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Contemplating the ultimate accelerator

F. Winterberg
Desert Research Institute, University of Nevada System, Reno, Nevada 89506
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γ-ray laser action from a relativistic electron-positron plasma radiatively collapsed to nuclear densities holds the potential to accelerate particles to ultrahigh energies at high luminosities. It is shown that particle energies of $\sim 10^{16}$ GeV, where the electroweak and strong interactions are to be united, can be reached by the acceleration over a distance well within the limits set by technical feasibility. With a proton-antiproton relativistic plasma, it may even be possible to reach the Planck energy at $\sim 10^{19}$ GeV, where the unification of all forces with gravity is believed to take place.

I. INTRODUCTION

The acceleration of particles to ever higher energies is of principal importance for the future of physics. It is the tool to test new fundamental theories. In the past history of physics, the attainment of the particle energies needed to test the theories which are now part of our knowledge was always possible. With the recent advent of theories attempting to unify all the known forces of nature, the situation has completely changed. In one class of these theories, called grand unified theories, which try to unify the electroweak with the strong interaction, the energy needed for a decisive test is about $\sim 10^{16}$ GeV. To verify theories which try to unify all forces with gravity, the energy needed, called the Planck energy, is $\sim 10^{19}$ GeV. Because these energies are so large, most physicists doubt that they can ever be reached. If in fact these energies should remain unattainable forever, it would have serious consequences for the future of physics. There would then be two very different branches of physics: an applied branch, making use of all the known laws of physics in a multitude of ever increasing technologies; and a second speculative theoretical branch with an ever increasing number of theories never tested to see whether they are right or wrong (even though most of them are probably wrong). A tendency towards this rift is already clearly visible. In one area of applied-physics research, for example, the latest solid-state physics knowhow is used in the development of supercomputers. And in the theoretical speculative branch we may cite quantum gravity as a good example. If this division of physics should take place, then the first branch is going to become pure technology. And with a lack of any experimental verification, the second branch can hardly be called physics. It would become more like pure mathematics with its unlimited number of possibilities, pretending without proof to describe physical reality.

It is the purpose of this communication to suggest that the widespread belief that the enormous energies needed to test these theories can never be reached appears to be the result of a prejudice derived from our present technical inability and lack of inventive imagination. Of course, present accelerator technologies are completely unsuitable to ever reach these energies, in the same way as railroad technologies would have been completely unsuitable to reach the moon. But then the moon could be reached, though, of course, not with railroads, contradicting the wisdom and long-held majority view of the scientific community.

II. THE LIKELY PROPERTIES OF AN ULTIMATE ACCELERATOR

The only known way to reach ever higher particle energies while at the same time keeping the size of the accelerator within technical limits is the invention of new technologies with ever larger accelerating fields. Rather than asking how the application of newly available technologies, such as powerful laser beams, can lead to new accelerator concepts promising larger particle energies, we would like to ask what kind of characteristic properties an ultimate accelerator is likely to have. Knowing these properties we may then make a search for their realization.

If we assume that the accelerating field is electromagnetic, the accelerator is likely to use the largest electric fields known to exist in nature. We know they occur on the surface of an atomic nucleus, and are given by (A atomic weight, Z nuclear charge number)

$$E \simeq (Z/A^{2/3})E_0 , \tag{1}$$

where $E_0 = e/r_0{}^2 \simeq 2\times 10^{18}$ V/cm is the electric field in the vicinity of an electron at the classical electron radius $r_0 = e^2/mc^2$. For a uranium nucleus one has $E \simeq 15_0$. With an accelerating voltage of the order of E_0 assumed for the hypothetical ultimate accelerator, its length to reach $\sim 10^{16}$ GeV would be ~ 50 km, well within the limits of technical feasibility.

To have a large accelerating field gradient is not enough. The accelerator must also be able to hold the accelerated particles on a prescribed trajectory. This property is of great importance if two particle beams, accelerated in an opposite direction, are to collide head on. Furthermore, to provide for an interesting number of high-energy collisions, the particle beams must be confined in channels as narrow as possible. The radial field needed for this confinement should therefore be as large as possible. Regardless of whether the confining field is either an electric or a magnetic field, we assume that in the ultimate accelerator it too should be of the order of E_0.

III. A POSSIBLE CONFIGURATION FOR THE HYPOTHETICAL ULTIMATE ACCELERATOR

As stated, two principal properties the ultimate accelerator should be (1) a field gradient of the order $E_0 \simeq 2\times 10^{18}$ V/cm, and (2) a guide field of the same magnitude. An idea proposed by the author[1] several years ago and analyzed in considerable detail by Meierovich[2] offers itself as a solution to the second problem. The idea was to form a pinch by the coalescence of two equal counter-streaming intense relativistic electron and positron beams, with each beam having a current in excess of 17 000 A. As it turns out, such a relativistic electron-positron plasma pinch can shrink down in its diameter to nuclear dimensions. The final minimum radius is determined by the laws of quantum mechanics, with the collapsed electron-positron plasma forming a kind of linear atom. To make this collapse possible requires only a prescribed choice for the initial electron and positron particle energy, and the total electric current. The collapse itself is caused by the radiative loss of heat, from the plasma by synchrotron radiation.

The electrons and positrons form two long strings moving parallel to each other in an opposite direction. In a typical example the pinch would have a radius of the order r_0, with the distance of separation between the individual electrons and positrons to be of the order r_0. If this configuration is stable, as the theoretical analysis by Meierovich suggests, it is immediately clear that it has a large radially confining magnetic field. The collapsed electron-positron plasma string therefore provides a large confining field permitting the acceleration of particles within this string to ultrahigh energies.

To solve the first problem, that is, to provide an accelerating field of magnitude E_0, acting at the position of the bunch of particles to be accelerated, would have to be accomplished by the excitation of some kind of wave propagating down the string. In order to accelerate a bunch of particles, the phase velocity of this wave must at each moment precisely match the particle velocity. And for the wave to accelerate a particle bunch with the highest possible field gradient, its wavelength must be of the same order as the string radius.

It was previously found[3] that the relativistic electron-positron plasma can act as a γ-ray laser, provided a number of conditions are met. One condition for γ-ray laser action to take place was that it required for the plasma pinch to shrink down to a certain minimum radius. At this radius the forward and radial velocity of the electrons and positrons within the pinch channel are approximately equal. The radius at which this condition is reached depends upon (1) the initial beam radius, (2) the energy of electrons and positrons, and (3) the total current. Under a proper choice of these initial conditions it is possible to make the pinch radius at which longitudinal and transverse velocities become comparable equal to the smallest radius permitted by quantum theory. Laser action would then only set in at this smallest radius, resulting in the highest possible γ-ray flux.

Consider now a situation where the positron current is chosen smaller than the electron current, such that the electric charge density of a proton bunch being placed inside the electron-positron plasma would make up (at this position) for the deficiency in the electric charge density of the positrons, to produce (at this position) complete charge neutrality. Under complete charge neutralization, the speed of the plasma collapse is solely determined by the magnetic forces. Under incomplete charge neutralization, the force causing the collapse is reduced. Therefore, the presence of the proton bunch accelerates (at its position) the ongoing pinch collapse, and if the proton bunch moves inside the electron-positron plasma channel it will leave in its wake a more collapsed plasma pinch. If in front of the proton bunch the collapse has not reached a pinch radius at which the transverse and the longitudinal electron and positron velocities are approximately equal, but has reached this state in its wake, laser action can commence in the wake of the proton bunch, but not in its front.

The laser action is depleting the electron-positron plasma by pair annihilation, and as it appears the plasma frequency is slightly larger than the laser frequency in the region of the proton bunch where the plasma attains its highest density. The laser beam therefore acts on this region with its total radiation pressure. The radiation pressure exerts a force only on the plasma, but through the electrostatic force acting between the plasma and the proton bunch, a force is transmitted onto the bunch.

To have laser action in the region behind the proton bunch, the plasma frequency in that region must be, of course, smaller than the laser frequency. This is very likely the case, since in the absence of the space-charge neutralizing effect of the proton bunch, and also the disintegration of the electron-positron plasma by stimulated emission, the plasma density is expected to be less. And because the plasma frequency in the region of the proton bunch is only slightly larger than the laser frequency, a

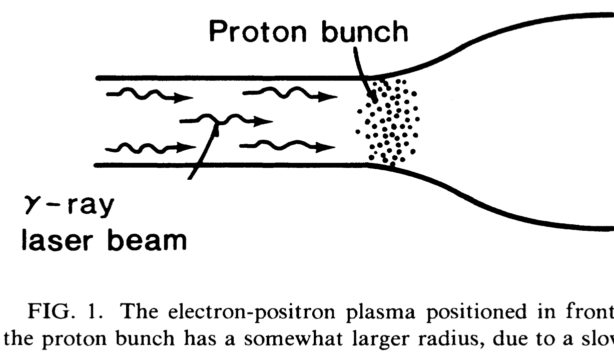

FIG. 1. The electron-positron plasma positioned in front of the proton bunch has a somewhat larger radius, due to a slower collapse. The electron-positron plasma behind the bunch assumes the right radius for laser action to commence. In the wake of the bunch the electron-positron plasma is coming apart by stimulated pair annihilation, resulting in a γ-ray laser beam which exerts its full radiation pressure on the not yet annihilated plasma, holding the proton bunch by electrostatic forces.

small decrease in the plasma density is sufficient to make the plasma transparent.

Figure 1 attempts to illustrate the suggested acceleration mechanism. The importance of this concept is that the proton bunch to be accelerated is precisely phase locked with the accelerating laser plasma wave. The phase locking is caused by the action of the proton bunch to trigger a laser wave in its wake.

In the following sections we supplement these ideas with some estimates. A final verdict regarding the feasibility of the proposed acceleration mechanism (or possible similar ones not yet found) can only be made after a much more detailed study. Nevertheless the estimates which can be presented already demonstrate that the laws of physics appear to be wide enough to permit an experimental verification of all physics. If this conjecture is true, the above-mentioned split of physics can be avoided. Physics would then remain what it always should be: an empirical science.

IV. SOME QUANTITATIVE ESTIMATES

In presenting the following estimates we rely on the various equations given in Refs. 1 and 3.

The smallest radius $r_{\min}$ the electron-positron pinch can assume is (in the ultrarelativistic limit) given by

$$\gamma m c r_{\min} \simeq \hbar \ . \tag{2}$$

To get γ-ray laser action, the total pinch current (the sum of the electron and positron current) must be

$$I = \gamma I_A \ , \tag{3}$$

where $I_A = 17\,000$ A. No currents $I > \gamma I_A$ are possible, and at $I = \gamma I_A$ the axial drift velocity of the electrons and positrons is reduced well below c, making a photon avalanche possible, moving with c along the pinch channel.

To reach $I = \gamma I_A$ at $r = r_{\min}$ requires the initial energies of the electron and positron beams to be larger, because during the pinch collapse to $r = r_{\min}$, a substantial amount of the initial beam energy is used to build up the large magnetic field surrounding the pinch. If the initial plasma radius is r_i, with the plasma formed by an electron and positron beam of the same radius, the initial γ value γ_0 needed to reach $I = \gamma I_A$ at $r = r_{\min}$, is approximately given by

$$\gamma_0 / \gamma = 0.693 + 0.384 \ln(r_i / r_{\min}) \ . \tag{4}$$

The plasma frequency ω_p of the electron-positron plasma is at its smallest radius, where $I/I_A = \gamma$, given by

$$\omega_p = \left[\frac{4\pi n e^2}{\gamma m} \right]^{1/2} = \sqrt{2} \left[\frac{c}{r_{\min}} \right] . \tag{5}$$

The frequency ω_L of the γ-ray laser beam is

$$\omega_L = \gamma m c^2 / \hbar \ . \tag{6}$$

From Eqs. (5) and (6) it follows that

$$\omega_L / \omega_p = 1/\sqrt{2} < 1 \ . \tag{7}$$

The electron-positron plasma is therefore opaque with regard to the γ-ray beam. However, only a more than twofold decrease in the plasma density is sufficient to make the plasma transparent, a condition needed for laser action. This condition is likely to be reached in the region where the space-charge neutralizing effect of the proton bunch is absent, and where also the plasma is depleted by electron-positron annihilation.

The maximum magnetic field strength at the minimum pinch radius is

$$H = 2I / r_{\min} c = 2\gamma I_A / r_{\min} c \ . \tag{8}$$

It can be expressed through the electric field $E_0 = e / r_0{}^2$ as

$$H / E_0 = 2\gamma^2 \alpha, \quad \alpha \equiv e^2 / \hbar c \ . \tag{9}$$

The length λ_0 over which the γ-ray laser beam is saturated is computed by the cross section for pair annihilation. One finds

$$\lambda_0 = 2 r_B / \gamma \ln(2\gamma) \ , \tag{10}$$

where $r_B = \hbar^2 / m e^2$ is the Bohr radius. From Eq. (10) it is clear that the laser beam can be seen as fully saturated over the entire length of the many kilometer long accelerator.

The radiation pressure p exerted by the γ-ray laser beam on the overdense ($\omega_p > \omega_L$) electron-positron plasma is computed to be

$$p = S / c \ , \tag{11}$$

where

$$S = 2 n c \gamma m c^2 \tag{12}$$

is the Poynting vector of the saturated γ-radiation flux. To compute the equivalent effective electric field acting on the plasma, caused by this radiation pressure, and hence acting the proton bunch to be accelerated, one must take into account that the γ ray penetrates the distance c / ω_p into the overdense plasma. One has therefore to put $\nabla p \simeq p \omega_p / 2c$. The effective electric field E_{eff} is then given by

$$e E_{\text{eff}} = \gamma m c^2 (\omega_p / c) \tag{13}$$

for which one can also write

$$E_{\text{eff}} / E_0 = \sqrt{2} \gamma^2 \alpha \ . \tag{14}$$

Comparing this result with Eq. (9) we see that

$$E_{\text{eff}} / H = 1/\sqrt{2} < 1 \ . \tag{15}$$

It therefore follows that the magnetic field is sufficiently strong to keep the accelerated proton bunch confined inside the pinch channel.

To make $E_{\text{eff}} = E_0$ and which would make the accelerator 50 km long to reach a proton energy of $\sim 10^{16}$ GeV, we must choose $\gamma \simeq 10$, which means the electrons and positrons must have a kinetic energy at $r = r_{\min} \simeq 4 \times 10^{-12}$ cm, of about 5 MeV, with a total pinch current equal to $I = \gamma I_A \simeq 170\,000$ A. According to Eq. (4), setting $r_i \simeq 1$ cm, this would mean that one would have to make $\gamma_0 \simeq 100$. One therefore would have to start with two 50-MeV electron and positron beams.

The electrons and positrons within the pinch channel of any radius r_b are radially confined by a magnetic force which is

$$F=-\gamma m \omega_p{}^2 r, \quad r<r_b \ . \tag{16}$$

Because of this force the electrons and positrons make radial oscillations of the frequency ω_p. These oscillations cause the emission of synchrotron radiation at the rate

$$P_e=\frac{2}{9}\frac{e^2}{c^3}\gamma^4 r^2 \omega_p{}^4 \tag{17}$$

and which results in the pinch to collapse. The collapse time is given as

$$\tau_c=\frac{9}{8}\frac{r_b{}^2}{r_0 c\gamma} \ . \tag{18}$$

To make the proton bunch accelerate the pinch collapse at the position of the bunch, the positron current has to be made smaller than the electron current which remains unchanged. If this imbalance reduces the total current by the factor $(1-\delta)$, where $\delta>0$, the attractive magnetic force is reduced by the same factor. Furthermore, for $\delta>0$, there is now also an imbalance in the electric charge neutralization, and a repulsive electrostatic force has to be added. As a result of both, the total radial force is changed from F to F' as

$$F'=(1-2\delta)F \ , \tag{19}$$

where F is given by Eq. (16). The reduction in the radial force is accompanied by a reduction in the radial oscillation frequency by the same factor $(1-2\delta)$. Clearly for $\delta=\frac{1}{2}$, there is only an electron beam left, for which (in the ultrarelativistic limit) the attractive magnetic force is compensated by the repulsive electric force. The synchrotron radiation which is proportional to $\omega_p{}^4$ is reduced by the factor $(1-2\delta)^2$, and the collapse time increased by the factor $(1-2\delta)^{-2}$.

The reduction in the positron current will of course also result in a reduction of the γ-ray laser beam flux by the factor $(1-2\delta)$, because there are now only that fewer of electron-positron pairs for annihilation available within the pinch channel. Consequently, the effective accelerating electric field is reduced by the same factor. To sustain the same effective electric field could then only be done by increasing γ.

The number of electrons per unit length of the pinch channel is

$$\pi r_b{}^2 n=\gamma/r_0 \ . \tag{20}$$

The same number of positrons would be needed to achieve complete space-charge neutralization.

If, for example, the positron beam would be reduced by making $(1-2\delta)=\frac{1}{5}$ (for which $\delta=0.4$), and γ increased $\sim$5-fold (to keep the same effective accelerating field), a proton bunch of $0.9\gamma/r_0$ protons per unit pinch length would be needed to compensate the remaining space charge. For $\gamma\simeq 100$, this is about 1.6×10^{14} protons. If the proton bunch shall be confined within the length

$$c/\omega_p\simeq r_{\min}=\hbar/\gamma mc\simeq 0.8\times10^{-12}\ \text{cm} \ ,$$

this could be done by placing within the pinch channel a heavy nucleus with a diameter of $\sim10^{-12}$ cm and having $\sim$100 protons. For $(1-2\delta)=\frac{1}{5}$, the pinch collapse time in the region outside the proton bunch would be increased $\sim$25-fold. This example shows that the proton bunch acts as a very effective catalyst to enhance the pinch collapse precisely at the position needed.

If two proton bunches are launched from two ends of a 25-km-long electron-positron plasma, they are going to collide in the middle with a center-of-mass energy equal to the energy one bunch would attain along a 50-km-long accelerator. Furthermore, since both bunches are strongly confined within the pinch channel of radius $r=r_{\min}=\hbar/\gamma mc$, the luminosity will be large.

The luminosity of two colliding bunches consisting of N protons is

$$\begin{aligned} L&=\frac{N^2\nu}{\pi r_{\min}{}^2} \\ &=(1/\pi)(mc/\hbar)^2 N^2\gamma^2\nu \\ &\simeq 2.15\times10^{20} N^2\gamma^2\nu \ , \end{aligned} \tag{21}$$

where ν is the frequency of the collisions between the bunches. If the proton bunches are made out of uranium nuclei, one has $N=92$. With $\gamma=10^2$, for example, we would have $L\simeq 2\times10^{28}\nu$. With $\gamma=10^5$, corresponding to $\sim$50-GeV electron and positron beams, and which is at the limit of what seems technically feasible, one would have $L\simeq 2\times10^{34}\nu$. Perhaps $\nu\sim 1/\text{sec}$ will be possible, which would bring the luminosity to the levels demanded by high-energy physics.

V. POSITRON FACTORIES

The number of positrons, contributing to half of the total current, is given by

$$N=\gamma l/2r_0 \ , \tag{22}$$

where l is the length of the electron-positron plasma. For $l=50$ km, $\gamma=10^2$, one finds that $N\simeq 10^{21}$ positrons would be needed.

We believe that this large number of positrons can be produced by the interaction of strongly focused laser radiation with dense matter.[4] In the focused radiation, electrons are accelerated by the strong oscillating electric fields of the laser radiation. If the electrons thereby gain an energy $\epsilon>3mc^2$, positrons can be produced by a trident process with the numbers of positrons produced given by[5]

$$N=(Z/\pi)(r_0\alpha)^2 n^2 c[\ln^3(\epsilon/mc^2)]V\tau \ , \tag{23}$$

where Z is the nuclear charge of the irradiated substance, having an atomic number density n, V the volume irradiated, and τ the irradiation time. To penetrate a solid substance, an UV laser is needed with a frequency $\omega>\omega_p\sim10^{16}\ \text{sec}^{-1}$, where ω_p is the electron plasma frequency of the solid. For the example $V\sim1\ \text{cm}^3$, $Z=90$, and $\tau \sim 10^{-8}$ sec, one would find $N \sim 10^{20}$. This is just a factor of 10 short of the number of positrons needed. To produce that many positrons would in turn require an energy of $E \sim 100$ MJ. The laser energy must be at least that large.

To produce $N \sim 10^{21}$ positrons an energy of probably several hundred gigajoules would be needed. Laser energies of this magnitude are conceivable, employing thermonuclear microexplosions.

VI. RELATIVISTIC QUARK-ANTIQUARK PLASMA AND THE PLANCK ENERGY

At the energy of $\sim 10^{16}$ GeV the unification of all interactions is still incomplete because it leaves out gravity. A unification with the gravitational interaction is believed to take place at the $\sim 10^3$ times larger energy of $\sim 10^{19}$ GeV, called the Planck energy. (It was predicted by Planck, even before his discovery of quantum theory.)

To reach the Planck energy with the hypothetical acceleration mechanism, making use of a completely collapsed relativistic electron-positron plasma, the accelerator would become 5×10^4 km long. If, however, large quantities of antiprotons can be produced, one could also make a relativistic proton-antiproton plasma. It would collapse down to a very small diameter by the same radiative loss mechanism. However, unlike electron and positrons, protons and antiprotons are not pointlike particles. The smallest diameter a relativistic proton-antiproton plasma could assume, would therefore not be determined by the uncertainty principle but rather by the radius of a nucleon, and which is of the order of $\sim 10^{-13}$ cm. At very high particle energies though, the magnetic binding energy in between the protons and antiprotons within the collapsed proton-antiproton plasma can become larger than the binding energy of the quarks and antiquarks, of which the protons and antiprotons consist. The magnetic binding energy will be larger than the internal binding energy holding the nucleons together if $\gamma > 1$, which is true for multi-GeV proton and antiproton beams.

If we introduce a strong coupling constant g, with $g^2/\hbar c \sim 1$, we have to replace $E_0 = e/r_0{}^2$ by $E_s = g/r_s{}^2$, r_0 by $r_s = g^2/Mc^2$, and the electron mass m by the proton mass M. We find

$$H/E_s = 2\gamma^2(g^2/\hbar c)(g/e) \simeq 2\gamma^2(g/e) \simeq 2\gamma^2/\sqrt{\alpha}\ . \quad (24)$$

Here, as before $\gamma = I/I_A$ but with $I_A = Mc^3/e = 3.1\times 10^7$ A. For $H/E_s > 1$, the magnetic binding energy exceeds the interquark binding energy and the collapsed state degenerates into a relativistic quark-antiquark plasma. Since the quarks are point particles, the smallest radius of the collapsed state is again determined by Heisenberg's uncertainty relation Eq. (2), replacing the electron mass m by the quark mass $m_q \simeq M/3$.

The field E_s of the strong interaction is related to E_0 by

$$E_s/E_0 = (e/g)^3(M/m)^2 = \alpha^{3/2}(M/m)^2\ . \quad (25)$$

We furthermore have, for the effective accelerating field of the strong force (with $m_q \simeq M/3$),

$$E_{s\mathrm{eff}}/E_s = (\sqrt{2}/3)\gamma^2(g^2/\hbar c) \simeq 0.47\gamma^2 \quad (26)$$

in analogy to Eq. (14). In a quark-antiquark plasma, the particles to be accelerated should also be quarks. If the strong charge of a quark of mass $M/3$ is $g/3$, then the comparable ratio of the accelerating forces is

$$gE_{s\mathrm{eff}}/eE_{\mathrm{eff}} = (M/m)^2 \simeq 3\times 10^6\ . \quad (27)$$

It therefore follows that with a collapsed proton-antiproton superpinch, the Planck energy could be already reached over a distance less than 1 km, provided a quark-antiquark plasma behaves in a similar way as an electron-positron plasma. Because the acceleration length is about ~ 1000 times shorter compared with the length of the electron-positron plasma and to reach a thousand times larger particle energy, the energy needed to generate the antiprotons is about the same. An x-ray laser driven by nuclear microexplosions could conceivably generate the number of antiprotons needed.

VII. CONCLUSION

In asking what the likely property of the ultimate accelerator might be, we have concluded that it would have an accelerating field comparable to the largest electric fields known to be realized in nature, which occur in nuclear matter. Producing these fields over long distances, a necessary requirement for an accelerator, is in principle possible with a relativistic matter-antimatter plasma, radiatively collapsed to nuclear densities. The laws of physics do not seem to prohibit that such relativistic plasmas can one day be made. More speculative is the conjectured acceleration mechanism proposed. To make a decision regarding its feasibility requires, of course, a much more detailed analysis. At the moment this question is of minor importance. More important is that relativistic matter-antimatter plasmas can assume ultrahigh densities in a configuration which might allow for a large variety of plasma wave modes suitable for acceleration. And because the expected accelerating fields are so large, the eventual attainment of energies required to test the very fundamental theories presently proposed cannot easily be denied.

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