Test of the transverse magneticity of the $\xi(2.23)$

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We propose the Jacob-Wick helicity-amplitude ratios $\bar{x}=A_1/A_0$ and $\bar{y}=A_2/A_0$ for $\psi/J\to\gamma\xi$, $\xi\to K^+K^-$, as a test of the transverse magneticity of the two gluon constituents of the $\xi(2.23)$ under the assumption that the latter state is in fact a spin-2 bound state of two constituent gluons. Here A_j is the amplitude for ξ helicity j, $j=0,1,2$. We therefore encourage experimentalists to measure these ratios.

Recently, the Mark III Collaboration, operating at the SLAC e^+e^- annihilation ring SPEAR, has reconfirmed[1] their initial observation[2] of the state $\xi(2.23)$ in $\psi/J\to\gamma\xi$, $\xi\to K^+K^-,K_SK_S$. In Ref. 3, we have presented a relatively detailed discussion of the particular possibility that the $\xi(2.23)$ is in fact a TM^2 glueball in the notation of Ref. 4, where the transverse magneticity is that of a massive constituent gluon in the context of the MIT bag model.[5] In Ref. 3, we considered both the spin-0 and the spin-2 scenarios for the ξ on this TM^2 hypothesis. In what follows, we wish to propose a test of the spin-2 aspect of the TM^2 ξ scenario.

Specifically, what we wish to describe is the extension to the ξ of the analysis which we presented in Ref. 6 of the $\theta(1700)$ values[7] of $\bar{x}=A_1/A_0$ and $\bar{y}=A_2/A_0$, where A_j, $j=0,1,2$, are the three Jacob-Wick θ helicity amplitudes for the process $\psi/J\to\gamma\theta$, $\theta\to K^+K^-$. We recall from Ref. 6 that we were in fact able to show that the Mark III values of $\bar{x}$ and $\bar{y}$ for the θ are consistent with the popular interpretation of the θ as a TE^2 glueball. Consequently, we believe that it is appropriate to apply the methods used in our θ analysis to the production and the decay of the $\xi(2.23)$. We shall begin with a brief review of these methods (we refer the reader to Ref. 6 for the details of these methods).

The key inputs to our computation of $\bar{x}$ and $\bar{y}$ for a tensor glueball of two constituent gluons are the perturbative amplitude for $\psi/J\to\gamma T$, $T\to m\bar{m}$, and the nonperturbative[8] amplitude for the same process due to T-$\chi(3.555)$ mixing. Here, $m=\pi,K$. We consider each amplitude in turn.

Insofar as the perturbative amplitude is concerned, the relevant diagrams are shown in Fig. 1 and have been evaluated in Refs. 3 and 6. The resulting expressions were used[3,6] to determine the effective Lagrangians for $\psi/J\to T\gamma$ and $T\to K^+K^-$, where T is a spin-2 glueball. For the specific case of the ξ, we have (in the notation of Refs. 3 and 6) the amplitudes

$$A_P(\psi/J\to\xi\gamma)=\frac{-ieg^2}{2N_c}f_{\psi/J}f_2 m_{\psi/J}\{[m_{\psi/J}(E_\xi^{\mathrm{lab}}-E_\gamma^{\mathrm{lab}})-2m_G{}^2]/(m_{\psi/J}E_\xi^{\mathrm{lab}}/2-m_G{}^2)^2\}\epsilon^{\nu}_{\psi/J}(s_z)\epsilon^{*\mu}_{\gamma}(\lambda_\gamma)\epsilon^{*}_{\nu\mu}(\lambda_\xi) \tag{1}$$

and

$$A(\xi\to K^+K^-)=\frac{4ig^2(m_\xi{}^2)}{N_c m_\xi{}^2}f_2F_2\epsilon^{\alpha_1\alpha_2}(\lambda_\xi)P_{K^+\alpha_1}P_{K^+\alpha_2}\,, \tag{2}$$

where we take the photon to have helicity 1 for definiteness in a kinematical setup which is illustrated in Fig. 2. The quantity $F_2(m_\xi{}^2)$ is the relevant $\xi\to K^+K^-$ decay function[3] and has been computed in Ref. 3 to be $-14F_K/3$ within the framework of the methods of Lepage and Brodsky,[9] where $F_K(m_\xi{}^2)$ is the kaon electromagnetic form factor; f_2 is the ξ decay constant in the convention that

$$\langle 0\,|\,A^a_{G\lambda_1}(0)A^a_{G\lambda_2}(0)\,|\,\xi\rangle=f_2\epsilon_{\lambda_1\lambda_2}/[2E_\xi(2\pi)^3]^{1/2}\,,$$

in an obvious notation where $A^a_{G\lambda_1}$ is the gluon field. Since we are only interested in $\bar{x}$ and $\bar{y}$, f_2 and F_2 and their attendant uncertainties will cancel out of our work in this paper.

Turning now to the nonperturbative amplitude for $\psi/J\to\xi\gamma$ which is illustrated in Fig. 3, we note that there is an important difference in the evaluation of this ampli-

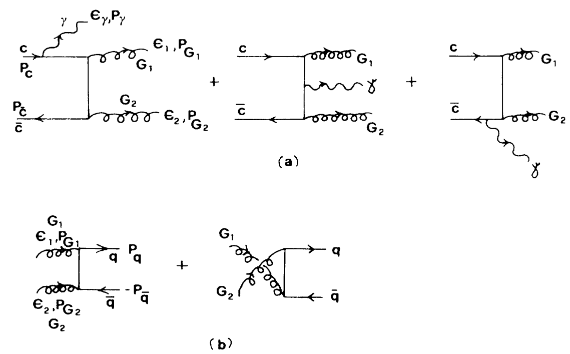

FIG. 1. (a) The process $c+\bar{c}\to G_1+G_2+\gamma$ to lowest order in g and e, where g is the QCD coupling and e is the electric charge of the positron. G_1 and G_2 are gluons, ϵ_γ and ϵ_j are polarization four-vectors, and P_A is (and will always be) the four-momentum of A, $A=c,\bar{c},\ldots$. (b) The process $G_1+G_2\to q\bar{q}$, $q=u,d,s$, to lowest order in g.

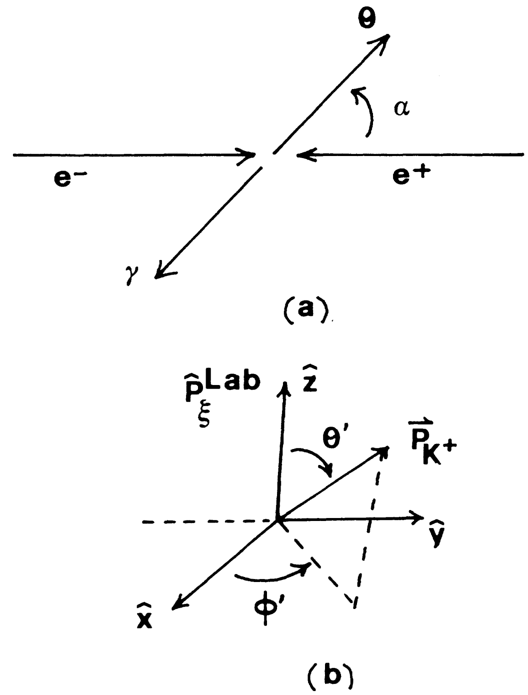

FIG. 2. Kinematics for $e^+e^- \to \psi/J$, $\psi/J \to \xi\gamma$, $\xi \to K^+K^-$. The laboratory frame is the ψ/J rest frame so that α, in (a), is the ξ production angle in this frame. In (b), the spherical angles of the K^+ momentum $\mathbf{P}_{K^+}$ in the ξ rest frame are shown. Thus, $\hat{\mathbf{P}}_\xi^{\text{lab}} = \hat{\mathbf{z}}$ is the direction of the ξ three-momentum in the laboratory frame. Thus, our kinematical conventions here follow those in Ref. 6.

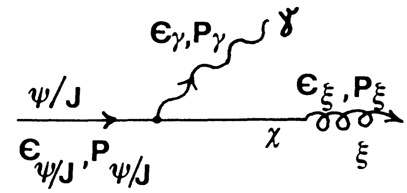

FIG. 3. The process $\psi/J \to \chi\gamma$, $\chi \to \xi$. ϵ_ξ is the ξ spin-2 polarization tensor. ϵ_γ and $\epsilon_{\psi/J}$ are the γ and ψ/J polarizations.

tude for the ξ on the one hand and for the θ on the other. The difference arises from the characteristics of the respective gluon polarizations in the relevant forward direction ($P_{G_j\perp}/P_{G_j\|} \to 0$) in the Van Royen–Weisskopf[10] limit for the χ-ξ mixing vertex in Fig. 3. It is this particular point at which the χ-ξ mixing vertex is evaluated in the spirit of the planar model of QCD.[11] We recall from Ref. 6 that, for a bound-state transverse-electric gluon G_1, the respective polarizations at this point are for $m = \pm 1$ and 0, respectively,

$$\hat{P}_{G_1 z}(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2} + (-\hat{P}_{G_1 x} \mp i\hat{P}_{G_1 y})\hat{\mathbf{z}}/\sqrt{2}\ ,$$

$$-i(\hat{P}_{G_1 y}\hat{\mathbf{x}} - \hat{P}_{G_1 x}\hat{\mathbf{y}})\ .$$

For a bound-state transverse-magnetic gluon G_1, the analogous polarizations are, for $m = \pm 1$ and 0, respectively,

$$\mp(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2},\ \ \hat{\mathbf{z}}\ .$$

On referring to the analysis in Ref. 6, we see that, due to this difference in polarization, both $\lambda_\xi = 1$ and $\lambda_\xi = 0$ amplitudes will receive a contribution from the nonperturbative process whereas only the $\lambda_\theta = 0$ amplitude was affected by our nonperturbative process.

On evaluating the process in Fig. 3 in complete analogy with the computations in Ref. 6 we find

$$A_{\text{NP}}(\psi/J \to \gamma\chi \to \gamma\xi) = \begin{cases} -3.51 A_P(\psi/J \to \xi\gamma), & \lambda_\xi = 0,\ \ \lambda_\gamma = 1\ , \\ -3.33 A_P(\psi/J \to \xi\gamma), & \lambda_\xi = 1,\ \ \lambda_\gamma = 1\ . \end{cases} \tag{3}$$

Thus, from Eq. (28) of Ref. 6 we see that the respective values of $\bar{x}$ and $\bar{y}$ are

$$\bar{x} \simeq 1.7 \ \text{ and } \bar{y} \simeq -0.98\ . \tag{4}$$

Clearly, these are different from the values $\bar{x} \simeq -0.85$ and $\bar{y} \simeq -1.0$ which we found in Ref. 6 for $\psi/J \to \theta\gamma$, $\theta \to K^+K^-$ on the TE^2 view of the θ. We therefore encourage experimentalists to try to measure the quantities $\bar{x}$ and $\bar{y}$ for the ξ. They appear to provide a clear check on the transverse magneticity of the gluons in the $\xi(2.23)$ if it is indeed a spin-2 bound state of transverse-magnetic constituent gluons.

It is interesting to recall the recent results of Bugg[12] for $\bar{x}$ and $\bar{y}$ for a tensor meson T which is composed of $q\bar{q}$, $q = u,d,s$. Specifically, working to leading order in QCD perturbation theory and to leading order in $\langle P_\perp{}^2 \rangle^{1/2}/E_\gamma$ where $\langle P_\perp{}^2 \rangle$ is the average value of the squared transverse momentum in the T, Bugg finds, for $L = 1$,

$$\bar{y} = 2\sqrt{2}\bar{x} - \sqrt{6} \tag{5}$$

and, for $L = 3$,

$$\sqrt{2}\bar{y} + \bar{x} + 1/\sqrt{3} = 0\ . \tag{6}$$

Here, L is the total-orbital-momentum eigenvalue.

We see that our results for the ξ, like those for the θ, do not satisfy the relations of Bugg. This is consistent with the fact that our theoretical models for these two states are not $q\bar{q}$ states with $L \leq 3$, and, if (4) are verified, this would support the idea that the ξ (as well as the θ) is a glueball. Clearly, more analysis will be required to rule out other scenarios in a definitive way.

In summary, we have calculated the values of $\bar{x}$ and $\bar{y}$ for the ξ in the process $\psi/J \to \gamma\xi$, $\xi \to K^+K^-$ on the view that the ξ is a TM^2 glueball. The results are sufficiently different from the analogous result for the θ and for $L = 1,3$ $q\bar{q}$ tensor mesons, $q = u,d,s$, that we feel they provide a nontrivial test of the TM^2 nature of the ξ. We await the experimental implementation of this test.

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