Effective chiral Lagrangian with baryons

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We use Ioffe's method of constructing (local) baryon field operators in terms of quarks to derive an effective chiral Lagrangian with U(3)$\times$U(3) symmetry consisting of the $J^P=\frac{1}{2}^+$ baryon octet, the $J^P=\frac{1}{2}^-$ baryon nonet, and the pseudoscalar-meson nonet. This formalism leads to a set of generalized Goldberger-Treiman relations of the form $F_\pi g_{\pi B_\pm B_\pm} \sim (M_\pm + M_\pm)$, $F_\pi g_{\pi B_+ B_-} \sim (M_+ - M_-)$. We enumerate all nonvanishing three-point pseudoscalar-baryon couplings and use the $g_{\pi B_+ B_-}$ couplings to study the decays $B_- \to B_+ \pi$. We explain why the negative-parity isodoublet $N(1535)$ decays predominantly into $N\eta$, as opposed to $N\pi$, even though relatively suppressed by phase space.

I. INTRODUCTION

It is widely believed that quantum chromodynamics[1] (QCD) constitutes the underlying theory of the strong interactions. Unfortunately, however, the dynamics of this theory is in terms of quarks and gluons, and despite recent progress in lattice gauge theories[2] little is understood about hadrons and their properties (e.g., masses, couplings, etc.) in relation to QCD.

In this paper we further exploit the consequences of the underlying symmetries of QCD (e.g., Lorentz invariance, chiral symmetry, and parity[3] invariance) on the physical hadron sector. This approach leads to relations among hadron masses and couplings, a well-known example of which is the Goldberger-Treiman relation.[4] A useful technique for implementing this program is that of effective chiral Lagrangians,[5] which amounts to nothing more than the construction of the most general interaction terms (between a set of prechosen fields) consistent with the assumed symmetries.

In the limit when L of the quark-mass parameters vanish QCD has a global flavor-U(L)$\times$U(L) chiral symmetry. In the real world, however, none of these quark-mass parameters vanish and the symmetry is explicitly broken. Nevertheless, there is considerable evidence to support the hypothesis that these symmetries are very good approximate symmetries[6] for $L=2$ and 3. [We stress that QCD has a genuine U(1) axial symmetry[7,8] (which is contained in the more general chiral invariance) irrespective of the anomaly and/or any topological gauge-field configurations (such as instantons). Even though the anomaly results in no explicit U(1) axial breaking it can still provide a means by which the unwanted preanomaly current-algebra conclusions (regarding, e.g., η-η', mixing and the $\eta\to\pi\pi\pi$ decays) can be avoided.[8]]

In this work we consider chiral U(3)$\times$U(3) symmetry and construct the terms of the effective chiral Lagrangian, consisting of the lowest-lying $J^P=\frac{1}{2}^\pm$ baryons and the $J^P=0^\pm$ mesons, relevant to the baryon masses and the three-point pseudoscalar-baryon couplings. The present work extends the U(2)$\times$U(2) case considered earlier.[9] Our considerations are based on the exact chiral limit, although the incorporation of explicit chiral-symmetry breaking, due to nonzero quark masses, is also possible. The techniques used can also be extended to study other baryons and their pseudoscalar interactions.

We begin by constructing local spin-$\frac{1}{2}$ baryon field operators directly in terms of the quark fields,[10] *in a relativistic quantum-field-theory language.* From these we deduce the chiral transformation properties of the baryons and use them to incorporate the baryons into an effective Lagrangian with the spin-0 meson fields. This procedure removes any assumptions about what the chiral transformation properties of the baryons might be, as in earlier investigations.[11]

In enumerating all possible spin-$\frac{1}{2}$ (octet) baryon field operators we discover that there are two possible inequivalent configurations, which we assign to opposite-parity baryons. Furthermore, we find that in this scheme there exists a flavor-singlet baryon. We show that the chiral transformations mix these fields and consequently that a linear effective Lagrangian requires the inclusion of *both* sets of parities. This is analogous to formulations with spin-0 meson fields. This nontrivial mixing leads to some new Goldberger-Treiman-type relations. One of these relates the mass splitting of the parity octets to certain mixed-parity meson-baryon couplings. We also show how these relations can be understood, in the traditional Goldberger-Treiman relation style, by sandwiching the axial-vector currents between opposite-parity baryon states.

We enumerate all nonvanishing three-point pseudoscalar-baryon couplings and use the $g_{\pi B_+ B_-}$ couplings to study the decays $B_- \to B_+\pi$. We explain why the negative-parity isodoublet $N(1535)$ decays predominantly into $N\eta$, as opposed to $N\pi$, even though relatively suppressed by phase space. We predict the decay modes and branching ratios of the parity partners of the Σ, Λ, and the Ξ.

II. THE SPIN-$\frac{1}{2}$ BARYON FIELD OPERATORS

The baryons are the color-singlet objects obtained from three quarks by totally antisymmetrizing their colors:[12]

$$B \sim q_a q_b q_c \epsilon_{abc} , \tag{1}$$

where a, b, and c are color indices (running from 1 to 3) and ϵ_{abc} is the usual three-dimensional epsilon tensor. In constructing the baryons explicitly in terms of quarks there are two other indices we need to consider: the flavor and the Dirac indices. Our considerations below are concerned with the spin-$\frac{1}{2}$ baryon flavor octet(s) (plus a possible singlet) although the techniques can also be extended to other higher-spin, decuplet and exotic baryons. We also consider three (light or massless) quark flavors, $L=3$.

The possible flavor representations of the baryons are specified by the triplet product

$$3\times3\times3=1+8+8+10 .$$

The mixed symmetry flavor-octet baryons are obtained by antisymmetrizing in the flavor of two of the quarks. We write

$$B_{kl} \sim q_{a,i} q_{b,j} q_{c,k} \epsilon_{abc} \epsilon_{ijl} , \tag{2}$$

where i, j, k, and l are flavor indices (running from 1 to 3). Note that (2) is deemed to include the possible flavor-singlet baryon corresponding to the trace of B.

This leaves the Dirac structure. Before proceeding we need to understand how to form Lorentz scalars, pseudoscalars, vectors, etc., from two quarks, as opposed to the usual situation of a quark and an antiquark. Under an infinitesimal proper Lorentz transformation

$$x^\mu \to x^\mu + \Delta\omega^\mu{}_\nu x^\nu, \quad \Delta\omega_{\mu\nu} = -\Delta\omega_{\nu\mu} ,$$

the quark and antiquark fields transform as

$$q(x) \to Sq(x), \quad \bar{q}(x) \to \bar{q}(x)S^{-1} , \tag{3}$$

where

$$S(\Delta\omega) = 1 - \frac{i}{4}\sigma_{\mu\nu}\Delta\omega^{\mu\nu}$$

and

$$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] .$$

It follows from (3) and the property of the usual charge-conjugation matrix $C=-i\gamma^2\gamma^0[S^TC=CS^{-1}]$ that q^TC transforms in the same way as $\bar{q}$ under proper Lorentz transformations. Combined with the parity transformation[13]

$$q \to \gamma^0 q \tag{4}$$

it can be shown that q^TCq behaves as a pseudoscalar, $q^TC\gamma_5 q$ as a scalar, $q^TC\gamma_\mu\gamma_5 q$ as a vector, $q^TC\gamma_\mu q$ as a pseudovector, and $q^TC\sigma_{\mu\nu}q$ as a tensor.

Suppose now that we were to pair the two quarks[14] $q_{a,i}$ and $q_{b,j}$, which are antisymmetric under the interchange of their flavor indices, into a quark dilinear $q^TC\Gamma q$, $\Gamma=[1,\gamma_5,\gamma_\mu,i\gamma_\mu\gamma_5,\sigma_{\mu\nu}(\mu<\upsilon)]$. The set of possible spin-$\frac{1}{2}$ baryon field operators (no free Lorentz indices) is given by[15]

$$\begin{aligned}
B^1_{kl} &= \mathcal{N}_B{}^{-3}(q^T_{a,i}C\gamma_5 q_{b,j})q_{c,k}\epsilon_{abc}\epsilon_{ijl} ,\\
B^2_{kl} &= \mathcal{N}_B{}^{-3}(q^T_{a,i}Cq_{b,j})q_{c,k}\epsilon_{abc}\epsilon_{ijl} ,\\
B^3_{kl} &= \mathcal{N}_B{}^{-3}(q^T_{a,i}C\gamma_\mu q_{b,j})\gamma^\mu q_{c,k}\epsilon_{abc}\epsilon_{ijl} ,\\
B^4_{kl} &= \mathcal{N}_B{}^{-3}(q^T_{q,i}C\gamma_\mu\gamma_5 q_{b,j})\gamma^\mu q_{c,k}\epsilon_{abc}\epsilon_{ijl} ,\\
B^5_{kl} &= \mathcal{N}_B{}^{-3}(q^T_{a,i}C\sigma_{\mu\nu}q_{b,j})\sigma^{\mu\nu}q_{c,k}\epsilon_{abc}\epsilon_{ijl} ,
\end{aligned} \tag{5}$$

where $\mathcal{N}_B$ is some normalization constant having the dimensions of mass.

It is easily seen that B^3 and B^5 vanish identically because of the flavor and color antisymmetries in the first two quarks. Of the remaining possibilities, B^4 can be excluded because it can be expressed in terms of B^1 and B^2 by carrying out a number of Fierz transformations:[16]

$$B^4 = 2B^1 - 2\gamma_5 B^2 .$$

This leaves the only possibilities as B^1 and B^2. [The other set of operators obtained by combining say the quarks $q_{a,i}$ and $q_{c,k}$ into quark dilinears can be Fierz transformed into the combinations (5), so need not be considered.]

Under parity (4)

$$B^1 \to \gamma^0 B^1, \quad B^2 \to -\gamma^0 B^2 , \tag{6}$$

which means that the B^1 nonet field operators have positive parity and the B^2 nonet field operators have negative parity. In the nonrelativistic limit, B^1 consists principally of s-wave quarks while B^2 contains terms which connect the "small" and "large" components of q and consequently involve quark-pair p-wave excitations. There is however a problem. We know from arguments in the nonrelativistic quark model that it is impossible to construct a spin-$\frac{1}{2}$ flavor-singlet baryon made out of s-wave quarks only. [Since the flavor-singlet baryon is totally antisymmetric under the interchange of any two quark colors (or flavors) and since the baryon must be antisymmetric under the total interchange of any two quarks, it must also be totally antisymmetric in spin + space. This is clearly impossible for a spin-$\frac{1}{2}$ baryon made of s-wave quarks.] This should be borne out in our formalism.

It can be shown,[9,17] by a number of Fierz transformations, that the singlets B^1_s and B^2_s are related to each other, thus reducing the number of operators (particles) by one:

$$B^2_s = -\gamma_5 B^1_s . \tag{7}$$

The parity of the remaining singlet is unspecified in our formalism, depending on whether we eliminate B^1_s or B^2_s, but we will choose it to have a negative parity, in correspondence with the quark model. (Under the assumption that our formalism corresponds to the lowest-lying baryon states and since the mass of the most likely $\frac{1}{2}^-$ single candidate [the $\Lambda(1405)$] is less than the $\frac{1}{2}^+$ singlet candidate [the $\Lambda(1600)$] this seems a more natural choice.)

We are then left with one positive-parity octet, one negative-parity octet, and one negative-parity flavor-singlet baryon. We associate the $\frac{1}{2}^+$ octet with the usual lowest-lying states $(p,n,\Sigma^{\pm},\Sigma^0,\Xi^0,\Xi^-,\Lambda)$. The assignments for the $\frac{1}{2}^-$ octet are not so straightforward as we believe that some of them have not been observed as yet. In Sec. VII we investigate the branching ratios for the decay of the negative-parity baryons into a positive-parity baryon plus a pseudoscalar meson. The results suggest that the $N(1535)$ and the $\Lambda(1670)$ are most likely the parity partners to the nucleon isodoublet (p,n) and the Λ, respectively. We are unable to identify the parity partners to the Σ and the Ξ. Assuming approximately equal intramultiplet mass splittings in the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ octets we expect these particles to have masses of order $M_\Sigma+(M_{N(1535)}-M_N)\simeq 1770$ MeV and $M_\Xi+(M_{N(1535)}-M_N)\simeq 1900$ MeV, respectively.

The technique developed here for enumerating all possible baryon field operators made of three quarks is inspired by the work of Ioffe.[10] Ioffe also noticed that there are two possible (inequivalent) representations for the spin-$\frac{1}{2}$ baryons but did not interpret them, as we have done, as two opposite-parity octets.

The technique can easily be extended to other higher-spin, decuplet and exotic baryons. Consider for instance the spin-$\frac{3}{2}$ decuplet. These states are totally symmetric under the interchange of any quark flavors. There are only two nonvanishing possible operators. Eliminating one of these by a Fierz transformation leaves the only possibility

$$\Delta_\mu=\sum_{\text{permutations of }(i,j,k)} \mathcal{N}_\Delta{}^{-3}(q_{a,i}^T C\gamma_\mu q_{b,j})q_{c,k}\epsilon_{abc} .$$

It is interesting to note that the spin-$\frac{3}{2}^+$ decuplet does not have a set of parity partners in the same way as the spin-$\frac{1}{2}$ octets.

III. CHIRAL TRANSFORMATION PROPERTIES

Consider the two spin-$\frac{1}{2}$ baryon nonet operators [for the moment we do not impose the constraint (7)]:

$$\begin{aligned} B_{kl}^1&=\mathcal{N}_B{}^{-3}(q_{a,i}^T C\gamma_5 q_{b,j})q_{c,k}\epsilon_{abc}\epsilon_{ijl} ,\\ B_{kl}^2&=\mathcal{N}_B{}^{-3}(q_{a,i}^T C q_{b,j})q_{c,k}\epsilon_{abc}\epsilon_{ijl} . \end{aligned} \tag{8}$$

Under *infinitesimal* U(3) vector and U(3) axial-vector transformations[18] $(\alpha,\beta \ll 1)$

$$\begin{aligned} &\mathrm{U}(3)_V:\ q\to(1+i\boldsymbol{\alpha}\cdot\boldsymbol{\lambda})q ,\\ &\mathrm{U}(3)_A:\ q\to(1+i\boldsymbol{\beta}\cdot\boldsymbol{\lambda}\gamma_5)q , \end{aligned} \tag{9}$$

$$\begin{aligned} B_{kl}^{1,2}\to B_{kl}^{1,2}&+i\boldsymbol{\alpha}\cdot\boldsymbol{\lambda}_{kk'}B_{k'l}^{1,2}-i\boldsymbol{\alpha}\cdot\boldsymbol{\lambda}_{l'l}B_{kl'}^{1,2}\\ &+i\sqrt{6}\alpha^9 B_{kl}^{1,2}+i\boldsymbol{\beta}\cdot\boldsymbol{\lambda}_{kk'}\gamma_5 B_{k'l}^{1,2}\\ &-i\boldsymbol{\beta}\cdot\boldsymbol{\lambda}_{l'l}B_{kl'}^{2,1}+i\sqrt{6}\beta^9 B_{kl}^{2,1} . \end{aligned} \tag{10}$$

The second and fifth terms on the right-hand side (RHS) of (10) arise on transforming the quark field $q_{c,k}$ in (8). The other terms arise from the chiral transformations on the quark dilinear $(q^T C\gamma_5 q)$. The minus sign in the third and sixth terms can be understood by the fact that, in many ways, the quantity $(q^T C\gamma_5 q)\epsilon\epsilon$ behaves like an antiquark except for the U(1) (vector and axial-vector) transformations *whose phases add coherently.* Notice that the U(3) axial-vector transformations mix the opposite-parity baryon nonets B^1 and B^2. [This should be compared with the two-flavor case[9] where only the U(1) axial-vector transformations mix B^1 and B^2. In this case the quark dilinear $(q^T C\gamma_5 q)\epsilon\epsilon$ is a SU(2) singlet.]

Instead of (9) it proves more convenient to use the left and right transformations $[q_{L,R}=\frac{1}{2}(1+\gamma_5)q]$

$$\begin{aligned} &U(3)_L:\ q_L\to e^{i\mathbf{l}\cdot\boldsymbol{\lambda}}q_L\equiv U_L q_L;\ q_R\to q_R ,\\ &U(3)_R:\ q_L\to q_L;\ q_R\to e^{i\mathbf{r}\cdot\boldsymbol{\lambda}}q_R\equiv U_R q_R , \end{aligned} \tag{11}$$

and the left and right components of the baryon fields

$$\begin{aligned} B_{L,R}^1&=\mathcal{N}_B{}^{-3}(-q_L^T C q_L+q_R^T C q_R)q_{L,R}\epsilon\epsilon ,\\ B_{L,R}^2&=\mathcal{N}_B{}^{-3}(q_L^T C q_L+q_R^T C q_R)q_{L,R}\epsilon\epsilon . \end{aligned} \tag{12}$$

The combinations $(B^1\pm B^2)_{L,R}$ transform homogeneously:

$$\begin{aligned} (B_L^1+B_L^2)&\to U_L(B_L^1+B_L^2)U_R^\dagger e^{i\sqrt{6}r^9} ,\\ (B_L^1-B_L^2)&\to U_L(B_L^1-B_L^2)U_L^\dagger e^{i\sqrt{6}l^9} ,\\ (B_R^1+B_R^2)&\to U_R(B_R^1+B_R^2)U_R^\dagger e^{i\sqrt{6}r^9} ,\\ (B_R^1-B_R^2)&\to U_R(B_R^1-B_R^2)U_L^\dagger e^{i\sqrt{6}l^9} . \end{aligned} \tag{13}$$

Note that certain combinations in (13) transform as $(3,3^*)+(3^*,3)$ and others as $(1,8)+(8,1)$. These transformation properties should be contrasted with those assumed in earlier investigations.[11] There is also the subtlety of the additional U(1) phases in (13) which will be of some importance later.

The chiral transformation properties of the $J^P=0^\pm$ meson fields

$$\mathcal{M}_{ij}\equiv\mathcal{N}_m{}^{-2}\bar{q}_j(1-\gamma_5)q_i=(\sigma^a+i\pi^a)\frac{\lambda_{ij}^a}{\sqrt{2}} \tag{14}$$

are given by

$$\begin{aligned} &\mathcal{M}\to U_L\mathcal{M}U_R^\dagger ,\\ &\det\mathcal{M}\to e^{i\sqrt{6}(l^9-r^9)}\det\mathcal{M} . \end{aligned} \tag{15}$$

IV. EFFECTIVE CHIRAL LAGRANGIAN

It is clear from Eq. (13) that one cannot construct chiral-invariant baryon mass terms of the form $\bar{B}_R B_L$ and $\bar{B}_L B_R$ directly. Instead they arise, from possible $\bar{B}\mathcal{M}B$-type terms, *after* assuming spontaneous chiral-symmetry breaking (SCSB) with $\langle\mathcal{M}\rangle\neq 0$. Keeping those terms in the effective Lagrangian which contribute to the baryon masses after SCSB,

$$\begin{aligned}\mathscr{L}_{\rm eff}=&\,{\rm tr}(\bar B^1 i\partial\!\!\!/ B^1)+{\rm tr}(\bar B^2 i\partial\!\!\!/ B^2)+g\{{\rm tr}[(\bar B^1_L+\bar B^2_L)\mathscr{M}(B^1_R+B^2_R)]+{\rm tr}[(\bar B^1_L-\bar B^2_L)\mathscr{M}(B^1_R-B^2_R)]\\&+{\rm tr}[(\bar B^1_R+\bar B^2_R)\mathscr{M}^\dagger(B^1_L+B^2_L)]+{\rm tr}[(\bar B^1_R-\bar B^2_R)\mathscr{M}^\dagger(B^1_L-B^2_L)]\}\\&+\kappa_1\{(\det\mathscr{M}){\rm tr}[(\bar B^1_L-\bar B^2_L)\mathscr{M}(B^1_R+B^2_R)\mathscr{M}^\dagger]+(\det\mathscr{M}^\dagger){\rm tr}[(\bar B^1_R+\bar B^2_R)\mathscr{M}^\dagger(B^1_L-B^2_L)\mathscr{M}]\}\\&+\kappa_2\{(\det\mathscr{M}){\rm tr}[(\bar B^1_R-\bar B^2_R)\mathscr{M}^\dagger(B^1_L+B^2_L)\mathscr{M}^\dagger]+(\det\mathscr{M}^\dagger){\rm tr}[(\bar B^1_L+\bar B^2_L)\mathscr{M}(B^1_R-B^2_R)\mathscr{M}]\}\\&+\kappa_3\{(\det\mathscr{M}^\dagger){\rm tr}[(\bar B^1_L+\bar B^2_L)\mathscr{M}]{\rm tr}[(B^1_R-B^2_R)\mathscr{M}]+(\det\mathscr{M}){\rm tr}[(\bar B^1_R-\bar B^2_R)\mathscr{M}^\dagger]{\rm tr}[(B^1_L+B^2_L)\mathscr{M}^\dagger]\}+\cdots\ .\end{aligned}\tag{16}$$

The traces in (16) are over flavor indices. The determinant factors are required to take into account the extra U(1) phase factors in the baryon transformation properties (13). Terms such as $(\det\mathscr{M}){\rm tr}(\bar B^1_L-\bar B^2_L){\rm tr}(B^1_R+B^2_R)$ are excluded because $(B^1_R+B^2_R)_s=0=(B^1_L-B^2_L)_s$ by virtue of the constraint equation (7). Kinetic energy cross terms coupling B^1 to B^2 have been eliminated by appealing to the diagonalization theorem of Feinberg, Kabir, and Weinberg.[19]

We assume SCSB with

$$\langle\mathscr{M}_{ij}\rangle=\langle\mathscr{M}^\dagger_{ij}\rangle=-\frac{F_\pi}{\sqrt2}\delta_{ij}\ .\tag{17}$$

This corresponds to the usual scheme ($\langle\bar q q\rangle\neq0$) with chiral U(3)×U(3) symmetry spontaneously broken down to U(3) vector symmetry. The quantity F_π appearing in (17) can be identified with the pion decay constant ($\simeq$93 MeV in our normalization) defined by

$$\langle 0|\mathscr{F}^a_{\mu5}(x)|\pi^b(\mathbf{q})\rangle=iq_\mu F_\pi\delta_{ab}e^{-iq\cdot x}\ ,\tag{18}$$

where $\mathscr{F}^a_{\mu5}$ is the axial-vector current [in QCD, $\mathscr{F}^a_{\mu5}=\bar q\gamma_\mu\gamma_5(\lambda^a/2)q$]. In the presence of (17) we redefine the meson fields so that

$$\mathscr{M}_{ij}=\langle\mathscr{M}_{ij}\rangle+\tilde{\mathscr{M}}_{ij}=\frac{-F_\pi}{\sqrt2}\delta_{ij}+(\sigma^a+i\pi^a)\frac{\lambda^a_{ij}}{\sqrt2}\ .\tag{19}$$

Substituting this into (16) and keeping only the baryon mass terms and the three-point pseudoscalar-baryon couplings,

$$\begin{aligned}\mathscr{L}_{\rm eff}=&\,{\rm tr}(\bar B^1 i\partial\!\!\!/ B^1)+{\rm tr}(\bar B^2 i\partial\!\!\!/ B^2)+{\rm tr}\left[(\bar B^1\ \bar B^2)\begin{bmatrix}M+X & Y\gamma_5\\ -Y\gamma_5 & M-X\end{bmatrix}\begin{bmatrix}B^1\\B^2\end{bmatrix}\right]\\&+\frac{i\pi^a}{F_\pi}{\rm tr}\left[(\bar B^1\ \bar B^2)\lambda^a\gamma_5\begin{bmatrix}M+X & Y\gamma_5\\ -Y\gamma_5 & M-X\end{bmatrix}\begin{bmatrix}B^1\\B^2\end{bmatrix}\right]+\frac{i\pi^a}{F_\pi}{\rm tr}\left[(\bar B^1\ \bar B^2)\gamma_5\begin{bmatrix}-Y & -X\gamma_5\\ X\gamma_5 & Y\end{bmatrix}\begin{bmatrix}B^1\\B^2\end{bmatrix}\lambda^a\right]\\&+\sqrt6\frac{i\pi^9}{F_\pi}{\rm tr}\left[(\bar B^1\ \bar B^2)\gamma_5\begin{bmatrix}Y & X\gamma_5\\ -X\gamma_5 & -Y\end{bmatrix}\begin{bmatrix}B^1\\B^2\end{bmatrix}\right]+Z\,{\rm tr}[(\bar B^1\ \bar B^2)]\begin{bmatrix}1 & -\gamma_5\\ \gamma_5 & -1\end{bmatrix}{\rm tr}\left[\begin{bmatrix}B^1\\B^2\end{bmatrix}\right]\\&+Z\frac{i\pi^a}{F_\pi}{\rm tr}[(\bar B^1\ \bar B^2)]\gamma_5\begin{bmatrix}1 & -\gamma_5\\ \gamma_5 & -1\end{bmatrix}{\rm tr}\left[\begin{bmatrix}B^1\\B^2\end{bmatrix}\lambda^a\right]+Z\frac{i\pi^a}{F_\pi}{\rm tr}[(\bar B^1\ \bar B^2)\lambda^a]\gamma_5\begin{bmatrix}1 & -\gamma_5\\ \gamma_5 & -1\end{bmatrix}{\rm tr}\left[\begin{bmatrix}B^1\\B^2\end{bmatrix}\right]\\&+Z\sqrt6\frac{i\pi^9}{F_\pi}{\rm tr}[(\bar B^1\ \bar B^2)]\gamma_5\begin{bmatrix}-1 & \gamma_5\\ -\gamma_5 & 1\end{bmatrix}{\rm tr}\left[\begin{bmatrix}B^1\\B^2\end{bmatrix}\right]+\cdots\ ,\end{aligned}\tag{20}$$

where

$$M\equiv\sqrt2 gF_\pi,\ \ X\equiv(\kappa_1+\kappa_2)\frac{F_\pi^5}{4\sqrt2},\ \ Y\equiv(\kappa_1-\kappa_2)\frac{F_\pi^5}{4\sqrt2},\ \ Z\equiv\kappa_3\frac{F_\pi^5}{4\sqrt2}\ .\tag{21}$$

We write

$$B^1=\mathscr{B}^1+\frac{1}{\sqrt3}B^1_sI,\ \ B^2=\mathscr{B}^2+\frac{1}{\sqrt3}B^2_sI\ ,\tag{22}$$

where $\mathscr{B}^1$ and $\mathscr{B}^2$ are traceless 3×3 matrices which represent the octet parity-even and -odd baryons

$$\mathscr{B}^{1,2}_{ij}=\sum_{a=1}^{8}\mathscr{B}^{1,2}_a\frac{\lambda^a_{ij}}{\sqrt2}$$

and B^1_s and B^2_s are the flavor-singlet baryon fields.

Substituting (22) into (20) and expressing B^1_s and B^2_s in terms of $C=(1/\sqrt2)(B^2_s-\gamma_5B^1_s)$ [note that the combination $B^2_s+\gamma_5B^1_s$ vanishes by (7)]

$$\mathcal{L}_{\rm eff}=\mathrm{tr}(\overline{\mathcal{B}}^{1} i\partial\mathcal{B}^{1})+\mathrm{tr}(\overline{\mathcal{B}}^{2} i\partial\mathcal{B}^{2})+\bar{C}i\partial C+\mathrm{tr}\left[(\overline{\mathcal{B}}^{1}\ \ \overline{\mathcal{B}}^{2})\begin{bmatrix} M+X & Y\gamma_5 \\ -Y\gamma_5 & M-X\end{bmatrix}\begin{bmatrix}\mathcal{B}^{1}\\ \mathcal{B}^{2}\end{bmatrix}\right]$$
$$+(Y-X-6Z)\bar{C}C+\frac{i\pi^a}{F_\pi}\mathrm{tr}\left[(\overline{\mathcal{B}}^{1}\ \ \overline{\mathcal{B}}^{2})\lambda^a\gamma_5\begin{bmatrix} M+X & Y\gamma_5 \\ -Y\gamma_5 & M-X\end{bmatrix}\begin{bmatrix}\mathcal{B}^{1}\\ \mathcal{B}^{2}\end{bmatrix}\right]$$
$$+\frac{i\pi^a}{F_\pi}\mathrm{tr}\left[(\overline{\mathcal{B}}^{1}\ \ \overline{\mathcal{B}}^{2})\gamma_5\begin{bmatrix} -Y & -X\gamma_5 \\ X\gamma_5 & Y\end{bmatrix}\begin{bmatrix}\mathcal{B}^{1}\\ \mathcal{B}^{2}\end{bmatrix}\lambda^a\right]+\sqrt{2/3}(-Y+X+6Z)\frac{i\pi^9}{F_\pi}\bar{C}\gamma_5 C$$
$$+\sqrt{6}\frac{i\pi^9}{F_\pi}\mathrm{tr}\left[(\overline{\mathcal{B}}^{1}\ \ \overline{\mathcal{B}}^{2})\gamma_5\begin{bmatrix} Y & X\gamma_5 \\ -X\gamma_5 & -Y\end{bmatrix}\begin{bmatrix}\mathcal{B}^{1}\\ \mathcal{B}^{2}\end{bmatrix}\right]$$
$$+\frac{i\pi^a}{\sqrt{6}F_\pi}\bar{C}(M+2X-2Y+6Z,(M+2Y-2X-6Z)\gamma_5)\mathrm{tr}\left[\begin{bmatrix}\mathcal{B}^{1}\\ \mathcal{B}^{2}\end{bmatrix}\lambda^a\right]$$
$$+\frac{i\pi^a}{\sqrt{6}F_\pi}\mathrm{tr}[(\overline{\mathcal{B}}^{1}\ \ \overline{\mathcal{B}}^{2})\lambda^a]\begin{bmatrix} -M-2X+2Y-6Z \\ (M+2Y-2X-6Z)\gamma_5\end{bmatrix}C+\cdots\ . \tag{23}$$

The sum over a runs from 1 to 9. The additional baryon couplings for the ninth pseudoscalar meson [eighth and ninth terms in (23)] are a consequence of the determinant factors in (16):

$$\det\mathcal{M}\simeq\frac{F_\pi^3}{2\sqrt{2}}[1+\sqrt{6}(\sigma^9+i\pi^9)+\cdots]\ .$$

The effective Lagrangian (23) contains γ_5-dependent $\mathcal{B}^1\mathcal{B}^2$ mass terms which should be removed. The baryon mass squared matrix in (23) is diagonalized in terms of the new fields $\mathcal{B}_+$ and $\mathcal{B}_-$, defined by

$$\begin{bmatrix}\mathcal{B}_+\\ \mathcal{B}_-\end{bmatrix}=U\begin{bmatrix}\mathcal{B}^{1}\\ \mathcal{B}^{2}\end{bmatrix},\quad (\overline{\mathcal{B}}_+\ \ \overline{\mathcal{B}}_-)=(\overline{\mathcal{B}}^{1}\ \ \overline{\mathcal{B}}^{2})U\ , \tag{24}$$

where[20]

$$U=\frac{\mathrm{sgn}(Y)}{[2(M^2+Y^2)-2M(M^2+Y^2)^{1/2}]^{1/2}}\begin{bmatrix} Y & [-M+(M^2+Y^2)^{1/2}]\gamma_5 \\ [M-(M^2+Y^2)^{1/2}]\gamma_5 & Y\end{bmatrix}\ . \tag{25}$$

Note that $\mathcal{B}_+$ has positive parity and $\mathcal{B}_-$ has negative parity. In terms of these new fields $\mathcal{L}_{\rm eff}$ (23) becomes [for simplicity we use the shorthand notation $\mu=(M^2+Y^2)^{1/2}$]

$$\mathcal{L}_{\rm eff}=\mathrm{tr}(\overline{\mathcal{B}}_+ i\partial\mathcal{B}_+)+\mathrm{tr}(\overline{\mathcal{B}}_- i\partial\mathcal{B}_-)+\bar{C}i\partial C+\mathrm{tr}\left[(\overline{\mathcal{B}}_+\ \ \overline{\mathcal{B}}_-)\begin{bmatrix} \mu+X & 0 \\ 0 & \mu-X\end{bmatrix}\begin{bmatrix}\mathcal{B}_+\\ \mathcal{B}_-\end{bmatrix}\right]$$
$$+(Y-X-6Z)\bar{C}C+\frac{i\pi^a}{F_\pi}\mathrm{tr}\left[(\overline{\mathcal{B}}_+\ \ \overline{\mathcal{B}}_-)\lambda^a\gamma_5\begin{bmatrix} \mu+X & 0 \\ 0 & \mu-X\end{bmatrix}\begin{bmatrix}\mathcal{B}_+\\ \mathcal{B}_-\end{bmatrix}\right]$$
$$+\frac{i\pi^a}{F_\pi}\mathrm{tr}\left[(\overline{\mathcal{B}}_+\ \ \overline{\mathcal{B}}_-)\gamma_5\begin{bmatrix} -Y-\frac{XY}{\mu} & -\frac{MX\gamma_5}{\mu} \\ \frac{MX\gamma_5}{\mu} & Y-\frac{XY}{\mu}\end{bmatrix}\begin{bmatrix}\mathcal{B}_+\\ \mathcal{B}_-\end{bmatrix}\lambda^a\right]+\sqrt{2/3}(-Y+X+6Z)\frac{i\pi^9}{F_\pi}\bar{C}\gamma_5 C$$
$$+\sqrt{6}\frac{i\pi^9}{F_\pi}\mathrm{tr}\left[(\overline{\mathcal{B}}_+\ \ \overline{\mathcal{B}}_-)\gamma_5\begin{bmatrix} Y+\frac{XY}{\mu} & \frac{MX\gamma_5}{\mu} \\ -\frac{MX\gamma_5}{\mu} & -Y+\frac{XY}{\mu}\end{bmatrix}\begin{bmatrix}\mathcal{B}_+\\ \mathcal{B}_-\end{bmatrix}\right]$$
$$+\frac{i\pi^a}{\sqrt{6}F_\pi}\bar{C}(M+2X-2Y+6Z,(M+2Y-2X-6Z)\gamma_5)U^\dagger\mathrm{tr}\left[\begin{bmatrix}\mathcal{B}_+\\ \mathcal{B}_-\end{bmatrix}\lambda^a\right]$$
$$+\frac{i\pi^a}{\sqrt{6}F_\pi}\mathrm{tr}[(\overline{\mathcal{B}}_+\ \ \overline{\mathcal{B}}_-)\lambda^a]U^\dagger\begin{bmatrix} -M-2X+2Y-6Z \\ (M+2Y-2X-6Z)\gamma_5\end{bmatrix}C+\cdots\ . \tag{26}$$

V. BARYON MASSES AND COUPLINGS

There are two distinct pseudoscalar-baryon (octet) interactions in (26)—corresponding to the two ways of forming an SU(3) singlet from three octets:

$$\mathscr{L}_{\text{eff}}=g^{(1)}_{\pi\mathscr{B}\mathscr{B}}i\sqrt{2}\text{tr}(\overline{\mathscr{B}}\xi\pi\mathscr{B})+g^{(2)}_{\pi\mathscr{B}\mathscr{B}}i\sqrt{2}\text{tr}(\overline{\mathscr{B}}\xi\mathscr{B}\pi)+\cdots\ , \tag{27}$$

where $\mathscr{B}$ is a traceless 3×3 matrix representing a baryon octet (even or odd parity)

$$\pi_{ij}=\pi^b\frac{\lambda^b_{ij}}{\sqrt{2}}$$

is a 3×3 matrix representing the pseudoscalar octet (or nonet), and $\xi=\gamma_5$ or 1 depending on whether or not the two baryons are of the same parity. These interaction terms can also be written in the form

$$\mathscr{L}_{\text{eff}}=g^{d}_{\pi\mathscr{B}\mathscr{B}}d_{abc}i\overline{\mathscr{B}}^a\xi\pi^b\mathscr{B}^c+ig^{f}_{\pi\mathscr{B}\mathscr{B}}f_{abc}i\overline{\mathscr{B}}^a\xi\pi^b\mathscr{B}^c+\cdots\ , \tag{28}$$

where d_{abc} and f_{abc} are the SU(3) symbols (defined by $[\lambda^a,\lambda^b]=2if_{abc}\lambda^c$, $\{\lambda^a,\lambda^b\}=\frac{4}{3}\delta_{ab}I+2d_{abc}\lambda^c$) and $g^{d,f}_{\pi\mathscr{B}\mathscr{B}}\equiv g^{(1)}_{\pi\mathscr{B}\mathscr{B}}\pm g^{(2)}_{\pi\mathscr{B}\mathscr{B}}$. A parameter which is often used to gauge the ratio of the d to f couplings is

$$\alpha_{\pi\mathscr{B}\mathscr{B}}\equiv\frac{g^{d}_{\pi\mathscr{B}\mathscr{B}}}{g^{d}_{\pi\mathscr{B}\mathscr{B}}+g^{f}_{\pi\mathscr{B}\mathscr{B}}}=\frac{g^{(1)}_{\pi\mathscr{B}\mathscr{B}}+g^{(2)}_{\pi\mathscr{B}\mathscr{B}}}{2g^{(1)}_{\pi\mathscr{B}\mathscr{B}}}\ . \tag{29}$$

From (26) we find that

$$\begin{aligned}&M_\pm=\mu\pm X\ ,\\ &M_C=Y-X-6Z\ ,\\ &\alpha_{\pi\mathscr{B}_+\mathscr{B}_+}\equiv\alpha=\frac{1}{2}\left[1-\frac{Y}{F_\pi\mu}\right]\ .\end{aligned} \tag{30}$$

We can use these equations to express the four Lagrangian parameters M, X, Y, and Z in terms of M_+, M_-, M_C, and $\alpha\equiv\alpha_{\pi\mathscr{B}_+\mathscr{B}_+}$. Taking[21] $M_+=M_\Sigma\simeq1190$ MeV, $M_-=M_{\Sigma}\simeq1770$ MeV, $M_C=M_{\Lambda(1405)}\simeq1405$ MeV, and[22] $\alpha=0.5$, 0.6, and 0.7 we find that

$$X=\tfrac{1}{2}(M_+-M_-)\simeq-290\text{ MeV}\ , \tag{31a}$$

$$\mu=\tfrac{1}{2}(M_++M_-)\simeq1480\text{ MeV}\ , \tag{31b}$$

$$Y=-\tfrac{1}{2}(M_++M_-)(2\alpha-1)\simeq\begin{cases}0, & \alpha=0.5\ ,\\ -296\text{ MeV}, & \alpha=0.6\ ,\\ -592\text{ MeV}, & \alpha=0.7\ ,\end{cases} \tag{31c}$$

$$M=(M_++M_-)\sqrt{\alpha(1-\alpha)}\simeq\begin{cases}1480\text{ MeV}, & \alpha=0.5\ ,\\ 1450\text{ MeV}, & \alpha=0.6\ ,\\ 1356\text{ MeV}, & \alpha=0.7\ ,\end{cases} \tag{31d}$$

$$Z=-\tfrac{1}{6}\alpha(M_++M_-)+\tfrac{1}{6}M_--\tfrac{1}{6}M_C\simeq\begin{cases}-186\text{ MeV}, & \alpha=0.5\ ,\\ -235\text{ MeV}, & \alpha=0.6\ ,\\ -285\text{ MeV}, & \alpha=0.7\ .\end{cases} \tag{31e}$$

The pseudoscalar-baryon couplings can be expressed in terms of M_+, M_-, M_C, and α. The results are[23,24]

$$g^{(1)}_{\pi\mathscr{B}_+\mathscr{B}_+}=\frac{M_+}{F_\pi}\simeq12.8\ , \tag{32a}$$

$$g^{(2)}_{\pi\mathscr{B}_+\mathscr{B}_+}=(2\alpha-1)\frac{M_+}{F_\pi}\simeq\begin{cases}0, & \alpha=0.5\ ,\\ 2.56, & \alpha=0.6\ ,\\ 5.12, & \alpha=0.7\ ,\end{cases} \tag{32b}$$

$$g^{(1)}_{\pi\mathscr{B}_-\mathscr{B}_-}=\frac{M_-}{F_\pi}\simeq19.0 \tag{32c}$$

$$g^{(2)}_{\pi\mathscr{B}_-\mathscr{B}_-}=-(2\alpha-1)\frac{M_-}{F_\pi}\simeq\begin{cases}0, & \alpha=0.5\ ,\\ -3.8, & \alpha=0.6\ ,\\ -7.6, & \alpha=0.7\ ,\end{cases} \tag{32d}$$

$$\alpha_{\pi\mathscr{B}_-\mathscr{B}_-}=1-\alpha=\begin{cases}0.5, & \alpha=0.5\ ,\\ 0.4, & \alpha=0.6\ ,\\ 0.3, & \alpha=0.7\ ,\end{cases} \tag{32e}$$

$$g^{(1)}_{\pi\mathscr{B}_+\mathscr{B}_-}=0\ , \tag{32f}$$

$$g^{(2)}_{\pi\mathscr{B}_+\mathscr{B}_-}=\frac{-1}{F_\pi}(M_+-M_-)\sqrt{\alpha(1-\alpha)}\simeq\begin{cases}3.12, & \alpha=0.5\ ,\\ 3.06, & \alpha=0.6\ ,\\ 2.86, & \alpha=0.7\ ,\end{cases} \tag{32g}$$

$$\alpha_{\pi\mathscr{B}_+\mathscr{B}_-}{}^{-1}=0\ , \tag{32h}$$

$$g_{\pi^9\mathscr{B}_+\mathscr{B}_+}=\left[\frac{2}{3}\right]^{1/2}\frac{M_+}{F_\pi}(3-4\alpha)\simeq\begin{cases}10.5, & \alpha=0.5\ ,\\ 6.27, & \alpha=0.6\ ,\\ 2.09, & \alpha=0.7\ ,\end{cases} \tag{32i}$$

$$g_{\pi^9\mathscr{B}_-\mathscr{B}_-}=\left[\frac{2}{3}\right]^{1/2}\frac{M_-}{F_\pi}(4\alpha-1)\simeq\begin{cases}15.5, & \alpha=0.5\ ,\\ 21.7, & \alpha=0.6\ ,\\ 27.9, & \alpha=0.7\ ,\end{cases} \tag{32j}$$

$$g_{\pi^9\mathscr{B}_+\mathscr{B}_-}=\frac{2}{F_\pi}\left[\frac{2}{3}\right]^{1/2}(M_+-M_-)\sqrt{\alpha(1-\alpha)}\simeq\begin{cases}-5.09, & \alpha=0.5\ ,\\ -5.00, & \alpha=0.6\ ,\\ -4.67, & \alpha=0.7\ ,\end{cases} \tag{32k}$$

$$g_{\pi C\mathscr{B}_+}=\frac{\mathrm{sgn}(Y)}{\sqrt{6}F_\pi[2(M^2+Y^2)-2M\mu]^{1/2}}[(M+2X-2Y+6Z)Y+(-M+\mu)(M-2X+2Y-6Z)]$$
$$\simeq\begin{cases}-0.948, & \alpha=0.5\ ,\\ -1.03, & \alpha=0.6\ ,\\ -1.09, & \alpha=0.7\ ,\end{cases} \tag{32l}$$

$$g_{\pi C\mathscr{B}_-}=\frac{\mathrm{sgn}(Y)}{\sqrt{6}F_\pi[2(M^2+Y^2)-2M\mu]^{1/2}}[(M-\mu)(M+2X-2Y+6Z)+Y(M-2X+2Y-6Z)]$$
$$\simeq\begin{cases}13.9, & \alpha=0.5\ ,\\ 12.4, & \alpha=0.6\ ,\\ 10.8, & \alpha=0.7\ ,\end{cases} \tag{32m}$$

$$g_{\pi^9CC}=-\left[\frac{2}{3}\right]^{1/2}\frac{M_C}{F_\pi}\simeq-12.3\ , \tag{32n}$$

where the couplings $g_{\pi^9\mathscr{B}_\pm\mathscr{B}_\pm}$, $g_{\pi C\mathscr{B}_\pm}$, and g_{π^9CC} are defined by

$$\mathscr{L}_{\rm eff}=\cdots+g_{\pi^9\mathscr{B}_+\mathscr{B}_+}i\pi^9\mathrm{tr}(\overline{\mathscr{B}}_+\gamma_5\mathscr{B}_+)+g_{\pi^9\mathscr{B}_-\mathscr{B}_-}i\pi^9\mathrm{tr}(\overline{\mathscr{B}}_-\gamma_5\mathscr{B}_-)+g_{\pi^9\mathscr{B}_+\mathscr{B}_-}[i\pi^9\mathrm{tr}(\overline{\mathscr{B}}_+\mathscr{B}_-)+\mathrm{H.c.}]$$
$$+g_{\pi C\mathscr{B}_+}[i\pi^a\overline{C}\,\mathrm{tr}(\mathscr{B}_+\lambda^a)+\mathrm{H.c.}]+g_{\pi C\mathscr{B}_-}[i\pi^a\overline{C}\gamma_5\mathrm{tr}(\mathscr{B}_-\lambda^a)+\mathrm{H.c.}]+g_{\pi^9CC}i\pi^9\overline{C}\gamma_5C+\cdots\ . \tag{33}$$

A more extensive list of the individual pseudoscalar-baryon couplings can be found in the Appendix.

VI. GENERALIZED GOLDBERGER-TREIMAN RELATIONS

There are a number of interesting relations among the baryon masses and couplings which follow from (26). Many of these have already been listed in (32) and (A3)—(A10). Equation (A3a), for example, corresponds to the usual SU(2) Goldberger-Treiman (GT) relations[4,23]

$$G_{N,\pi N}=g_{p,\pi^0p}=-g_{n,\pi^0n}=\frac{1}{\sqrt{2}}q_{p,\pi^+n}=\frac{1}{\sqrt{2}}g_{n,\pi^-p}=\frac{M_+}{F_\pi}\ . \tag{34}$$

The other set of relations can be classified as follows.

(i) The SU(3) generalizations of (34). These are listed in (A3) and in more general terms summarized by (32a) and (32b).

(ii) The extension of (i) to include the η'-baryon couplings. See (32i) and (A6).

(iii) The generalization of (i) and (ii) to negative-parity baryons. See (32c), (32d), (32j), (A4), and (A7).

(iv) Relations which involve the pseudoscalar couplings to an even- and odd-parity baryon. See (32f), (32g), (A5), and (A8).

(v) Relations involving the flavor-singlet baryon couplings. See (32l), (32m), (32n), (A9), and (A10).

All of these relations can be derived in a similar manner as the GT relation (34), i.e., by sandwiching the axial-vector currents between two (even- and/or odd-parity) baryon states. By Lorentz invariance and $SU(3)_V$ symmetry these matrix elements can be written in the form $[q=p_f-p_i, \mathcal{F}^b_{\mu 5}=\bar{q}\gamma_\mu\gamma_5(\lambda^b/2)q]$.

$$\begin{aligned}\langle B^a_\pm(p_f)\,|\,\mathcal{F}^b_{\mu 5}\,|\,B^c_\pm(p_i)\rangle &= \bar{U}^a_\pm(p_f)[\gamma_\mu\gamma_5 C^{abc}_{1\pm\pm}(q^2)+q_\mu\gamma_5 C^{abc}_{2\pm\pm}(q^2)]U^c_\pm(p_i)\ ,\\ \langle B^a_+(p_f)\,|\,\mathcal{F}^b_{\mu 5}\,|\,B^c_-(p_i)\rangle &= \bar{U}^b_+(p_f)[\gamma_\mu C^{abc}_{1+-}(q^2)+q_\mu C^{abc}_{2+-}(q^2)]U^c_-(p_i)\ ,\end{aligned} \tag{35}$$

where U and $\bar{U}$ are Dirac spinors. Each of the coefficients C^{abc}_{1++}, C^{abc}_{1--}, etc., can be split up into the sum of two terms—corresponding to the two ways of forming an SU(3) singlet, i.e., of the $g^{(1)}$ and $g^{(2)}$ or f and d types. There is no sum over a, b, and c in (35).

Multiplying (35) by $q^\mu=(p_f-p_i)^\mu$, the LHS's become proportional to $\partial^\mu\mathcal{F}^b_{\mu 5}$ and hence vanish, in the chiral limit. This gives

$$\begin{aligned} 0&=\bar{U}^a_\pm(p_f)[\not{q}\gamma_5 C^{abc}_{1\pm\pm}(q^2)+q^2\gamma_5 C^{abc}_{2\pm\pm}(q^2)]U^c_\pm(p_i)\ ,\\ 0&=\bar{U}^a_+(p_f)[\not{q} C^{abc}_{1+-}(q^2)+q^2 C^{abc}_{2+-}(q^2)]U^c_-(p_i)\ .\end{aligned} \tag{36}$$

Setting $\not{q}=\not{p}_f-\not{p}_i$ and using the spinor properties

$$\not{p}_i U^c_\pm(p_i)=M^c_\pm U^c_\pm(p_i)\ ,$$
$$\bar{U}^a_\pm(p_f)\not{p}_f=\bar{U}^a_\pm(p_f)M^a_\pm\ ,$$

gives

$$\begin{aligned} 0=\bar{U}^a_\pm(p_f)[&(M^a_\pm+M^c_\pm)\gamma_5 C^{abc}_{1\pm\pm}(q^2)\\ &+q^2\gamma_5 C^{abc}_{2\pm\pm}(q^2)]U^c_\pm(p_i)\ ,\\ 0=\bar{U}^a_+(p_f)[&(M^a_+-M^c_-)C^{abc}_{1+-}(q^2)\\ &+q^2 C^{abc}_{2+-}(q^2)]U^c_-(p_i)\ ,\end{aligned} \tag{37}$$

In the first equation above we have $M^a_\pm+M^c_\pm$ because the $\not{p}_i$ has to be passed through γ_5 to act on $U^c_\pm(p_i)$, while in the last equation we have $M^a_+-M^c_-$. When B^a and B^c are members of the same octet we can set $M^a_+=M^c_+$ and $M^a_-=M^c_-$. We have not imposed this in (37) to allow for the inclusion of the flavor-singlet baryon ($M_C \neq M_+, M_-$).

In the limit $q^2\to 0$ the second terms on the RHS's of (37) receive a contribution from the π^b pseudoscalar pole. Therefore[25]

$$\begin{aligned}(M^a_\pm+M^c_\pm)C^{abc}_{1\pm\pm}(0)&=2F_\pi g_{\pi^b B^a_\pm B^c_\pm}\ ,\\ (M^a_+-M^c_-)C^{abc}_{1+-}(0)&=2F_\pi g_{\pi^b B^a_+ B^c_-}\ .\end{aligned} \tag{38}$$

These equations hold for the f and d (or $g^{(1)}$ and $g^{(2)}$) couplings, *separately*

The coefficients $C^{abc}_{1++}(0)$, $C^{abc}_{1--}(0)$, and $C^{abc}_{1+-}(0)$ appearing in (38) are the axial-vector transition constants (generalizations of g_A, the axial-vector coupling constant) and can be easily determined by finding the $\bar{B}_\pm\gamma_\mu\gamma_5 B_\pm$ and $\bar{B}_+\gamma_\mu B_-$ pieces of the effective axial-vector currents.

Under the infinitesimal *local* axial-vector transformations (9) $[\beta=\beta(x)]$ $\mathcal{L}_{\rm QCD}$ is not invariant because of the $\bar{q}i\not{\partial}q$ kinetic term:

$$\mathcal{L}_{\rm QCD}(x)\to\mathcal{L}_{\rm QCD}(x)-2[\partial^\mu\beta^b(x)]\mathcal{F}^b_{\mu 5}(x)\ , \tag{39}$$

where the $\mathcal{F}^b_{\mu 5}$ are the axial-vector currents $\bar{q}\gamma_\mu\gamma_5(\lambda^b/2)q$. $\mathcal{L}_{\rm eff}$ is expected to transform similarly. Starting with the baryon kinetic terms in (20) and performing the infinitesimal axial-vector transformations (13) (with $\mathbf{r}=\boldsymbol{\beta}=-\mathbf{l}\ll 1$) we find that

$$\begin{aligned}\mathcal{F}^b_{\mu 5}=&\,{\rm tr}\left[\bar{B}^1\gamma_\mu\gamma_5\frac{\lambda^b}{2}B^1\right]+{\rm tr}\left[\bar{B}^2\gamma_\mu\gamma_5\frac{\lambda^b}{2}B^2\right]\\ &-\left[{\rm tr}\left[\bar{B}^2\gamma_\mu B^1\frac{\lambda^b}{2}\right]\right.\\ &\left.+{\rm tr}\left[\bar{B}^1\gamma_\mu B^2\frac{\lambda^b}{2}\right]\right](1-3\delta_{b9})\ .\end{aligned} \tag{40}$$

For $Y\neq 0$ we need to perform the diagonalization transformations (24) and (25).

In the analysis of this section leading up to the η' baryon couplings we have deliberately ignored the U(1) axial anomaly:

$$\partial^\mu\mathcal{F}^b_{\mu 5}=\delta_{b9}\sqrt{6}\frac{g^2}{32\pi^2}F\cdot\tilde{F}=\delta_{b9}\sqrt{6}\,\partial^\mu K_\mu\ .$$

With the inclusion of the anomaly the LHS's of Eqs. (36) and (37) contain a piece proportional to $\delta_{b9}\langle B_\pm\,|\,\sqrt{6}\,\partial_\mu K^\mu\,|\,B_\pm\rangle$. Assuming that the η' is now no longer massless there is also no pole in the C_2 terms of Eqs. (37). It turns out however that this is a compensated by the Veneziano pole[26] contribution to the matrix element $\langle B_\pm\,|\,\sqrt{6}\,\partial_\mu K^\mu\,|\,B_\pm\rangle$ and that the end result is precisely the same as if we had ignored the anomaly from the very outset.

VII. THE DECAYS $B_-\to B_+\pi$

Assuming that the dominant contribution to the decays $B_-\to B_+\pi$ occurs through the three-point interaction, we can use the results for the $g_{\pi B_+ B_-}$ couplings (listed in the Appendix) to determine the decay modes and the branching ratios of these decays. We have

$$\Gamma_{B_-\to B_+\pi}\sim g^2_{\pi B_+ B_-}\frac{kM_{B_+}}{M_{B_-}}\ , \tag{41}$$

where k is the center-of-mass momentum for the channel being considered:

$$k=\frac{1}{2M_{B_-}}\{[M^2_{B_-}-(M_{B_+}+m_\pi)^2]\times[M^2_{B_-}-(M_{B_+}-m_\pi)^2]\}^{1/2}\ . \tag{42}$$

We consider each negative-parity baryon in turn. The[27] $\tilde{p}$ has nonzero couplings to Σ^0K^+, ΛK^+, Σ^+K^0, $p\eta$, and $p\eta'$ [see (A5)] but if the $\tilde{p}$ belongs to the lowest-lying negative-parity isodoublet, the $N(1535)$, mass considerations rule out all decay modes except $p\eta$. Therefore $\tilde{p}$ decays *only* into $p\eta$ *in the chiral limit.* Similarly $\tilde{n}\to n\eta$:

$$\tilde{N}\to N\eta\ \ (100\%)\ \ ({\rm theory})\ . \tag{43a}$$

Experimentally the $N(1535)$ decays into both $N\eta$ and $N\pi$ with

$$N(1535) \to N\eta \quad (40\text{—}60\,0005)$$
$$N\pi \quad (35\text{—}50\,\%) \ . \tag{43b}$$

It is heartening to note that the predominant decay mode is into $N\eta$, even though the phase space would favor the decay into $N\pi$ (by a factor of 2.4). We expect that the inclusion of $g_A \neq 1$ effects and explicit chiral-symmetry breaking (due to nonzero quark masses) generates a $g^{(1)}$-type $\pi B_+ B_-$ interaction which allows the decay $\tilde{N} \to N\pi$. With the relatively large phase-space factor we hope this can account for (43b).

Similarly, using the couplings (A5) and calculating the phase-space factors[28] we find that

	$\phi=0°$	$\phi=10°$	$\phi=18°$	
$\tilde{\Lambda} \to \Sigma\pi$	62%	60%	58%	
$N\bar{K}$	35%	33%	32%	(theory)
$\Lambda\eta$	3%	7%	10%	
				(44a)
$\tilde{\Sigma} \to \Lambda\pi$	8%	8.5%	8.5%	
$\Sigma\pi$	48%	49%	49.5%	(theory)
$N\bar{K}$	41%	41.5%	42%	
$\Sigma\eta$	3%	1%	0%	
				(44b)
$\tilde{\Xi} \to \Xi\pi$	71%	72%	73%	
$\Lambda\bar{K}$	26.5%	27%	27%	(theory)
$\Xi\eta$	2.5%	1%	0%	
				(44c)
$C \to \Sigma\pi$	(100%)	(theory),		(44d)

where ϕ is the η-η' mixing angle. The $\tilde{\Lambda}$ decay modes and branching ratios compare favorable with the $\Lambda(1670) \to \Sigma\pi$ (20—60 %), $N\bar{K}$ (15—30 %), $\Lambda\eta$ (15—35 %). On the other hand, the decay modes and branching ratios of the $\tilde{\Sigma}$ do not correspond to the observed lowest-lying $\frac{1}{2}^-$ candidate the $\Sigma(1750) \to N\bar{K}$ (10—40 %), $\Lambda\pi$ (seen), $\Sigma\pi$ ($<8\%$), $\Sigma\eta$ (15—55 %). Perhaps the $\tilde{\Sigma}$ is one of the bumps close to this resonance. There is no $\frac{1}{2}^-$ candidate for the $\tilde{\Xi}$. The state $\Xi(2030)$ whose spin-parity is not known is observed to decay into $\Lambda\bar{K}$ ($\sim 20\%$), $\Sigma\bar{K}$ ($\sim 80\%$), $\Xi\pi$ (small) cannot be the $\tilde{\Xi}$. The $\tilde{\Xi}$ does not decay into $\Sigma\bar{K}$ in the $g_A = 1$ chiral limit. Secondary effects due to $g_A \neq 1$ and explicit chiral-symmetry breaking can never induce such a large coupling as what is required for this identification. Furthermore the $\Xi\pi$ branching ratio is much too small. Since the negative-parity singlet $\Lambda(1405)$ has a comparitively small mass it can only decay into $\Sigma\pi$.

VIII. BEYOND

The present work was limited to a study of the πBB interactions but the techniques can be extended to study the implications of chiral symmetry for other interactions, e.g., the four nucleon interaction $(NN)^2$, the $\Delta\Delta\pi$, $\Delta N\pi$, and $\pi\pi NN$ interactions, etc. In connection with the $\Delta N\pi$ system it seems that, by sandwiching the axial-vector currents between a Δ and a N that

$$g_{\Delta N\pi} \sim (2F_\pi)^{-1}(M_\Delta + M_N) \simeq 11.6 \ .$$

It would also be interesting to include explicit chiral-symmetry breaking and $g_A \neq 1$ effects into this formalism.

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APPENDIX: COMPILATION OF PSEUDOSCALAR-BARYON COUPLING CONSTANTS

The usual charge eigenstate pseudoscalar-meson and -baryon fields are defined by[27,29]

$$\pi = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta}{\sqrt{6}} + \frac{\eta'}{\sqrt{3}} \end{bmatrix} , \tag{A1a}$$

$$\mathscr{B}_+ = \begin{bmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & \frac{-2\Lambda}{\sqrt{6}} \end{bmatrix} , \tag{A1b}$$

$$\mathscr{B}_- = \begin{bmatrix} \frac{\tilde{\Sigma}^0}{\sqrt{2}} + \frac{\tilde{\Lambda}}{\sqrt{6}} & \tilde{\Sigma}^+ & \tilde{p} \\ \tilde{\Sigma}^- & \frac{-\tilde{\Sigma}^0}{\sqrt{2}} + \frac{\tilde{\Lambda}}{\sqrt{6}} & \tilde{n} \\ \tilde{\Xi}^- & \tilde{\Xi}^0 & \frac{-2\tilde{\Lambda}}{\sqrt{6}} \end{bmatrix} . \tag{A1c}$$

1. $g_{\pi\mathscr{B}_+\mathscr{B}_+}$ couplings

Substituting (A1a) and (A1b) into the $g^{(1)}$-type interaction $\mathscr{L}_{g^{(1)}}$ in (27) gives

$$\begin{aligned}
\mathscr{L}_{g^{(1)}} &\sim i\sqrt{2}\mathrm{tr}(\overline{\mathscr{B}}_+\gamma_5\pi\mathscr{B}_+) \\
&= i\pi^0\left[\overline{\Sigma}{}^+\gamma_5\Sigma^+ - \overline{\Sigma}{}^-\gamma_5\Sigma^- + \frac{1}{\sqrt{3}}\overline{\Sigma}{}^0\gamma_5\Lambda + \frac{1}{\sqrt{3}}\overline{\Lambda}\gamma_5\Sigma^0 + \overline{p}\gamma_5 p - \overline{n}\gamma_5 n\right] \\
&\quad + i\pi^+\left[\overline{\Sigma}{}^0\gamma_5\Sigma^- - \overline{\Sigma}{}^+\gamma_5\Sigma^0 + \frac{1}{\sqrt{3}}\overline{\Lambda}\gamma_5\Sigma^- + \frac{1}{\sqrt{3}}\overline{\Sigma}{}^+\gamma_5\Lambda + \sqrt{2}\overline{p}\gamma_5 n\right] \\
&\quad + i\pi^-\left[\overline{\Sigma}{}^-\gamma_5\Sigma^0 - \overline{\Sigma}{}^0\gamma_5\Sigma^+ + \frac{1}{\sqrt{3}}\overline{\Sigma}{}^-\gamma_5\Lambda + \frac{1}{\sqrt{3}}\overline{\Lambda}\gamma_5\Sigma^+ + \sqrt{2}\overline{n}\gamma_5 p\right] \\
&\quad + iK^+\left[\overline{\Sigma}{}^0\gamma_5\Xi^- + \sqrt{2}\overline{\Sigma}{}^+\gamma_5\Xi^0 + \frac{1}{\sqrt{3}}\overline{\Lambda}\gamma_5\Xi^- - \frac{2}{\sqrt{3}}\overline{p}\gamma_5\Lambda\right] \\
&\quad + iK^-\left[\overline{\Xi}{}^-\gamma_5\Sigma^0 + \sqrt{2}\overline{\Xi}{}^0\gamma_5\Sigma^+ + \frac{1}{\sqrt{3}}\overline{\Xi}{}^-\gamma_5\Lambda - \frac{2}{\sqrt{3}}\overline{\Lambda}\gamma_5 p\right] \\
&\quad + iK^0\left[2\overline{\Sigma}{}^-\gamma_5\Xi^- - \overline{\Sigma}{}^0\gamma_5\Xi^0 + \frac{1}{\sqrt{3}}\overline{\Lambda}\gamma_5\Xi^0 - \frac{2}{\sqrt{3}}\overline{n}\gamma_5\Lambda\right] \\
&\quad + i\overline{K}{}^0\left[\sqrt{2}\overline{\Xi}{}^-\gamma_5\Sigma^- - \overline{\Xi}{}^0\gamma_5\Sigma^0 + \frac{1}{\sqrt{3}}\overline{\Xi}{}^0\gamma_5\Lambda - \frac{2}{\sqrt{3}}\overline{\Lambda}\gamma_5 n\right] \\
&\quad + \frac{i\eta}{\sqrt{3}}(\overline{\Sigma}{}^0\gamma_5\Sigma^0 + \overline{\Sigma}{}^+\gamma_5\Sigma^+ + \overline{\Sigma}{}^-\gamma_5\Sigma^- + \overline{p}\gamma_5 p + \overline{n}\gamma_5 n - 2\overline{\Xi}{}^-\gamma_5\Xi^- - 2\overline{\Xi}{}^0\gamma_5\Xi^0 - \Lambda\gamma_5\Lambda) \\
&\quad + (\tfrac{2}{3})^{1/2}i\eta'(\overline{\Sigma}{}^0\gamma_5\Sigma^0 + \overline{\Sigma}{}^+\gamma_5\Sigma^+ + \overline{\Sigma}{}^-\gamma_5\Sigma^- + \overline{p}\gamma_5 p + \overline{n}\gamma_5 n + \overline{\Xi}{}^0\gamma_5\Xi^0 + \overline{\Xi}{}^-\gamma_5\Xi^- + \overline{\Lambda}\gamma_5\Lambda) .
\end{aligned} \tag{A2a}$$

For the $g^{(2)}$-type interaction $\mathscr{L}_{g^{(2)}}$ we obtain

$$\begin{aligned}
\mathscr{L}_{g^{(2)}} &\sim i\sqrt{2}\mathrm{tr}(\overline{\mathscr{B}}_+\gamma_5\mathscr{B}_+\pi) \\
&= i\pi^0\left[\overline{\Sigma}{}^-\gamma_5\Sigma^- - \overline{\Sigma}{}^+\gamma_5\Sigma^+ + \frac{1}{\sqrt{3}}\overline{\Sigma}{}^0\gamma_5\Lambda + \frac{1}{\sqrt{3}}\overline{\Lambda}\gamma_5\Sigma^0 + \overline{\Xi}{}^-\gamma_5\Xi^- - \overline{\Xi}{}^0\gamma_5\Xi^0\right] \\
&\quad + i\pi^+\left[\overline{\Sigma}{}^+\gamma_5\Sigma^0 - \overline{\Sigma}{}^0\gamma_5\Sigma^- + \frac{1}{\sqrt{3}}\overline{\Sigma}{}^+\gamma_5\Lambda + \frac{1}{\sqrt{3}}\overline{\Lambda}\gamma_5\Sigma^- + \sqrt{2}\overline{\Xi}{}^0\gamma_5\Xi^-\right] \\
&\quad + i\pi^-\left[\overline{\Sigma}{}^0\gamma_5\Sigma^+ - \overline{\Sigma}{}^-\gamma_5\Sigma^0 + \frac{1}{\sqrt{3}}\overline{\Lambda}\gamma_5\Sigma^+ + \frac{1}{\sqrt{3}}\overline{\Sigma}{}^-\gamma_5\Lambda + \sqrt{2}\overline{\Xi}{}^-\gamma_5\Xi^0\right] \\
&\quad + iK^+\left[\overline{p}\gamma_5\Sigma^0 + \sqrt{2}\overline{n}\gamma_5\Sigma^- + \frac{1}{\sqrt{3}}\overline{p}\gamma_5\Lambda - \frac{2}{\sqrt{3}}\overline{\Lambda}\gamma_5\Xi^-\right] \\
&\quad + iK^-\left[\overline{\Sigma}{}^0\gamma_5 p + \sqrt{2}\overline{\Sigma}{}^-\gamma_5 n + \frac{1}{\sqrt{3}}\overline{\Lambda}\gamma_5 p - \frac{2}{\sqrt{3}}\overline{\Xi}{}^-\gamma_5\Lambda\right] \\
&\quad + iK^0\left[\sqrt{2}\overline{p}\gamma_5\Sigma^+ - \overline{n}\gamma_5\Sigma^0 + \frac{1}{\sqrt{3}}\overline{n}\gamma_5\Lambda - \frac{2}{\sqrt{3}}\overline{\Lambda}\gamma_5\Xi^0\right]
\end{aligned}$$

$$+i\bar{K}^0\left[\sqrt{2}\bar{\Sigma}^+\gamma_5 p-\bar{\Sigma}^0\gamma_5 n+\frac{1}{\sqrt{3}}\bar{\Lambda}\gamma_5 n-\frac{2}{\sqrt{3}}\bar{\Xi}^0\gamma_5\Lambda\right]$$

$$+\frac{i\eta}{\sqrt{3}}(\bar{\Sigma}^0\gamma_5\Sigma^0+\bar{\Sigma}^+\gamma_5\Sigma^++\bar{\Sigma}^-\gamma_5\Sigma^-+\bar{\Xi}^0\gamma_5\Xi^0+\bar{\Xi}^-\gamma_5\Xi^--2\bar{p}\gamma_5 p-2\bar{n}\gamma_5 n-\bar{\Lambda}\gamma_5\Lambda)$$

$$+(\tfrac{2}{3})^{1/2}i\eta'(\bar{\Sigma}^0\gamma_5\Sigma^0+\bar{\Sigma}^+\gamma_5\Sigma^++\bar{\Sigma}^-\gamma_5\Sigma^-+\bar{p}\gamma_5 p+\bar{n}\gamma_5 n+\bar{\Xi}^0\gamma_5\Xi^0+\bar{\Xi}^-\gamma_5\Xi^-+\bar{\Lambda}\gamma_5\Lambda)\ . \tag{A2b}$$

From (A2a), (A2b), (32a), and (32b) we read off the following couplings:

$$G_{N,\pi N}\equiv g_{p,\pi^0 p}=-g_{n,\pi^0 n}=\frac{1}{\sqrt{2}}g_{p,\pi^+ n}=\frac{1}{\sqrt{2}}g_{n,\pi^- p}=g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+}\ , \tag{A3a}$$

$$G_{\Sigma,\pi\Lambda}\equiv g_{\Sigma^0,\pi^0\Lambda}=g_{\Lambda,\pi^0\Sigma^0}=g_{\Lambda,\pi^+\Sigma^-}=g_{\Sigma^+,\pi^+\Lambda}=g_{\Sigma^-,\pi^-\Lambda}=g_{\Lambda,\pi^-\Sigma^+}=\frac{1}{\sqrt{3}}(g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+}+g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+})=\frac{2\alpha}{\sqrt{3}}G_{N,\pi N}\ , \tag{A3b}$$

$$G_{\Sigma,\pi\Sigma}\equiv g_{\Sigma^+,\pi^0\Sigma^+}=-g_{\Sigma^-,\pi^0\Sigma^-}=g_{\Sigma^0,\pi^+\Sigma^-}=-g_{\Sigma^+,\pi^+\Sigma^0}=g_{\Sigma^-,\pi^0\Sigma^0}=-g_{\Sigma^0,\pi^-\Sigma^+}=g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+}-g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+}$$
$$=(2-2\alpha)G_{N,\pi N}\ , \tag{A3c}$$

$$G_{\Xi,\pi\Xi}\equiv g_{\Xi^-,\pi^0\Xi^-}=-g_{\Xi^0,\pi^0\Xi^0}=\frac{1}{\sqrt{2}}g_{\Xi^0,\pi^+\Xi^-}=\frac{1}{\sqrt{2}}g_{\Xi^-,\pi^-\Xi^0}=g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+}=(2\alpha-1)G_{N,\pi N}\ , \tag{A3d}$$

$$G_{N,K\Lambda}\equiv g_{p,K^+\Lambda}=g_{\Lambda K^-p}=g_{n,K^0\Lambda}=g_{\Lambda,\bar{K}^0 n}=\frac{-2}{\sqrt{3}}(g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+}-\tfrac{1}{2}g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+})=\frac{-2}{\sqrt{3}}(\tfrac{3}{2}-\alpha)G_{N,\pi N}\ , \tag{A3e}$$

$$G_{N,K\Sigma}\equiv g_{p,K^+\Sigma^0}=\frac{1}{\sqrt{2}}g_{n,K^+\Sigma^-}=g_{\Sigma^0,K^-p}=\frac{1}{\sqrt{2}}g_{\Sigma^-,K^-n}=\frac{1}{\sqrt{2}}g_{p,K^0\Sigma^+}=g_{n,K^0\Sigma^0}=\frac{1}{\sqrt{2}}g_{\Sigma^+,\bar{K}^0 p}=g_{\Sigma^0,\bar{K}^0 n}$$
$$=g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+}=(2\alpha-1)G_{N,\pi N}\ , \tag{A3f}$$

$$G_{\Lambda,K\Xi}\equiv g_{\Lambda,K^+\Xi^-}=g_{\Xi^-,K^-\Lambda}=g_{\Lambda,K^0\Xi^0}=g_{\Xi^0,\bar{K}^0\Lambda}=\frac{1}{\sqrt{3}}(g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+}-2g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+})=\frac{1}{\sqrt{3}}(3-4\alpha)G_{N,\pi N}\ , \tag{A3g}$$

$$G_{\Sigma,K\Xi}\equiv g_{\Sigma^0,K^+\Xi^-}=\frac{1}{\sqrt{2}}g_{\Sigma^+,K^+\Xi^0}=g_{\Xi^-,K^-\Sigma^0}=\frac{1}{\sqrt{2}}g_{\Xi^0,K^-\Sigma^+}=\frac{1}{\sqrt{2}}g_{\Sigma^-,K^0\Xi^-}=-g_{\Sigma^0,K^0\Xi^0}=\frac{1}{\sqrt{2}}g_{\Xi^-,\bar{K}^0\Sigma^-}$$
$$=-g_{\Xi^0,\bar{K}^0\Sigma^0}=g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+}=G_{N,\pi N}\ , \tag{A3h}$$

$$G_{\Sigma,\eta\Sigma}\equiv g_{\Sigma^0,\eta\Sigma^0}=g_{\Sigma^+,\eta\Sigma^+}=g_{\Sigma^-,\eta\Sigma^-}=g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+}+g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+}=2\alpha G_{N,\pi N}\ , \tag{A3i}$$

$$G_{N,\eta N}\equiv g_{p,\eta p}=g_{n,\eta n}=\frac{1}{\sqrt{3}}(g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+}-2g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+})=\frac{1}{\sqrt{3}}(3-4\alpha)G_{N,\pi N}\ , \tag{A3j}$$

$$G_{\Xi,\eta\Xi}\equiv g_{\Xi^-,\eta\Xi^-}=g_{\Xi^0,\eta\Xi^0}=\frac{1}{\sqrt{3}}(g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+}-2g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+})=\frac{1}{\sqrt{3}}(2\alpha-3)G_{N,\pi N}\ , \tag{A3k}$$

$$G_{\Lambda,\eta\Lambda}=g_{\Lambda,\eta\Lambda}=-\frac{1}{\sqrt{3}}(g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+}+g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+})=\frac{-2\alpha}{\sqrt{3}}G_{N,\pi N}\ . \tag{A3l}$$

The $\eta'\mathcal{B}_+\mathcal{B}_+$ couplings are listed in subsection (4). Numerical values for the couplings (A3) can be obtained from (32).

2. $g_{\pi\mathcal{B}_-\mathcal{B}_-}$ couplings

These can be obtained from (A2a) and (A2b) by replacing all of the $\frac{1}{2}^+$ baryons by their parity partners. The couplings are as in (A3) with a tilde placed over the baryon fields and with the appropriate $g^{(1)}$ and $g^{(2)}$ couplings, i.e., $g^{(1)}_{\pi\mathcal{B}_-\mathcal{B}_-}$ and $g^{(2)}_{\pi\mathcal{B}_-\mathcal{B}_-}$, e.g.,

$$G_{\tilde{N},\pi\tilde{N}}\equiv g_{\tilde{p},\pi^0\tilde{p}}=-g_{\tilde{n},\pi^0\tilde{n}}=\frac{1}{\sqrt{2}}g_{\tilde{p},\pi^+\tilde{n}}=\frac{1}{\sqrt{2}}g_{\tilde{n},\pi^-\tilde{p}}=g^{(1)}_{\pi\mathcal{B}_-\mathcal{B}_-}\ . \tag{A4}$$

3. $g_{\pi\mathcal{B}_+\mathcal{B}_-}$ couplings

As $g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_-}=0$ [see (26) and (32)], there are only $\pi\mathcal{B}_+\mathcal{B}_-$ interactions of the $g^{(2)}$ type. These can be obtained from (A2b) (without the γ_5) by replacing each baryon and antibaryon field separately by their corresponding parity partners. When an antibaryon is changed there is extra minus sign. We find that

$$G_{\Sigma,\pi\tilde\Lambda} \equiv g_{\Sigma^0,\pi^0\tilde\Lambda} = g_{\Lambda,\pi^0\tilde\Sigma^0} = g_{\Sigma^+,\pi^+\tilde\Lambda} = g_{\Lambda,\pi^+\tilde\Sigma^-} = g_{\Lambda,\pi^-\tilde\Sigma^+} = g_{\Sigma^-,\pi^-\tilde\Lambda} = -g_{\tilde\Sigma^0,\pi^0\Lambda} = -g_{\tilde\Lambda,\pi^0\Sigma^0}$$
$$= -g_{\tilde\Sigma^+,\pi^+\Lambda} = -g_{\tilde\Lambda,\pi^+\Sigma^-} = -g_{\tilde\Lambda,\pi^-\Sigma^+} = -g_{\tilde\Sigma^-,\pi^-\Lambda} = \frac{1}{\sqrt{3}} g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} , \tag{A5a}$$

$$G_{\Sigma,\pi\tilde\Sigma} \equiv g_{\Sigma^-,\pi^0\tilde\Sigma^-} = -g_{\Sigma^+,\pi^0\tilde\Sigma^+} = g_{\Sigma^+,\pi^+\tilde\Sigma^0} = -g_{\Sigma^0,\pi^+\tilde\Sigma^-} = g_{\Sigma^0,\pi^-\tilde\Sigma^+} = -g_{\Sigma^-,\pi^-\tilde\Sigma^0} = -g_{\tilde\Sigma^-,\pi^0\Sigma^-}$$
$$= g_{\tilde\Sigma^+,\pi^0\Sigma^+} = -g_{\tilde\Sigma^+,\pi^+\Sigma^0} = g_{\tilde\Sigma^0,\pi^+\Sigma^-} = -g_{\tilde\Sigma^0,\pi^-\Sigma^+}$$
$$= g_{\tilde\Sigma^-,\pi^-\Sigma^0} = g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} , \tag{A5b}$$

$$G_{\Xi,\pi\tilde\Xi} \equiv g_{\Xi^-,\pi^0\tilde\Xi^-} = -g_{\Xi^0,\pi^0\tilde\Xi^0} = \frac{1}{\sqrt{2}} g_{\Xi^0,\pi^+\tilde\Xi^-} = \frac{1}{\sqrt{2}} g_{\Xi^-,\pi^-\tilde\Xi^0} = -g_{\tilde\Xi^-,\pi^0\Xi^-} = \cdots = g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} , \tag{A5c}$$

$$G_{N,K\tilde\Sigma} \equiv g_{p,K^+\tilde\Sigma^0} = \frac{1}{\sqrt{2}} g_{n,K^+\tilde\Sigma^-} = g_{\Sigma^0,K^-\tilde p} = \frac{1}{\sqrt{2}} g_{\Sigma^-,K^-\tilde n} = \frac{1}{\sqrt{2}} g_{p,K^0\tilde\Sigma^+} = -g_{n,K^0\tilde\Sigma^0} = \frac{1}{\sqrt{2}} g_{\Sigma^+,\bar K^0\tilde p} = -g_{\Sigma^0,\bar K^0\tilde n}$$
$$= -g_{\tilde p,K^+\Sigma^0} = \cdots = g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} , \tag{A5d}$$

$$G_{N,K\tilde\Lambda} \equiv g_{p,K^+\tilde\Lambda} = g_{\Lambda,K^-\tilde p} = g_{n,K^0\tilde\Lambda} = g_{\Lambda,\bar K^0\tilde n} = -g_{\tilde p,K^+\Lambda} = \cdots = \frac{1}{\sqrt{3}} g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} , \tag{A5e}$$

$$G_{\Lambda,K\tilde\Xi} \equiv g_{\Lambda,K^+\tilde\Xi^-} = g_{\Xi^-,K^-\tilde\Lambda} = g_{\Lambda,K^0\tilde\Xi^0} = g_{\Xi^0,\bar K^0\tilde\Lambda} = -g_{\tilde\Lambda,K^+\Xi^-} = \cdots = \frac{-2}{\sqrt{3}} g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} , \tag{A5f}$$

$$G_{\Sigma,\eta\tilde\Sigma} = g_{\Sigma^0,\eta\tilde\Sigma^0} = g_{\Sigma^-,\eta\tilde\Sigma^-} = g_{\Sigma^+,\eta\tilde\Sigma^+} = -g_{\tilde\Sigma^0,\eta\Sigma^0} = \cdots = \frac{1}{\sqrt{3}} g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} , \tag{A5g}$$

$$G_{\Xi,\eta\tilde\Xi} \equiv g_{\Xi^0,\eta\tilde\Xi^0} = g_{\Xi^-,\eta\tilde\Xi^-} = -g_{\tilde\Xi^0,\eta\Xi^0} = -g_{\tilde\Xi^-,\eta\Xi^-} = \frac{1}{\sqrt{3}} g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} , \tag{A5h}$$

$$G_{N,\eta\tilde N} \equiv g_{p,\eta\tilde p} = g_{n,\eta\tilde n} = -g_{\tilde p,\eta p} = -g_{\tilde n,\eta n} = \frac{-2}{\sqrt{3}} g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} , \tag{A5i}$$

$$G_{\Lambda,\eta\tilde\Lambda} \equiv g_{\Lambda,\eta\tilde\Lambda} = -g_{\tilde\Lambda,\eta\Lambda} = -\frac{1}{\sqrt{3}} g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} . \tag{A5j}$$

4. $g_{\eta'\mathcal{B}_+\mathcal{B}_+}$ couplings

We have

$$g_{\Sigma^0,\eta'\Sigma^0} = g_{\Sigma^-,\eta',\Sigma^-} = g_{\Sigma^+,\eta'\Sigma^+} = g_{\Xi^0,\eta'\Xi^0} = g_{\Xi^-,\eta'\Xi^-} = g_{p,\eta'p} = g_{n,\eta'n} = g_{\Lambda,\eta'\Lambda} = (\tfrac{2}{3})^{1/2} (g^{(1)}_{\pi\mathcal{B}_+\mathcal{B}_+} - 2g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_+})$$
$$= (\tfrac{2}{3})^{1/2} (3-4\alpha) G_{N,\pi N} . \tag{A6}$$

5. $g_{\eta'\mathcal{B}_-\mathcal{B}_-}$ couplings

We have

$$g_{\tilde\Sigma^0,\eta'\tilde\Sigma^0} = g_{\tilde\Sigma^-,\eta'\tilde\Sigma^-} = g_{\tilde\Sigma^+,\eta'\tilde\Sigma^+} = g_{\tilde\Xi^0,\eta'\tilde\Xi^0} = g_{\tilde\Xi^-,\eta'\tilde\Xi^-} = g_{\tilde p,\eta'\tilde p} = g_{\tilde n,\eta'\tilde n} = g_{\tilde\Lambda,\eta'\tilde\Lambda} = (\tfrac{2}{3})^{1/2} (g^{(1)}_{\pi\mathcal{B}_-\mathcal{B}_-} - 2g^{(2)}_{\pi\mathcal{B}_-\mathcal{B}_-})$$
$$= (\tfrac{2}{3})^{1/2} (4\alpha-1) G_{\tilde N,\pi\tilde N} . \tag{A7}$$

6. $g_{\eta'\mathcal{B}_+\mathcal{B}_-}$ couplings

We have

$$g_{\Sigma^0,\eta'\tilde\Sigma^0} = g_{\Sigma^-,\eta'\tilde\Sigma^-} = g_{\Sigma^+,\eta'\tilde\Sigma^+} = g_{\Xi^0,\eta'\tilde\Xi^0} = g_{\Xi^-,\eta'\tilde\Xi^-} = g_{p,\eta'\tilde p} = g_{n,\eta'\tilde n} = g_{\Lambda,\eta'\tilde\Lambda} = -g_{\tilde\Sigma^0,\eta'\Sigma^0} = \cdots = -2(\tfrac{2}{3})^{1/2} g^{(2)}_{\pi\mathcal{B}_+\mathcal{B}_-} . \tag{A8}$$

Note that the $\eta'\mathcal{B}\mathcal{B}$ couplings in 4, 5, and 6 do not involve any new parameters other than those of the $\pi\mathcal{B}\mathcal{B}$ sector because nonet symmetry ($F_{\eta'} = F_\pi$) has been assumed in (14). This is justified in the large-N_c (N_c = number of colors) limit.[26]

7. $g_{\pi C \mathscr{B}_\pm}$ couplings

We have

$$g_{C,\pi^0\Sigma^0}=g_{C,\pi^-\Sigma^+}=g_{C,\pi^+\Sigma^-}=g_{C,K^-p}=g_{C,\bar{K}^0 n}=g_{C,K^+\Xi^-}=g_{C,K^0\Xi^0}=g_{C,\eta\Lambda}=-g_{\Sigma^0,\pi^0 C}=\cdots=\sqrt{2}g_{\pi C\mathscr{B}_+}\,, \tag{A9}$$

$$g_{C,\pi^0\tilde{\Sigma}^0}=g_{C,\pi^-\tilde{\Sigma}^+}=g_{C,\pi^+\tilde{\Sigma}^-}=g_{C,K^-\tilde{p}}=g_{C,\bar{K}^0\tilde{n}}=g_{C,K^+\tilde{\Xi}^-}=g_{C,K^0\tilde{\Xi}^0}=g_{C,\eta\tilde{\Lambda}}=g_{\tilde{\Sigma}^0,\pi^0 C}=\cdots=\sqrt{2}g_{\pi C\mathscr{B}_-}\,, \tag{A10}$$

where $g_{\pi C\mathscr{B}_+}$ and $g_{\pi C\mathscr{B}_-}$ are given in (32l) and (32m).

Those pseudoscalar-baryon couplings which are not listed above vanish in the chiral limit, e.g., $g_{N,\pi\tilde{N}}=0$.

The "experimental values" of only a handful of these couplings are known.[30] They include $G_{N,\pi N}$, $G_{N,K\Lambda}$, $G_{\Sigma,\pi\Lambda}$, $G_{\Sigma,\pi\Sigma}$, $G_{N,K\Sigma}$, $G_{C,\pi\Sigma}$, and $G_{C,\bar{K}N}$. Our chiral limit $g_A=1$ results are in basic agreement with these values.

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[2]See, for instance, M. Creutz, L. Jacobs, and C. Rebbi, Phys. Rep. **95**, 201 (1983).

[3]We consider $\Theta=0$, where Θ is the usual *CP*- and *P*-violating coupling that can be introduced into QCD by adding the term $-\Theta(g^2/32\pi^2)F\cdot\tilde{F}$ to the conventional Lagrangian density. Phenomenologically Θ is constrained to be $\lesssim 10^{-9}$. See V. Baluni, Phys. Rev. D **19**, 2227 (1979); R. J. Crewther, P. Di Vecchia, G. Veneziano, and E. Witten, Phys. Lett. **88B**, 123 (1979).

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[12]Our results are unaffected by taking instead of (1), nonlocal baryon operators, such as

$$B\sim q_{a'}(x_1)q_{b'}(x_2)q_{c'}(x_3)$$
$$\times\left[P\exp\left[ig\int_{x_1}^{z}A_\mu(y_1)dy_1^\mu\right]\right]_{a'a}$$
$$\times\left[P\exp\left[ig\int_{x_2}^{z}A_\nu(y_2)dy_2^\nu\right]\right]_{b'b}$$
$$\times\left[P\exp\left[ig\int_{x_3}^{z}A_\lambda(y_3)dy_3^\lambda\right]\right]_{c'c}\epsilon_{abc}\,.$$

[13]Assuming for definiteness that the quarks have an even intrinsic parity.

[14]The other possibilities are considered below.

[15]We have excluded operators with derivatives, e.g., $(q_{a,i}^T C\partial^\mu q_{b,j})\gamma_\mu q_{c,k}\epsilon_{abc}\epsilon_{ijl}$. There is good reason to believe that these operators are associated with baryons with higher internal orbital angular momentum since in the nonrelativistic reduction more derivatives imply more factors of $(\boldsymbol{\sigma}\cdot\mathbf{p})$.

[16]A Fierz transformation is an expansion in terms of a complete set of matrices Γ^a, in which the indices of a product of two matrices A and B are interchanged as

$$A_{rs}B_{r's'}=\sum_{a,b}\Gamma^a_{rs'}\Gamma^b_{r's}\frac{1}{N^2}\mathrm{tr}(A\Gamma_b B\Gamma_a)$$

with $N\delta_{ab}=\mathrm{tr}(\Gamma_a\Gamma_b)$. In the case of Dirac indices $\Gamma_a=[1,\gamma_5,\gamma_\mu,i\gamma_\mu\gamma_5,\sigma_{\mu\nu}(\mu<\nu)]$ and $N=4$.

[17]For a detailed derivation of this result see G. A. Christos, Australian National University Report No. ANU/TP/043, 1984 (unpublished).

[18]$\boldsymbol{\alpha}\cdot\boldsymbol{\lambda}=\sum_{a=1}^{9}\alpha^a\lambda^a$ where $\lambda^1,\lambda^2,\ldots,\lambda^8$ are the usual SU(3) Gell-Mann matrices and $\lambda^9=(\frac{2}{3})^{1/2}\mathrm{diag}(1,1,1)$, normalized so that $\mathrm{tr}(\lambda^a\lambda^b)=2\delta^{ab}$.

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[20]The factor sgn(Y) in (25) is required so that it reduces to the identity matrix when $Y\to 0$, i.e., when there is no required diagonalization:

$$\lim_{Y\to 0}U\sim\frac{\mathrm{sgn}(Y)}{|Y|}\begin{bmatrix}Y & O(Y^2)\\ O(Y^2) & Y\end{bmatrix}.$$

[21]For M_+ and M_- we use the values of M_Σ and $M_{\tilde{\Sigma}}$, respectively, where $\tilde{\Sigma}$ is the parity partner to the Σ. This is appropriate because these values represent the average values for the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ octets. When chiral-symmetry-breaking corrections are included M_Σ does not change by much while M_N and M_Ξ are, respectively, shifted down and up by an amount which is proportional to the strange-quark mass.

[22]The value of α obtained from experiment is subject to some

uncertainty—ranging from 0.5 to 0.75. See, for instance, M. M. Nagels *et al.*, Nucl. Phys. **B147**, 189 (1979). There is a heuristic argument in our formalism which suggests that $\alpha \simeq \frac{1}{2}$. Although there is no (symmetry) reason why the couplings κ_1 and κ_2 appearing in (16) should be equal, we do not expect that two practically identical couplings should differ by all that much, at least in magnitude. That they should have the same sign is evident from (31a) since $M_+ - M_- \sim X \sim \kappa_1 + \kappa_2$. This corresponds to a small value of Y and consequently to $\alpha \simeq \frac{1}{2}$.

[23]In our approach the usual axial-vector renormalization constant g_A is set equal to one. A contribution to $g_A \neq 1$ can come from derivative coupling terms: see V. De Alfaro, S. Fubini, G. Furlan, and C. Rossetti, *Currents in Hardon Physics* (North-Holland, Amsterdam, 1973). The effect of these contributions is to rescale the right-hand sides of Eqs. (32a)—(32d), (32g), and (32i)—(32n) by the corresponding factors of g_A.

[24]The expressions for $g_{\pi C \mathscr{B}_\pm}$ take on particularly interesting forms at $Y=0$:

$$g_{\pi C \mathscr{B}_\pm}\,|_{Y=0} = \frac{1}{\sqrt{6}F_\pi}(M \pm 2X \pm 6Z)\bigg|_{\alpha=1/2} = \frac{1}{\sqrt{6}F_\pi}(M_\pm \mp M_C)\ .$$

[25]The factors of 2 on the RHS's of (38) are a consequence of our normalization of the $\pi\mathscr{B}\mathscr{B}$ couplings as in (27). Some extra care must be taken with the η' and C couplings.

[26]G. Veneziano, Nucl. Phys. **B159**, 213 (1979); E. Witten, *ibid.* **B156**, 269 (1979). See also G. A. Christos, Phys. Rep. **116**, 251 (1984); Z. Phys. C **21**, 83 (1983).

[27]The negative-parity partners of the $J^P = \frac{1}{2}^+$ (octet) baryons are designated with a tilde.

[28]We assume approximately equal intramultiplet mass splittings in the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ octets, with $M_{\tilde{\Lambda},\tilde{\Sigma},\tilde{\Xi}} = M_{\Lambda,\Sigma,\Xi} + (M_{\tilde{N}} - M_N)$. For the purposes of the phase-space calculations we take $M_{\tilde{\Lambda}} \simeq 1670$ MeV, $M_{\tilde{\Sigma}} \simeq 1770$ MeV, and $M_{\tilde{\Xi}} \simeq 1900$ MeV.

[29]Particle mixings are ignored. These are absent in the chiral limit.

[30]M. M. Nagels *et al.*, Nucl. Phys. **B147**, 189 (1979).