) \ll c$ due to sextet dominance.

In Tables I and II we have listed all the decay amplitudes with and without final-state-interaction phases. We notice that apart from the phases all the decay amplitudes can be expressed in terms of four parameters:

$$A = c + d + \frac{e}{3}, \quad B = c - d + \frac{e}{3}, \quad r = \frac{4e}{3A}, \quad p = \frac{4e}{3B} \quad (2)$$

We are omitting the details but these parameters arise naturally from the isospin structure of the decay amplitudes. Our analysis is summarized in the following sections.

TABLE I. Cabibbo-angle-favored decays. Symbols: $A \equiv c + d + e/3, B \equiv c - d + e/3, r = (4e/3A), p = (4e/3B), \delta^{\pi K} \equiv \delta_{1/2}^{\pi K} - \delta_{3/2}^{\pi K}, \delta^{\pi\pi} \equiv \delta_0^{\pi\pi} - \delta_2^{\pi\pi}, \delta^{K\bar{K}} \equiv \delta_0^{K\bar{K}} - \delta_1^{K\bar{K}},$ FSI \equiv final-state interactions. A factor of $\cos^2\theta_C$ is omitted.

Mode	SU(3)-symmetric amplitudes	Amplitudes with FSI
$D^0 \rightarrow \pi^+ K^-$	$A(1+r/2)$	$A \exp(i\delta_{1/2}^{\pi K})[1+(r/2)\exp(i\delta^{\pi K})]$
$\rightarrow \pi^0 \bar{K}^0$	$-(1/\sqrt{2})A(1-r)$	$-(1/\sqrt{2})A \exp(i\delta_{1/2}^{\pi K})[1-r \exp(i\delta^{\pi K})]$
$\rightarrow \eta \bar{K}^0$	$-(1/\sqrt{6})A(1-r)$	$-(1/\sqrt{6})A \exp(i\delta_{1/2}^{\eta \bar{K}^0})(1-r)$
$\rightarrow \eta' \bar{K}^0$ ^a	$(2/\sqrt{3})A(1+r/8)$	$(2/\sqrt{3})A \exp(i\delta_{1/2}^{\eta' \bar{K}^0})(1+r/8)$
$D^+ \rightarrow \pi^+ \bar{K}^0$	$2e$	$2e \exp(i\delta_{3/2}^{\pi K})$
$D_s^+ \rightarrow \pi^+ \pi^0$	0	0 (forbidden by isospin)
$\rightarrow K^+ \bar{K}^0$	$-B(1-p)$	$-B \exp(i\delta_1^{K\bar{K}})(1-p)$
$\rightarrow \eta \pi^+$	$-(2/\sqrt{6})B(1+p/2)$	$-(2/\sqrt{6})B \exp(i\delta_1^{\eta\pi})(1+p/2)$
$\rightarrow \eta' \pi^+$ ^a	$-(2/\sqrt{3})B(1-\frac{5}{8}p)$	$-(2/\sqrt{3})B \exp(i\delta_1^{\eta'\pi})(1-\frac{5}{8}p)$

^aThis assumes a nonet symmetry. η - η' mixing is ignored.

TABLE II. Cabibbo-angle-suppressed decays. Symbols: As in Table I. A factor of $\sin\theta_C\cos\theta_C$ is omitted.

Mode	SU(3)-symmetry amplitudes	Amplitudes with FSI
$D^0 \rightarrow \pi^+ \pi^-$	$A(1+r/2)$	$A \exp(i\delta_0^{\pi\pi})[1+(r/2)\exp(i\delta^{\pi\pi})]$
$\rightarrow \pi^0 \pi^0$ ^a	$A(1-r)$	$A \exp(i\delta_0^{\pi\pi})[1-r \exp(i\delta^{\pi\pi})]$
$\rightarrow K^+ K^-$	$-A(1+r/2)$	$-\frac{1}{2}A(1+r/2)[\exp(i\delta_0^{K\bar{K}})+\exp(i\delta_1^{K\bar{K}})]$
$\rightarrow K^0 \bar{K}^0$	0	$-\frac{1}{2}A(1+r/2)[\exp(i\delta_0^{K\bar{K}})-\exp(i\delta_1^{K\bar{K}})]$
$\rightarrow \eta\eta$	$-A(1-r)$	$-A(1-r)\exp(i\delta_0^{\eta\eta})$
$\rightarrow \eta\pi^0$	$-(1/\sqrt{3})A(1-r)$	$-(1/\sqrt{3})A(1-r)\exp(i\delta_1^{\eta\pi})$
$\rightarrow \eta'\pi^0$ ^b	$-(2/\sqrt{6})A(1+r/8)$	$-(2/\sqrt{6})A(1+r/8)\exp(i\delta_1^{\eta'\pi})$
$\rightarrow \eta\eta'$ ^b	$\sqrt{2}A(1+r/8)$	$\sqrt{2}A(1+r/8)\exp(i\delta_0^{\eta\eta'})$
$D^+ \rightarrow \pi^+ \pi^0$	$-\sqrt{2}e$	$-\sqrt{2}e \exp(i\delta_2^{\pi\pi})$
$\rightarrow K^+ \bar{K}^0$	$-B(1+p/2)$	$-B(1+p/2)\exp(i\delta_1^{K\bar{K}})$
$\rightarrow \pi^+ \eta$	$-(2/\sqrt{6})B(1-\frac{7}{4}p)$	$-(2/\sqrt{6})B(1-\frac{7}{2}p)\exp(i\delta_1^{\pi\eta})$
$\rightarrow \pi^+ \eta'$ ^b	$-(2/\sqrt{3})B(1-\frac{5}{8}p)$	$-(2/\sqrt{3})B(1-\frac{5}{8}p)\exp(i\delta_1^{\pi\eta'})$
$D_s^+ \rightarrow \pi^0 K^+$	$(1/\sqrt{2})B(1-p)$	$(1/\sqrt{2})B \exp(i\delta_{1/2}^{\pi K})[1-p \exp(i\delta^{\pi K})]$
$\rightarrow \pi^+ K^0$	$B(1+p/2)$	$B \exp(i\delta_{1/2}^{\pi K})[1+\frac{1}{2}p \exp(i\delta^{\pi K})]$
$\rightarrow \eta K^+$	$-(1/\sqrt{6})B(1-4p)$	$-(1/\sqrt{6})B(1-4p)\exp(i\delta_{1/2}^{\eta K})$
$\rightarrow \eta' K^+$	$(2/\sqrt{3})B(1+p/8)$	$(2/\sqrt{3})B(1+p/8)\exp(i\delta_{1/2}^{\eta' K})$

^aThe phase-space is to be divided by 2.

^bThis assumes a nonet symmetry. η - η' mixing is ignored.

I. THE CABIBBO-ANGLE-ALLOWED D^0 AND D^+ DECAYS

These decays determine the parameters r and $\delta^{\pi K} \equiv \delta_{1/2}^{\pi K} - \delta_{3/2}^{\pi K}$. We use

$$R_{00} \equiv \frac{\Gamma(D^0 \rightarrow \bar{K}^0 \pi^0)}{\Gamma(D^0 \rightarrow K^- \pi^+)} = 0.455 \pm 0.086 \pm 0.049 \quad (3)$$

and

$$R_{0+} \equiv \frac{\Gamma(D^0 \rightarrow K^- \pi^+)}{\Gamma(D^+ \rightarrow \bar{K}^0 \pi^+)} = 3.316 \pm 0.405 \pm 0.445 \quad (4)$$

In evaluating R_{0+} we have used the data in Ref. 2 with $\sigma_{D^0} = 4.9 \pm 0.3 \pm 0.4$ nb and $\sigma_{D^+} = 3.6 \pm 0.4 \pm 0.4$ nb with $\sigma_{D^0}/\sigma_{D^+} = 1.36 \pm 0.136$ (systematic only) and (Ref. 13) $\tau_{D^+}/\tau_{D^0} = 2.4 \pm 0.16$ (statistical only). We find two solutions:¹⁴

$$r > 0, \quad \delta^{\pi K} = (59^\circ - 96^\circ), \quad (5)$$

$$r < 0, \quad \delta^{\pi K} = (84^\circ - 121^\circ). \quad (6)$$

In Table III we have shown the allowed range of r for different values of $\delta^{\pi K}$. Note that the new range of $\delta^{\pi K}$ in (6) replaces the earlier determination¹⁴ of $\delta^{\pi K} = (100^\circ - 145^\circ)$. The change is due to the new data² from Mark III. The sign of r is not determined by $D \rightarrow K_\pi$ data. Let us now look at other Cabibbo-angle-favored decays.

With (Ref. 13) $\tau_{D^+}/\tau_{D^0} = 2.4$ we derive

$$B(D^0 \rightarrow \eta \bar{K}^0)/B(D^+ \rightarrow \pi^+ \bar{K}^0) = 0.028[(1-r)/r]^2. \quad (7)$$

TABLE III. Parameter r as determined from R_{00} and R_{0+} . There is another solution with $r \rightarrow -r$ and $\delta^{\pi K} \rightarrow (180^\circ - \delta^{\pi K})$.

$\delta^{\pi K}$ (degrees)	r
59.6	0.36 ± 0.0
65.0	0.41 ± 0.06
70.0	0.41 ± 0.06
75.0	0.40 ± 0.06
80.0	0.40 ± 0.06
90.0	0.38 ± 0.05
95.9	0.32 ± 0.0

For $r=(0.3,0.4,0.5)$ we obtain $(0.15,0.06,0.03)$ for this ratio. For $r=(-0.3,-0.4,-0.5)$ we get $(0.52,0.34,0.25)$. Experimentally (Ref. 2), $B(D^0 \rightarrow \eta \bar{K}^0) = 1.8 \pm 0.8 \pm 0.3$ and $B(D^+ \rightarrow \pi^+ \bar{K}^0) = 3.5 \pm 0.5 \pm 0.4$. The data do not make a clear choice since the errors are large. Similarly in *nonet symmetry* $B(D^0 \rightarrow \eta' \bar{K}^0)/B(D^+ \rightarrow \pi^+ \bar{K}^0)$ for $r=(0.3,0.4,0.5)$ is $(1.91,1.1,0.72)$ while for $r=(-0.3,-0.4,-0.5)$ it is $(1.65,0.90,0.56)$. Since¹⁵ $B(D^+ \rightarrow \pi^+ \bar{K}^0)$ is $\approx 4\%$, *nonet symmetry* implies a branching ratio $B(D^0 \leftarrow \eta' \bar{K}^0) \approx 2-8\%$.

II. CABIBBO-ANGLE-SUPPRESSED D^0 DECAYS

We find, with $\tan\theta_C = 0.23$,

$$\frac{B(D^0 \rightarrow K^+ K^-)}{B(D^0 \rightarrow K^- \pi^+)} = 0.0243 \frac{(1+r/2)^2(1+\cos\delta^{K\bar{K}})}{1+r^2/4+r\cos\delta^{\pi K}}, \quad (8)$$

where $\delta^{K\bar{K}} \equiv \delta_0^{K\bar{K}} - \delta_1^{K\bar{K}}$. Experimentally¹⁶ this ratio is $0.122 \pm 0.018 \pm 0.012$. In order to reach anywhere near the experimental number one has to choose $\delta^{K\bar{K}} \approx 0$, $r > 0$, and $\delta^{\pi K}$ in the second quadrant. The choice of $\delta^{K\bar{K}} \approx 0$ is not unreasonable since the S -wave $K\bar{K}$ channel resonates¹⁵ in both $I=0$ [$f_0(975)$] and $I=1$ [$a_0(980)$] states below the D mass. Some sample values for the ratio in (8) are with $\delta^{\pi K} = 100^\circ$ and $r=(0.3,0.4,0.5)$ the ratio is $(0.066,0.072,0.077)$ while with $\delta^{\pi K} = 120^\circ$ and $r=(0.3,0.4,0.5)$ this ratio is $(0.074,0.083,0.094)$. Recall, however, that $D \rightarrow \pi K$ data had yielded $59^\circ \leq \delta^{\pi K} \leq 96^\circ$ for $r > 0$. There is thus a small mismatch in the parameters determined by $D \rightarrow K\pi$ data and those needed to fit the ratio in (8). However the mismatch is not large and could be indicative of a *small* SU(3) breaking. On the other hand, if the $D \rightarrow K\pi$ data were to change in the future such that R_{00} were to rise above the value in (3), the data would allow a larger incursion into the second quadrant for $\delta^{\pi K}$ (with $r > 0$) and thereby remove this mismatch.

We also find

$$\frac{B(D^0 \rightarrow \pi^+ \pi^-)}{B(D^0 \rightarrow K^- \pi^+)} = 0.057 \frac{1+r^2/4+r\cos\delta^{\pi\pi}}{1+r^2-2r\cos\delta^{\pi K}}, \quad (9)$$

where $\delta^{\pi\pi} = \delta_0^{\pi\pi} - \delta_2^{\pi\pi}$.

In the presence of inelasticity the phase of the decay amplitude need not be the same as the final-state scattering phase shift. However, for small inelasticities the scattering phase may serve as a guide to the phase of the decay amplitude. Some π - π phase-shift analyses¹⁷ suggest that at 1.8 GeV δ_0 is close to 180° and δ_2 close to zero. Thus a choice of $\delta_0 \approx 180^\circ$, which we make, may not be unreasonable. With $\delta^{\pi K} = 100^\circ$ and $r=(0.3,0.4,0.5)$ we get $(0.035,0.028,0.022)$ for the ratio in (9). The experimental¹⁶ value for this ratio is $0.033 \pm 0.010 \pm 0.006$. From (8) and (9) we find, as a corollary, that the ratio $B(D^0 \rightarrow K^+ K^-)/B(D^0 \rightarrow \pi^+ \pi^-)$ can be made larger than unity by choosing $r > 0$, $\delta^{K\bar{K}} \approx 0$, and $\delta^{\pi\pi} \approx 180^\circ$. With $\delta^{K\bar{K}} = 0$, $\delta^{\pi\pi} = 180^\circ$, and $r=(0.3,0.4,0.5)$ we get $(1.55,1.91,2.36)$ for this ratio. Experimentally¹⁵ this ratio is 3.5 ± 1.1 . This has been assumed in the past to signal a large SU(3) violation. Clearly within SU(3) symmetry one can go a long way toward understanding the larger branching ratio for $D^0 \rightarrow K^+ K^-$ compared to $D^0 \rightarrow \pi^+ \pi^-$. SU(3) breaking plays no role in the ratio

$B(D^0 \rightarrow \pi^0 \pi^0)/B(D^0 \rightarrow \pi^+ \pi^-)$. For $\delta^{\pi\pi} \approx 180^\circ$ and $r=(0.3,0.4,0.5)$ we get $(1.17,1.53,2.0)$ for this ratio. Furthermore for the ratio $B(D^0 \rightarrow \pi^+ \pi^-)/B(D^+ \rightarrow \pi^+ \pi^0)$ with $\tau_{D^+}/\tau_{D^0} = 2.4$, $\delta^{\pi\pi} \approx 180^\circ$ and $r=(0.3,0.4,0.5)$ we get $(0.3,0.27,0.23)$. We emphasize that the measurement of $(D^0, D^+) \rightarrow \pi\pi$ in all charged modes will determine $\delta^{\pi\pi}$ and r . A comparison of r determined from the $\pi\pi$ mode with that determined from the πK mode would serve as a measure of SU(3) breaking.

With *nonet symmetry* the branching ratios for the decay of D^0 into $\pi^0 \eta'$ and $\eta \eta'$ are surprisingly large. For $r=(0.3,0.4,0.5)$,

$$B(D^0 \rightarrow \pi^0 \eta')/B(D^0 \rightarrow \bar{K}^0 \eta) = (0.4, 0.57, 0.85), \quad (10)$$

$$B(D^0 \rightarrow \eta \eta')/B(D^0 \rightarrow \bar{K}^0 \eta) = (0.94, 1.34, 1.96). \quad (11)$$

The following ratios are independent of the parameter r :

$$B(D^0 \rightarrow \pi^0 \eta)/B(D^0 \rightarrow \bar{K}^0 \eta) = 0.11, \quad (12)$$

$$B(D^0 \rightarrow \eta \eta)/B(D^0 \rightarrow \bar{K}^0 \eta) = 0.15. \quad (13)$$

III. CABIBBO-ANGLE-SUPPRESSED D^+ DECAYS

In our model with $\tan\theta_C = 0.23$ we derive

$$\frac{B(D^+ \rightarrow \bar{K}^0 K^+)}{B(D^+ \rightarrow \pi^+ \bar{K}^0)} = 0.0216 \frac{(1+p/2)^2}{p^2}, \quad (14)$$

where p is defined in (2). Since we expect $(d,e) \ll c$ due to sextet dominance, we expect p to be approximately equal to r . For $p=(0.3,0.4,0.5)$ we get $(0.31,0.19,0.13)$ for the ratio in (14). Experimentally¹⁶ this ratio is $0.317 \pm 0.086 \pm 0.048$. For $p=(0.3,0.4,0.5)$ we also find

$$\frac{B(D^+ \rightarrow \pi^+ \eta)}{B(D^+ \rightarrow \bar{K}^0 K^+)} = (0.12, 0.04, 0.007), \quad (15)$$

$$\frac{B(D^+ \rightarrow \pi^+ \eta')}{B(D^+ \rightarrow \bar{K}^0 K^+)} = (0.56, 0.44, 0.34), \quad (16)$$

$$\frac{B(D^+ \rightarrow \pi^+ \pi^0)}{B(D^+ \rightarrow \bar{K}^0 K^+)} = (0.09, 0.15, 0.21). \quad (17)$$

Thus for $p > 0$, $D^+ \rightarrow \bar{K}^0 K^+$ is the strongest Cabibbo-angle-suppressed mode. A parameter-independent prediction is

$$B(D^+ \rightarrow \pi^+ \pi^0)/B(D^+ \rightarrow \bar{K}^0 \pi^+) = 0.028. \quad (18)$$

Combining this with the experimental determination¹⁶

$$\frac{B(D^+ \rightarrow \bar{K}^0 K^+)}{B(D^+ \rightarrow \bar{K}^0 \pi^+)} = 0.317 \pm 0.086 \pm 0.048, \quad (19)$$

we get (only the central value)

$$B(D^+ \rightarrow \pi^+ \pi^0)/B(D^+ \rightarrow \bar{K}^0 K^+) = 0.09 \quad (20)$$

in agreement with (17) with $p=0.3$

IV. CABIBBO-ANGLE-FAVORED D_s^+ DECAYS

D_s^+ decay amplitudes depend on the parameter p as do the Cabibbo-angle-suppressed D^+ decays. For $p=(0.3,0.4,0.5)$ we obtain

$$\frac{B(D_s^+ \rightarrow \bar{K}^0 K^+)}{B(D^+ \rightarrow \bar{K}^0 \pi^+)} = (0.73, 0.30, 0.13), \quad (21)$$

$$\frac{B(D_s^+ \rightarrow \pi^+ \eta)}{B(D^+ \rightarrow \bar{K}^0 \pi^+)} = (1.38, 0.85, 0.59), \quad (22)$$

$$\frac{B(D_s^+ \rightarrow \pi^+ \eta')}{B(D^+ \rightarrow \bar{K}^0 \pi^+)} = (1.13, 0.54, 0.29), \quad (23)$$

where we have used (Ref. 15) $\tau_{D_s^+}/\tau_{D^+} = 0.3$. For $p > 0$, $D_s^+ \rightarrow \pi^+ \eta$ would appear to be the strongest mode with a branching ratio of 2–6%.

V. CABIBBO-ANGLE-SUPPRESSED D_s^+ DECAYS

For $p = (0.3, 0.4, 0.5)$ we obtain, with $\tau_{D_s^+}/\tau_{D^+} = 0.3$,

$$\frac{B(D_s^+ \rightarrow \eta K^+)}{B(D^+ \rightarrow K^+ \bar{K}^0)} = (0.0016, 0.013, 0.034), \quad (24)$$

$$\frac{B(D_s^+ \rightarrow \eta' K^+)}{B(D^+ \rightarrow K^+ \bar{K}^0)} = (0.27, 0.25, 0.24), \quad (25)$$

and, with $\delta^{\pi K} = 100^\circ$,

$$\frac{B(D_s^+ \rightarrow \pi^+ K^0)}{B(D^+ \rightarrow K^+ \bar{K}^0)} = (0.26, 0.24, 0.22), \quad (26)$$

$$\frac{B(D_s^+ \rightarrow \pi^0 K^+)}{B(D^+ \rightarrow K^+ \bar{K}^0)} = (0.16, 0.16, 0.16). \quad (27)$$

Since (19) with the known branching ratio (Ref. 15) $B(D^+ \rightarrow \bar{K}^0 \pi^+) \approx 4\%$ implies $B(D^+ \rightarrow K^+ \bar{K}^0) \approx 1.3\%$, all the above four Cabibbo-angle-suppressed D_s^+ -decay modes are predicted to have branching ratios $< 1\%$.

In summary, we find that the known

$(D^0, D^+, D_s^+) \rightarrow PP$ data are quite consistent with a small SU(3) breaking. Almost all the data can be understood within an SU(3) scheme with final-state interactions. We make some parameter-independent predictions such as those in Eqs. (12), (13), and (18). We emphasize, as do the authors of Ref. 7, the need for $D \rightarrow \pi\pi$ measurements in all charged modes. This will allow a determination of $\delta^{\pi\pi}$ and the parameter r . A comparison with the same parameter r determined from $D \rightarrow K\pi$ decays would be a measure of SU(3) breaking. As we have used $\delta_0^{KK} = \delta_1^{K\bar{K}}$, $D^0 \rightarrow K^0 \bar{K}^0$ will stay strictly suppressed. $D^0 \rightarrow K^0 \bar{K}^0$ is allowed in our scheme through a mismatch $\delta_0^{KK} \neq \delta_1^{K\bar{K}}$. Among the Cabibbo-angle-suppressed $D^+ \rightarrow PP$ we find $D^+ \rightarrow K^+ \bar{K}^0$ to be the strongest mode. Among the Cabibbo-angle-allowed $D_s^+ \rightarrow PP$ we predicted $D_s^+ \rightarrow \pi^+ \eta$ to be the strongest decay channel. All our predictions involving η' are within the context of the *nonet symmetry*. Our approach is complementary to that of Refs. 7 and 18. The relation between our parameters c, d, e , and a, b, c of Chau and Cheng⁷ (identified by a subscript CC) and E, S, T of Quigg¹⁸ is

$$\begin{aligned} c &= \frac{1}{2}(a-b)_{CC} = -S, \\ d &= (c)_{CC} = E - T/2, \\ e &= \frac{1}{2}(a+b)_{CC} = \frac{5}{2}T. \end{aligned} \quad (28)$$

In writing (28) we have invoked SU(3) symmetry and set \bar{c} of Chau and Cheng⁷ equal to their c .

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¹E. H. Thorndike, in *Proceedings of the 1985 International Symposium on Lepton and Photon Interactions at High Energies*, Kyoto, Japan, 1985, edited by M. Konuma and K. Takahashi (Research Institute for Fundamental Physics, Kyoto University, Kyoto, 1986), p. 406; M. G. D. Gilchriese, in *Proceedings of the Twenty-Third International Conference on High Energy Physics*, Berkeley, California, 1986, edited by S. Loken (World Scientific, Singapore, to be published).

²R. H. Schindler, in *Supersymmetry*, Proceedings of the Thirteenth Annual SLAC Summer Institute on Particle Physics, Stanford, 1985, edited by E. C. Brennan (SLAC Report No. 296, 1986), p. 625.

³J. M. Prentice, in *Supersymmetry* (Ref. 2), p. 555.

⁴R. Rückl, in *Proceedings of the 22nd International Conference on High Energy Physics*, Leipzig, 1984, edited by A. Meyer and E. Wieczorek (Akademie der Wissenschaften, Zeuthen, East Germany, 1984), Vol. 1, p. 135.

⁵M. Bauer and B. Stech, Phys. Lett. **152B**, 380 (1985); B. Stech, in *Perspectives of Electroweak Interactions*, proceedings of the XXth Rencontre de Moriond, Les Arcs, France, 1985, edited by J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, France, 1985); Heidelberg Report No. HD-THEP-86-7 (unpublished).

⁶A. J. Buras, J.-M. Gérard, and R. Rückl, Nucl. Phys. **B268**, 16 (1986); A. J. Buras, in *Proceedings of the International Europhysics Conference on High Energy Physics*, Bari, Italy, 1985, edited by L. Nitti and G. Preparata (Laterza, Bari, 1985), p. 1035.

⁷L.-L. Chau and H.-Y. Cheng, Phys. Rev. Lett. **56**, 1655 (1986).

⁸A. N. Kamal, Phys. Rev. D **33**, 1344 (1986); A. N. Kamal and M. D. Scadron, *ibid.* **32**, 1164 (1985).

⁹M. B. Einhorn and C. Quigg, Phys. Rev. D **12**, 2015 (1975).

¹⁰A. N. Kamal and R. C. Verma, Can. J. Phys. **58**, 1734 (1980).

¹¹H. J. Lipkin, Phys. Rev. Lett. **44**, 710 (1980).

¹²J. H. Donoghue and B. R. Holstein, Phys. Rev. D **21**, 1334 (1980); A. N. Kamal and E. D. Cooper, Z. Phys. C **8**, 67 (1981); C. Sorensen, Phys. Rev. D **23**, 2618 (1981).

¹³M. G. D. Gilchriese, in *Proceedings of the Twenty-Third International Conference on High Energy Physics* (Ref. 1).

¹⁴A. N. Kamal, J. Phys. G **12**, L43 (1986); the relation between the parameter r in this paper and the parameter A_3/A_1 in Ref. 14 is $r = -\sqrt{2}A_3/A_1$.

¹⁵Particle Data Group, Phys. Lett. **170B**, 1 (1986).

¹⁶R. M. Baltrusaitis *et al.*, Phys. Rev. Lett. **55**, 151 (1985).

¹⁷C. D. Froggatt and J. L. Petersen, Nucl. Phys. **B91**, 451 (1975).

¹⁸C. Quigg, Z. Phys. C **4**, 55 (1980).