Remarks concerning the $O(Z\alpha^2)$ corrections to Fermi decays, conserved-vector-current predictions, and universality

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(Received 2 December 1986; revised manuscript received 18 February 1987)

Finite-nuclear-size contributions to the $O(Z\alpha^2)$ corrections to Fermi decays are studied for realistic nuclear-charge distributions. In conjunction with the results of Koslowsky *et al.* and recent papers by the author and Zucchini and by Jaus and Rasche, these refinements lead to an average value $\mathcal{F}t = 3070.6 \pm 1.6$ s for the accurately measured superallowed Fermi transitions. Correspondingly, $V_{ud} = 0.9744 \pm 0.0010$ and $V_{ud}{}^2 + V_{us}{}^2 + V_{ub}{}^2 = 0.9979 \pm 0.0021$ in good agreement with the three-generation standard model at the level of its quantum corrections. The agreement with conserved-vector-current predictions is very good, with each of the eight transitions differing from the average by $< 1\sigma$. The consequences of using two other calculations of the nuclear mismatch correction δ_c, Wilkinson's microscopic analysis and the recent results of Ormand and Brown, are briefly discussed. A useful upper bound on $\mathcal{F}t$, independent of the δ_c calculation, is given.

After a 14-yr hiatus, significant progress has been achieved recently in the analysis of the $O(Z\alpha^2)$ corrections to Fermi decays. This has brought a large number of highly accurate experiments into close agreement with the predictions of the conserved-vector-current (CVC) hypothesis and the standard model (SM), at the level of the quantum corrections.[1]

The $O(Z\alpha^2)$ corrections are defined as the contributions of this order to the positron-nucleus interaction not contained in the product $F(Z,E)(1+\delta_1)$, where $F(Z,E)$ is the Fermi function and δ_1 the $O(\alpha)$ correction.[2] An ingenious formalism to treat perturbatively these residual $O(Z\alpha^2)$ corrections in the framework of the independent-particle model of β decay was developed by Jaus and Rasche in the early 1970s.[3,4] In this framework the $O(Z\alpha^2)$ corrections were evaluated relying heavily on numerical, computer-based calculations. In recent years, experiments on the superallowed Fermi transitions have reached great accuracy, of $\sim 0.1\%$, and at this level a sharp discrepancy between the low-Z and the high-Z decays was found.[5,6]

The first recent theoretical progress on this subject occurred when an approximate analytic calculation, supported by general theorems of perturbative quantum field theory, revealed a sharp difference with the older numerical results. Further, when the $\mathcal{F}t$ values of the eight accurately measured superallowed Fermi transitions were reexamined in the light of the new $O(Z\alpha^2)$ corrections, they were found to be in very good agreement with CVC and SM predictions.[1] The second recent significant development on this subject occurred when Jaus and Rasche reanalyzed the complicated two-loop calculations by computer-based methods, found an error in the older work, and obtained new answers which are completely consistent with the analytical work of Ref. 1.[7] Some relatively small differences remain between Refs. 1 and 7 (to be referred henceforth as I and II, respectively), having to do with the treatment of finite-nuclear-size effects and the small $O(Z^2\alpha^3)$ corrections. It is the main aim of this article to provide explicit expressions for the finite-nuclear-size effects in terms of realistic nuclear-charge distributions. As a second objective we will compare the results of I and II and slightly modify the treatment of the $O(Z^2\alpha^3)$ corrections. Finally, the effect of these refinements on the verification of CVC and SM and the determination of V_{ud} will be discussed.

In order to compare papers I and II, a brief qualitative description of the $O(Z\alpha^2)$ corrections is desirable. They involve the exchange of a photon of momentum k and a Coulombic photon of momentum q. The diagrams fall into two classes: (a) in which the k photon is interchanged between the decaying nucleons and the positron and (b) in which the k photon is fully attached to the positron (the two types of diagrams are depicted in Figs. 1 and 2 of I, respectively). The sum of diagrams in each class is ultraviolet and infrared convergent. In turn, the contributions of type (a) can be classified according to the dependence on the nucleon mass M before the k and q integrals are carried out. The terms of leading order in M, called $Z\alpha^2\Delta_1$ in II, are complicated and difficult to evaluate. Calling $G(Q^2)$ the charge form factor of the daughter nucleus ($Q^2 \equiv -q^2 \geq 0$) and writing $G(Q^2) = 1 + G(Q^2) - 1$ it is convenient to split $\Delta_1 = \Delta_1^0(E) + \Delta_1^F$ where Δ_1^0 corresponds to a point nucleus and Δ_1^F arises from $G(Q^2) - 1$. Δ_1^0 depends on the positron energy E while, neglecting terms of $O(E/\Lambda)$ ($\Lambda \equiv \sqrt{6}/a$ and a is the rms nuclear charge radius), Δ_1^F is energy independent and much easier to evaluate. On the other hand, Δ_1^F depends on the charge distribution of the daughter nucleus and, in general, involves terms of logarithmic, zeroth, and higher orders in Λ/M. The contributions of class (a) which are not of leading order in M are also easier to evaluate. Neglecting again terms of $O(E/\Lambda)$ they become constants of first and higher orders in Λ/M; the contributions of this type arising from the vector and axial-vector currents are called in II $Z\alpha^2\Delta_2$ and $Z\alpha^2\Delta_3$, respectively. Finally the diagrams of class (b) represent a pure "QED" correction, depend on E, and are denoted by

$Z\alpha^2\Lambda_4(E)$ in II.

Analytic expressions for $\Delta_1^0(E)$ and $\Delta_4(E)$, obtained in the extreme relativistic approximation (ERA) for the positrons, are given in Eqs. (3) and (4) of paper I, respectively. In the case of the dominant contribution $\Delta_1^0(E)$ the opposite, nonrelativistic limit, was also calculated to verify the smoothness of the extrapolation between the two domains. According to Eq. (5) of I one finds, in the ERA,

$$\Delta_1^0(E)+\Delta_4(E)\doteq\ln(M/m)-\tfrac{5}{3}\ln(2E/m)+\tfrac{43}{18}\ , \qquad (1)$$

where m is the positron mass. Table I compares Eq. (1) with the new computer-based calculations of paper II; the fourth column gives the percentual difference while the fifth shows the percentual shift induced by this difference on the decay probability in the case $Z=26$ (the largest Z among the accurately measured transitions). It is clear that the numerical calculation agrees remarkably well with the simple analytic expression in Eq. (1) and the difference has a very small effect on the decay probability and $\mathcal{F}t$ values.

We turn now to the evaluation of the finite-nuclear-size effects represented by Δ_1^F, Δ_2, and Δ_3. After neglecting terms of $O(E/\Lambda)$ and performing the k and one of the Feynman-parameter integrations, Δ_1^F can be cast in the form

$$\Delta_1^F=\frac{4}{\pi^2}\int_0^1\left[\frac{u}{1-u}\right]^{1/2}du\int_0^1 dv\int_0^\infty\frac{dQ}{Q}[G(Q^2)-1]\arctan\left[\frac{Mu^{1/2}v}{Q(1-u)^{1/2}}\right]. \qquad (2)$$

It is convenient to separate the integrals proportional to $G(Q^2)$ and 1 by introducing a temporary infrared cutoff ϵ. The first term involves

$$\int_\epsilon^\infty (dQ/Q)G(Q^2)\{\pi/2-\arctan[Q(1-u)^{1/2}/Mu^{1/2}v]\}\ .$$

To evaluate $\int_\epsilon^\infty (dQ/Q)G(Q^2)$ for a spherically symmetry charge distribution $\rho(r)$ one considers the Fourier transform

$$G(Q^2)=4\pi\int_0^\infty \rho(r)r\sin(Qr)dr/Q\ .$$

$\rho(r)$ is normalized so that $\int\rho(r)d^3r=1$. Performing the Q integration in the small-ϵ limit leads to

$$\int_\epsilon^\infty dQ\,G(Q^2)/Q=1-\gamma-4\pi\int_0^\infty\rho(r)r^2\ln(r\epsilon)dr\ ,$$

where $\gamma=0.5772\ldots$. Combining with $-\int_\epsilon^\infty (dQ/Q)\arctan[Mu^{1/2}v/Q(1-u)^{1/2}]$ from Eq. (2), one obtains

$$\frac{4}{\pi^2}\int_0^1\left[\frac{u}{1-u}\right]^{1/2}du\int_0^1 dv\int_\epsilon^\infty\frac{dQ}{Q}\left[\frac{\pi}{2}G(Q^2)-\arctan\left[\frac{M}{Q}\frac{u^{1/2}v}{(1-u)^{1/2}}\right]\right]=1-\gamma-4\pi\int_0^\infty\rho(r)r^2\ln(Mr)dr\ . \qquad (3)$$

Equation (3) contains the contributions to Δ_1^F of logarithmic and zeroth order in Λ/M and were already given in paper I (where they are called $\Delta/Z\alpha^2$). Instead, the term proportional to $G(Q^2)\arctan[Q(1-u)^{1/2}/Mu^{1/2}v]$ leads to contributions of $O(\Lambda/M)$. To evaluate it one sets $\epsilon=0$, performs the v integration in (2) and then expands the integrand in powers of Q/M. Expressing once more $G(Q^2)$ in terms of $\rho(r)$ all the integrals of $O(\Lambda/M)$ can be readily done and one finds

$$\Delta_1^F=1-\gamma-4\pi\int_0^\infty\rho(r)r^2\ln(Mr)dr$$
$$-(8/M)\int_0^\infty\rho(r)r[1+\gamma+\ln(Mr)]dr\ , \qquad (4)$$

where we have neglected terms of $O((\Lambda/M)^3)$. The contributions from Δ_2 and Δ_3 can be evaluated with very similar methods. The only novel point is that they contain terms of $O(\Lambda^2/M^2)$ which we wish to retain; in our derivation we assume that $\int_0^\infty\rho(r)dr$ exists, which is the case for all realistic nuclear charge distributions. The

TABLE I. Comparison of Eq. (1) with computer-based calculation in paper II. Data in the second column are from Eq. (1) and in the third column from II; p is the positron momentum. Diff. is the difference (in %) between Eq. (1) and the calculation in II and $\delta P(E)$ is the shift (in %) induced by this difference on the decay probability in the case $Z=26$.

p/m	$\Delta_1^0+\Delta_4$ [Eq. (1)]	$\Delta_1^0+\Delta_4$ (II)	Diff.	$\delta P(E)$ ($Z=26$)
0.1	8.74	8.99	−2.8	−0.03
0.5	8.56	8.48	+0.9	+0.01
1.0	8.17	7.98	+2.4	+0.03
5.0	6.03	6.01	+0.3	0.00
10.0	4.90	4.93	−0.6	0.00
16.0	4.12	4.18	−1.4	−0.01

analysis leads to

$$\Delta_2=(4/M)\int_0^{\infty}\rho(r)r\,dr(1-\pi/4Mr)\ , \tag{5}$$

$$\Delta_3=\frac{8g_A(1+\mu_V)}{M}\int_0^{\infty}\rho(r)r\left[\gamma+\ln(Mr)-\frac{1}{2}+\frac{\pi}{8Mr}\right]dr\ , \tag{6}$$

where $g_A=1.25$, $1+\mu_V=4.70$ arises from the isovector magnetic moment of the nucleon and we have again neglected terms of $O((\Lambda/M)^3)$.

We now consider specific models for $\rho(r)$. For the uniformly charged sphere of radius $R=(\frac{5}{3})^{1/2}a=10^{1/2}/\Lambda$, Eqs. (4)—(6) become

$$\Delta_1^F=\ln\left[\frac{\Lambda}{M}\right]-\kappa_2-\frac{3}{10^{1/2}\pi}\frac{\Lambda}{M}\left[\frac{1}{2}+\gamma+\ln10^{1/2}+\ln\left[\frac{M}{\Lambda}\right]\right]\ , \tag{7}$$

$$\Delta_2=\frac{3}{2\times10^{1/2}\pi}\frac{\Lambda}{M}\left[1-\frac{\pi}{2\times10^{1/2}}\frac{\Lambda}{M}\right]\ , \tag{8}$$

$$\Delta_3=\frac{3}{10^{1/2}\pi}g_A(1+\mu_V)\frac{\Lambda}{M}\left[\gamma-1+\ln10^{1/2}+\ln\left[\frac{M}{\Lambda}\right]+\frac{\pi}{4\times10^{1/2}}\frac{\Lambda}{M}\right]\ , \tag{9}$$

where $\kappa_2\equiv\gamma-\frac{4}{3}+\ln10^{1/2}=0.395$. For the modified Gaussian model $\rho(r)=\text{const}\times(1+\alpha k^2r^2/a^2)e^{-r^2k^2/a^2}$ with $\alpha=(Z-2)/3$, $k^2=(\frac{3}{2})(2+5\alpha)/(2+3\alpha)$ (Ref. 8) the corresponding expressions are

$$\Delta_1^F=\ln\left[\frac{\Lambda}{M}\right]-\kappa_1(Z)-\frac{8}{(6\pi^3)^{1/2}}\frac{k}{2+3\alpha}\frac{\Lambda}{M}\left\{\left[1+\frac{\gamma}{2}+\ln\left[\frac{6^{1/2}}{k}\frac{M}{\Lambda}\right]\right](1+\alpha)+\frac{\alpha}{2}\right\}\ , \tag{10}$$

$$\Delta_2=\frac{4}{(6\pi^3)^{1/2}}\frac{k}{2+3\alpha}\frac{\Lambda}{M}\left[1+\alpha-\left[\frac{\pi^3}{96}\right]^{1/2}k\frac{\Lambda}{M}\left[1+\frac{\alpha}{2}\right]\right]\ , \tag{11}$$

$$\Delta_3=\frac{8}{(6\pi^3)^{1/2}}g_A(1+\mu_V)\frac{k}{2+3\alpha}\frac{\Lambda}{M}\left\{\left[\frac{\gamma-1}{2}+\ln\left[\frac{6^{1/2}}{k}\frac{M}{\Lambda}\right]\right](1+\alpha)+\frac{\alpha}{2}+\left[\frac{\pi^3}{6}\right]^{1/2}\frac{k}{8}\frac{\Lambda}{M}\right\}\ , \tag{12}$$

where $\kappa_1(Z)\equiv\frac{1}{2}[\gamma+\ln(3/2k^2)+2\alpha/(2+3\alpha)]$. The form factor $G(Q^2)=\Lambda^2/(\Lambda^2+Q^2)$ employed in II corresponds to a Yukawa-type distribution $\rho(r)=(\Lambda^2/4\pi)e^{-\Lambda r}/r$. In this case Eq. (4) reduces to the expression for Δ_1^F given in paper II; the same is true of the terms of $O(\Lambda/M)$ in Eqs. (5) and (6). The contributions of $O((\Lambda/M)^2)$ in Δ_2 and Δ_3 in the case of the Yukawa distribution are given in II [they cannot be obtained from Eqs. (5) and (6) of this paper as the integral $\int_0^{\infty}\rho(r)dr$ diverges].

Table II lists the values for $\Delta^F\equiv\Delta_1^F+\Delta_2+\Delta_3$ for the uniformly charged sphere, modified Gaussian and Yukawa models. The first two are calculated using $M/\Lambda=r_0A^{1/3}/0.665$ with r_0 as given in Ref. 9; the latter are evaluated with $M/\Lambda=2A^{1/3}$ as in paper II. One sees that the uniformly charged sphere and modified Gaussian models give very similar answers but both differ significantly from the Yukawa results. This is due to the fact that $\Lambda^2/(\Lambda^2+Q^2)$ is not a realistic representation of nuclear charge form factors and decreases too slowly as Q^2 increases. Correspondingly, the Yukawa distribution has a singularity at $r=0$ while the phenomenologically derived distributions are finite and flat near $r=0$ (Ref. 8). The table also gives the values of $\langle\delta_2(E)\rangle$ corresponding to the Yukawa and modified Gaussian models [$\langle\ \rangle$ denotes an average over the energy spectrum including the effect of $F(Z,E)$]. As the averaging was done in detail in paper II, the values for $\langle\delta_2\rangle$ obtained in that work are listed in the Yukawa entry; those corresponding to the

TABLE II. Finite-nuclear-size contributions evaluated with Yukawa (Y), uniformly-charged-sphere (UCS), and modified Gaussian (MG) distribution, and $O(Z\alpha^2)$ corrections.

	Δ^F Y	Δ^F UCS	Δ^F MG	$\langle\delta_2\rangle$ (from II) Y (%)	$\langle\delta_2\rangle$ (present paper) MG (%)
^{14}O	−0.68	−1.39	−1.31	0.24	0.22
^{26}Al^m	−0.99	−1.60	−1.53	0.35	0.32
^{34}Cl	−1.12	−1.69	−1.64	0.43	0.39
^{38}K^m	−1.17	−1.73	−1.73	0.46	0.41
^{42}Sc	−1.22	−1.77	−1.72	0.50	0.45
^{46}V	−1.26	−1.80	−1.75	0.53	0.47
^{50}Mn	−1.30	−1.83	−1.79	0.55	0.49
^{54}Co	−1.33	−1.86	−1.82	0.57	0.50

TABLE III. Fractional radiative corrections (in %) and $\mathcal{F}t$ values.

Decay	$\langle\delta_1^{ou}\rangle$[a]	$\langle\delta_2\rangle$[b]	$\langle\delta_3^{he}\rangle$[c]	$\mathcal{F}t(s)$[d]
^{14}O	1.29	0.22	0.01	3074.0±3.9
^{26}Al^m	1.11	0.32	0.02	3068.1±3.7
^{34}Cl	1.00	0.39	0.03	3069.0±4.7
^{38}K^m	0.96	0.41	0.04	3066.6±4.6
^{42}Sc	0.94	0.45	0.04	3077.5±7.5
^{46}V	0.90	0.47	0.05	3074.7±4.3
^{50}Mn	0.87	0.49	0.05	3069.6±5.7
^{54}Co	0.84	0.50	0.06	3069.0±4.4
Ave				3070.6±1.6
χ^2/ν				0.57
C.L.				78%

[a]From Ref. 9.
[b]Calculated in the modified Gaussian model (Table II).
[c]From paper I and Ref. 10.
[d]Obtained by modifying the $\mathcal{F}t$ values of Ref. 6 on the basis of $\langle\delta_1^{ou}+\delta_2+\delta_3^{he}\rangle$.

modified Gaussian model can be simply obtained by taking into account the change in Δ^F between the two models. Alternatively, good approximations can be obtained by averaging Eq. (1) with a weighting factor $S(E)=pE(E_m-E)^2$ (E_m is the end-point energy and p the positron momentum) and adding the appropriate values for Δ^F. The results for the modified Gaussian distribution will be regarded as a realistic calculation of $\langle\delta_2(E)\rangle$. They represent a refinement relative to the results reported in paper I in that terms of $O(\Lambda/M)$ and $O(\Lambda^2/M^2)$ have been retained, and relative to those of paper II in that a realistic charge distribution has been employed. As noted in I, the dominant terms of logarithmic and zeroth order in Λ bear no reference to the decaying nucleon in contraposition with the nucleus. On the other hand, the term of first and higher order in Λ do involve nucleon properties such as M, g_A, μ_V; it is clear that the independent-particle model plays an important role in the evaluation of these smaller terms proportional to powers of Λ/M.

Table III lists the average $\langle\delta_1^{ou}\rangle$ of the outer correction of $O(\alpha)$ as given in Ref. 9, $\langle\delta_2\rangle$ as evaluated in Table II for the modified Gaussian distribution and $\langle\delta_3^{he}\rangle$ where δ_3^{he} is the heuristic estimate of $O(Z^2\alpha^3)$ contributions discussed in I (Ref. 10). The values $\langle\delta_3^{he}\rangle$ coincide with those given in paper II for the four low-Z nuclei but are smaller by 0.01–0.02 % for the high-Z ones, due mainly to differences in the analysis of the positron mass singularities. The last column presents the $\mathcal{F}t$ values obtained by modifying the results of Ref. 6 on the basis of the corrections $\langle\delta_1^{ou}+\delta_2+\delta_3^{he}\rangle$ listed in Table III.

As was the case in I, the $\mathcal{F}t$ values of Table III agree very well with CVC predictions: now each of the eight accurately measured Fermi transitions stands within 1σ of the average. The χ^2/ν is 0.57 corresponding to 78% confidence level. The difference of 2 sec between the average $\mathcal{F}t=3070.6\pm1.6$ sec in Table III and the corresponding result of paper I is due to the inclusion of the terms of $O(\Lambda/M)$ ($\simeq$0.9 sec) and the $O(Z^2\alpha^3)$ terms ($\simeq$1.1 sec). The difference of 1.8 sec with paper II is mainly due to the use of realistic nuclear charge distributions. Repeating the analysis of paper I and Ref. 11,[12] Table IV lists the values of V_{ud} and $V_{ud}{}^2+V_{us}{}^2+V_{ub}{}^2$ obtained on the basis of various combinations of the $\mathcal{F}t$ values given in Table III, in conjunction with $V_{us}=0.220\pm0.002$ (Ref. 13). It is apparent that the results of Table IV are in very good agreement with the third-generation SM, at the level of its quantum corrections. As demonstrated in Ref. 14, such accurate agreement can be used to derive interesting constraints on new physics.

To illustrate the dependence of these conclusions on specific calculations of the nuclear mismatch corrections δ_c we consider, aside from the Towner-Hardy-Harvey (THH) results[12] employed above, the microscopic analysis due to Wilkinson (W) (Ref. 9) and the more recent results of Ormand and Brown (OB) (Ref. 15). Employing the same radiative corrections and errors as in Table III, one obtains on the basis of these calculations $(\mathcal{F}t)_{ave}=3069.8\pm1.6$ s ($\chi^2/\nu=0.97$) and $(\mathcal{F}t)_{ave}=3076.1\pm1.6$ s ($\chi^2/\nu=1.01$), respectively. Thus, as far as CVC is concerned, although these two fits are not as good as the one presented in Table III, they are certainly quite acceptable. It is worth noting that the new $O(Z\alpha^2)$ results are very significant in achieving good agreement with CVC, as in

TABLE IV. Values of V_{ud} obtained from various combinations of $\mathcal{F}t$ values listed in Table III. The last column tests the third-generation SM at the level of its quantum corrections.

	V_{ud}	$V_{ud}{}^2+V_{us}{}^2+V_{ub}{}^2$
^{14}O	0.9739±0.0015	0.9969±0.0031
Average of ^{14}O and ^{26}Al^m	0.9744±0.0012	0.9979±0.0025
Average of 4 low-Z decays	0.9746±0.0011	0.9982±0.0023
Average of 8 decays	0.9744±0.0010	0.9979±0.0021

all cases the fits with the older corrections were very poor. There remains the problem that although the THH and W calculations lead to averages that are very close to each other (3070.6±1.6 s and 3069.8±1.6 s) the OB result (3076.1±1.6 s) differs from THH by +0.18% (implying $V_{ud}=0.9735\pm0.0010$ and $V_{ud}{}^2+V_{us}{}^2+V_{ub}{}^2=0.9962\pm0.0021$).

We now observe that there is a simple argument that leads to a useful upper bound on the $\mathscr{F}t$ values: if one assumes that all corrections except δ_c are well understood, using only $\delta_c>0$ one concludes that the lowest $\mathscr{F}t$ value calculated with $\delta_c=0$ should be an upper bound to the real value. From the data on $^{26}\mathrm{Al}^m$ one finds in this way $\mathscr{F}t\leq 3078.6\pm1.4$ s, independent of the details of the δ_c calculation. It is worth noting that all the $\mathscr{F}t$ values calculated with the THH or W δ_c's essentially obey this bound but the central values of the ^{14}O, ^{34}Cl, ^{42}Sc, and ^{46}V $\mathscr{F}t$ values calculated with the OB δ_c's are somewhat larger. Thus, at present the THH and W δ_c's are in better agreement with the model-independent upper bound than the OB results, although the argument is not conclusive in discriminating among those calculations because of the errors involved. Until nuclear theorists and β-decay experimentalists resolve this discrepancy, two reasonable procedures for the important V_{ud} determination are (i) use the result in the last row of Table IV on the grounds that the THH δ_c's provide at present the best CVC fit or (ii) employ the $\mathscr{F}t$ value for $^{26}\mathrm{Al}^m$ calculated with $\delta_c=(0.34\pm0.08)\%$, the average of the corresponding THH, W, and OB δ_c's with the error chosen to cover the range of the three values, on the grounds that the three calculations of δ_c are close for this decay and, furthermore, it is the most accurate experimentally. This second procedure leads to $(\mathscr{F}t)_{\mathrm{Al}}=3068.1\pm2.8$ s corresponding to $V_{ud}=0.9748\pm0.0010$ and $V_{ud}{}^2+V_{us}{}^2+V_{ub}{}^2=0.9986\pm0.0021$ (no doubling of errors was done here in extracting V_{ud}). The two determinations are compatible within the stated errors.

Another matter that ought to be clarified is the recent claim by Drukarev and Strikman[16] that the effect of screening due to atomic electrons is considerably larger than provided by the conventional theory.[9] We note that inclusion of their additional screening correction would worsen the very good fit of Table III.

On the positive side one should stress that the discrepancies mentioned above are at the 0.1–0.2% level, i.e., accuracies seldom attained in particle physics, they are much smaller than the overall quantum corrections which are of ~4%, and moreover most of the calculations of these subtle effects were carried out at a time when the difficult $O(Z\alpha^2)$ corrections were marred by computational errors and, as a consequence, comparison with the experimental results was somewhat misleading.

It is to be hoped that nuclear and atomic theorists and β-decay experimentalists will reexamine these small corrections in the light of the new $O(Z\alpha^2)$ calculations and that progress will be made in settling whatever discrepancy remains. In this connection the accurate measurements of C and $\pi\beta$ decays would be most welcome.

The author is indebted to W. Jaus, G. Rasche, and D. Wilkinson for very stimulating private communications and to W. J. Marciano and J. Napolitano for useful conversations. This research was partially supported by the National Science Foundation under Grant No. PHY-8413569.

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