Vortices on the string world sheet and constraints on toral compactification

B. Sathiapalan
Department of Physics, University of California, Los Angeles, California 90024
(Received 30 October 1986)

The consistency of a bosonic string theory in flat spacetime with one of the internal dimensions compactified on a circle is analyzed by studying the phase structure of the corresponding regularized two-dimensional field theory. It is shown that the radius of compactification must be larger than $2\sqrt{\alpha'}$. One can also derive in this manner the limiting temperature for strings.

Nonperturbative effects on the string world sheet have been the subject of some recent papers.[1,2] In particular in Ref. 1 it was shown that an infinite perturbation series in α' (the loop-expansion parameter) for the generalized β functions of the two-dimensional field theory on the world sheet has to be summed in order to derive the equations of motion of the tachyon and other massive fields. In this paper we study strings propagating in a space-time manifold where one of the dimensions has been compactified into a circle. We will see that there are nonperturbative effects associated with vortices on the world sheet that give rise to constraints on the compactification radius. This could be of some interest in view of recent attempts to relate bosonic and fermionic string theories.[3,4]

We start with the first-quantized action for a bosonic string in flat 26-dimensional space-time:

$$S=\frac{1}{4\pi\alpha'}\int d\sigma\, d\tau(\partial_\alpha x^\mu\partial^\alpha x_\mu+\partial_\alpha Y\partial^\alpha Y)\ , \tag{1}$$

where $\mu=1-25$ denotes noncompact directions and $Y(\sigma,t)$ is the coordinate along the compactified dimension. Note that Y is a noncompact variable but with points Y and $Y+2\pi R$ identified. (Equivalently one can impose $0\leq Y\leq 2\pi R$ and introduce an extra integer-valued variable to represent the winding number.) The mode expansion for $Y(\sigma,\tau)$ $(0\leq\sigma\leq\pi)$ is[5]

$$\begin{aligned}Y(\sigma,\tau)=&y_0+2\alpha'\frac{M}{R}\tau+2NR\sigma\\&+i\frac{\sqrt{\alpha}'}{2}\sum_{n\neq 0}\frac{1}{n}(\alpha_n^i e^{-2in(\tau-\sigma)}+\tilde{\alpha}_n^i e^{-2in(\tau+\sigma)})\ ,\end{aligned} \tag{2}$$

where M and N are integers and are the quantized momentum and winding number, respectively. The $(\text{mass})^2$ operator for the string is usually written as[5]

$$\tfrac{1}{4}(\text{mass})^2=\frac{1}{4}\left[\frac{M^2}{R^2}+\frac{R^2}{(\alpha')^2}N^2\right]+(N+\tilde{N})-2\ , \tag{3}$$

where $N,\tilde{N}$ are the number operators and -2 is the normal-ordering constant which gives the ground-state energy. Now in quantum field theory there is always an implicit regularization scheme to control the ultraviolet divergences (e.g., to precisely define concepts such as normal ordering). Therefore, we define the theory on a lattice—since we are studying nonperturbative effects, a nonperturbative regularization scheme is required. We will focus exclusively on the action for Y. The point Y being, as mentioned before, identified with the point $Y+2\pi R$ this theory is nothing but the periodic Gaussian ("Villain") model well known in statistical mechanics as a low-temperature approximation to the X-Y model.

Because of the periodicity, one has to include multivalued vortex configurations for $Y(\sigma,\tau)$. The action of a vortex is roughly $\sim \ln R/a$, where R is the linear dimension of the statistical mechanics system being studied (we can think of it as an infrared cutoff) and a is the lattice spacing. Clearly, this diverges as $a\to 0$ and one might think they are irrelevant in the continuum. However, the entropy also has the same form $\sim \ln R/a$ and hence above a certain temperature there is a phase transition (discovered by Kosterlitz and Thouless)[6] when these vortices condense.[7] The phase diagram and the coupling-constant flow pattern (see Fig. 1) have been worked out in Ref. 7 to lowest order and to higher order in Ref. 8 where no qualitative difference was found. We will assume here that this remains true to all orders. (The existence of the phase transition, however, has been rigorously proved in Ref. 9.)

The y axis corresponds to the fugacity [exp($-$chemical potential)] and the x axis to β (=inverse temperature) which is related to the radius of compactification by $\beta=R^2/2\pi\alpha'$. In the periodic Gaussian model there is a

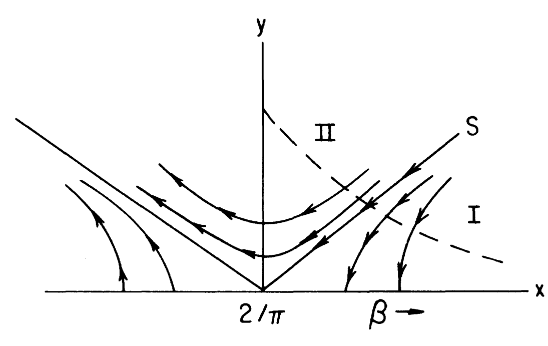

FIG. 1. Kosterlitz flow scheme for periodic Gaussian model. The region (I) to the right of S is the low-temperature insulating phase. The entire x axis is a line of fixed points. The initial point must lie on the dashed curve; hence only the fixed points corresponding to $\beta>2/\pi$ can be reached. The y axis is fugacity and the x axis is β.

definite relation[10,8] between these two quantities shown by the dashed curve in Fig. 1. If one starts at $T < T_c$, one approaches one of a line of (trivial) fixed points on the x axis. This is the low-temperature (insulating) phase where there are no vortices. For $T > T_c$, one is driven away from the x axis. It is not known whether these are nontrivial fixed points, but the crucial observation for us is that one cannot reach the trivial fixed points for $T > T_c$. What does this imply for string theories? For a consistent string theory the action has to be conformally invariant which in turn means that one has to sit at a fixed point of the renormalization group. If we assume that the only fixed points are the trivial ones then one is forced to conclude that a consistent string theory can be defined only in the low-temperature phase, i.e., $\pi\beta \geq 2$ or $R \geq 2\sqrt{\alpha'}$. If there are nontrivial fixed points, then $R \leq 2\sqrt{\alpha'}$ is allowed. However, this theory is not described at all by the Hamiltonian (3). This is because the high-temperature phase corresponds to a vacuum that consists of a condensate of vortices. This would correspond to a string propagating in a nontrivial background of the type described in Ref. 1 for which the action (1) is not the correct one. Indeed, in such a background, winding number is not well defined and the expression (3) for the Hamiltonian is not appropriate. To summarize, while naively one expects the entire line of fixed points along the x axis to represent consistent vacuum states for the string, in fact we find here that only the half-line $R \geq 2\sqrt{\alpha'}$ is allowed. For $R \leq 2\sqrt{\alpha'}$ we do not know if these are nontrivial fixed points (and therefore consistent string theories). If there are nontrivial fixed points, these are not described by the simple quadratic Hamiltonian (3) or the action (1) since these fixed points would correspond to nontrivial backgrounds. As we will see below, these backgrounds, even if consistent would contribute to the energy-momentum tensor and therefore to curvature. Thus, we conclude that for flat backgrounds, we are restricted to $R \geq 2\sqrt{\alpha'}$ (Refs. 11 and 12). This is the main result of this paper.

In the context of string theories, it is possible to interpret this phase transition in a different way. It is easy to convince oneself that a vortex can be interpreted as the emission of a soliton by the string with a concomitant change in its winding number. $R = 2\sqrt{\alpha'}$ is the radius for which the soliton is massless and for $R < 2\sqrt{\alpha'}$ it becomes tachyonic and thus tends to acquire an expectation value. One would have thought that even though the trivial vacuum is unstable (for $R < 2\sqrt{\alpha'}$) it should still be possible to describe a consistent string theory in it since it is a solution (albeit an unstable one) of the equations of motion. In fact, one can start with a modified lattice action for the X-Y model[13] in which the vortices are suppressed, i.e., the vortices correspond to relevant operators whose coefficients can be tuned to zero. In this case we do have a continuum theory for $R < 2\sqrt{\alpha'}$ at the trivial fixed point. However, this is separated by a phase transition from the $R > 2\sqrt{\alpha'}$ theory. In fact, if one tried to reach $R < 2\sqrt{\alpha'}$ continuously from $R > 2\sqrt{\alpha}$ one would expect this phase transition to show up as a singularity: the one-loop amplitudes may be expected to diverge due to soliton emission into the vacuum in analogy with what happens with the dilaton in the ordinary bosonic string. To compensate for this, one would need to give an expectation value to the soliton field (in the manner of Fischler and Susskind[14]) and this takes you away from the trivial fixed point for $R > 2\sqrt{\alpha'}$. We believe it is in this sense that the inconsistency is to be interpreted. It should be noted that the equation of motion obeyed by this (tachyonic) soliton field is exactly the same as those of the tachyon in Ref. 1 (restricted to a single frequency mode) and, in particular, therefore, one expects contributions to the right-hand side of the Einstein equations for the gravitational field.[1] Thus, indeed the nontrivial fixed points require curved space-time backgrounds.

If the compactified dimension is a Euclidean time coordinate, then this action describes a string in equilibrium with a finite-temperature heat bath,[15] with the identification $2\pi R = \beta = 1/T$. Thus, the constraint $R \geq 2\sqrt{\alpha'}$ becomes $T \leq 1/4\pi\sqrt{\alpha'}$—an intriguing way of obtaining the well-known value of the limiting temperature for the bosonic string.[16,17] It would be very interesting to understand what a vortex or soliton corresponds to physically in this case.

Finally, a word on the supersymmetric case. The ground-state energy in the fermionic string is half of that in the bosonic string and this would then give $R \geq \sqrt{2\alpha'}$. This seems to agree with the analysis of the supersymmetric Kosterlitz-Thouless phase transition in Ref. 18. This also gives a limiting temperature $1/\sqrt{8\pi}\sqrt{\alpha'}$. One can also see this in the supersymmetric sine-Gordon theory where the operator $\cos(\beta\phi)$ of the bosonic theory is replaced by the operator $\bar{\psi}\psi\cos(\beta\phi)$. Thus, the phase transition occurs when the anomalous dimension of $\cos(\beta\phi)$ is one rather than two, i.e., at $\beta^2 = 4\pi$ rather than at $\beta^2 = 8\pi$. Now in the superstring this operator has vanishing matrix elements between physical states because of the Gliozzi-Scherk-Olive (GSO) projection (if the index on ψ correspond to the transverse directions). Thus, there is no phase transition and we do not get any constraint on the radius. Another way to see this is that since the ground state has zero mass, all of the solitonic states have positive $(\text{mass})^2$ for all R and hence are never tachyonic. However, if the index on ψ corresponds to the time direction then the previous analysis goes through and the limiting temperature is $1/\sqrt{8\pi}\sqrt{\alpha'}$ as in the fermionic string.

In conclusion we have shown that $R > 2\sqrt{\alpha'}$ (or $R < \sqrt{\alpha'}/2$) are the allowed values of the radius for compactification of the internal dimensions in a bosonic string and $R < 2\sqrt{\alpha'}$ cannot be reached continuously from this region. However, we should emphasize that the origin of this constraint was the existence of a tachyon. We expect that once one shifts the tachyon field to its minimum (if it exists at all), one would then get a bosonic theory in a nontrivial background. This theory can presumably have $R < 2\sqrt{\alpha'}$. It would also be interesting to see whether these ideas generalized to situations where the internal dimensions are compactified on group manifolds.[19,20]

I would like to acknowledge useful discussions with S. R. Das and E. T. Tomboulis. I would also like to acknowledge a conversation with E. Martinec. This work was supported in part by the National Science Foundation under Grant No. PHY/83-14188.

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