Ages of the Universe for decreasing cosmological constants

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The age of the Universe is calculated for background energy densities that decrease in time, using models that generalize the constant vacuum energy and pressure equivalent to a cosmological constant. Ages old enough to agree with observations are obtained if the energy density decreases slowly. Some models of faster decrease may be ruled out. The age can be quite different when the energy density of the background is the same but the pressure is changed.

Various lines of thought lead to a time-dependent energy density that acts as a background for cosmology in the Universe of radiation and matter. They generalize the idea of a Lorentz-invariant vacuum state[1,2] which gives constant energy density and pressure corresponding to a cosmological constant. In the most familiar example, the energy of a scalar field is assumed to produce inflation; it acts as a cosmological constant during inflation but then it vanishes.[3,4] In recent models, the background energy density decreases as an inverse power of the scale factor.[5–10] For a background of torsion[5] or texture,[6] it decreases as the inverse second power. For a wide class of conformally invariant quantum field backgrounds,[7] it decreases as the inverse third power.

One of the most direct arguments for including a vacuum or background energy at any recent time in cosmology is that without it the calculated age of the Universe tends to be less than the age deduced from observations. This is particularly clear when the overall energy density has the critical value implied by inflation (nearly just enough to close the Universe).[11–15] Here we show that the same observational data rule out some models of background energy density that decreases in time. We point out that an energy density decreasing in time is generally less effective than a constant energy density in making the age of the Universe large enough to agree with observations. It can make the age large enough if it decreases slowly. If it decreases as the inverse second power of the scale factor, it has no effect on the age. If it decreases as the inverse third power, and the overall energy density has the critical value implied by inflation, the age may not be large enough. The models we consider show that the age can be quite different when the energy density of the background is the same but the pressure is changed.

We use Einstein's equations for an isotropic homogeneous universe, so space-time is described by a Robertson-Walker metric and the energy-momentum tensor is that of a perfect fluid with energy density $\rho(t)$ and pressure $p(t)$ which are functions only of the time coordinate t that is proper time for observers moving with the fluid. Then Einstein's equations are equivalent to

$$\frac{1}{a^2}\left[\frac{da}{dt}\right]^2=\tfrac{8}{3}\pi G\rho-\frac{k}{a^2}\ , \tag{1}$$

$$a\frac{d\rho}{da}=-3(\rho+p)\ , \tag{2}$$

where $a(t)$ is the scale factor of the Robertson-Walker metric, G is Newton's gravitational constant, and k is 1, 0 or -1 for spherical, flat or hyperbolic three-dimensional geometry. In the second equation we have canceled a factor da/dt because we will consider ρ and p as functions of the scale factor rather than t. We do not consider a case where da/dt is zero at any time in the past history of the Universe.[16] We use units in which the speed of light c is 1.

Using the Hubble parameter

$$H=\left[\frac{1}{a}\right]\frac{da}{dt}\ ,$$

and using the subscript 0 to denote the present values of H, a, and ρ, we get k in terms of H_0, a_0, and ρ_0 from the first equation and thus rewrite that equation as

$$\frac{1}{a^2}\left[\frac{da}{dt}\right]^2=\tfrac{8}{3}\pi G(\rho-\rho_0 a_0{}^2/a^2)+H_0{}^2a_0{}^2/a^2\ . \tag{1'}$$

From this we find that the present age of the Universe is

$$t_0=\int_0^1\left[\tfrac{8}{3}\pi G\left[\rho\frac{a^2}{a_0{}^2}-\rho_0\right]+H_0{}^2\right]^{-1/2}d\left[\frac{a}{a_0}\right]\ . \tag{3}$$

We see that the age t_0 is larger if $\rho a^2-\rho_0 a_0{}^2$ is smaller. A part of ρ makes t_0 larger if it decreases in time slower than $1/a^2$. If it decreases as $1/a^2$ it makes no difference. A part of ρ that is constant contributes more to t_0 than a part that decreases. If it were to increase it would contribute even more. If it decreases more, it contributes less. If it decreases faster than $1/a^2$, it makes t_0 smaller.

To be more specific we consider particular models for the dependence of ρ on a. For matter alone we would have

$$\rho=\rho_0(a_0/a)^3\ ,\quad p=0\ ;$$

for radiation alone we would have

$$\rho=\rho_0(a_0/a)^4\ ,\quad p=\tfrac{1}{3}\rho\ ;$$

and for a Lorentz-invariant vacuum state we would have

$$\rho=\rho_0\,,\quad p=-\rho\,.$$

Equation (2) is satisfied in each case, so it would be satisfied also for any sum of matter, radiation, and vacuum. Since matter is dominant compared to radiation through most of the history of the Universe, we neglect the contribution of radiation. To generalize from a Lorentz-invariant vacuum to a time-dependent background, we have a choice of two simple models that combine matter and background in

$$\rho=\rho_m+\rho_b$$

with

$$\rho_b=\rho_{b0}(a_0/a)^b\,.$$

The first model is

$$\rho_m=\rho_{m0}(a_0/a)^3$$

with

$$p=\tfrac{1}{3}b\rho_b-\rho_b\,. \tag{4}$$

In this model the energy densities are particularly simple. The second model is

$$\rho_m=\left[\rho_{m0}-\frac{b}{3-b}\rho_{b0}\right]\left[\frac{a_0}{a}\right]^3+\frac{b}{3-b}\rho_{b0}\left[\frac{a_0}{a}\right]^b$$

for $b\neq3$ and

$$\rho_m=\rho_{m0}\left[\frac{a_0}{a}\right]^3+3\rho_{b0}\left[\frac{a_0}{a}\right]^3\ln\left[\frac{a}{a_0}\right]$$

for $b=3$ with

$$p=-\rho_b\,. \tag{5}$$

In this model the relation between pressure and density (and thus the form of the energy-momentum tensor) is the same as for matter and vacuum. Both models satisfy Eq. (2). In both models ρ_{m0}, the matter density in the Universe now, is a positive number to be obtained from observations, and ρ_{b0} and b are real parameters. When b is 0, both models describe matter and vacuum.

We can write Eq. (2) as

$$d\rho a^2+pda^3=0 \tag{2$'$}$$

and interpret it as expressing conservation of energy. The first term is the change of the energy in a volume and the second is the work done by the pressure as the volume expands. Let $p_m=0$. In the first model, Eq. (2) holds for ρ_m and p_m alone, so there is conservation of energy for the matter alone. This does not hold for the second model. There is transfer of energy between the background and matter. It is from the background to the matter if $0<b<3$ and $\rho_{b0}>0$ and from the matter to the background if $b>3$ and $\rho_{b0}<0$.

Requiring either ρ_m or ρ to be non-negative puts restrictions on ρ_{b0} for each b. From Eq. (1) we see that ρ must be non-negative if k is 0 or 1. For $0<b<3$ in the second model [Eq. (5)], requiring either ρ_m or ρ to be non-negative in the limit of small a/a_0 implies

$$\rho_{b0}\leq\frac{3-b}{b}\rho_{m0}\,. \tag{6}$$

For $b\geq3$ in the second model, the same requirement implies ρ_{b0} is negative or zero. Requiring ρ to be non-negative at the present time implies

$$\rho_{b0}\geq-\rho_{m0}\,.$$

For $b>3$ in the first model [Eq. (4)], requiring ρ to be non-negative in the limit of small a/a_0 implies ρ_{b0} is non-negative.

The range of ρ_{b0} is restricted also by knowledge of the deceleration parameter q_0, which is the present value of

$$-a\left[\frac{d^2a}{dt^2}\right]\Big/\left[\frac{da}{dt}\right]^2\,.$$

From Eqs. (1) and (2) we get

$$q=\frac{4\pi G}{3H^2}(\rho+3p)\,.$$

Let $\rho_c=3H_0{}^2/8\pi G$; this is the critical density, the value the density must have if k is 0, which is predicted by inflation. Suppose ρ is ρ_c. Then for the first model

$$2q_0=1+(b-3)\frac{\rho_{b0}}{\rho_c}$$

and for the second model

$$2q_0=1-3\frac{\rho_{b0}}{\rho_c}\,.$$

The tightest limits on q_0 obtained from observations[15] are $-1.27\lesssim q_0\lesssim2$. For the first model they imply

$$-\frac{3.54}{3-b}\lesssim\frac{\rho_{b0}}{\rho_c}\lesssim\frac{3}{3-b}$$

for $b\neq3$. There is no restriction for $b=3$. In the second model the restriction is

$$-1\lesssim\frac{\rho_{b0}}{\rho_c}\lesssim1.18\,.$$

For small b in the first model and all b in the second model, these restrictions are not helpful. They would be even less so for more conservative limits on q_0.

In a paper we received after this was finished[17] we find several constraints on the background energy density of our second model. To preserve the agreement between big-bang nucleosynthesis predictions and observations of element abundances it is required that the background energy density be less than one-tenth the radiation energy density at the time of nucleosynthesis.[17] What this implies for ρ_{b0}/ρ_{m0} depends on whether the energy density of the background decreases faster or slower than that of radiation and matter between then and now. More stringent constraints are obtained if assumptions are made about the form of the energy transferred from the background to radiation and matter.[17]

Ages of the Universe calculated from Eq. (3) for $k=0$ are shown in Figs. 1 and 2. Both show ages t_0 as func-

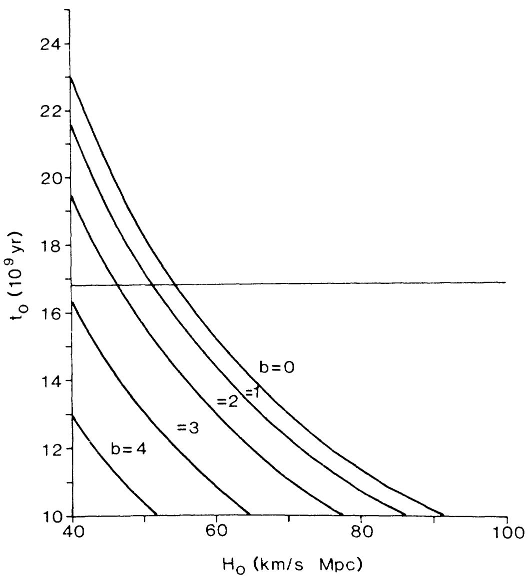

FIG. 1. Ages of the Universe calculated from Eq. (3) with $k=0$ for the first model [Eq. (4)] with $\rho_{b0}/\rho_{m0}=2$, shown as functions of possible values of the Hubble parameter H_0. The curves show ages calculated for different values of b. The horizontal line is claimed to be the lower limit of ages that could agree with observations.

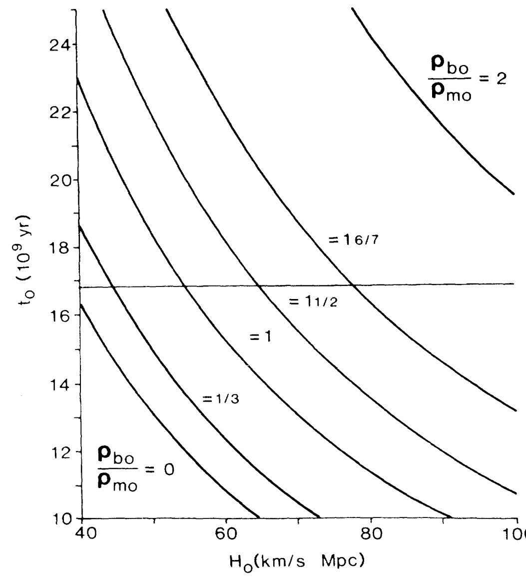

FIG. 2. Ages of the Universe calculated from Eq. (3) with $k=0$ for the second model [Eq. (5)] with $b=1$, shown the same as Fig. 1 except that the curves show ages calculated for different values of ρ_{b0}/ρ_{m0}.

tions of H_0 (Refs. 18 and 19). The horizontal line is claimed to be the lower limit of the ages that could agree with observations.[15,20,21] Figure 1 is for the first model with $\rho_{b0}/\rho_{m0}=2$. (Since $\rho_{m0}+\rho_{b0}$ depends only on H_0 when k is 0 [as we can see from Eq. (1)], both ρ_{m0} and ρ_{b0} are specified.) The curves show ages t_0 for various values of b. The ages are higher when b is smaller, as expected. We have similar curves for other values of ρ_{b0}/ρ_{m0}. Increasing ρ_{b0}/ρ_{m0} gives higher ages for $b<3$ and lower ages for $b>3$, as expected. Ages old enough to agree with observations are obtained when the background energy decreases slowly, with b less than about 2.9. Faster decreases, with b around 3 or larger, are ruled out.

Figure 2 is for the second model [Eq. (5)] with $b=1$. The curves show ages t_0 for various values of ρ_{b0}/ρ_{m0}. Again, increasing ρ_{b0}/ρ_{m0} gives higher ages. The highest curve is for the maximum value of ρ_{b0}/ρ_{m0} allowed by Eq. (6). The ages for $b<3$ are higher for this model than for the first model because part of the mass density decreases as a^{-b} rather than a^{-3}. Thus, the curve for $\rho_{b0}/\rho_{m0}=2$ in Fig. 2 is higher than the curve for $b=1$ in Fig. 1. For these cases where k is zero, the dependence of t_0 on H_0 is just that t_0 is proportional to H_0^{-1} so the curves are easily obtained.

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