Production of spin-one resonances in $\gamma\gamma^*$ collisions

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The decay of a $J^{PC}=1^{++}$ resonance into one real and one virtual photon is studied in a $q\bar{q}$ bound-state model. The decay rate is proportional to Q^2/M^2 if Q^2, the mass squared of the virtual photon, is small compared to M^2, the mass squared of the resonance. The cross section for production in e^+e^- collisions of a resonance with $J^{PC}=1^{++}$ is derived in terms of the partial width of the resonance to decay into one real and one virtual photon. The angular distributions for the three-body decays of 1^{++} and 1^{-+} states produced by the two-photon process are presented.

I. INTRODUCTION

Reports[1,2] of the observation of a resonance seen in $e^+e^-\rightarrow e^+e^-X$ only when, in addition to the hadronic system ($X=K\bar{K}\pi$), a final e^+ or e^- is observed away from the forward direction, are strong evidence for a spin-one particle with even charge conjugation. Such a resonance could be produced by the two-photon mechanism only if at least one photon were virtual since Yang's theorem prohibits the production of a spin-one particle by two real photons. Observation of a final e^+ or e^- with substantial transverse momentum guarantees that one of the bremsstrahlung photons is quite virtual.

The possibility of such production has been considered earlier,[3] especially by Renard,[4] who gave a prediction for the hypothetical decay into one real and one virtual photon of a spin-one resonance composed of a nonrelativistic quark-antiquark pair in a $^3P_1$ state. The present recomputation of the rate is in partial disagreement with the results of Renard.[4] A derivation of the result is presented in Secs. II and III. The production cross section of a spin-one resonance in e^+ and e^- collisions via the two-photon mechanism is calculated in the equivalent photon approximation in Sec. IV and without approximation in the Appendix. Estimates for the partial widths of spin-one resonances to decay into one real and one virtual photon are presented in Sec. V.

The possibility that the state observed by the TPC Collaboration[1] and the Mark II Collaboration[2] has $J^{PC}=1^{-+}$ has been raised by Chanowitz.[5] Assuming that the effect reported[1,2] is indeed a spin-one resonance we note in Sec. VI a simple test to determine the parity of the state.

II. PRELIMINARIES

The matrix element $\mathscr{M}_{\rm BS}$ for the decay of a nonrelativistic bound state is obtained by calculating the matrix element $\mathscr{M}_{\rm scat}$ for the corresponding free-quark process to the same final state.[6] The precise prescription is

$$\mathscr{M}_{\rm BS}=\int d^3p\,\tilde{\phi}(p)\mathscr{M}_{\rm scat}\frac{(2\pi)^{3/2}(2M)^{1/2}}{[(2\pi)^{3/2}(2m)^{1/2}]^2}\ , \qquad (2.1)$$

where $\tilde{\phi}$ is the momentum-space wave function, M is the bound-state mass, and $m=M/2$ is the quark mass. The final factor in Eq. (2.1) arises from the relation between the normalization of the S matrix and the invariant amplitude. The relation between the configuration-space and momentum-space wave functions is

$$\tilde{\phi}(\mathbf{p})=\int\frac{d^3r}{(2\pi)^{3/2}}e^{-i\mathbf{p}\cdot\mathbf{r}}\phi(\mathbf{r})\ ,$$
$$\phi(\mathbf{r})=\int\frac{d^3p}{(2\pi)^{3/2}}e^{i\mathbf{p}\cdot\mathbf{r}}\tilde{\phi}(\mathbf{p})\ . \qquad (2.2)$$

More explicitly, we write

$$\phi(\mathbf{r})=\mathscr{R}_L(r)\chi^J_{LM}(\hat{\mathbf{r}})\ , \qquad (2.3)$$

where $\chi^J_{LM}(\hat{\mathbf{r}})$ is the angular part of the wave function for total angular momentum J, orbital angular momentum L, and $J_z=M$. The spin dependence is not displayed. The radial wave function is normalized as

$$\int dr\,r^2R_L(r)^2=1 \qquad (2.4)$$

and the momentum-space wave function is

$$\tilde{\phi}(\mathbf{p})=\tilde{\mathscr{R}}_L(p)\tilde{\chi}^J_{LM}(\hat{\mathbf{p}})\ . \qquad (2.5)$$

It is easy to derive the relation between the decay of a $q\bar{q}$ s-wave bound state and $q\bar{q}$ the cross section at threshold into the same final state. From Eqs. (2.1) and (2.2)

$$\mathscr{M}_{\rm BS}=\left[\frac{2}{M}\right]^{1/2}\phi(0)\mathscr{M}_{\rm scat}\ , \qquad (2.6)$$

while the decay rate is

$$\Gamma=\frac{1}{8\pi}\frac{p_f}{M^2}\,|\,\mathscr{M}_{\rm BS}\,|^2 \qquad (2.7)$$

and the threshold behavior of the cross section is

$$\sigma(p\rightarrow 0)\rightarrow\frac{1}{16\pi M^2}\frac{p_f}{p}\,|\,\mathscr{M}_{\rm scat}\,|^2\ , \qquad (2.8)$$

where p_f is the final-state c.m. momentum and p is the initial-state c.m. momentum. Thus

$$\Gamma=\frac{4p}{M}\,|\,\phi(0)\,|^2\sigma(p\rightarrow 0)$$
$$=|\,\phi(0)\,|^2v\sigma(p\rightarrow 0)\ , \qquad (2.9)$$

where $v=2p/m=4p/M$ is the relative velocity of the quark and antiquark in the bound state. The product $v\sigma(p)$ is nonzero as $v,p\to 0$.

For bound states with $L>0$ we have, from Eqs. (2.2), (2.3), and (2.5),

$$R_L(r)=\int \frac{dp\,p^2}{(2\pi)^{3/2}} i^l 4\pi j_L(pr)\tilde{R}_L(p)\ , \tag{2.10}$$

so that for small r

$$R_L(r)\to \frac{i^L r^L}{(2L+1)!!}\int \frac{4\pi\,dp\,p^2}{(2\pi)^{3/2}} p^L \tilde{R}_L(p)\ . \tag{2.11}$$

Thus Eq. (2.6) generalizes to

$$\mathcal{M}_{\rm BS}=\left[\frac{2}{M}\right]^{1/2}\frac{(2L+1)!!}{L!}\left[\frac{d}{dr}\right]^L R_L(r)\Bigg|_{r=0} \times\int\frac{d\Omega_p}{4\pi}\tilde{\chi}^J_{LM}(\hat{\mathbf{p}})\overline{\mathcal{M}}(\hat{\mathbf{p}})\ , \tag{2.12}$$

where we have ignored an overall phase and written $\overline{\mathcal{M}}$ for $\lim_{p\to 0}(\mathcal{M}_{\rm scat}/p^L)$.

III. THE DECAY RATE FOR $^3P_1\to\gamma\gamma^*$

The scattering amplitude for $q\bar{q}\to\gamma^*\gamma^*$ is

$$\mathcal{M}_{\rm scat}=e^2\langle e_q^2\rangle\bar{v}(p')\left[\frac{\not\epsilon_2\not k_1\not\epsilon_1-2p\cdot\epsilon_1\not\epsilon_2}{2k_1\cdot p-k_1^2}+\frac{\not\epsilon_1\not k_2\not\epsilon_2-2p\cdot\epsilon_2\not\epsilon_1}{2k_2\cdot p-k_2^2}\right]u(p)\ , \tag{3.1}$$

where e is the charge of the positron and $\langle e_q^2\rangle$ is the effective quark charge squared in the bound state in units of e^2. Thus for an $s\bar{s}$ state $\langle e_q^2\rangle=\frac{1}{9}$, while for an isoscalar $(u\bar{u}+d\bar{d})/\sqrt{2}$ state it is $5\sqrt{2}/18$, and for an isovector $(u\bar{u}-d\bar{d})/\sqrt{2}$, $\langle e_q^2\rangle=\sqrt{2}/6$. The initial quark and antiquark momenta are p and p', with $\mathbf{p}'=-\mathbf{p}$, while the photons have polarizations and momenta (ϵ_1,k_1) and (ϵ_2,k_2). We allow $k_1^2\neq 0$ and $k_2^2\neq 0$, but in any event $\epsilon_1\cdot k_1=\epsilon_2\cdot k_2=0$.

The appropriate choice of spinors depends on the values of J, L, S, and M. The spins of the quark and antiquark are combined in the usual fashion to make a state of fixed S and S_z, then coupled to the appropriate spherical harmonics to produce the required state. For $L=1$, $S=1$, $J=1$, $M=0$, we have, up to an inconsequential phase,

$$u(p)\bar{v}(p')=\left[\frac{3}{16\pi}\right]^{1/2}(\not p+m)\gamma_0(\hat{\mathbf{p}}\times\hat{\mathbf{z}}\cdot\boldsymbol{\gamma})\ . \tag{3.2}$$

From Eq. (3.1) it is clear that the term proportional to m in Eq. (3.2) does not contribute to the bound-state decay amplitude. Denoting by η^μ the four-vector that is $(0,\hat{\mathbf{p}}\times\hat{\mathbf{z}})$ in the c.m., we have

$$\mathcal{M}_{\rm scat}=e^2\langle e_q^2\rangle\left[\frac{3}{16\pi}\right]^{1/2}{\rm Tr}\not p\gamma_0\not\eta \times\left[\frac{\not\epsilon_2\not k_1\not\epsilon_1-2p\cdot\epsilon_1\not\epsilon_2}{2p\cdot k_1-k_1^2}+(1\leftrightarrow 2)\right]\ . \tag{3.3}$$

The angular dependence on $\hat{\mathbf{p}}$ is contained in Eq. (3.3). After integrating over $d\Omega_{\hat{\mathbf{p}}}$ [see Eq. (2.12)] there will be an explicit factor of p in the amplitude. It suffices to expand Eq. (3.3) in powers of p/m, retaining the linear terms. Thus we write

$$2p\cdot k_1-k_1^2\simeq 2m^2-2\mathbf{p}\cdot\mathbf{k}_1-\tfrac{1}{2}(k_1^2+k_2^2)\simeq D\left[1-\frac{2\mathbf{p}\cdot\mathbf{k}_1}{D}\right] \tag{3.4}$$

with

$$D=2m^2-\tfrac{1}{2}k_1^2-\tfrac{1}{2}k_2^2\ . \tag{3.5}$$

Retaining only terms that will survive the angular integration,

$$\mathcal{M}_{\rm scat}=e^2\langle e_q^2\rangle\left[\frac{3}{16\pi}\right]^{1/2}\times{\rm Tr}\left[m\not\eta\frac{2\mathbf{p}\cdot\epsilon_1\not\epsilon_2}{D}+(\not\epsilon_2\not k_1\not\epsilon_1-2m\epsilon_1^0\not\epsilon_2)\left[\frac{2m\not\eta\,\mathbf{p}\cdot\mathbf{k}_1}{D^2}-\frac{\mathbf{p}\cdot\boldsymbol{\gamma}\gamma_0\not\eta}{D}\right]+(1\leftrightarrow 2)\right]\ . \tag{3.6}$$

Carrying out the traces and averaging over the orientation of $\hat{\mathbf{p}}$ gives

$$\langle\mathcal{M}_{\rm scat}\rangle_{\rm angular}=\frac{8e^2\langle e_q^2\rangle}{3D}\left[\frac{3}{16\pi}\right]^{1/2}\times\left[\frac{k_1^2-k_2^2}{2m}\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_1\times\hat{\mathbf{z}}-\frac{k_1^2+k_2^2}{D}(\epsilon_1^0\boldsymbol{\epsilon}_2-\epsilon_2^0\boldsymbol{\epsilon}_1)\cdot\hat{\mathbf{z}}\times\mathbf{k}_1\right]\ . \tag{3.7}$$

It is easy to check that Eq. (3.7) satisfies gauge invariance by substituting $\epsilon_1\to k_1$, $k_1^2=0$ and verifying that the result vanishes. Inserting Eq. (3.7) into Eq. (2.12) gives the decay amplitude for $^3P_1\to\gamma^*\gamma^*$:

$$\mathcal{M}_{\rm BS}=\left[\frac{2}{M}\right]^{1/2}3R'(0)\frac{8e^2\langle e_q^2\rangle}{3D}\left[\frac{3}{16\pi}\right]^{1/2}\times\left[\frac{k_1^2-k_2^2}{2m}\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_1\times\hat{\mathbf{z}}-\frac{k_1^2+k_2^2}{D}(\epsilon_1^0\boldsymbol{\epsilon}_2-\epsilon_2^0\boldsymbol{\epsilon}_1)\cdot\hat{\mathbf{z}}\times\mathbf{k}_1\right]\ . \tag{3.8}$$

If photon 1 is real,

$$\mathcal{M}_{\rm BS}=\frac{2e^2\langle e_q^2\rangle}{D}R'(0)\left[\frac{6}{\pi M}\right]^{1/2}\frac{k_2^2}{2m}\times(-\boldsymbol{\epsilon}_1\cdot\hat{\mathbf{z}}\times\boldsymbol{\epsilon}_2+\epsilon_2^0\boldsymbol{\epsilon}_1\cdot\hat{\mathbf{z}}\times\hat{\mathbf{k}}_1)\ . \tag{3.9}$$

For the polarizations of photon 1 we take

$$\hat{\boldsymbol{\epsilon}}_1^a=\frac{\hat{\mathbf{z}}\times\hat{\mathbf{k}}_1}{|\hat{\mathbf{z}}\times\hat{\mathbf{k}}_1|}\,,\qquad \hat{\boldsymbol{\epsilon}}_1^b=\hat{\mathbf{k}}_1\times\hat{\boldsymbol{\epsilon}}_1^a\,. \tag{3.10}$$

For the polarizations of photon 2 we take

$$\hat{\boldsymbol{\epsilon}}_2^a=\hat{\boldsymbol{\epsilon}}_1^a\,,\qquad \hat{\boldsymbol{\epsilon}}_2^b=\hat{\boldsymbol{\epsilon}}_1^b\,,\qquad \epsilon_2^L=\frac{(k_1^0,-k_2^0\hat{\mathbf{k}}_1)}{(k_2{}^2)^{1/2}}\,. \tag{3.11}$$

The various decay amplitudes are

$$\begin{aligned}\mathcal{M}(\hat{\boldsymbol{\epsilon}}_1^a,\epsilon_2^L)&=\frac{e^2\langle e_q{}^2\rangle R'(0)}{mk}\left[\frac{3}{\pi m}\right]^{1/2}(k_2{}^2)^{1/2}|\hat{\mathbf{z}}\times\hat{\mathbf{k}}_1|\,,\\ \mathcal{M}(\hat{\boldsymbol{\epsilon}}_1^b,\epsilon_2^L)&=0\,,\\ \mathcal{M}(\hat{\boldsymbol{\epsilon}}_1^a,\hat{\boldsymbol{\epsilon}}_2^a)&=0=\mathcal{M}(\hat{\boldsymbol{\epsilon}}_1^b,\hat{\boldsymbol{\epsilon}}_2^b)\,,\\ \mathcal{M}(\hat{\boldsymbol{\epsilon}}_1^a,\hat{\boldsymbol{\epsilon}}_2^b)&=-\mathcal{M}(\hat{\boldsymbol{\epsilon}}_1^b,\hat{\boldsymbol{\epsilon}}_2^a)\\ &=\frac{e^2\langle e_q{}^2\rangle}{mk}R'(0)\left[\frac{3}{\pi m}\right]^{1/2}\frac{k_2{}^2}{2m}\hat{\mathbf{z}}\cdot\hat{\mathbf{k}}_1\,,\end{aligned} \tag{3.12}$$

where $k=k_1^0=(4m^2-k_2{}^2)/(4m)$ is the momentum of each virtual photon in the rest frame of the decaying resonance. The decay rate for ${}^3P_1\to\gamma\gamma^*(k_2{}^2)$ from $\mathcal{M}(\hat{\boldsymbol{\epsilon}}_1^a,\epsilon_2^L)$ alone is

$$\Gamma({}^3P_1\to\gamma\gamma^*(k_2{}^2))=\frac{96\alpha^2|R'(0)|^2}{kM^5}\langle e_q{}^2\rangle^2|k_2{}^2|\,, \tag{3.13}$$

where we have included the factor of 3 arising from the color wave function, $(\mathrm{Tr}\delta_{ij}/\sqrt{3})^2=3$. For $k_2{}^2$ not too far from zero,

$$\Gamma({}^3P_1\to\gamma\gamma^*(k_2{}^2))=\frac{192\alpha^2|R'(0)|^2}{M^4}\langle e_q{}^2\rangle^2\frac{|k_2{}^2|}{M^2}\,, \tag{3.14}$$

which is a factor of 3 larger than the result of Ref. 4.

IV. PRODUCTION OF A 1^{++} STATE IN PHOTON-PHOTON COLLISIONS IN THE EQUIVALENT-PHOTON APPROXIMATION

The coupling of the $J^{PC}=1^{++}$ particle given in Eq. (3.8) may be written as

$$\begin{aligned}&\mathcal{M}(1^{++}\to\gamma_1^*\gamma_2^*)\\ &\quad=\mathcal{A}(k_1{}^2,k_2{}^2)\epsilon_{\alpha\beta\gamma\delta}(k_2{}^2k_1^\alpha-k_1{}^2k_2^\alpha)\xi^\beta\epsilon_1^\gamma\epsilon_2^\delta\,.\end{aligned} \tag{4.1}$$

Here ξ indicates the polarization vector of the spin-one resonance, and ϵ_1 and ϵ_2 are the polarization vectors of the final-state photons. For the bound-state model of the previous section

$$\mathcal{A}=\frac{e^2\langle e_q{}^2\rangle R'(0)}{D^2}2\left[\frac{6}{\pi M}\right]^{1/2}\sqrt{3}\,, \tag{4.2}$$

where we have chosen to display explicitly the $\sqrt{3}$ arising from color. The partial decay rate for the axial vector into one real photon γ_1 and one virtual photon γ_2^* is obtained from Eq. (4.1):

$$\mathcal{M}=\mathcal{A}(0,k_2{}^2)k_2{}^2\epsilon_{\alpha\beta\gamma\delta}k_1^\alpha\xi^\beta\epsilon_1^\gamma\epsilon_2^\delta\,. \tag{4.3}$$

The partial decay rate if the virtual photon is longitudinal is

$$\Gamma_{TS}=\frac{k^3|k_2{}^2|\mathcal{A}(0,k_2{}^2)^2}{12\pi} \tag{4.4}$$

while if it is transverse, the result is

$$\Gamma_{TT}=\frac{k^3|k_2{}^2|\mathcal{A}(0,k_2{}^2)^2}{12\pi}\frac{|k_2{}^2|}{M^2}\,, \tag{4.5}$$

where we have indicated by $k=(M^2-k_2{}^2)/(2M)$ the momenta of the virtual photons in the rest frame of the resonance. With the ansatz Eq. (4.3) there is only one independent amplitude so Γ_{TS} and Γ_{TT} are necessarily related. Since Γ_{TS} dominates at small values of $|k_2{}^2|$, it is sensible to regard it as the independent quantity. In principle, Γ_{TS} and Γ_{TT} can be measured separately. In practice, this is not likely to be achieved since it would require the determination of the detailed dependence of the cross section on the kinematic variables.

The partial widths Γ_{TS} and Γ_{TT} are functions of $k_2{}^2$. It is convenient to introduce related quantities that are independent of $k_2{}^2$,

$$\tilde{\Gamma}_{TS}=\frac{M^5}{96\pi}\mathcal{A}(0,0)^2=\tilde{\Gamma}_{TT}\,, \tag{4.6}$$

so that, for small $k_2{}^2$,

$$\Gamma_{TS}=\frac{|k_2{}^2|}{M^2}\tilde{\Gamma}_{TS}\,,\qquad \Gamma_{TT}=\left[\frac{|k_2{}^2|}{M^2}\right]^2\tilde{\Gamma}_{TT}\,. \tag{4.7}$$

We can use these results to predict the cross section for $e^+e^-\to e^+e^-R$, where R is the spin-one resonance, using the equivalent-photon approximation. Alternatively, we can do a complete Feynman-diagrammatic calculation. This is done in the Appendix. For an electron beam of energy E, the flux of equivalent transverse photons of energy ω and virtual mass q^2 is[7]

$$\begin{aligned}dn_T&=\frac{d\omega}{\omega}\frac{dq^2}{q^2}\frac{\alpha}{\pi}\left[1-\frac{\omega}{E}+\frac{\omega^2}{2E^2}\right]\\ &=\frac{dx}{x}\frac{dq^2}{q^2}\frac{\alpha}{\pi}\left[1-x+\frac{x^2}{2}\right]\,,\end{aligned} \tag{4.8}$$

where $x=\omega/E$. For longitudinal photons, the result is[7]

$$dn_S=\frac{dx}{x}\frac{dq^2}{q^2}\frac{\alpha}{\pi}(1-x)\,. \tag{4.9}$$

The cross section for the resonance production can be taken to be the narrow-width limit of a Breit-Wigner form;

$$\sigma(\hat{s})=\frac{4\pi}{k^2}\frac{2J+1}{(2S_1+1)(2S_2+1)}\Gamma_{R\to\gamma\gamma^*}\pi M\delta(M^2-\hat{s}) , \tag{4.10}$$

where k is the c.m. momentum of the photon in the decay $R\to\gamma\gamma^*$. For the case when γ^* is longitudinal, we take $2S_1+1=2$, $2S_2+1=1$. For the present, both k_1^2 and k_2^2 will be treated as small quantities. This limitation is removed in the Appendix. If the incident electron and positron momenta are p_1 and p_2, the virtual-photon momenta can be written

$$k_1=x_1p_1 ,$$
$$k_2=x_2p_2 . \tag{4.11}$$

The photon momentum in the rest frame of the resonance is $k=M/2$ and $M^2=x_1x_2s$, where s is the e^+e^- c.m. energy squared. Combining Eqs. (4.8), (4.9), and (4.10),

$$d\sigma_{e^+e^-\to e^+e^-R}=dn_{T1}dn_{S2}\sigma_{TS}+dn_{T1}dn_{T2}\sigma_{TT}$$
$$=\frac{dx_1}{x_1}\frac{dk_1^2}{k_1^2}\frac{dx_2}{x_2}\frac{dk_2^2}{k_2^2}\left[\frac{\alpha}{\pi}\right]^2\frac{16\pi^2}{M}\delta(M^2-x_1x_2\delta)[\tfrac{3}{2}\Gamma_{TS}(1-x_2)+\tfrac{3}{4}\Gamma_{TT}(1-x_2+x_2^2/2)]$$
$$\times(1-x_1+x_1^2/2) . \tag{4.12}$$

Our interest is in the case where one electron is deflected very little, so photon 1 is nearly real and collinear with the beam. The partial widths, Eqs. (4.4) and (4.5), contain at least one power of k_2^2. Thus the integral over k_2^2 is finite, while the integral over k_1^2 is rendered finite by the nonzero electron mass:

$$\int\frac{dk_1^2}{k_1^2}\to\ln\left[\frac{s}{4m_e^2}\right] . \tag{4.13}$$

The differential cross section in terms of the observed electron's momentum is

$$d\sigma_{e^+e^-\to e^+e^-R}=\left[\frac{\alpha}{\pi}\right]^2\ln\left[\frac{s}{4m_e^2}\right]\frac{16\pi^2}{M^3}\int\frac{dx_2}{x_2}\frac{dk_2^2}{k_2^2}[\tfrac{3}{2}\Gamma_{TS}(1-x_2)+\tfrac{3}{4}\Gamma_{TT}(1-x_2+x_2^2/2)](1-x_1+x_1^2/2) , \tag{4.14}$$

where $x_1=(M^2/s)/x_2\equiv\tau/x_2$. If we integrate over a range of k_2^2 with $|k_2^2|\ll M^2$ and use Eqs. (4.6) and (4.7), we find

$$\sigma_{e^+e^-\to e^+e^-R}=\left[\frac{\alpha}{2\pi}\right]^2\ln\left[\frac{s}{4m_e^2}\right]\frac{16\pi^2}{M^3}\frac{3}{2}\tilde{\Gamma}_{TS}$$
$$\times\left[4[(1+\tau)\ln 1/\tau-(1-\tau)(\tfrac{7}{4}+\tau/4)]\int\frac{dk_2^2}{M^2}\right.$$
$$\left.+\tfrac{1}{2}\times4[(1+\tau/2)^2\ln(1/\tau)-\tfrac{1}{2}(1-\tau)(3+\tau)]\int\frac{dk_2^2}{M^2}\frac{|k_2^2|}{M^2}\right] , \tag{4.15}$$

where in the spirit of this section we have assumed $k_2^2\ll M^2$ and have taken $k\approx M/2$. The factors

$$\int\frac{dk_2^2}{M^2} \tag{4.16}$$

and

$$\int\frac{dk_2^2}{M^2}\frac{|k_2^2|}{M^2} \tag{4.17}$$

depend on the range of transverse momentum of the detected electron accepted by the detector. Since we have dropped some terms of order k_2^2/M^2, the portion of Eq. (4.15) with an extra power of k_2^2/M^2 cannot be trusted. For values of the transverse momentum that are not small, it is reasonable to expect that $\tilde{\Gamma}_{TS}$ and $\tilde{\Gamma}_{TT}$ will depend on k_2^2. A vector-meson-dominance picture would suggest a form factor of the sort

$$F(k_2^2)=\frac{1}{1+|k_2^2|/M_V^2} \tag{4.18}$$

with the particular vector meson (ρ,ω,ϕ) depending on the quark composition of the resonance. This factor, squared, would thus appear inside the integrals (4.16) and (4.17). Since, for the moment $k_2{}^2$ is assumed to be small, the form factor is ignored.

The expression in Eq. (4.15) is similar to the standard expression for the production of resonances with $J\neq 1$:

$$\sigma_{e^+e^-\to e^+e^-R}=\left[\frac{\alpha}{2\pi}\ln\left[\frac{s}{4m_e{}^2}\right]\right]^2(2J+1)\frac{8\pi^2\Gamma(R\to\gamma\gamma)}{M^3}4[(1+\tau/2)^2\ln(1/\tau)-\tfrac{1}{2}(1-\tau)(3+\tau)]\ . \tag{4.19}$$

The correspondence between Eq. (4.15) and Eq. (4.19) is clear. If the produced resonance is not spin one, there are two factors of $\int dk^2/k^2$ multiplying a nearly k^2-independent width, Γ. For the spin-one case, for the longitudinal polarization of the virtual photon, one factor of $\Gamma\int dk^2/k^2$ becomes essentially $\int dk^2(\Gamma_{TS}/k^2)$ where now Γ_{TS}/k^2 is nearly independent of k^2. The difference between the factors in square brackets results from the small difference between the flux for transverse and longitudinal virtual photons: $(1-x+x^2/2)$ vs $(1-x)$. Equation (4.15) permits an estimate of the observable cross section for the two-photon mechanism. Suppose $\sqrt{s}=30$ GeV, $M=1.4$ GeV, so $\tau=2.18\times 10^{-3}$. Then, from Eq. (4.15),

$$\sigma_{e^+e^-\to e^+e^-R}=(16.3\text{ pb})[\tilde{\Gamma}_{TS}\text{ (keV)}]\left[\int\frac{dk_2{}^2}{M^2}+0.53\int\frac{dk_2{}^2}{M^2}\frac{|k_2{}^2}{M^2}\right]. \tag{4.20}$$

The more complete treatment given in the Appendix yields

$$\sigma_{e^+e^-\to e^+e^-R}=(16.3\text{ pb})[\tilde{\Gamma}_{TS}\text{ (keV)}]\left\{\int\frac{dk_2{}^2}{M^2}F(k_2{}^2)^2\left[1+0.53\frac{|k_2{}^2|}{M^2}+0.23\left[1+\frac{|k_2{}^2|}{2M^2}\right]\ln\frac{M^2}{M^2+|k_2{}^2|}\right]\right\}, \tag{4.21}$$

where F is the form factor of Eq. (4.18). This result, and the similar equations such as (4.14), (4.15), and (4.20), must be multiplied by two if the detected lepton is allowed to be either the electron or the positron. For comparison, a spin-zero resonance of the same mass and at the same c.m. energy with no electron tagging, Eq. (4.19), gives

$$\sigma_{e^+e^-\to e^+e^-R(J=0)}=(120\text{ pb})[\Gamma(R\to\gamma\gamma)\text{ (keV)}]\ . \tag{4.22}$$

V. ESTIMATED PARTIAL WIDTHS

Renard[4] has used the measured width for $f_2\to\gamma\gamma$ to estimate the $\gamma\gamma^*$ widths of the ${}^3P_1$ quark-antiquark state with the same quark content. If we suppose the $f_2(1270)$ and the $f_1(1285)$ [formerly the $D(1285)$] to have the same quark content and the same radial wave functions, then, in the nonrelativistic bound-state model[8] of Sec. III,

$$\Gamma(f_2\to\gamma\gamma)=\tfrac{576}{5}\alpha^2\langle e_q{}^2\rangle^2\frac{|R'(0)|^2}{M^4}\ ,$$
$$\Gamma(f_1\to\gamma\gamma^*(k_2{}^2))=192\alpha^2\langle e_q{}^2\rangle^2\frac{|R'(0)|^2}{M^4}\frac{|k_2{}^2|}{M^2}\ . \tag{5.1}$$

Using the average value $\Gamma(f_2\to\gamma\gamma)=2.70$ keV cited by the Particle Data Group,[9]

$$\Gamma(f_1\to\gamma\gamma^*)=\tfrac{5}{3}\times(2.70\text{ keV})\left[\frac{|k_2{}^2|}{M^2}\right]$$
$$=(4.50\text{ keV})\left[\frac{|k_2{}^2|}{M^2}\right]. \tag{5.2}$$

Of course, the prediction depends dramatically on the quark content. The ratios of partial widths to $\gamma\gamma^*$ for the corresponding isovector $[(u\bar{u}-d\bar{d})/\sqrt{2}]$, nonstrange isoscalar $[(u\bar{u}+d\bar{d})/\sqrt{2}]$, and strange isoscalar $[s\bar{s}]$, are $\frac{1}{18}:\frac{25}{162}:\frac{1}{81}$. Thus the nonstrange isoscalar has the largest width, if there is ideal mixing.

VI. ANGULAR DISTRIBUTIONS FOR SPIN-ONE PARTICLES PRODUCED IN $\gamma\gamma^*$ COLLISIONS

A resonance seen in $\gamma\gamma^*$ collisions but absent in $\gamma\gamma$ collisions is quite probably spin one. The production mechanism requires it to have $C=+1$. Bound $q\bar{q}$ states with $J=1$ and $C=+1$ have $P=+1$. There is, however, the possibility of states outside the $q\bar{q}$ model.[5,10] It is of interest, therefore, to have a means of distinguishing 1^{++} from 1^{-+}.

Consider the $\gamma\gamma^*$ collision in the rest frame of the produced resonance. It the virtual photon has momentum squared $k_2{}^2\ll M^2$, the production is dominated by the collision of a transverse real photon and a longitudinal virtual photon. With the photon direction chosen as the z axis, the polarization of the resonance is

$$\hat{\zeta}=\frac{1}{\sqrt{2}}(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}})\ . \tag{6.1}$$

Suppose the resonance has $J^{PC}=1^{++}$ and decays into three pseudoscalars, with momenta p_1,p_2,p_3. The decay amplitude in the rest frame of the resonance has the general form

$$(a\mathbf{p}_1+b\mathbf{p}_2)\cdot\zeta\equiv\mathbf{w}\cdot\zeta\ . \tag{6.2}$$

For a fixed position of the normal to the decay plane

$$\hat{\mathbf{n}}=\frac{\mathbf{p}_1\times\mathbf{p}_2}{|\mathbf{p}_1\times\mathbf{p}_2|} \tag{6.3}$$

and a fixed *relative* orientation of $\mathbf{p}_1$ and $\mathbf{p}_2$, $\mathbf{w}$ takes on locations azimuthally distributed about $\mathbf{n}$; and

$$\int d\phi(\mathbf{w}\cdot\hat{\xi})^2 \propto |\hat{\mathbf{n}}\times\hat{\xi}|^2 . \tag{6.4}$$

Summing over the polarizations ξ,

$$\frac{dN}{d\cos\theta} \propto 1+\cos^2\theta , \tag{6.5}$$

where θ is the angle between the normal $\mathbf{n}$ and the beam direction.

For a 1^{-+} particle, the decay amplitude is

$$\hat{\xi}\cdot(\mathbf{p}_1\times\mathbf{p}_2)\propto\hat{\xi}\cdot\hat{\mathbf{n}} . \tag{6.6}$$

Summing over the initial polarizations gives the angular distribution

$$\frac{dN}{d\cos\theta} \propto 1-\cos^2\theta . \tag{6.7}$$

VII. SUMMARY

Photon-photon collisions provide an effective means of studying spin-one resonances. Their spin-one character is signaled by their presence in events with one rather virtual photon and their absence in events with two nearly real photons. The angular distribution of the normal to the decay plane for a three-body decay separates the 1^{++} case from the 1^{-+}.

The partial width for a 1^{++} to decay into one real photon and one virtual photon can be computed in a nonrelativistic $q\bar{q}$ model of the spin-one resonance. On the basis of the measured width of f_2 (1270)$\rightarrow\gamma\gamma$, the width of $f_1(1285)\rightarrow\gamma\gamma^*[D(1285)\rightarrow\gamma\gamma^*]$ is predicted to be (4.5 keV) ($|k_2{}^2|/M^2$), where M is the mass of the $f_1(1285)$, assuming the quark content of the $f_1(1285)$ is the same as that of the $f_2(1270)$. The partial width of an $s\bar{s}$ state in this mass range would be much smaller if it were part of the same multiplet.

The production cross section of a $J^{PC}=1^{++}$ state via the two-photon mechanism in e^+e^- collisions can be computed in the equivalent photon approximation or, alternatively, by a full Feynman-diagrammatic calculation. The full calculation contains corrections of order $k_2{}^2/M^2$ where $k_2{}^2$ is the mass of the virtual photon and M is the mass of the produced resonance.

Note added in proof. The decays of a ${}^3P_1$ $q\bar{q}$ state into two virtual photons has been considered by J. H. Kuhn, J. Kaplan, and E.G.O. Safiani, Nucl. Phys. **B157**, 125 (1979). Corrections to the approximation given in Eq. (4.13) are discussed by J. H. Field, Nucl. Phys. **B168**, 477 (1980); **B176**, 545 (E) (1980).

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APPENDIX: PRODUCTION OF A 1^{++} STATE IN PHOTON-PHOTON COLLISIONS WITHOUT APPROXIMATION

The results of Sec. IV can be compared with a complete calculation. The production amplitude is

$$\mathscr{M}=e^2\bar{u}(p_3)\gamma_\mu u(p_1)\bar{u}(p_4)\gamma_\nu u(p_2) \times\frac{1}{k_1{}^2}\frac{1}{k_2{}^2}\mathscr{A}(k_1{}^2,k_2{}^2)\epsilon^{\alpha\beta\mu\nu}(k_2{}^2k_{1\alpha}-k_1{}^2k_{2\alpha})\xi_\beta \tag{A1}$$

$$\equiv\frac{e^2\mathscr{A}(k_1{}^2,k_2{}^2)}{k_1{}^2k_2{}^2}\epsilon^{\alpha\beta\mu\nu}K_\alpha\xi_\beta L_\mu L'_\nu , \tag{A2}$$

where ξ is the polarization vector of the produced $J^{PC}=1^{++}$ state, p_1 and p_2 are the incident lepton momenta, p_3 and p_4 are the final lepton momenta, $k_1=p_1-p_3$ and $k_2=p_2-p_4$ are the virtual-photon momenta, and

$$K=k_2{}^2k_1-k_1{}^2k_2 , \quad L_\mu=\bar{u}(p_3)\gamma_\mu u(p_1) , \quad L'_\nu=\bar{u}(p_4)\gamma_\nu u(p_2) . \tag{A3}$$

Summing over final spins and averaging over initial spins gives the replacements

$$\xi^\beta\xi^{\beta'}\rightarrow -g^{\beta\beta'}+\frac{P^\beta P^{\beta'}}{M^2} , \quad L^\mu L^{\mu'}\rightarrow 2(p_1{}^\mu p_3{}^{\mu'}+p_3{}^\mu p_1{}^{\mu'}-g^{\mu\mu'}p_1\cdot p_3) , \quad L'^\nu L'^{\nu'}\rightarrow 2(p_2{}^\nu p_4{}^{\nu'}+p_4{}^\nu p_2{}^{\nu'}-g^{\nu\nu'}p_2\cdot p_4) , \tag{A4}$$

with

$$P=k_1+k_2 . \tag{A5}$$

Defining

$$Q=k_1-k_2 , \quad A=\tfrac{1}{2}(k_2{}^2-k_1{}^2) , \quad S=\tfrac{1}{2}(k_2{}^2+k_1{}^2) , \tag{A6}$$

we have

$$\sum|\mathscr{M}|^2=\Big[-A^2\epsilon_{\alpha\beta\gamma\delta}P^\beta L^\gamma L'^\delta\epsilon^\alpha{}_{\beta'\gamma'\delta'}P^{\beta'}L^{\gamma'}L'^{\delta'} -2AS\epsilon_{\alpha\beta\gamma\delta}P^\beta L^\gamma L'^\delta\epsilon^\alpha{}_{\beta'\gamma'\delta'}Q^{B'}L^{\gamma'}L'^{\delta'} -S^2\epsilon_{\alpha\beta\gamma\delta'}Q^\beta L^\gamma L'^\delta\epsilon^\alpha{}_{\beta'\gamma'\delta'}Q^{\beta'}L^{\gamma'}L'^{\delta'} +\frac{S^2}{M^2}(\epsilon_{\alpha\beta\gamma\delta}P^\alpha Q^\beta L^\gamma L'^\delta)^2\Big] \times\frac{(4\pi\alpha)^2\mathscr{A}^2(k_2{}^2,k_2{}^2)}{(k_1{}^2)^2(k_2{}^2)^2} . \tag{A7}$$

We define analogues of the usual Mandelstam variables, treating the leptons as massless:

$$s=2p_1\cdot p_2,\quad s'=2p_3\cdot p_4\,,$$
$$t=-2p_1\cdot p_3,\quad t'=-2p_2\cdot p_4\,, \tag{A8}$$
$$u=-2p_1\cdot p_4,\quad u'=-2p_2\cdot p_3\,.$$

These are related to the mass M of the resonance by

$$s+s'+t+t'+u+u'=M^2\,. \tag{A9}$$

The sum of the first three terms in the large parentheses of Eq. (A7) is

$$tt'[(ss'-uu')(s+s'-u-u') + tt'(2t+2t'-3s-3s'-3u-3u') + 2sut'+2su't+2s'ut+2s'u't']\,. \tag{A10}$$

The final term gives

$$\frac{(t+t')^2}{M^2}\{16(\epsilon_{\alpha\beta\gamma\delta}p_1^\alpha p_2^\beta p_3^\gamma p_4^\delta)^2 + \tfrac{1}{2}tt'[(s-s')^2+(u-u')^2]\}\,. \tag{A11}$$

Taking the incident electron momentum p_1 along the z axis,

$$p_1=(\sqrt{s}/2,0,0,\sqrt{s}/2)\,,$$
$$p_2=(\sqrt{s}/2,0,0,-\sqrt{s}/2) \tag{A12}$$

from Eqs. (A8)

$$p_3=(2(-u'-t)/\sqrt{s},\mathbf{n}_3\sqrt{u't/s},2(-u'+t)/\sqrt{s})\,,$$
$$p_4=(2(-u-t')/\sqrt{s},\mathbf{n}_4\sqrt{ut'/s},-2(-u+t')/\sqrt{s})\,. \tag{A13}$$

It follows that

$$(\epsilon_{\alpha\beta\gamma\delta}p_1^\alpha p_2^\beta p_3^\gamma p_4^\delta)^2=\tfrac{1}{4}uu'tt'\sin^2\phi\,, \tag{A14}$$

where

$$\mathbf{n}_3\cdot\mathbf{n}_4=\cos\phi\,. \tag{A15}$$

Our primary interest is in a single tagging, where p_3 is nearly along the z axis and

$$p_3\approx(1-x)p_1\,,$$
$$k_1\approx xp_1\,, \tag{A16}$$
$$t\approx 0\,,$$

where x is the fraction of the initial electron momentum given to the virtual photon whose momentum is k_1. We wish to drop all terms with more than the minimal number of factors of t. From Eqs. (A8) and (A13),

$$ss'-uu'=tt'+2\sqrt{uu'tt'}\cos\phi\,, \tag{A17}$$

so Eq. (A11) reduces to

$$2tt'^2(su+s'u') \tag{A18}$$

and Eq. (A12) reduces to

$$\frac{t'^2}{M^2}\{4uu'tt'\sin^2\phi+\tfrac{1}{2}tt'[(s-s')^2+(u-u')^2]\}\,. \tag{A19}$$

At this point it is safe to use the replacements

$$s'=-(1-x)u\,,$$
$$u'=-(1-x)s\,, \tag{A20}$$

which ignore the transverse components of p_3. The result is

$$\mathscr{M}^2=2tt'^2su[1+(1-x)^2]+\frac{tt'^3}{M^2}\times\{-2us(1-x)\cos2\phi+\tfrac{1}{2}(s^2+u^2)[1+(1-x)^2]\}\,. \tag{A21}$$

The differential cross section is obtained from

$$d\sigma=\frac{(2\pi)^4}{2s}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}\frac{\delta(M^2-s-s'-t'-t'-u-u')}{(2\pi)^3}\frac{(4\pi\alpha)^2\mathscr{A}^2(k_1{}^2,k_2{}^2)}{(k_1{}^2)^2(k_2{}^2)^2}\mathscr{M}^2\,. \tag{A22}$$

Substituting Eq. (A21) into Eq. (A22),

$$d\sigma=\frac{\alpha^2}{16\pi^3 st}\mathscr{A}(0,t')^2\delta(M^2-x(s+u)-t')\frac{d^3p_3}{E_3}\frac{d^3p_4}{E_4}\left[-2su[1+(1-x)^2] + \frac{-t'}{M^2}\{-2us(1-x)\cos2\phi+\tfrac{1}{2}(s^2+u^2)[1+(1-x)^2]\}\right]. \tag{A23}$$

This can be simplified by noting that

$$\frac{d^3p_3}{E_3}=\pi\,dx\,dt\,,$$
$$\frac{d^3p_4}{E_4}=\frac{du\,dt'\,d\phi}{2s}\,. \tag{A24}$$

Doing the integral over the azimuthal angle and making the usual replacement, $\int dk^2/k^2\to\ln(s/4m_e{}^2)$,

$$d\sigma=\frac{\alpha^2}{16\pi s^2}\frac{du\,dt'}{s+u}\ln\left[\frac{s}{4m_e^2}\right]\mathscr{A}(0,t')^2\left[-2su+\frac{-t'}{2M^2}(s^2+u^2)\right][1+(1-x)^2]\;. \tag{A25}$$

Integrating u from $-s+M^2-t'$ to 0 gives

$$d\sigma=\left[\frac{\alpha}{2\pi}\right]^2\ln\left[\frac{s}{4m_e^2}\right]\frac{\pi}{4}\int dt'\,\mathscr{A}(0,t')^2\left[4[(1+\tau')\ln(1/\tau')-(1-\tau')(7+\tau')/4]+\left[\frac{-t'}{2M^2}\right]4[(1+\tfrac{1}{2}\tau')^2\ln(1/\tau')-(1-\tau')(3+\tau')/2]\right]\;, \tag{A26}$$

where $\tau'=(M^2-t')/s$.

This complete result may be put in a more practical form. First, since in realistic situations $\tau'\ll 1$, τ' can be dropped except in the $\ln(1/\tau')$ term. Second, we write, using Eqs. (4.6) and (4.7),

$$\mathscr{A}(0,t')^2=\frac{96\pi}{M^5}\tilde{\Gamma}_{TS}F(t')^2\;, \tag{A27}$$

where $F(t')$ is a form factor. We then have

$$d\sigma=\left[\frac{\alpha}{2\pi}\right]^2\ln\left[\frac{s}{4m_e^2}\right]\frac{16\pi^2}{M^3}\frac{3}{2}\tilde{\Gamma}_{TS}\times\int\frac{dt'}{M^2}F(t')^2\left[4\left[\ln\frac{1}{\tau'}-\frac{7}{4}\right]+2\left[\frac{-t'}{M^2}\right]\left[\ln\frac{1}{\tau'}-\frac{3}{2}\right]\right]\;. \tag{A28}$$

This is in agreement with the result of Sec. IV, except that $\tau=M^2/s$ has been replaced by $\tau'=(M^2-t')/s$ and a form factor has been included.

The general treatment of the two-photon process by Budnev *et al.*[7] prescribes for the case in which photon 1 is nearly real

$$d\sigma=\frac{\alpha^2}{16\pi^4k_1{}^2k_2{}^2}\frac{k_1\cdot k_2}{s/2}\frac{d^3p_3}{E_3}\frac{d^3p_4}{E_4}\times(4\rho_1^{++}\rho_2^{++}\sigma_{TT}+2\,|\,\rho_1^{+-}\rho_2^{+-}\,|\,\tau_{TT}\cos2\phi+2\rho_1^{++}\rho_2^{00}\sigma_{TS})\;, \tag{A29}$$

where

$$\begin{aligned}&2\rho_1^{++}=X^{-1}(2p_1\cdot k_2-k_1\cdot k_2)+1\;,\\&\rho_1^{00}=2\rho_1^{++}-2\;,\\&\rho_1^{+-}=\rho_1^{++}-1\;,\\&X=(k_1\cdot k_2)^2\;,\end{aligned} \tag{A30}$$

and similarly for ρ_2^{ab}. In the above, $k_1{}^2$ has been set to zero. In terms of the variables s,t,u,s',t',u', with $k_1=xp_1$,

$$\begin{aligned}&\rho_1^{++}=\frac{1+(1-x)^2}{x^2}\;,\\&\rho_2^{++}=\frac{s^2+u^2}{(s+u)^2}\;,\end{aligned} \tag{A31}$$

where we have used $s'=-(1-x)u$, $u'=-(1-x)s$. Further, we have

$$\begin{aligned}&\tfrac{1}{2}\rho_1^{00}=\rho_1^{+-}=\frac{2-2x}{x^2}\;,\\&\tfrac{1}{2}\rho_2^{00}=\rho_2^{+-}=\frac{-2us}{(s+u)^2}\;.\end{aligned} \tag{A32}$$

Inserting these into Eq. (A29),

$$d\sigma=\frac{\alpha^2}{16\pi^4st}\frac{8}{xt'(s+u)}\frac{d^3p_3}{E_3}\frac{d^3p_4}{E_4}\times\{-us[1+(1-x)^2]\sigma_{TS}+(-us)(1-x)\tau_{TT}\cos2\phi+\tfrac{1}{2}(s^2+u^2)[1+(1-x)^2]\sigma_{TT}\}\;. \tag{A33}$$

This form can be compared to the result (A23). The two are equivalent if

$$\begin{aligned}&\sigma_{TS}(\hat{s})=\frac{\pi x(s+u)}{4}(-t')\mathscr{A}(0,t')^2\delta(M^2-\hat{s})\;,\\&\sigma_{TT}(\hat{s})=\frac{\pi x(s+u)}{8}(-t')\left[\frac{-t'}{M^2}\right]\mathscr{A}(0,t')^2\delta(M^2-\hat{s})\;,\\&\tau_{TT}(\hat{s})=\frac{\pi x(s+u)}{4}(-t')\left[\frac{-t'}{M^2}\right]\mathscr{A}(0,t')^2\delta(M^2-\hat{s})\;.\end{aligned} \tag{A34}$$

The cross-section terms in Eq. (A34) can be compared to the forms used in Sec. IV. From Eqs. (4.5), (4.8), and (4.10),

$$\begin{aligned}&\sigma_{TS}(\hat{s})=\frac{\pi kM}{2}(-t')\mathscr{A}(0,t')^2\delta(M^2-\hat{s})\;,\\&\sigma_{TT}(\hat{s})=\frac{\pi kM}{4}(-t')\left[\frac{-t'}{M^2}\right]\mathscr{A}(0,t')^2\delta(M^2-\hat{s})\;,\end{aligned} \tag{A35}$$

where p is the momentum of the virtual photons in the rest frame of the decaying resonance. The agreement between Eq. (A34) and Eq. (A35) follows from the kinematic relation

$$x(s+u)=2kM\;. \tag{A36}$$

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