CP violation through Higgs-boson exchange in left-right-symmetric models

D. Cocolicchio and G. L. Fogli
Dipartimento di Fisica, Università di Bari, Bari, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Bari, Italy
(Received 6 August 1986)

A chiral electroweak model which accounts for spontaneous CP breaking is proposed. Under the requirement of realistic quark mixing matrices and as a result of a detailed study of the Higgs potential, CP-violating relative phases among the vacuum expectation values of the scalars coupled to the fermions are seen to emerge. The strong CP problem is solved without any restrictive assumptions. The phenomenological implications of the neutral-kaon system are analyzed, and a new physics in the Higgs sector is proposed to render $|\Delta\mathscr{S}|=2$ enhanced interactions consistent with a not too high value of parity-breaking scale.

I. INTRODUCTION

The trail blazers for the new physics in the Higgs-boson sector pay much attention to the still unresolved thorny problem of the origin of CP violation. In several approaches CP is not supposed to be an exact symmetry of the Lagrangian (general complex Yukawa couplings) and the diagonalizing transformation of the quark mass matrix M_Q contains phases introducing CP violation.

CP noninvariance at the Lagrangian level is, however, a source of several difficulties. Going from the QFD to the QCD sector, in connection with the solution of the $U_A(1)$ problem given by 't Hooft,[1] the diagonalization of M_Q afflicts $\mathscr{L}_{\rm QCD}$ with a term of Pontrjagin form,

$$\mathscr{L}_{\rm QCD}^{CP}=\frac{\theta_{\rm QFD}}{64\pi^2}\,{\rm tr}\epsilon^{\mu\nu\rho\sigma}\mathscr{G}_{\mu\nu}\mathscr{G}_{\rho\sigma}\,,$$

where $\mathscr{G}$ is the covariant curl of color-gluon field matrix and $\theta_{\rm QFD}=\arg(\det M_Q)$. Without affecting the Euler-Lagrange field equations, it conserves C but not P and T and then CP.

To control the amount of the nontrivial effects we may add to $\mathscr{L}_{\rm QCD}^{\rm tot}$ a counterterm of the same form with a parameter $\theta_{\rm QCD}$ leading to a redefinition of the $\theta_{\rm eff}$ such that $\theta_{\rm eff}=\theta_{\rm QCD}+\theta_{\rm QFD}$. It appears indeed quite difficult to justify in a natural way the smallness $\sim 10^{-8}$ of $\theta_{\rm eff}$ as can be deduced by using the present experimental limit of the neutron electric dipole moment.[2] This mainly because in this approach, referred as "hard" CP violation, the $\theta_{\rm eff}$ parameter is subject to infinite renormalization,[3] being so a free parameter of the theory.

In the framework of the "hard" approach several proposals have been made, which use some concomitant assumptions in order to keep θ under control. Automatic solutions can be realized in Peccei-Quinn models,[4] where a cancellation $\arg(\det M_Q)\simeq-\theta_{\rm QCD}$ at least one part in 10^5 is determined by instanton effects. The realization of the same global U(1) invariance in Wigner-Weyl scenario requires unphysical massless quarks.[5] However, these models are less and less appealing for the unseen pseudoscalar axion particle[6] and for cosmological difficulties.[7]

A spontaneous-CP-breaking approach emerges as an alternative way out: a bare true value $\theta_{\rm QCD}$ as a symmetry requirement is imposed, while CP effects are likely to arise not from $\theta_{\rm QFD}$ but as higher-order corrections.

This last point of view appears quite attractive at a speculative level, if analyzed in the spirit of attributing to the symmetry-breaking mechanism those violation effects typical of the weak-interaction physics at the energy scale which characterizes the current experimental results. Within this approach the problem to obtain $\theta_{\rm QFD}=\arg(\det M_Q)<10^{-8}$ can be solved in general by imposing extra symmetries on the model to restrict the form of the quark mass matrix at the tree level. Moreover, an electroweak left-right-symmetric (LRS) extension of the standard Glashow-Weinberg-Salam model based on the gauge group

$$\mathscr{G}_{LR}^{w}={\rm SU}(2)_L\otimes{\rm SU}(2)_R\otimes{\rm U}(1)$$

can easily supply a Hermitian quark mass matrix leading to $\theta_{\rm QFD}^{\rm tree}=0$ in the framework of the so-called manifest LRS (MLRS) models.[8] Unfortunately the charge conjugation is explicitly broken in this scheme.

In the further case of pseudomanifest LRS (PMLRS) the total Lagrangian is invariant not only under parity but also under charge conjugation; so, besides gauge, all other symmetries can be set spontaneously broken. In this context, our motivation is to ascribe CP violation to the presence of right-handed (RH) currents, taking under control their potentially dangerous $\Delta\mathscr{S}=2$ enhancements due mainly to LR gauge box diagrams and flavor-changing Higgs-boson exchange.

A final comment before entering the details of the approach in the next sections. As it will appear clear in the following, the scheme involves additional discrete symmetries, whose effect[9] is to select as the most general solution of the Higgs potential $\langle\Phi_i\rangle={\rm diag}\{k_i,0\}$. The requirement $k_i'=0$ implies no W_L-W_R mixing as a zeroth-order relation with nonvanishing Cabibbo angles. A similar solution, if maintained at all orders, should account also for a zero W_L-W_R mixing at all orders, in such a way that the absence of dangerous tadpole fermion-loop contributions is guaranteed.[10] However, this is not the case in the present scheme. As we already pointed out,[9] a first-order phase transition in the parameter space takes place in the limit $k_i' \to 0$. The tree-level relations $k_i'=0$ are then unstable under renormalization because of the dynamical effects of radiative corrections. Radiative corrections at one-loop level give birth to a new symmetry-breaking pattern $0 < k_i' \ll k_i$ (a hierarchy stable under the higher-order corrections[11]). Of course, the dynamical generation of a small $W_L - W_R$ mixing does not modify the tree-level implications of the present approach.

In the next section we recall some conditions in LRS theories which have already been discussed before.[9] These conditions rule out the minimal model which cannot give nontrivial Cabibbo matrices and, as a result, we are led to consider a more complicated Higgs sector. In Sec. III the detailed study of the enlarged potential is connected with the quark mixing sector. In this investigation more than two scalars coupled to fermions are required to obtain *CP*-violating vacuum and in turn the source of *CP* violation can be ascribed also to Higgs-boson exchange. The non-Peccei-Quinn approach to solve the strong *CP* problem is described in Sec. IV. Section V contains some remarks on the consistency of not too high parity-breaking scale with the *CP* parameters in the kaon system. The last part of the paper deals with the issue of introducing six-colored triplet Higgs bosons to enhance $|\Delta\mathscr{S}|=2$ processes without imposing a high value of their intermediate M_{W_R} mass scale.

II. SPONTANEOUS *CP* VIOLATION IN LRS MODELS

Several attempts along the line of spontaneous *CP* violation can be found in the literature, the main part of them indeed based on LRS models,[12] even though the possibility of obtaining spontaneous *CP* breaking directly in $G_{ws}=\mathrm{SU}(2)_L \otimes \mathrm{U}(1)$ through the introduction of suitable discrete[13] or gauge symmetries[14] cannot be excluded. LRS models, however, can account for the appealing feature of describing both *P*-, and *CP*-violating effects as due to the same mechanism of spontaneous symmetry breaking. *CP*-violation models in LRS theories based on $\mathscr{G}_{LR}^{w}$ can be resumed in Table I.

We already saw that their most appreciable quality is that in the case of MLRS the quark mass matrix is constrained to be Hermitian at skeleton level whatever the vacuum expectation values (VEV's) of the Higgs bosons coupled to the fermions are, i.e.,

$$M_Q^{\mathrm{tree}} = (M_Q^{\mathrm{tree}})^{\dagger} , \tag{1}$$

so that

$$\theta_{\mathrm{QFD}}^{\mathrm{tree}} = \frac{i}{2} \ln \left[\frac{\det M_Q^{\mathrm{tree}}}{\det M_Q^{\mathrm{tree}\dagger}} \right] = 0 . \tag{2}$$

On the other hand, a similar solution is hopefully to be obtained in the case of PMLRS, where the mass matrix is only symmetric, under the further condition of a left-handed (LH) Cabibbo matrix real. In this case, $\theta^{1\ \mathrm{loop}}$ will

TABLE I. The status of quark mixing in chiral electroweak models is here summarized. The notation is clairvoyant though explained in the paper. The imposed parity is the usual one. The number of independent *CP*-violating phases gets reduced in MLRS because angles and phases in the RH sector are identically equal to those in the LH sector. $A_{u,d}$ are diagonal matrices of rank N with elements ± 1. *CP* as well as *P* is really spontaneously broken only in the PMLRS. In this case, in order to have positive eigenvalues for diagonalized quark mass matrices $D_{u,d}$, the matrices I_u, I_d [diagonal unitary matrices such that $I_q = \mathrm{diag}(e^{i\phi_q})$] are introduced: a large number of *CP*-violating phases follows, equal to $\frac{1}{2}N(N+1)$ for N generations.

Yukawa couplings Γ and Δ matrices	VEV's $\langle \Phi_i \rangle$	Mass matrices M_u and M_d	Diagonalizing matrices $U_{uL,R}, U_{dL,R}$	Generalized Cabibbo matrices	Kind of LRS	Relevant relations (to absorb extra phases)	*CP*-violating phases for N generations
Real	Real	Real symmetric					No *CP* violation
Hermitian	Real	Hermitian	$U_{u,dL}=U_{u,dR}$	$U_{cl}=U_{cR}$	Manifest	$U_{cR}=A_u U_{cL} A_d$	$\frac{1}{2}(N-1)(N-2)$
Real symmetric	Complex	Complex symmetric	$U_{u,dL}=U_{u,dR}^*$	$U_{cl}=U_{cR}^*$	Pseudomanifest	$U_{cR}=I_u^* U_{cR} I_d$	$\frac{1}{2}N(N-1)$
Complex symmetric	Complex	Complex			Nonmanifest		$N(N-1)$

be hierarchical suppressed at the order of RH currents, differently from both the questionable Weinberg model[15] and the case of MLRS (Ref. 16) (where the first nonvanishing contribution to $\theta_{\rm QFD}$ comes from two-loop graphs). It is tantalizing to look for this alternative approach to the CP problem in which the following occurs.

(a) CP is spontaneously broken (i.e., it is a good symmetry at the Lagrangian level) with real Yukawa couplings and complex scalar VEV's. Then the generalized Cabibbo mixing matrices satisfy $U_{cR}=I_u^\dagger U_{cL}^* I_d$ and we may choose U_{cL} real and $U_{cR}=O_{cL}I_d$ where $I_{u,d}$ are diagonal unitary matrices. Since CP is spontaneously broken, the phases in U_{cR} can be also given in terms of the single relative phase between the two VEV's present in this extended model.

(b) The symmetry-breaking pattern assures a correct strength to the CP-violating parameter $\theta_{\rm QFD}$.

(c) The difficulties inherent in Weinberg's model can be overcome without specific assumptions about the parameters.

The aim of this paper is to show that the previous points can be achieved in the framework of a recently proposed approach[9] to the problem of natural flavor conservation (NFC) restricted to the lower-energy sector of the theory. Let us require the total Lagrangian invariant under a discrete symmetry (to be seen as possible relics of higher unification gauge symmetries)

$$D:\begin{cases}\Phi_1\to e^{+i\pi/2}\Phi_1\ ,\\ \Phi_2\to e^{-i\pi/2}\Phi_2\ ,\end{cases} \tag{3}$$

the appropriate transformation for fermions being $Q_{iL}\to e^{i\pi/2}Q_{iL}, Q_{iR}\to Q_{iR}$ $(i=1,\ldots,N)$. When D is added to the usual LR symmetry

$$\begin{aligned}&R\text{ fields}\to L\text{ fields}\ ,\\ &\Phi_i\to\Phi_i^\dagger\ ,\\ &(\widetilde{\Phi}_i\to\widetilde{\Phi}_i^\dagger)\ ,\end{aligned} \tag{4}$$

the minimal Higgs content which does not make meaningless the Cabibbo mixing is represented by two $(\frac{1}{2},\frac{1}{2},0)$ Higgs multiplets Φ_1,Φ_2 coupled to the quarks, to be added to the usual $\chi_L(\frac{1}{2},0,1)$, $\chi_R(0,\frac{1}{2},1)$ (whose role is of generating the spontaneous breakdown of parity). Let us recall that,[9] because of the symmetries imposed to the Lagrangian, the solution

$$\langle\Phi_i\rangle=\begin{bmatrix}k_i & 0\\ 0 & 0\end{bmatrix},\quad i=1,2 \tag{5}$$

appears as the most general one, since it corresponds to the largest arbitrariness of the parameters present in the potential. More specifically, solution (5) is the only solution which, preserving electromagnetic gauge invariance, is able to introduce the typical observed $V-A$ structure as a consequence of the spontaneous symmetry breaking, without imposing specific restrictions to the parameters of the potential. On the other hand, it is possible to show that (5) is just the solution which (a) ensures NFC in the lower sector, since it preserves the Higgs-induced flavor-changing neutral current (FCNC) from appearing to the same degree of suppression which characterizes RH currents, (b) leads to a vanishing $W_L^\pm - W_R^+$ mixing at the tree level, where the charged gauge vector mesons $W_L^\pm, W_R^\pm$ correspond then to the physical states, and (c) does not despoil of meaning the generalized Cabibbo mixing.

A simple assumption about the origin of the masses of the lightest quarks allows for a reasonable determination of the mixing angles, both for the two- and three-generation case, preserving Cabibbo universality (once CP is supposed to be spontaneously broken).

The main point we will discuss in the following sections is the possibility of accounting for a reasonable pattern of spontaneous CP violation in the framework of the previous approach.

III. *CP*-CONSERVING AND *CP*-VIOLATING SOLUTIONS

Let us analyze the possible solutions to the CP-violation problem in the framework of the approach we have proposed.

As usual the Higgs-induced CP-violating effects can be parametrized in terms of transition propagators[16] which describe the Higgs-boson exchange in an effective interaction picture of the Yukawa part of the Lagrangian. Accordingly with the imposed symmetries

$$\mathscr{L}_Y=\overline{Q}_{iL}(\Gamma_1^{ij}\Phi_1+\Delta_2^{ij}\widetilde{\Phi}_2)Q_{jR}+\text{H.c.}\ , \tag{6}$$

where, if suitable transformation properties under CP for all the involved fields are assumed, the Yukawa constant matrices Γ_1, Δ_2 can be taken as real and symmetric because of CP invariance of the total Lagrangian. The fermion mass matrices, generated as usual by spontaneous symmetry breaking (SSB), become

$$\begin{aligned}M_u&=k_1\Gamma_1\ ,\\ M_d&=k_2^*\Delta_2\ .\end{aligned} \tag{7}$$

In general these matrices are diagonalized by biunitary transformations

$$\begin{aligned}U_{uL}^\dagger M_u U_{uR}&=D_u\ ,\\ U_{dL}^\dagger M_d U_{dR}&=D_d\ .\end{aligned} \tag{8}$$

The physical states are then given by

$$Q_{u,dL,R}^{(s)}=U_{u,d\,L,R}\,Q_{u,d\,L,R}^{(w)}\ . \tag{9}$$

The generalized LH and RH Cabibbo mixing matrices are defined as

$$\begin{aligned}U_{cL}&=U_{uL}^\dagger U_{dL}\ ,\\ U_{cR}&=U_{uR}^\dagger U_{dR}\end{aligned} \tag{10}$$

and they are present in the part of the Lagrangian which describes the interaction of the gauge bosons with fermions:

$$\mathscr{L}_{fW}=i\frac{g}{\sqrt{2}}(\bar{Q}^{(w)}_{uL}\gamma^\mu Q^{(w)}_{dL}W^{(+)}_{L\mu}+\bar{Q}^{(w)}_{uR}\gamma^\mu Q^{(w)}_{dR}W^{(+)}_{R\mu})+\text{H.c.}$$
$$=i\frac{g}{\sqrt{2}}(\bar{Q}^{(s)}_{uL}\gamma^\mu U_{cL}Q^{(s)}_{dL}W^{(+)}_{L\mu}+\bar{Q}^{(s)}_{uR}\gamma^\mu U_{cR}Q^{(s)}_{dR}W^{(+)}_{R\mu})+\text{H.c.} \quad (11)$$

The divergent contributions of the renormalized coupling constants g_L and g_R are not different, and a single g can be used, provided that quartic couplings like tr $(\Phi\Phi^\dagger)(\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R)$ are chosen in the LRS form. This prevents that quartic couplings could induce a horizontal Ward identity between the SU(2)'s and makes infinite renormalizations to the couplings. By removing the maximal number of phases[17] from the LH mixing matrix U_{cL}, the number of independent and arbitrary phases in $U_{cL,R}$ is[18,19]

$$\tfrac{1}{2}(N-1)(N-2) \text{ for } U_{cL} \,,$$
$$\tfrac{1}{2}N(N+1) \text{ for } U_{cR} \quad (12)$$

(no W_L-W_R mixing eliminates an overall phase for U_{cR}).

Having assumed spontaneous P and CP violation (PMLRS), in general the following relations hold:

$$U_{cR}=I_u^\dagger U_{cL}^* I_d \quad (13)$$

and

$$U_{uR}=U_{uL}^* I_u \,,$$
$$U_{dR}=U_{dL}^* I_d \,, \quad (14)$$

where I_u,I_d are unitary diagonal matrices with elements $\exp(i\phi_q)$, $q=u,c,t,d,s,b$: here I_u,I_d cannot be rotated away via a suitable redefinition of RH quark phases like in SU(2)⊗U(1) (Ref. 20).

Real U_{cL} follows easily imposing the standard condition for CP invariance:[21]

$$\det[M_uM_u^\dagger,M_dM_d^\dagger]=0 \,, \quad (15)$$

though, in this case, phases, determined by the external legs in the usual CP-violating $\Delta\mathscr{S}=2$ gauge interactions cannot be excluded. In fact, even if a real U_{cL} allows to remove all complex phases in LH charged currents by changing the phases of the quark fields and reducing in this way the Kobayashi-Maskawa (KM) mechanism only to the RH sector, CP violation cannot be prevented to arise through W_R gauge interactions. Moreover, if U_{cL} is real, as we shall consider deeper afterwards, the contributions $\Delta\mathscr{S}=1$ arising from gauge interactions are characterized by a vanishing relative phase between $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ amplitudes: the well-known CP-violating parameter ϵ' satisfies $\epsilon'=0$.

Since in PMLRS $M_{u,d}$ are symmetric matrices, the relations $M_{u,d}M_{u,d}^T=M_{u,d}^TM_{u,d}$ yield $[\text{Re}M_{u,d},\text{Im}M_{u,d}]=0$. Moreover, $MM^\dagger=(M^\dagger M)^*$ holds instead of the usual MLRS relation $MM^\dagger=M^\dagger M$. For mass matrices (7) then it follows $U_{cL}=O_{cL}$ and the RH phases can be factorialized. Further, when one is able to reduce M_u to be real, without any lack of generality the following remarkable form is obtained:

$$O_u^TM_uO_u=D_u \,,$$
$$O_d^TM_dO_dI_d=D_d \,, \quad (16)$$

with I_d again diagonal unitary.

On the other hand, the Yukawa part of the theory can be rewritten explicitly in terms of the couplings of the "strong" quarks with neutral and charged Higgs bosons. Both neutral and charged Higgs bosons are involved in

$$\mathscr{L}_Y=\mathscr{L}_Y^{(0)}+\mathscr{L}_Y^{(\pm)} \,,$$

where

$$\mathscr{L}_Y^{(0)}=\left[\bar{Q}^{(s)}_{uL}D_uQ^{(s)}_{uR}\frac{\phi_1^{(0)}}{k_1}+\bar{Q}^{(s)}_{dL}D_dQ^{(s)}_{dR}\frac{\psi_1^{(0)*}}{k_2^*}\right]_{\text{NFC}}$$
$$+\left[\bar{Q}^{(s)}_{uL}U_{cL}D_dU_{cR}^\dagger Q^{(s)}_{uR}\frac{\psi_2^{(0)*}}{k_2^*}+\bar{Q}^{(s)}_{dL}U_{cL}^\dagger D_uU_{cR}Q^{(s)}_{dR}\frac{\phi_2^{(0)}}{k_1}\right]_{\text{FCNC}}+\text{H.c.} \,, \quad (17)$$

$$\mathscr{L}_Y^{(\pm)}=\bar{Q}_{uL}D_uU_{cR}Q_{dR}\frac{\phi_1^{(+)}}{k_1}+\bar{Q}_{dL}U_{cL}^\dagger D_uQ_{uR}\frac{\phi_2^{(-)}}{k_1}-\bar{Q}_{uL}U_{cL}D_dQ_{dR}\frac{\psi_2^{(+)}}{k_2^*}-\bar{Q}_{dL}D_dU_{cR}^\dagger Q_{uR}\frac{\psi_1^{(-)}}{k_2^*}+\text{H.c.} \quad (18)$$

The total Lagrangian is still CP conserving, but a CP-violating effect may enter into play if the charged-Higgs-boson mixing is not real (and the phases cannot be reabsorbed by a redefinition of the fields) as a consequence of the complex values of the VEV's. In the last formula $U_{cL}=O_{cL}$ and $U_{cR}=O_cI_d$. Starting from $\mathscr{L}_Y^{(\pm)}$ each charged-Higgs-boson exchange can be described as an effective four-fermion interaction multiplied by a transition propagator depending on the charged-Higgs-boson mixing which, as said above, may introduce a CP-violating imaginary part.

If the Higgs content is restricted to two Higgs fields Φ_1,Φ_2 alone (besides χ_L,χ_R responsible of the spontaneous P violation), then the symmetry-breaking pattern required by the D and L-R symmetries and ensuring NFC in the low-energy sector of theory, does not allow for an Higgs-induced CP-violating term. The most general renormalizable potential compatible with gauge invariance, D and L-R symmetries can be written in the form

$$V=V_0(\chi_L,\chi_R,\Phi_1,\Phi_2)+\sum_{i=1,2}V_i(\Phi_i)+V_{12}(\Phi_1,\Phi_2)\ , \tag{19}$$

where

$$V_0(\chi_L,\chi_R,\Phi_1,\Phi_2)=-\mu^2(\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R)+\tfrac{1}{4}\rho_1(\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R)^2-\tfrac{1}{4}\rho_2(\chi_L^\dagger\chi_L-\chi_R^\dagger\chi_R)^2$$
$$+\sum_{i=1,2}[\alpha_i\mathrm{tr}(\Phi_i^\dagger\Phi_i)(\chi_L^\dagger\chi_L+\chi_R^\dagger\chi_R)+\beta_i(\chi_L^\dagger\Phi_i\Phi_i^\dagger\chi_L+\chi_R^\dagger\Phi_i^\dagger\Phi_i\chi_R)]\ , \tag{20}$$

$$V_i(\Phi_i)=-\mu_i{}^2\mathrm{tr}(\Phi_i^\dagger\Phi_i)+\tfrac{1}{2}\lambda_1^{(i)}[\mathrm{tr}(\Phi_i^\dagger\Phi_i)]^2+\tfrac{1}{16}\lambda_2^{(i)}[\mathrm{tr}(\Phi_i^\dagger\widetilde{\Phi}_i)+\mathrm{tr}(\widetilde{\Phi}_i^\dagger\Phi_i)]^2+\tfrac{1}{16}\lambda_3^{(i)}[\mathrm{tr}(\Phi_i^\dagger\widetilde{\Phi}_i)-\mathrm{tr}(\widetilde{\Phi}_i^\dagger\Phi_i)]^2\ , \tag{21}$$

$$V_{12}(\Phi_1,\Phi_2)=\sigma_1\mathrm{tr}(\Phi_1\Phi_1^\dagger)\mathrm{tr}(\Phi_2\Phi_2^\dagger)+\tfrac{1}{4}\sigma_2[\mathrm{tr}(\Phi_1\widetilde{\Phi}_2^\dagger)+\mathrm{tr}(\Phi_1^\dagger\widetilde{\Phi}_2)]^2+\tfrac{1}{4}\sigma_3[\mathrm{tr}(\Phi_1\widetilde{\Phi}_2^\dagger)-\mathrm{tr}(\Phi_1^\dagger\widetilde{\Phi}_2)]^2$$
$$+\tfrac{1}{4}\sigma_4[\mathrm{tr}(\Phi_1\Phi_2^\dagger)+\mathrm{tr}(\Phi_1^\dagger\Phi_2)]^2+\tfrac{1}{4}\sigma_5[\mathrm{tr}(\Phi_1\Phi_2^\dagger)-\mathrm{tr}(\Phi_1^\dagger\Phi_2)]^2$$
$$+\tfrac{1}{4}\sigma_6[\mathrm{tr}(\Phi_1\widetilde{\Phi}_1^\dagger)\mathrm{tr}(\Phi_2\widetilde{\Phi}_2^\dagger)+\mathrm{tr}(\Phi_1^\dagger\widetilde{\Phi}_1)\mathrm{tr}(\Phi_2^\dagger\widetilde{\Phi}_2)]$$
$$+\tfrac{1}{4}\sigma_7[\mathrm{tr}(\Phi_1\widetilde{\Phi}_1^\dagger)\mathrm{tr}(\Phi_2^\dagger\widetilde{\Phi}_2)+\mathrm{tr}(\Phi_1^\dagger\widetilde{\Phi}_1)\mathrm{tr}(\Phi_2\widetilde{\Phi}_2^\dagger)]+\sigma_8\mathrm{tr}(\Phi_1\Phi_1^\dagger\Phi_2\Phi_2^\dagger)\ . \tag{22}$$

The minimum conditions coming from V of Eq. (19) are easily written down. The one relevant to our purposes concerns the relative phase α of k_1,k_2. Without loss of generality we can assume (with h_1,h_2,α real parameters)

$$k_1=h_1e^{i\alpha},\quad k_2=h_2\ , \tag{23}$$

and so

$$\frac{\partial V}{\partial\alpha}=-\tfrac{1}{2}(\sigma_4+\sigma_5)\sin2\alpha\ , \tag{24}$$

leading to two possible sets of solutions (n integer):

$$\alpha=n\pi\ , \tag{25}$$

$$\alpha=(2n+1)\frac{\pi}{2}\ . \tag{26}$$

Solution (25) [(26)] corresponds to the absolute minimum if $\sigma_4+\sigma_5$ is positive [negative]. If both k_1, k_2 are real, solution (25), clearly CP violation is impossible. Let us then consider the case (26) as a possible source of spontaneous CP breaking. A detailed analysis is performed in Appendix A, where the Higgs-boson mass matrices are explicitly worked out. Here the main points are reported. The light and heavy Higgs sectors appear to be disconnected. For both sectors the (mass)^2 matrices are not real, and their diagonalization is obtained through unitary matrices. However, a proper redefinition on the Higgs fields allows to reabsorb the phase and make the (mass)^2 matrices real. Indeed, the only nonzero transition propagators (taken at zero-momentum transfer) which enter to characterize the charged-Higgs-boson exchange in $\mathscr{L}_Y^{(\pm)}$ of Eq. (18) are

$$A_1(0)=-\frac{1}{k_1k_2^*}\langle 0|T(\phi_1^{(+)}\psi_1^{(-)})|0\rangle_{q^2=0}\ , \tag{27a}$$

$$A_2(0)=-\frac{1}{k_1k_2^*}\langle 0|T(\phi_2^{(-)}\psi_2^{(+)})|0\rangle_{q^2=0}\ , \tag{27b}$$

$i=1$ ($i=2$) identifying the heavy (light) Higgs sector. Both are seen to satisfy

$$\mathrm{Im}A_i(0)=0\quad(i=1,2) \tag{28}$$

so that there is no Higgs-induced CP violation.

A different way of analyzing the two Higgs case, with the same conclusion, of course, is based on the following argument. As said before, it is possible to assume suitable transformation properties under CP for all the involved fields, accordingly with the choice of real Γ_1,Δ_2 in $\mathscr{L}_Y$ [Eq. (6)]: as far as the Higgs fields are concerned, we can consistently assume that, under CP,

$$\begin{matrix}\Phi_1\to\Phi_1^*\\ \Phi_2\to\Phi_2^*\end{matrix}\ \text{ if }\alpha=n\pi \tag{29}$$

and

$$\begin{matrix}\Phi_1\to-\Phi_1^*\\ \Phi_2\to+\Phi_2^*\end{matrix}\ \text{ if }\alpha=(2n+1)\frac{\pi}{2}\ , \tag{30}$$

by connecting in this way the transformation properties of the Higgs fields to the specific set of solutions we are dealing with: in both cases CP invariance is guaranteed at Lagrangian level. It is easy now to verify the following.

(i) In both cases [accordingly with the choice fixed by the relations (29) and (30)] the corresponding symmetry-breaking pattern is such that the VEV's do not violate the CP transformation properties of the Higgs fields: the Georgi-Pais theorem[22] can then be invoked in order to conclude that CP will be conserved to any order of the perturbation expansion.

(ii) A redefinition of the Φ_1 field ($\Phi_1\to i\Phi_1$) allows to reconduce (30) (and the corresponding symmetry-breaking pattern) to the simplest case given by (29).

In order to allow for a spontaneous CP violation, the Higgs content must be enlarged to at least three Higgs fields of Φ type. It is an easy matter to prevent the third

Higgs field Φ_3 from coupling to the quarks through a suitable reflection symmetry. Moreover, as far as the D symmetry is concerned, let us now suppose and then support $\Phi_3 \to e^{-i\pi/2}\Phi_3$.

In this way NFC in the low-energy sector is ensured despite the enlargement of the Higgs content: the most general symmetry-breaking pattern is still realized in the form (5), with $i=1,2,3,\ldots$. The most general Higgs potential can be written as

$$V = V_0(\chi_L,\chi_R,\Phi_1,\Phi_2,\Phi_3) + \sum_i V_i(\Phi_i) + \sum_{\text{cycl}} V_{ij}(\Phi_i,\Phi_j) \quad (i,j=1,2,3) , \tag{31}$$

which appears to be the obvious generalization of the form (19). A detailed analysis in given in Appendix B: here the main points are briefly summarized. D symmetry dictates the symmetry-breaking pattern

$$\langle \Phi_i \rangle = \begin{bmatrix} k_i & 0 \\ 0 & 0 \end{bmatrix} , \tag{32}$$

where $k_i = h_i e^{i\alpha_i}$ $(i=1,2,3)$, with h_i and α_i as real parameters.

By imposing the usual minimum conditions two sets of solutions can be found concerning the relative phases:

$$\sin 2(\alpha_i - \alpha_j) = 0 \quad \text{if } \sigma_{12}\sigma_{13}\sigma_{23} < 0 , \tag{33}$$

which does not lead to CP violation, and a CP-violating solution, which can be written in the form

$$\sigma_{12}\sigma_{13}{k_1}^2 + \sigma_{12}\sigma_{23}{k_2}^2 + \sigma_{13}\sigma_{23}{k_3}^2 = 0 , \tag{34}$$

corresponding to the case

$$\sigma_{12}\sigma_{13}\sigma_{23} > 0 . \tag{35}$$

In the above equations $\sigma_{ij} = \sigma_4^{(ij)} + \sigma_5^{(ij)}$ with $i,j=1,2,3$ and $i \neq j$, the $\sigma_k^{(ij)}$'s, defined in Appendix B, being a generalization of those appearing in Eq. (22). The second set of solutions [Eqs. (34) and (35)] induces an imaginary part in the transition propagators. The light and heavy Higgs sectors are disconnected, the main source of CP-violation arising obviously from the first one. The Yukawa part of the Lagrangian is still given by Eq. (26): by calculating the imaginary part of the light-Higgs-boson-exchange propagator $A_2(0)$, given by Eq. (27b), it is easily found that

$$\operatorname{Im} A_2(0) = -\frac{1}{{M_1}^2{M_2}^2}\frac{1}{2}\sigma_{12}\sin 2(\alpha_1-\alpha_2) , \tag{36}$$

with M_1 and M_2 being the light-Higgs-boson masses.

Let us conclude this section with the following remark. From the analysis of Appendix B, it may appear that the CP-violating factor (36) written in the form (B29) vanishes in the limit $M_1 = M_2$. The situation is, however, rather different: it is the specific solution (34) which at the same time introduces the CP violation and assures the absence of degeneracy in the Higgs sector.

IV. A POSSIBLE SOLUTION TO THE "STRONG *CP*" PROBLEM

As pointed out in the Introduction, the CP-violation mechanism must be able at the same time to satisfy the requirement of a negligible effect on the strong-interaction physics and to reproduce in a correct way the known effects in the weak-interacting sector. Along the line of a spontaneous CP approach, the chiral structure of the gauge group induces $\theta_{\text{QCD}}=0$. The requirement $\theta_{\text{QFD}} \sim 0$ at the order of 10^{-9} can be achieved only through specific assumptions: (a) An additional discrete symmetry imposed to $V(\chi_L,\chi_R;\Phi_1,\Phi_2,\Phi_3)$ by requiring its invariance under the symmetric group S_3 of the permutations of Φ_1,Φ_2,Φ_3; (b) the masses of the light Higgs bosons in Eq. (36) supposed to be at least of the order of the $W_L^{\pm}$ mass.

The assumed discrete symmetry S_3 can be interpreted as a remnant of a larger gauge group or can be ascribed to the topological defects accomplished by compactification of extra dimensional theories. S_3 implies a unique value, say σ, for the σ_{ij} of Eq. (33) and leads, through the general CP-violating solution (34), to a specific solution satisfying $h_1 = h_2 = h_3 = h/\sqrt{3}$. As a consequence, since Γ_1 and Δ_2 are real symmetric matrices, we have

$$\theta_{\text{QFD}} = -3(\alpha_2 - \alpha_1) = 3\alpha_{12}, \quad \alpha_i = \arg(k_i) . \tag{37}$$

This symmetric interchange of the indices of the scalar fields leads to $\alpha_{12} = \frac{2}{3}\pi$ so that, though $\operatorname{Im} A_2(0) \neq 0$, it holds

$$\theta_{\text{QFD}}^{\text{tree}} = 0, \pi \quad [\operatorname{mod}(2\pi)] , \tag{38}$$

which does not induce CP-violating effects at skeleton level. Going to one-loop corrections to θ_{QFD} they depend on the imaginary part induced by the radiative corrections to the quark mass matrix, according to the leading contribution

$$\delta\theta_{\text{QFD}}^{1\text{ loop}} \simeq -2 \sum_q^{\text{all flavors}} \frac{1}{m_q} \operatorname{Im}(\delta M_Q)_{qq} + O(\alpha^2) . \tag{39}$$

There are several contributions to (δM_Q), that we analyze in detail. Single loops involving γ and $Z_{L,R}^0$ bosons do not involve phases. Negligible contributions are introduced by the $W_L^{(\pm)} - W_R^{(\pm)}$ mixing, absent here at the tree level. Then any one-loop level diagrams involving only gauge bosons and fermions will never induce θ_{QFD}, because any corrections to M_Q are relatively real to the tree-level value of M_Q. A nonzero imaginary part is induced by one-Higgs-boson-loop corrections to M_Q: the largest part comes from the lightest Higgs sector, only the charged ones being involved because of the suppression of FCNC (NFC guaranteed in the light Higgs sector). The largest ΔM_Q corrections can be derived from typical

$$\operatorname{tr}\langle \Phi_i \rangle \langle \Phi_j^\dagger \rangle \Phi_k \Phi_l \tag{40}$$

terms originating graphs such as that of Fig. 1, whose imaginary part does not vanish. The main contribution comes presumably from the t quark, so that one can estimate

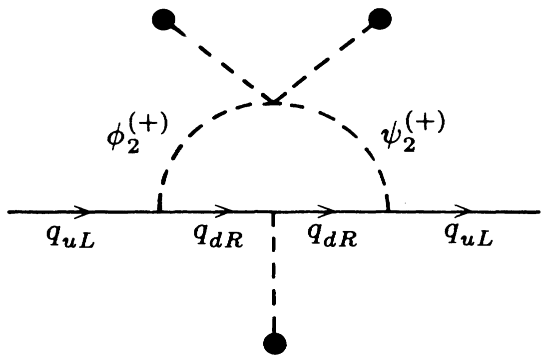

FIG. 1. A typical one-Higgs-boson-loop diagram giving a finite nonzero contribution to the imaginary part of the radiative corrections to the quark mass matrix.

$$\delta\theta_{\rm QFD}^{1\,\rm loop} \simeq \frac{{\rm Im}A_2(0)}{16\pi^2} m_{H_1}{}^2 m_t{}^2 \ln\frac{m_{H_2}{}^2}{m_{H_1}{}^2} \; . \tag{41}$$

The magnitude of the strong CP noninvariance effects is tolerably small if

$${\rm Im}A_2(0) \le 10^{-11} - 10^{-12}\ {\rm GeV}^{-2} \; . \tag{42}$$

From the upper bound (42) it can be argued that the masses of the lightest charged Higgs bosons are of the same order of magnitude of those of the light gauge bosons, or more.

V. CONSISTENCY IN THE NEUTRAL-KAON SYSTEM

A satisfactory description of the CP-violation phenomenon is constrained by the smallness of the K_L-K_S mass differences (ΔM_{LS}), mainly as far as the strength of flavor-changing neutral currents (FCNC's) is concerned.

It has been already pointed out[9] that in the case of the solution $k' \neq 0$ of the most general tree-level potential for the minimal Higgs sector, a physical-Higgs-boson inducing FCNC acquires a mass of the order M_L with a dangerously large contribution to ΔM_{LS}. The disconnected solution $k'=0$, obtained for a wider range of the parameters of the potential, yields the mass of the FC neutral Higgs boson of the same order as M_R but in turn guarantees only real VEV's and then in one-Φ models the source of CP cannot arise from Higgs-boson exchange (vacuum not CP violating). Moreover, trivial Cabibbo matrices are obtained if a more complicated Higgs sector is not considered. In our more realistic approach, the different gauge and scalar contributions to the $\bar{K}^0 - K^0$ mass mixing term need to be computed to check its numerical consistency.

Before all, we must stress that any theoretical computation of the nonvanishing ϵ'/ϵ parameter characterizing CP violation, is rather foggy. In fact, many uncertainties are involved, mainly due to long-distance (LD) effects and to the increased number of phases in this nonminimal LRS model. Further, any result depends sensitively on mass and couplings of the t quark.[23]

We begin our analysis with the lowest-order diagrams involving gauge contributions leading to direct $|\Delta\mathcal{S}|=1$ CP violation without forgetting the absence of $W_L - W_R$ mixing at the tree level. These contributions (Fig. 2) are already seen to vanish[19] since

$${\rm Im}\left\{\frac{\mathcal{A}_2}{\mathcal{A}_0}\right\} = 0 \; ; \tag{43}$$

i.e., the same imaginary part

$$\left\{\lambda_{uL} - \lambda_{uR}\frac{M_L{}^2}{M_R{}^2}\right\} = \left\{1 - e^{i(\phi_d - \phi_s)}\frac{M_L{}^2}{M_R{}^2}\right\} \tag{44}$$

dresses the isospin $I=2$ and $I=0$ amplitudes, where as usual

$$\lambda_{iL,R} = (U_{cL,R}^{\dagger})_{di}(U_{cL,R})_{is} \; . \tag{45}$$

Going to the gauge box diagrams involving RH gauge bosons, they have been already used to find the value of the M_R scale through a comparison with the experimental result of ΔM_{LS} (Refs. 24 and 25). The experimental constraints ϵ, ϵ', and d_n^e (the value of the electric dipole moment of the neutron), appear to be reproduced only in a several TeV M_R scenario,[24,25] even if the numerical results are sensitive to the kind of LRS and quark model adopted.[26]

In our scheme, even if the $\Delta\mathcal{S}=2$ CP-violating effect due to the box diagrams with one LH and one RH gauge boson is present, however, a suppression enters into play because of the specific character of the PMLRS approach. Once $\Delta\mathcal{S}=2$ effective Hamiltonian describing the box diagrams related to gauge-boson exchange is obtained, real and imaginary parts of $M_{12} = \langle \bar{K}^0 | H_w | K^0 \rangle$ can be extracted in a straightforward four-quark approximation which generalizes the result of Ref. 24 to the PMLRS case. At the first order in $\beta = (M_L/M_R)^2$, it follows that

$$\frac{{\rm Im}M_{12}}{{\rm Re}M_{12}} = -\frac{\beta C\left\{A\dfrac{\sin 2\theta_R}{\sin 2\theta_L}\sin\delta - B\dfrac{\sin^2\theta_R}{\sin^2\theta_L}\sin 2\delta\right\}}{\left\{1 - B\dfrac{\cos^2\theta_R}{\cos^2\theta_L}\right\} + \beta C\left\{A\dfrac{\sin 2\theta_R}{\sin 2\theta_L}\cos\delta - B\dfrac{\sin^2\theta_R}{\sin^2\theta_L}\cos 2\delta\right\}} \; . \tag{46}$$

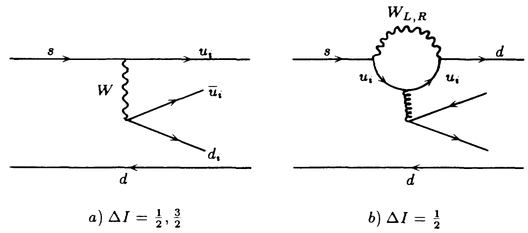

FIG. 2. Dominant gauge-boson diagrams contributing to the $|\Delta\mathscr{S}|=1$ Hamiltonian.

Here A and B, emerging from loop integration, are nearly constants up to $\beta\ln\beta$; δ_L and δ_R are the phases of the two-dimensional left and right Cabibbo mixing matrices, respectively, $C=8\rho_{\rm vac}$ arises from the $(S-P)(S+P)$ part of the matrix element $H_{\rm eff}^{\Delta\mathscr{S}=2}$, $\rho_{\rm vac}$ being

$$\rho_{\rm vac}e^{i\bar\delta}=\frac{\langle\bar K^0|\bar s(1-\gamma_5)d\bar s(1+\gamma_5)d\,|K^0\rangle_{\rm vac}}{\langle\bar K^0|[\bar s\gamma_\mu(1-\gamma_5)d^2]\,|K^0\rangle_{\rm vac}}$$

$$\text{with } \bar\delta=\arg(s_Ls_R)\ , \qquad (47)$$

while $\delta=\delta_R-\delta_L$ is the remaining phase after the redefinition of the s-quark argument ($s\to e^{i\delta_L}s$). Approximately it is

$$A\simeq1+\ln\left[\frac{m_c}{M_L}\right]^2,$$
$$B\simeq\frac{m_u}{m_c}\ln\left[\frac{m_c}{M_L}\right]^2, \qquad (48)$$
$$C=1+6\left[\frac{m_K}{m_s+m_d}\right]^2,$$

and then

$$\frac{{\rm Im}M_{12}}{{\rm Re}M_{12}}\approx\frac{430\beta\left[\frac{\sin2\theta_R}{\sin2\theta_L}\right]\sin\delta}{1-430\beta\left[\frac{\sin2\theta_R}{\sin2\theta_L}\right]\cos\delta}, \qquad (49)$$

where numerically the factor 430 comes from the use of reasonable values of the involved masses.[27] The ratio (49) may cover the expected range also for low M_R without giving a too apprehensive contribution: in fact in the case of PMLRS four-quark model it is plausible that the two rotation angles $\theta_{L,R}$ are nearly opposite up to the phase, and then if $\Delta m_K\simeq2({\rm Re}M_{12})_{\rm box}$ remains positive, a lower M_R can be allowed.[28]

Going now to the CP-violating effects coming from the Higgs sector, they enter into play only if the Higgs-boson masses belong to the lowest scale. As mentioned in Sec. III, in $\mathrm{SU}(2)_L\otimes\mathrm{U}(1)$ the Higgs-boson-exchange model of CP nonconservation guaranteeing NFC in the neutral sector[16] leads to a quark mixing matrix $U_{\rm KM}$ real[20] and CP violation arises only from the exchange of scalars. If in the Weinberg model the three VEV's are assumed almost degenerate, a discouraging value of ϵ'/ϵ (Ref. 29) is forecast: with $\mathscr{D}$ parametrizing long-distance contributions,

$$\frac{\epsilon'}{\epsilon}=-\frac{1}{20}\left[\frac{2\xi_H}{2\xi_H+\epsilon_m}\right]\left[\frac{1}{1+\mathscr{D}}\right]\simeq-0.015 \qquad (50)$$

being[30]

$$\left[\xi_H=\frac{{\rm Im}\mathscr{A}_0}{\mathscr{A}_0}\right]<\left[\epsilon_m=\frac{{\rm Im}M_{12}}{{\rm Re}M_{12}}\right]. \qquad (51)$$

Even if several possible solutions have been proposed,[31] however, the model is seen to suffer the risk of being ruled out unless experiments now in progress[32] soon find a nonvanishing result for ϵ'/ϵ. This trouble springs out because of the dominance of Higgs-boson-induced $\Delta\mathscr{S}=1$ CP-violating transitions (Fig. 3), in particular, the rather messy Higgs-boson—penguin diagram of Fig. 3(a), with respect to the $\Delta\mathscr{S}=2$ transitions (Fig. 4), dominance which involves ${\rm Im}M_{12}\le2\xi_H{\rm Re}M_{12}$.

Differently from these standard conclusions, in our model the dominant contributions to ΔM_{LS} and ϵ may come from tree-level neutral-Higgs-boson exchange (Fig. 5) (to be added, of course, to the already seen box diagrams with W_LW_R exchange). Indeed, as pointed out some time ago,[33] the neutral-Higgs-boson-induced $\Delta\mathscr{S}=2$ transitions do not introduce CP violation (only the heavy Higgs bosons have flavor-changing couplings) because of LRS and D symmetry. Neutral-Higgs-boson exchange in our limit, being $\langle0|T(\phi_{2r}^{(0)}\phi_{2i}^{(0)})|0\rangle=0$, contributes only to ${\rm Re}M_{12}$.

The effective $\Delta\mathscr{S}=2$ Hamiltonian is given by

$$H_{\rm eff}^{\Delta\mathscr{S}=2}=-\sqrt2\frac{G_F}{m_{\phi_2^{(0)}}^{\ 2}}\left[\frac{2}{k_1^{\ 2}}(U_{cL}^\dagger D_uU_{cR})^2\right](\bar s_Ld_R\bar s_Rd_L) \qquad (52)$$

(where $\phi_2^{(0)}$ nearly physical state and degenerate $\phi_{2i}^{(0)},\phi_{2r}^{(0)}$ masses are assumed without loss of generality). Moreover, PMLRS (with the Cabibbo matrix real, as discussed in Sec. III) leads to

$$(U_{cL}^\dagger D_uU_{cR})_{\bar sd}=e^{i(\phi_d-\phi_s)}(U_{cR}^\dagger D_uU_{cL})_{\bar sd}^*$$
$$=\sum_{q=u,c,t}m_q(I_d)_{\bar sd} \qquad (53)$$

and using the Glashow-Iliopoulos-Maiani cancellation, yields

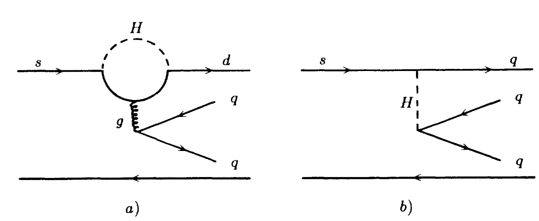

FIG. 3. Diagrams involving Higgs bosons contributing to the $|\Delta\mathscr{S}|=1$ effective Hamiltonian.

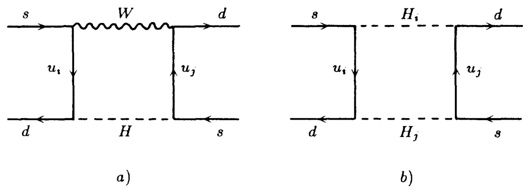

FIG. 4. Box diagrams for $\bar{K}^0-K^0$ with (a) a W gauge boson and a physical charged Higgs scalar, (b) two charged physical Higgs bosons.

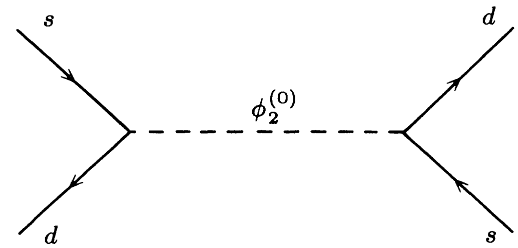

FIG. 5. A flavor-changing neutral-Higgs-boson contribution to the $|\Delta\mathscr{S}|=2$ Hamiltonian.

$$\mathrm{Re}M_{12}(\phi_2^{(0)})=\frac{1}{m_{\phi_2^{(0)}}{}^2}\left[\frac{2\sqrt{2}G_F}{k_1{}^2}\right]B(\tfrac{4}{3}f_K{}^2m_K)\times\left[\frac{3}{4}\left[\frac{m_K}{m_s+m_d}\right]^2+\frac{1}{8}\right]\sum_{q=u,c,t}m_q{}^2 \tag{54}$$

with $B=1$ in the case of vacuum-insertion saturation for the evaluation of hadronic matrix elements. It is worthwhile mentioning that this contribution is negligible relatively to $\mathrm{Re}M_{12}(WW)$, since it is suppressed at the order of $k^2/v^2 \sim M_L{}^2/M_R{}^2$ (Ref. 33).

Let us consider formally the charged-Higgs-boson contributions to the effective $\Delta\mathscr{S}=2$ Hamiltonian; they can be realistically approximated simply by the box diagram obtained starting from the usual one involving two-W-boson exchange by replacing one of them by a charged Higgs boson.[34] Then the typical CP-violating parameter of the K system

$$\begin{aligned}\epsilon&=\frac{e^{i\pi/4}}{\sqrt{2}(\Delta M_{LS})_{\mathrm{expt}}}(\mathrm{Im}M_{12}+2\xi\,\mathrm{Re}M_{12})_{LD}\\&\simeq\frac{e^{i\pi/4}}{2\sqrt{2}}(\epsilon_m+2\xi)\end{aligned} \tag{55}$$

shrinks on the whole in

$$|\epsilon|\simeq\tfrac{1}{64}m_K{}^2\frac{\mathrm{Im}A_2(0)}{G_F} \tag{56}$$

to be compared with the experimental value $|\epsilon|=2.3\times10^{-2}$.

By comparing Eqs. (42) and (56) we find the typical situation of a sheet too short: in order to satisfy (56), $\mathrm{Im}A/G_F\sim10^{-2}$, but this does not agree with Eq. (42). In other words, the lightest charged Higgs bosons are to be taken so heavy in order to suppress θ_{QFD}, that their exchange produces a CP-conserving $\Delta\mathscr{S}=1$, $\Delta\mathscr{C}=0$ interaction of several orders of magnitude under the strength required if a milliweak interaction is called to imitate the successful predictions of the superweak theory. A way out can be suggested by a deeper insight into the aspects of the weak CP violation as it appears in the K system.

VI. NEW PHYSICS IN THE HIGGS SECTOR

Alternatively to the standard KM mechanism[35] and to the Lee-Weinberg Higgs-boson-exchange effects,[16] there may be some other kinds of scalars[36] mediating $s\bar{d}\to d\bar{s}$ interactions and contributing to the $\bar{K}^0-K^0$ amplitude.

Besides the asymmetric triplet of color states,[37] the symmetric color-six states deserve much attention.[38] In the case of colored scalar bosons the violation of CP may rise softly by explicit mass terms.[37] If, however, CP is required to be broken spontaneously, a relative phase between the VEV's can be called to induce the complex feature of the effective coupling constants.

In this latter viewpoint, CP invariance is imposed on all dimension-4 terms of the Lagrangian so that, as usual, Yukawa couplings are all real. We use the Higgs scalars belonging to the six-dimensional representation of SU(3), whose origin rests in grand-unified theories[39] (GUT's), which under $\mathscr{G}_{LR}^{w}$ transform according to

$$\Delta_L\equiv(1,0,\tfrac{2}{3}),\quad \Delta_R\equiv(0,1,\tfrac{2}{3})\ . \tag{57}$$

The Yukawa couplings of these bosons,[33] after imposing the discrete invariance $\Delta_L\to-\Delta_L$, $\Delta_R\to\Delta_R$, can be rewritten in terms of scalars with definite electric charge

$$\mathscr{L}_Y^{(\Delta)}=g^{\alpha\beta}\left[\left[u_\alpha^T\mathscr{C}u_\beta\Delta^{(-4/3)}-d_\alpha^T\mathscr{C}d_\beta\Delta^{(2/3)}-\frac{\sqrt{2}}{2}(d_\alpha^T\mathscr{C}u_\beta+u_\alpha^T\mathscr{C}d_\beta)\Delta^{(-1/3)}\right]_L+(L\to R)\right]+\mathrm{H.c.}\ , \tag{58}$$

with α,β now running in generation space. It is easy to realize that $\Delta^{(2/3)}$ components contribute to $\bar{K}^0\to K^0$ transition (Fig. 6). The relevant most general renormalizable Higgs potential results a bit larger than (19), with the introduction of the further term

$$\begin{aligned}V(\Delta_L,\Delta_R,\Phi_i,\tilde{\Phi}_j)= &-\mu_\Delta{}^2\mathrm{tr}(\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R)+\rho_1\{[\mathrm{tr}(\Delta_L^\dagger\Delta_L)]^2+[\mathrm{tr}(\Delta_R^\dagger\Delta_R)]^2\}\\ &+\rho_2[\mathrm{tr}(\Delta_L^\dagger\Delta_L\Delta_L^\dagger\Delta_L)+\mathrm{tr}(\Delta_R^\dagger\Delta_R\Delta_R^\dagger\Delta_R)]+\rho_3[\mathrm{tr}(\Delta_L^\dagger\Delta_L)\mathrm{tr}(\Delta_R^\dagger\Delta_R)]\\ &+\alpha[\mathrm{tr}(\Phi_1^\dagger\Phi_1)+\mathrm{tr}(\Phi_2^\dagger\Phi_2)][\mathrm{tr}(\Delta_L^\dagger\Delta_L)+\mathrm{tr}(\Delta_R^\dagger\Delta_R)]\\ &+\beta[\mathrm{tr}(\Delta_L^\dagger\Delta_L\Phi_1\Phi_1^\dagger)+\mathrm{tr}(\Delta_R^\dagger\Delta_R\Phi_1^\dagger\Phi_1)+\mathrm{tr}(\Delta_L^\dagger\Delta_L\Phi_2\Phi_2^\dagger)+\mathrm{tr}(\Delta_R^\dagger\Delta_R\Phi_2^\dagger\Phi_2)]\\ &+\gamma_1[\mathrm{tr}(\Delta_L^\dagger\Phi_1\Delta_R\tilde{\Phi}_1^\dagger)+\mathrm{tr}(\Delta_L^\dagger\Phi_2\Delta_R\Phi_2^\dagger)]+\gamma_2[\mathrm{tr}(\Delta_L^\dagger\tilde{\Phi}_1\Delta_R\Phi_1^\dagger)+\mathrm{tr}(\Delta_L^\dagger\tilde{\Phi}_2\Delta_R\tilde{\Phi}_2^\dagger)]\\ &+\gamma_3[\mathrm{tr}(\Delta_L^\dagger\Phi_1\Delta_R\Phi_2^\dagger)+\mathrm{tr}(\Delta_L^\dagger\Phi_2\Delta_R\tilde{\Phi}_1^\dagger)]+\gamma_4[\mathrm{tr}(\Delta_L^\dagger\tilde{\Phi}_1\Delta_R\tilde{\Phi}_2^\dagger)+\mathrm{tr}(\Delta_L^\dagger\tilde{\Phi}_2\Delta_R\tilde{\Phi}_1^\dagger)]+\mathrm{H.c.}\end{aligned} \tag{59}$$

No sensible modifications are introduced to the symmetry-breaking pattern by the Δ contributions since color-unbroken invariance prevents new VEV's. The $\Phi_3-\Delta$ couplings are prevented if, as postulated in Sec. III, $\Phi_3\to e^{i\varphi}\Phi_3$ with $\varphi=(n-\frac{1}{2})\pi$ ($n=0,\pm1,\pm2,\ldots$). The $\chi-\Delta$ couplings can be written as

$$V(\chi-\Delta)=\delta_1[(\chi_L^\dagger\chi_L)+(\chi_R^\dagger\chi_R)]\mathrm{tr}(\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R)+\delta_2[(\chi_L^\dagger\chi_L)\mathrm{tr}(\Delta_L^\dagger\Delta_L)+(\chi_R^\dagger\chi_R)\mathrm{tr}(\Delta_R^\dagger\Delta_R)] \ . \tag{60}$$

Since isospin quantum numbers of Δ_L,Δ_R are different, this part of the potential generates the tadpoles involving scalars such as $\langle\chi_R\rangle$ able to break U(1) and $SU(2)_R$. It is worth while stressing that the $\Delta_{dd}^{(-2/3)}$ component is a symmetric $\{6\}$ representation of $SU(3)_C$, while $\Delta_{\bar{s}\bar{s}}^{(+2/3)}$ belongs to the $\{\bar{6}\}$. Looking at an extended scheme of flavor-color unification, both (10;1,0) and (10;0,1) under $G_{PS}=SU(4)_C\otimes SU(2)_L\otimes SU(2)_R$ are required, while prodigiously in a GUT approach only one $\{126\}$ of SO(10) is enough. The $\Delta_L^{(2/3)}$, $\Delta_R^{(2/3)}$ mass matrix reads

$$M_\Delta{}^2=\begin{array}{c} \\ \Delta_L^{(+2/3)}\\ \Delta_R^{(+2/3)}\end{array}\begin{array}{cc}\Delta_L^{(-2/3)} & \Delta_R^{(-2/3)}\\ \left[\begin{array}{c} M_L{}^2 \\ b^* \end{array}\right. & \left.\begin{array}{c} b \\ M_R{}^2\end{array}\right]\end{array} , \tag{61}$$

with

$$\begin{aligned}M_L{}^2&=-\mu_\Delta{}^2+(\alpha+\beta)(|k_1|^2+|k_2|^2)+\delta_1 v^2 \ ,\\ M_R{}^2&=-\mu_\Delta{}^2+(\alpha+\beta)(|k_1|^2+|k_2|^2)\\ &\quad+(\delta_1+\delta_2)v^2 \ ,\\ b&=\gamma_2(k_1{}^{*2}+k_2{}^{*2}) \ ,\end{aligned} \tag{62}$$

while the transition propagator (taken at zero-momentum transfer) which characterizes the $\Delta_L^{(-2/3)}\Delta_R^{(+2/3)}$ exchange looks like

$$\begin{aligned}\mathrm{Im}\langle 0|T(\Delta_L^{(-2/3)}\Delta_R^{(+2/3)})|0\rangle&=\frac{1}{M_1{}^2M_2{}^2}\mathrm{Im}(b)\\ &=\frac{\gamma_2}{M_1{}^2M_2{}^2}\mathrm{Im}(k_1^*+k_2^*) \ .\end{aligned} \tag{63}$$

Assuming that the v^2 terms are the dominant ones,

$$M_1{}^2M_2{}^2\simeq M_{\Delta_L}{}^2M_{\Delta_R}{}^2\simeq\delta_1(\delta_1+\delta_2)v^4=\delta\delta' v^4 \ , \tag{64}$$

with $\delta_1=\delta$ and $(\delta_1+\delta_2)=\delta'$. In conclusion, with α relative phase, $\gamma_2=\gamma$, $h_1=h_2=h$ (the requirement of degeneracy has been discussed in Sec. IV), we find

$$\mathrm{Im}\langle 0|T(\Delta_L^{(-2/3)}\Delta_R^{(+2/3)})|0\rangle\simeq\frac{\gamma}{\delta\delta'}\frac{h^2}{v^4}\sin 2\alpha \ . \tag{65}$$

The effective $\Delta\mathscr{S}=2$ interactions are described by

$$\mathscr{L}_{\mathrm{eff}}^{\Delta\mathscr{S}=2}=G(\bar{d}_L\gamma_\mu s_L)(\bar{d}_L\gamma^\mu s_L)+(L\to R)+\mathrm{H.c.} \ , \tag{66}$$

where

$$G=\tfrac{1}{2}(g)_{dd}(g^\dagger)_{ss}(M_\Delta{}^{-2}) \ . \tag{67}$$

The strongest constraint comes from the real part of the K_L-K_S mass difference, because of the direct $\Delta\mathscr{S}=2$ tree-level amplitudes describing the exchange of the diquark Δ_{dd} component. After evaluating the hadron matrix elements and for nearly degenerate masses we have

$$(\mathrm{Re}M_{12})_\Delta=\frac{2g_{dd}g_{ss}}{M_{\Delta_{dd}}{}^2}(\tfrac{1}{3}f_K{}^2m_K) \ . \tag{68}$$

Imposing the limit

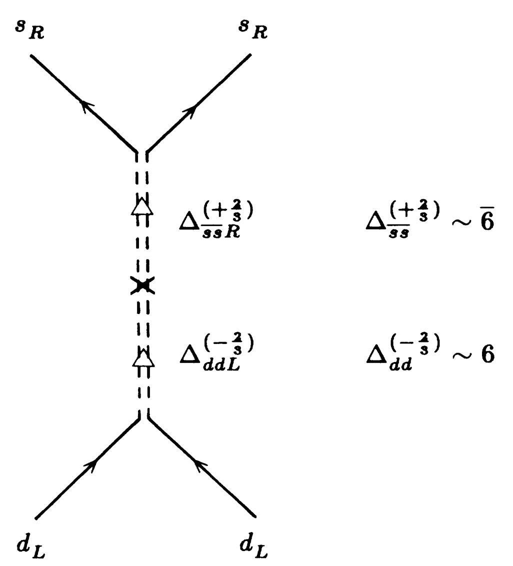

FIG. 6. Diagram for $|\Delta\mathscr{S}|=2$ contributions mediated by Δ bosons exchanges. The cross represents the insertion of an even number of relative CP-violating VEV's.

$$(\mathrm{Re}M_{12})_\Delta \le \frac{\Delta m_{\mathrm{expt}}}{2} = 1.7\times 10^{-15}\ \mathrm{GeV}\ , \tag{69}$$

it follows

$$\frac{g_{dd}g_{ss}}{\delta v^2} \le 10^{-13}\ \mathrm{GeV}^{-2}\ . \tag{70}$$

Thereby the relation

$$\mathrm{Im}M_{12} \sim \sqrt{2}\epsilon_{\mathrm{expt}}\Delta m_{\mathrm{expt}} \tag{71}$$

yields

$$\frac{\gamma}{\delta'}\left[\frac{h_2}{v^2}\right]\sin 2\alpha \simeq \frac{\gamma}{\delta'}\left[\frac{M_L}{M_R}\right]^2 \sin 2\alpha \sim 10^{-3}\ . \tag{72}$$

This result involves too many parameters to let more than a rough estimate of the mass scale of parity restoration. Without being restrictive, it is conceivable to have a low intermediate-mass scale M_R, a reasonable limit for $\beta=(M_L/M_R)^2$ being 10^{-3}–10^{-2}. It is worthwhile noting that, as discussed in Sec. V, this limit is, at the same time, compatible with the restrictions coming from the box diagram involving one L and one R gauge boson.

VII. CONCLUSIONS

In this paper we have analyzed the problems connected to the Higgs-boson-induced spontaneous *CP* violation in the framework of a left-right-symmetric approach to the gauge theories of weak interactions in which P is spontaneously broken [as a mere consequence of the LRS and spontaneous symmetry breaking (SSB)], and NFC is ensured in the light Higgs sector with the same degree of suppression which characterizes right-handed currents with respect to the left-handed ones.

The approach belongs to the most general so-called "spontaneous *CP*" case, which appears, however, as discussed in Sec. I, the more promising one. It is shown in detail how the approach we are proposing is able to satisfy the hard condition coming from strong *CP*. It is worth noting that, despite the higher content of the Higgs bosons, due to the higher symmetry we are starting with, the interaction is quite similar to that of the standard model, exactly reproduced in the low-energy limit. In this context the key role of NFC is to be emphasized. The charged Higgs sector must be characterized by relatively large mass, at least of the order of light gauge bosons: this is quite natural, and a different result would be unattractive. The enhancement of $\Delta\mathscr{S}=2$ contributions in LRS models is sizable only in a really high M_R scenario. So PMLRS models support the Weltanschauung of not-too-high-energy parity restoration, blooming the desert with an intermediate-mass scale. Of course $\bar{K}^0\to K^0$ effects are affected by the presence of right-handed currents. The chiral structure of the LRS models lead to an enhancement of $(V-A)\times(V+A)$ matrix elements (as for penguin contributions).[40] However, the $\Delta I=\frac{3}{2}$ parts of $|\Delta\mathscr{S}|=1$ effective theory are increased on the same foot as $\Delta I=\frac{1}{2}$ one; namely, in our scheme no new important phenomena of *CP* violation with respect to the standard left-left computation are induced without valuable W_L-W_R mixing.

Strong radiative corrections do not affect sizable *CP*-violating parameters[41] sharing odd matrix elements (namely, ϵ) and direct *CP* violation (i.e., ϵ') of a similar amount. The introduction of colored Higgs bosons may solve in general the difficulties of Weinberg-type models of *CP* violation, though their presence is mainly motivated in the hierarchical fashion of a $\mathscr{G}^w_{LR}$ electroweak model. A similar model can be reviewed in GUT context based on the gauge group SO(10) to be consistent with an intermediate-mass scale for low-parity restoration.[39]

APPENDIX A

Here the analysis is briefly reported concerning the two Higgs fields

$$\Phi_1=\begin{bmatrix}\phi_1^{(0)} & \phi_1^{(+)}\\ \phi_2^{(-)} & \phi_2^{(0)}\end{bmatrix},\quad \Phi_2=\begin{bmatrix}\psi_1^{(0)} & \psi_1^{(+)}\\ \psi_2^{(-)} & \psi_2^{(0)}\end{bmatrix}\ , \tag{A1}$$

interacting with the quarks through an usual Yukawa coupling and subjected to the potential (19). According to the analysis of the text [see Eqs. (22), (23), and (24)], the only possible source of *CP* violation is given by the solution (24). The corresponding VEV's are given by Eq. (5). Without any lack of generality we can assume the form (23) with $\alpha=\pi/2$. After some standard algebra one may calculate explicitly the (mass)^2 matrix of the charged Higgs sector $M_\pm{}^2$. The matrix is block diagonal, and the following Hermitian submatrices can be isolated:

$$M_\pm{}^2(\phi_2^{(\pm)},\psi_2^{(\pm)})=\begin{matrix} & \phi_2^{(-)} & \psi_2^{(-)}\\ \phi_2^{(+)} & (\sigma_4+\sigma_8)h_2{}^2 & i(\sigma_4+\sigma_8)h_1h_2\\ \psi_2^{(+)} & -i(\sigma_4+\sigma_8)h_1h_2 & (\sigma_4+\sigma_8)h_1{}^2\end{matrix} \tag{A2}$$

and

$$M_\pm{}^2(\chi_R^{(\pm)},\phi_1^{(\pm)},\psi_1^{(\pm)})=\begin{matrix} & \chi_R^{(-)} & \phi_1^{(-)} & \psi_1^{(-)}\\ \chi_R^{(+)} & \rho_1h_1{}^2+\beta_2h_2{}^2 & i\beta_1h_1v & \beta_2h_2v\\ \phi_1^{(+)} & -i\beta_1h_1v & \beta_1v^2+\sigma_5h_2{}^2 & i\sigma_5h_1h_2\\ \psi_1^{(+)} & \beta_2h_2v & -i\sigma_5h_1h_2 & \beta_2v^2+\sigma_5h_1{}^2\end{matrix} \tag{A3}$$

from which, together with two Goldstone bosons (one from each matrix), light and heavy physical Higgs bosons can be worked out, respectively, to be added to the further (mass)^2 eigenstate:

$$\chi_L^{(\pm)} \text{ with (mass)}^2 \ M_{\chi_L^{(\pm)}}{}^2=\rho_2 v^2+\beta_1 h_1{}^2+\beta_2 h_2{}^2 . \tag{A4}$$

It is an easy matter to diagonalize the matrices (A2) and (A3) through unitary transformations: by applying

$$U_2=\frac{1}{h}\begin{bmatrix} h_1 & -ih_2 \\ -ih_2 & h_1 \end{bmatrix} \text{ with } h^2=h_1{}^2+h_2{}^2 \tag{A5}$$

to the matrix (A2), the following decompositions in terms of (mass)^2 eigenstates can be obtained

$$\begin{aligned} \phi_2^{(\pm)}&=\frac{1}{h}(h_1 G_2^{(\pm)}+ih_2 H_2^{(\pm)}) , \\ \psi_2^{(\pm)}&=\frac{1}{h}(ih_2 G_2^{(\pm)}+h_1 H_2^{(\pm)}) , \end{aligned} \tag{A6}$$

where $G_2^{(\pm)},H_2^{(\pm)}$ are a Goldstone boson and a light physical Higgs boson, respectively.

As far as the matrix (A3) is concerned, the unitary transformation

$$U_1=\begin{bmatrix} -\frac{v}{\Delta} & \frac{h}{\Delta} & 0 \\ -i\frac{h_1}{\Delta} & i\frac{v}{\Delta}\frac{h_1}{h} & i\frac{h_2}{h} \\ \frac{h_2}{\Delta} & \frac{v}{\Delta}\frac{h_2}{h} & -\frac{h_1}{h} \end{bmatrix} \text{ with } \Delta^2=h^2+v^2 \tag{A7}$$

allows to separate the Goldstone boson $G_1^{(\pm)}$ from the heavy physical Higgs bosons $H_1^{(\pm)},H_1'^{(\pm)}$, whose remaining mixing requires a further rotation (the corresponding angle θ does not need to be explicitly specified here): one finds, finally ($c_\theta=\cos\theta, s_\theta=\sin\theta$),

$$\begin{aligned} \chi_R^{(\pm)}&=\frac{1}{\Delta}(-vG_1^{(\pm)}+hc_\theta H_1^{(\pm)}-hs_\theta H_1'^{(\pm)}) , \\ \phi_1^{(\pm)}&=\frac{i}{h\Delta}[-hh_1 G_1^{(\pm)}+(vh_1 c_\theta+\Delta h_2 s_\theta)H_1^{(\pm)} \\ &\quad +(\Delta h_2 c_\theta - vh_1 s_\theta)H_1'^{(\pm)}] , \\ \psi_1^{(\pm)}&=\frac{1}{h\Delta}[+hh_2 G_1^{(\pm)}+(vh_2 c_\theta-\Delta h_1 s_\theta)H_1^{(\pm)} \\ &\quad -(\Delta h_1 c_\theta - vh_2 s_\theta)H_1'^{(\pm)}] . \end{aligned} \tag{A8}$$

Equations (A3)—(A8) allows to verify Eq. (25).

For the sake of completeness let us consider the neutral Higgs sector: in this case the (mass)^2 matrix is block-diagonal, and the following Hermitian submatrices can be worked out:

$$M_0{}^2(\chi_{R,r}^{(0)};\phi_{1,i}^{(0)};\psi_{1,r}^{(0)})=\begin{bmatrix} (\rho_1-\rho_2)v^2 & 2\alpha_1 h_1 v & 2\alpha_2 h_2 v \\ 2\alpha_1 h_1 v & 2\lambda_1^{(1)} h_1{}^2 & 2h_1 h_2(\sigma_1-\sigma_5+\sigma_8) \\ 2\alpha_2 h_2 v & 2h_1 h_2(\sigma_1-\sigma_5+\sigma_8) & 2\lambda_1^{(2)} h_2{}^2 \end{bmatrix} , \tag{A9}$$

$$M_0{}^2(\phi_{1,r}^{(0)};\psi_{1,i}^{(0)})=\begin{bmatrix} (\sigma_4+\sigma_5)h_2{}^2 & (\sigma_4+\sigma_5)h_1 h_2 \\ (\sigma_4+\sigma_5)h_1 h_2 & (\sigma_4+\sigma_5)h_1{}^2 \end{bmatrix} , \tag{A10}$$

$$M_0{}^2(\phi_{2,r}^{(0)};\psi_{2,i}^{(0)})=\begin{bmatrix} \beta_1 v^2-\lambda_1^{(1)} h_1{}^2+(\sigma_2-\sigma_5-\sigma_8)h_2{}^2 & (\sigma_5+\sigma_7-\sigma_2-\sigma_6)h_1 h_2 \\ (\sigma_5+\sigma_7-\sigma_2-\sigma_6)h_1 h_2 & \beta_2 v^2-\lambda_3^{(2)} h_2{}^2+(\sigma_2-\sigma_5-\sigma_8)h_2{}^2 \end{bmatrix} , \tag{A11}$$

$$M_0{}^2(\psi_{2,r}^{(0)};\phi_{2,i}^{(0)})=\begin{bmatrix} \beta_2 v^2+\epsilon_2 h_2{}^2+(\sigma_5-\sigma_3-\sigma_8)h_1{}^2 & -(\sigma_3+\sigma_5+\sigma_6+\sigma_7)h_1 h_2 \\ -(\sigma_3+\sigma_5+\sigma_6+\sigma_7)h_1 h_2 & \beta_2 v^2+\lambda_2^{(1)} h_1{}^2+(\sigma_5-\sigma_3-\sigma_8)h_2{}^2 \end{bmatrix} . \tag{A12}$$

APPENDIX B

We consider here the relevant technical aspects of the Higgs-boson-induced *CP* violation in left-right-symmetric models, starting with an Higgs content including, together with Φ_1 and Φ_2 given by Eq. (A1), a third Higgs field of Φ type:

$$\Phi=\begin{bmatrix} \omega_1^{(0)} & \omega_1^{(+)} \\ \omega_2^{(-)} & \omega_2^{(0)} \end{bmatrix} . \tag{B1}$$

The Higgs potential is given by Eq. (31) [a pair of upper indices must be added to the σ_l's, $\sigma_l^{(ij)}$ being the coefficients entering into $V_{ij}(\Phi_i,\Phi_j)$], being $\langle \Phi_i \rangle$ given by Eq.

(32). The potential (31), by substituting the corresponding VEV's, takes on the form

$$\langle V\rangle = V_0+\tfrac{1}{2}\sum_{\text{cycl}}(\sigma_4^{(ij)}+\sigma_5^{(ij)})h_i^2h_j^2\cos 2(\alpha_i-\alpha_j) \quad (i,j=1,2,3)\ , \qquad \text{(B2)}$$

V_0 being independent of the α_i's. The minimum conditions reads then ($\sigma_{ij}=\sigma_4^{(ij)}+\sigma_5^{(ij)}$)

$$\sigma_{12}h_1^2h_2^2\sin 2(\alpha_1-\alpha_2)=\sigma_{23}h_2^2h_3^2\sin 2(\alpha_2-\alpha_3) =\sigma_{13}h_1^2h_3^2\sin 2(\alpha_3-\alpha_1) \qquad \text{(B3)}$$

with the restrictions

$$H_{12}+H_{23}+H_{13}<0\ , \qquad \text{(B4)}$$

$$H_{12}H_{13}+H_{12}H_{23}+H_{13}H_{23}>0\ , \qquad \text{(B5)}$$

being

$$H_{ij}=\sigma_{ij}\mathrm{Re}(k_i^2k_j^{*2})=\sigma_{ij}h_i^2h_j^2\cos 2(\alpha_i-\alpha_j) \quad (i,j=1,2,3;\ i\neq j)\ . \qquad \text{(B6)}$$

It is easy to identify the two following sets of solutions of Eqs. (B6).

1. *CP*-conserving solution

They are characterized by

$$\sin 2(\alpha_i-\alpha_j)=0 \quad (i,j=1,2,3;\ i\neq j) \qquad \text{(B7)}$$

and because of (B4) and (B5), they correspond to a minimum of the potential if and only if

$$\sigma_{12}\sigma_{13}\sigma_{23}<0\ . \qquad \text{(B8)}$$

More specifically it can be easily found that condition (B8) must be satisfied with the further correspondence

$$\sigma_{ij}<0 \ \text{ if } \alpha_i-\alpha_j=n\pi\ , \qquad \text{(B9)}$$

$$\sigma_{ij}>0 \ \text{ if } \alpha_i-\alpha_j=(2n+1)\frac{\pi}{2}\ . \qquad \text{(B10)}$$

It will be verified later that the solutions (B7) are indeed *CP* conserving.

2. *CP*-violating solutions

They can be easily obtained starting from (B3) and excluding the solutions (B7). In this case the following identity must be satisfied:

$$\sigma_{12}\sigma_{13}k_1^2+\sigma_{12}\sigma_{23}k_2^2+\sigma_{13}\sigma_{23}k_3^2=0 \qquad \text{(B11)}$$

involving the (complex) quantities k_i^2. The explicit solutions are well expressed in terms of the H_{ij}'s defined in Eq. (B6): one finds

$$H_{ij}=\frac{1}{2}\frac{1}{\sigma_{12}\sigma_{13}\sigma_{23}}(c_k^2-c_i^2-c_j^2) \quad (i,j,k=1,2,3;\ i\neq j\neq k) \qquad \text{(B12)}$$

with

$$c_i=\sigma_{ij}\sigma_{ik}h_i^2 \quad (i,j,k=1,2,3;\ i\neq j\neq k)\ . \qquad \text{(B13)}$$

It is worth noting that Eqs. (B12) allow us to express explicitly all relative phases in terms of the parameters. The solutions (B12) correspond to a minimum if the two conditions (B4) and (B5) are satisfied. Equation (B4) leads to the requirement

$$\sigma_{12}\sigma_{13}\sigma_{23}>0 \qquad \text{(B14)}$$

which, compared with the analogous (B8), allows us to separate the range of values of the σ_{ij}'s which give rise to two different solutions. Equation (B5) gives

$$\lambda(c_1^2,c_2^2,c_3^2)<0\ , \qquad \text{(B15)}$$

$\lambda(x,y,z)$ being the well-known triangular function. A restriction about the parameters follows, which is easily identified as the condition which makes $\cos 2(\alpha_i-\alpha_j)$ physically acceptable. Moreover, it is easy to show that all the quantities H_{ij} given by Eqs. (B12) must be negative, and this is incompatible with the *CP*-conserving solutions (B4). In conclusion, Eqs. (B13), (B14), and (B15) identify a set of *CP*-violating solutions.

In order to verify that the two distinct sets of solutions listed above are *CP* conserving and *CP* violating, respectively, let us consider the (mass)^2 matrix of the charged Higgs boson. After some algebra, it appears to be block diagonal in terms of heavy ($\sim v^2$) and light Higgs bosons. It is evident that the relevant (to induce *CP*-violation) sector is the light one, which involves $\phi_2^{(\pm)},\psi_2^{(\pm)},\omega_2^{(\pm)}$. In this basis the (mass)^2 matrix elements are given by

$$M_{ii}^2=-\frac{1}{2}\frac{1}{h_i^2}\sum_{j\neq i}(H_{ij}+h_i^2h_j^2\Sigma_{ij})\ , \qquad \text{(B16)}$$

$$M_{ij}^2=\tfrac{1}{2}(\sigma_{ij}k_ik_j^*+\Sigma_{ij}k_i^*k_j)=M_{ji}^* \quad (i,j=1,2,3;\ i<j)\ , \qquad \text{(B17)}$$

where

$$\Sigma_{ij}=2\sigma_8^{(ij)}+\sigma_4^{(ij)}-\sigma_5^{(ij)}\ . \qquad \text{(B18)}$$

The light Higgs sector (mass)^2 matrix can be easily diagonalized. The Goldstone boson (to be absorbed by $W_L^{(\pm)}$) is easily found

$$G^{(\pm)}=\frac{1}{h}(k_1\phi_2^{(\pm)}+k_2\psi_2^{(\pm)}+k_3\omega_2^{(\pm)}) \qquad \text{(B19)}$$

($h^2=\sum_i h_i^2$): a unitary matrix which allows to isolate it can be written in the form

$$U_1=\begin{pmatrix} \frac{k_1}{h} & \frac{k_2}{h} & \frac{k_3}{h} \\ \frac{\Delta}{h} & -\frac{k_1^*k_2}{h\Delta} & -\frac{k_1^*k_3}{h\Delta} \\ 0 & -\frac{k_3^*}{\Delta} & \frac{k_2^*}{\Delta} \end{pmatrix} \quad \text{with } \Delta^2=h_2^2+h_3^2\ . \qquad \text{(B20)}$$

This leads to

$$U_1M_H^2U_1^\dagger=\begin{pmatrix} 1 & 0 & 0 \\ 0 & & \\ 0 & & m_H^2 \end{pmatrix}\ , \qquad \text{(B21)}$$

$m_H{}^2$ being the (Hermitian) 2×2 (mass)^2 matrix of the light Higgs bosons, whose elements correspond to

$$(m_H{}^2)_{11}=\frac{h^2}{\Delta^2}M_{11} ,$$
$$(m_H{}^2)_{22}=M_{22}+M_{33}-\frac{h_1{}^2}{\Delta^2}M_{11} , \quad \text{(B22)}$$
$$(m_H{}^2)_{12}=\frac{1}{2}\frac{h}{\Delta^2}k_1^*k_2k_3(\sigma_{13}e^{2i(\alpha_1-\alpha_3)}-\sigma_{12}e^{2i(\alpha_1-\alpha_2)})=(m_H{}^2)_{21}^* .$$

The most general unitary matrix which diagonalize (B21) can be written as

$$U_2=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta e^{i\varphi}\\ 0 & \sin\theta e^{-i\varphi} & \cos\theta\end{bmatrix} ; \quad \text{(B23)}$$

$\cos\theta$ and φ can be, as usually, expressed in terms of the matrix elements (B22) and of the Higgs-boson masses M_1,M_2 according to

$$U_2m_H{}^2U_2^\dagger=\begin{bmatrix}M_1{}^2 & 0\\ 0 & M_2{}^2\end{bmatrix} . \quad \text{(B24)}$$

The relevant quantity to be calculated is the imaginary part of A_2 [Eq. (27b)], since the high mass of the heavy Higgs boson ($M^2\sim v^2$) makes A_1 negligible with respect to A_2. In conclusion the diagonal fields are obtained through

$$\begin{bmatrix}G^{(\pm)}\\ H_1^{(\pm)}\\ H_2^{(\pm)}\end{bmatrix}=U_2U_1\begin{bmatrix}\phi_2^{(\pm)}\\ \psi_2^{(\pm)}\\ \omega_2^{(\pm)}\end{bmatrix} , \quad \text{(B25)}$$

with

$$U=U_2U_1U' . \quad \text{(B26)}$$

We find then, starting from Eq. (27b),

$$\text{Im}A_2(0)=-\frac{1}{h_1h_2}\text{Im}\left[\frac{U_{21}U_{22}^*}{M_1{}^2}+\frac{U_{31}U_{32}^*}{M_2{}^2}\right] , \quad \text{(B27)}$$

i.e.,

$$\text{Im}A_2(0)=-\frac{1}{h}\frac{h_3}{h_1h_2}\sin\xi\cos\xi\left[\frac{1}{M_1{}^2}-\frac{1}{M_2{}^2}\right]\text{Im}(e^{-i(\alpha_1-\alpha_2-\alpha_3+\varphi)})$$
$$=-\frac{1}{h}\frac{h_3}{h_1h_2}\frac{1}{M_1{}^2M_2{}^2}\text{Im}(e^{-i(\alpha_1-\alpha_2-\alpha_3)}m_{12}{}^{2*}) . \quad \text{(B28)}$$

By making use of the last of Eqs. (B22), the result (36) of the text follows.

[1]G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976); Phys. Rev. D **14**, 3432 (1976).
[2]I. S. Altarev *et al.*, Phys. Lett. **102B**, 13 (1981).
[3]J. Ellis and M. K. Gaillard, Nucl. Phys. **B150**, 141 (1979).
[4]R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1971 (1977).
[5]A. Zepeda, Phys. Rev. Lett. **41**, 139 (1978); P. Langacker and H. Pagels, Phys. Rev. D **19**, 2070 (1979).
[6]S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, *ibid.* **40**, 279 (1978).
[7]D. Dicus, E. Kolb, V. Teplitz, and R. Wagoner, Phys. Rev. D **18**, 1829 (1978); L. Abbott and P. Sikivie, Phys. Lett. **120B**, 133 (1983).
[8]M. A. B. Bég and H.-S. Tsao, Phys. Rev. Lett. **41**, 278 (1978); M. A. B. Bég, D. J. Maloof, and H.-S. Tsao, Phys. Rev. D **19**, 2221 (1979).
[9]P. Cea and G. L. Fogli, Phys. Lett. **102B**, 22 (1981); D. Cocolicchio and G. L. Fogli, Phys. Rev. D **32**, 3020 (1985).
[10]G. C. Branco and G. Senjanovic, Phys. Rev. D **18**, 1621 (1978); P. Ramond and D. B. Reiss, Phys. Lett. **80B**, 87 (1978); G. C. Branco and J. M. Gerard, *ibid.* **124B**, 415 (1983).
[11]J. Oliensis, Phys. Lett. **165B**, 99 (1985).
[12]G. Ecker, W. Grimus, and H. Neufeld, Nucl. Phys. **B247**, 70 (1984), and references therein.
[13]H. Georgi, Hadronic J. **1**, 155 (1978); G. Segrè and H. A. Weldon, Phys. Rev. Lett. **42**, 1191 (1979).
[14]S. Barr and P. Langacker, Phys. Rev. Lett. **42**, 1291 (1979), following the same logic of Ref. 8.
[15]R. Akhoury and I. I. Bigi, Nucl. Phys. **B234**, 459 (1984).
[16]S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976).
[17]The phases of the generalized Cabibbo matrices can be easily connected to the relative phase between the VEV's in minimal models both for $N=2,3$ generations of quarks: R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975); D. Chang, Nucl. Phys. **B214**, 435 (1983).
[18]R. N. Mohapatra, F. E. Paige, and D. P. Sidhu, Phys. Rev. D **17**, 2462 (1978); R. N. Mohapatra and D. P. Sidhu, *ibid.* **17**, 1876 (1978).
[19]G. C. Branco, J. M. Frère, and J. M. Gerard, Nucl. Phys. **B221**, 317 (1983).
[20]G. C. Branco, Phys. Rev. Lett. **44**, 504 (1980); K. Shizuya and S. H. Tye, Phys. Rev. D **23**, 1613 (1981).
[21]C. Jarlskog, Z. Phys. C **29**, 491 (1985).
[22]H. Georgi and A. Pais, Phys. Rev. D **16**, 3520 (1977).
[23]A four-quark model becomes reliable only if the t-quark couplings to the light-quark sector is negligibly small: G. L. Fogli, Phys. Rev. D **28**, 1153 (1983).

[24]G. Beall, M. Bander, and A. Soni, Phys. Rev. Lett. **48**, 848 (1982) ($\Delta m_K < 0$ unless M_R in MLRS is very large).

[25]G. Ecker and W. Grimus, Nucl. Phys. **B258**, 328 (1985); Phys. Lett. **153B**, 279 (1985).

[26]P. Basak, A. Datta, and A. Raychaudury, Z. Phys. C **20**, 305 (1983); J. Trampetič, Phys. Rev. D **27**, 1565 (1983); L. Maharana, Phys. Lett. **144B**, 399 (1984).

[27]L. L. Chau, Phys. Rep. **95**, 1 (1983).

[28]H. Harari and M. Leurer, Nucl. Phys. **B233**, 221 (1981).

[29]F. J. Gilman and M. B. Wise, Phys. Rev. D **20**, 2392 (1979); J. M. Frère, J. Hagelin, and A. I. Sanda, Phys. Lett. **151B**, 161 (1985).

[30]A. Sanda, Phys. Rev. D **23**, 2647 (1981); N. G. Deshpande, *ibid.* **23**, 2654 (1981); J. F. Donoghue and B. R. Holstein, *ibid.* **32**, 1152 (1985).

[31]For details see A. J. Buras, in *Proceedings of the International Europhysics Conference on High Energy Physics,* Bari, Italy, 1985, edited by L. Nitti and G. Preparata (Laterza, Bari, 1985), p. 1037.

[32]For a recent review see I. Mannelli, Riv. Nuovo Cimento **8**, 1 (1985).

[33]G. L. Fogli, Phys. Lett. **123B**, 57 (1983).

[34]R. N. Mohapatra, G. Senjanovič, and M. D. Tran, Phys. Rev. D **28**, 546 (1983).

[35]A. J. Buras, W. Slominski, and H. Steger, Nucl. Phys. **B238**, 529 (1984); J. F. Donoghue, E. Golowich, and B. Holstein, Phys. Rep. **131**, 319 (1986).

[36]A list of these scalars and their couplings may be found in G. Costa and A. H. Zimmerman, Nuovo Cimento **64A**, 285 (1981); H. Nilles, Nucl. Phys. **B189**, 182 (1981).

[37]D. Chang, R. N. Mohapatra, and G. Senjanovič, Phys. Rev. Lett. **53**, 1419 (1984).

[38]L. E. Ibañez, Nucl. Phys. **B181**, 105 (1981); J. F. Nieves, *ibid.* **B189**, 182 (1981); A. Masiero, R. Mohapatra, and R. D. Peccei, *ibid.* **B192**, 66 (1981).

[39]For the relevant GUT implications see D. Cocolicchio and G. L. Fogli, Phys. Rev. D **30**, 2391 (1984).

[40]G. Altarelli and L. Maiani, Phys. Lett. **52B**, 35 (1974); M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Rev. D **18**, 2583 (1978).

[41]I. I. Bigi and J. M. Frère, Phys. Lett. **129B**, 469 (1983).