PHYSICAL REVIEW D VOLUME 35, NUMBER 10 15 MAY 1987

Properties of Z_2 strings

Mukunda Aryal and Allen E. Everett
Department of Physics, Tufts University, Medford, Massachusetts 02155
(Received 24 November 1986)

We discuss various properties of cosmic strings formed by symmetry breaking of the form $G \rightarrow K \times Z_2$. The unbroken Z_2 can be embedded in different U(1) subgroups of G, giving topologically equivalent but dynamically different strings. We obtain the string mass in terms of the Higgs-field vacuum expectation value for various values of the gauge and quartic Higgs-field coupling constants by numerical calculation. We find that in a wide range of realistic cases, unless there is an intermediate $K \times \mathrm{U}(1)$ phase, the lowest-mass embedding gives strings which are gauge equivalent to antistrings, in which case the beadlike solitons of Hindmarsh and Kibble do not occur.

I. INTRODUCTION

Recently there has been a good deal of interest in theories in which stringlike vacuum structures[1,2] (henceforth, strings) arise in phase transitions in the early Universe. Strings may arise in phase transitions associated with the spontaneous breaking of either a (local) gauge symmetry or a global symmetry. In this paper we shall be concerned with gauge strings. Of particular interest are strings which arise in some versions of grand unified theories of the strong, electromagnetic, and weak interactions, since strings formed in a phase transition occurring at a temperature of the order of 10^{16} GeV, the temperature scale expected to be associated with the breaking of grand unification, are possible sources of the density fluctuations needed to explain galaxy formation.[3]

Many realistic models leading to cosmologically interesting gauge strings which have been proposed involve the spontaneous breaking of a simple group G, leaving an unbroken subgroup which includes a discrete symmetry.[4] In models considered to date the discrete subgroup of the broken-symmetry group has been the group Z_2, so that the symmetry-breaking pattern producing strings is of the form

$$G \rightarrow H = K \times Z_2 \ . \tag{1}$$

Strings produced in this way will be called Z_2 strings. Z_2 strings may also arise in a symmetry-breaking pattern differing from that of (1) by the existence of an intermediate phase having an additional unbroken U(1) symmetry, so that the symmetry-breaking pattern is of the form

$$G \rightarrow K \times \mathrm{U}(1) \rightarrow K \times Z_2 = H \ . \tag{2}$$

We shall refer to the symmetry-breaking patterns (1) and (2) henceforth as types 1 and 2.

The purpose of the present paper is to consider some of the properties of Z_2 strings, and to compare these for symmetry-breaking patterns 1 and 2. We shall consider the kind of generalized magnetic flux carried by the strings, as well as the structure of the Higgs field within the string. In case 2 we shall find that these are determined essentially entirely by topology. However, in case 1 there are a number of possible string structures which are topologically equivalent, but differ dynamically; in case 1, physically occurring strings will presumably have the structure leading to the lowest mass per unit length μ.

All of this is discussed in Sec. II below. In Sec. III we study the dependence of μ on the string structure by solving the coupled differential equations for the radial dependence of the Higgs and gauge fields, and finding the configuration giving the lowest value of μ. As a by-product we obtain, for a range of parameters, the relation between μ and η, the asymptotic value of the Higgs-field vacuum expectation value. As expected,[1] one finds μ of the order of a few times η^2. Finally, in Sec. IV, we discuss the relation between the internal structure of Z_2 strings and some of their physical properties; in particular, we consider whether Z_2 strings can be distinguished from antistrings or whether the strings are self-conjugate, and the connection between the nature of the magnetic flux carried by the strings and the way in which they radiate photons. We conclude briefly in Sec. V.

II. STRUCTURE OF Z_2 STRINGS

For the sake of concreteness we will consider a specific example[4] of a symmetry-breaking scheme of type 1 which is of some physical interest: namely,

$$\mathrm{SO}(10) \rightarrow \mathrm{SU}(5) \times Z_2 \ . \tag{3}$$

In (3), or other processes of type 1, the group manifold of G is simply connected, while that of H is disconnected, containing two disjoint pieces because of the discrete symmetry. Strings arise in the following way. Suppose that after the phase transition there is a closed circle C in coordinate space along which the vacuum state at angle θ is obtained from that at angle 0 by acting with the element $g(\theta) \in G$. Letting H now represent the invariance group of the vacuum at $\theta = 0$, we must have $g(0)$ and $g(2\pi) \in H$. If the curve $g(\theta)$ runs from one of the two disconnected pieces of H to the other as θ goes from 0 to 2π, as with curve 1 in Fig. 1, then the curve cannot be smoothly contracted to the identity and a string will be present passing someplace through the interior of C.

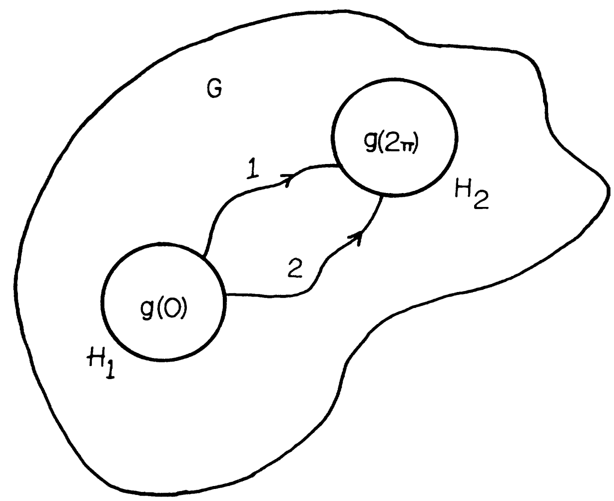

FIG. 1. Schematic representations of the group manifold of the group G and unbroken subgroup H of Eq. (1). Along a closed curve in coordinate space enclosing a string, the group element $g(\theta)$ relating the vacuum state at θ to that at $\theta=0$ varies along closed paths, such as curves 1 or 2, between the two disconnected pieces of the manifold of H.

One can also find a specific model, related to that of (3), giving a symmetry-breaking pattern of type 2: namely,

$$\mathrm{SO}(10)\rightarrow \mathrm{SU}(5)\times \mathrm{U}(1)_Y + \mathrm{SU}(5)\times Z_2 , \tag{4}$$

where $\mathrm{U}(1)_Y$ is the group whose generator is the operator Y whose matrix in the 16-dimensional spinor representation of SO(10) is the matrix

$$Y=\mathrm{diag}[(1)_{10},(-3)_5,(5)_1] , \tag{5}$$

where the notation means that the eigenvalues of Y are 1, -3, and 5 in the 10-, 5-, and 1-dimensional representations of SU(5), respectively. The discrete symmetry Z_2 is a rotation of $\pi/5$ generated by Y, which is not an element of SU(5), since the determinant of its submatrix within the 5-dimensional SU(5) subspace is -1. The first stage of symmetry breaking in (4) can be accomplished by giving a vacuum expectation value (VEV) to the member of a Higgs multiplet belonging to the 45-dimensional adjoint representation of SO(10) which transforms like Y. The final stage of symmetry breaking in both (3) and (4) involves a multiplet of Higgs fields ϕ_i belonging to the complex 126-dimensional representation of SO(10). The **126** is contained in the direct product of $\mathbf{16}\times\mathbf{16}$, and so the ϕ_i transform like appropriate linear combinations of the Cartesian products $u_j\times v_k$ (j, $k=1-16$) of two 16-dimensional spinors. We order the components of the 16 so that u_{16} is the SU(5)-singlet member of the 16-dimensional representation. Then the unbroken symmetry will be $\mathrm{SU}(5)\times Z_2$ if the component of the 126-plet transforming as $u_{16}\times v_{16}$ acquires a VEV. We label this component as ϕ_{16}. We shall adopt the notation $\phi=\langle\phi_{16}\rangle$, the VEV of ϕ_{16}.

The Lagrangian density for the system is given by

$$L=-F^a_{\mu\sigma}F^{\mu\sigma,a}/4+(D_\mu\phi_i)^*D^\mu\phi_i/2-V(\phi_i) , \tag{6}$$

$$D_\mu=-i\partial_\mu-eA^a{}_\mu\tau^a . \tag{7}$$

In Eqs. (6) and (7), $F_{\mu\sigma}$ are the components of the usual Yang-Mills gauge field tensor, the τ are the matrices representing the generators of SO(10) in the 126-dimensional representation, a and i are indices running from 1 to 45 and 1 to 126, respectively, e is the gauge coupling constant; and V the Higgs potential. [We omit the dependence of L on the Higgs 45-plet responsible for the first stage of symmetry breaking in (4) if present.] We have, of course, adopted the usual conventions of using units where $\hbar=c=1$, and summing over repeated indices. Equation (6) leads to the usual field equation

$$D_\mu D^\mu\phi-\partial V/\partial\phi=0 , \tag{8}$$

where, in Eq. (8) and from now on $A^a{}_\mu$ is to be interpreted as a classical field $\langle A^a{}_\mu\rangle$. In the cases we shall consider $A^a{}_\mu$ will be nonvanishing for only one value of μ and as a result the terms in $F^a{}_{\mu\sigma}$ which involve products $A^b{}_\mu A^c{}_\sigma$ for $\mu\neq\sigma$ all vanish. Taking this into account, the field equations for $A^a{}_\mu$ become simply

$$\Box A^a{}_\mu-\partial_\mu(\partial^\sigma A^a{}_\sigma)=j^a{}_\mu , \tag{9}$$

where $j^a{}_\mu$ is the ath component of the SO(10) current due to the fields ϕ_1, and is given by

$$j^a{}_\mu=ie[\partial_\mu\phi^{j*}(\tau^a\boldsymbol{\phi})^j-(\tau^a\boldsymbol{\phi})^{*j}\partial_\mu\phi^j] -e^2A^b{}_\mu\{\boldsymbol{\phi}^{*j}[(\tau^a\tau^b+\tau^b\tau^a)\boldsymbol{\phi}]^j\} , \tag{10}$$

where $\boldsymbol{\phi}$ is a vector whose components are the fields ϕ_i.

Let us consider first the symmetry-breaking pattern (4). We suppose we have a string lying along the z axis, and adopt a cylindrical coordinate system centered on the string with radial and azimuthal coordinates r, θ. We choose the set of generators τ^a to be mutually orthogonal, and to include the 24 generators of SU(5), as well as a generator which we label τ^{25} given by

$$\tau^{25}=NY=Q^{25} , \tag{11}$$

where N is a normalization constant. We normalize the matrices of τ^a in the 16-dimensional representation to which the fermions belong by

$$\mathrm{Tr}(\tau^{a\,2})=2 \tag{12}$$

which means that the eigenvalues of the SU(5) operator corresponding to, e.g., the weak isospin, have the conventional values $\pm\frac{1}{2}$. Equations (5) and (12) then imply that $N=(40)^{-1/2}$. This in turn means that the eigenvalue of the charge Q^{25} carried by the field ϕ_{16} with VEV ϕ is

$$q^{25}=10/(40)^{1/2} . \tag{13}$$

Then for a string arising from the symmetry-breaking model (4) we have the solution

$$\phi=\phi(r)e^{i\theta} , \tag{14a}$$

$$A^{25}{}_\theta=A(r) , \tag{14b}$$

$$A^a{}_\mu=0,\quad \mu\neq\theta \quad\text{or}\quad a\neq 25 , \tag{14c}$$

where $\phi(r)$ and $A(r)$ are subject to the boundary conditions

$$\phi(r),\ \ A(r)\to 0 \ \text{ for } r\to 0 \tag{15}$$

and

$$\phi(r)\to\eta,\ \ r\gg\eta^{-1}\ , \tag{16a}$$

$$A(r)\to 1/(eq^{25}r),\ \ r\gg\eta^{-1}\ . \tag{16b}$$

The conditions (15) are required in order that there not be an infinite contribution to the string energy at $r=0$. Equations (16) guarantee that ϕ and A^{25} satisfy the field Eqs. (8) and (9) in the asymptotic region, since they imply that, for $r\gg\eta^{-1}$, $D_\mu\phi=0$, and, using Eqs. (10), $j^a{}_\mu=0$. In the vicinity of the string, $r\leq\eta^{-1}$, $\phi(r)$, and $A(r)$ must be determined by solving the appropriate differential equations, obtained from Eqs. (8), (9), and (10), which we shall write down in detail in the next section. Equations (14b), (14c), and (16b), together with Stokes's theorem imply that the flux Φ^a of the generalized magnetic field B^a carried by the string is given by

$$\Phi^a=0,\ \ a\neq 25\ , \tag{17a}$$

$$\Phi^{25}=2\pi\int B(r)r\,dr=2\pi/eq^{25}\ . \tag{17b}$$

Equation (14c) is consistent with Eq. (9) since it follows from Eqs. (10), (14a), and (14b) that $j^a{}_\mu=0$ if $a\neq 25$ or $\mu\neq\theta$. Equation (14a) implies that $g(\theta)$, the operator relating the vacuum state at θ to that at $\theta=0$ is

$$g(\theta)=\exp(iY\theta/10)=\exp(i\tau^{25}\theta/10N)\ . \tag{18}$$

Thus, the direction in group space of the magnetic flux carried by the string is that of the generator which rotates the vacuum as one goes around the string. In fact, $g(\theta)$ is not uniquely determined to have the value given by Eq. (18), since one can add to the generator τ^{25} an arbitrary linear combination of generators of SU(5), which leaves the vacuum invariant. We shall discuss the consequences of this ambiguity later. In the present situation, where only ϕ_{16} has a nonzero VEV, the magnetic flux carried by the string is given, without ambiguity, by Eqs. (17), since the currents $j^a{}_\mu$, and therefore the gauge fields $A^a{}_\mu$, coupled to the SU(5) generators vanish.

Let us now compare the situation we have been discussing, which holds for the symmetry-breaking pattern (4), with that which obtains in the case of (3). There are an infinite number of different U(1) subgroups of SO(10) in which the discrete Z_2 symmetry can be embedded beside that generated by Y. This corresponds to the fact that, in passing around a curve C encircling the string, the group element $g(\theta)$ could follow, e.g., curve 2 in Fig. 1, rather than curve 1, in going from one of the disconnected pieces of H to the other. In the case of the symmetry-breaking pattern (4) which we have been discussing, $g(\theta)$ was forced to follow the path given by Eq. (18), apart from the previously mentioned ambiguity, because $g(\theta)$ must leave the VEV of the Higgs 45-plet responsible for the first stage of the symmetry breaking invariant. No such restriction is present in the case of (3).

Let us exhibit explicitly an alternative subgroup to $U(1)_Y$ in which Z_2 can be embedded. For the sake of specificity, we order the elements of the spinor representation of SO(10) so that the 15th element, u_{15}, is the left-handed positron. Let τ^{26} be the generator of the right-handed weak-isospin group $SU(2)_R$ [which, as is well known, is a subgroup of SO(10)] whose two-dimensional submatrix in the subspace spanned by u_{15} and u_{16} is $\sigma_1/2$, where σ_1 is the first Pauli matrix; i.e., τ^{26} is the "x component" of the right-handed weak isospin. For future reference, we order the SO(10) generators so that τ^{27} and τ^{28} are the y and z components of the right-handed weak isospin, with two-dimensional submatrices given by $\sigma_2/2$ and $\sigma_3/2$. Note that τ^{26} is already normalized in accordance with Eq. (12), so that an analog of the normalizing factor N in Eq. (11) is required. Under a rotation of 2π generated by τ^{26}, $u_{16}\to -u_{16}$, so that ϕ is left unchanged. Hence, the discrete symmetry can be embedded in $U(1)_{26}$, the U(1) subgroup generated by τ^{26}, with Eq. (18) replaced by

$$g(\theta)=\exp(i\tau^{26}\theta)\ . \tag{19}$$

To see the structure of the resulting string, we write ϕ as a linear combination of fields carrying definite charge q^{26} under rotations generated by τ^{26}; thus, we write

$$\phi=\phi_+ +\phi_- +\phi_0\ , \tag{20}$$

where ϕ_+, ϕ_0, and ϕ_- have $q_{26}=+1$, 0, and -1, respectively. [Thus, e.g., ϕ_+ transforms as the Cartesian product $(u_{15}+u_{16})\times(v_{15}+v_{16})/2$.] Then if $g(\theta)$ is given by Eq. (19), Eqs. (14) are replaced by Eq. (20) together with

$$\phi_\pm=\phi_+(r)e^{\pm i\theta}\ , \tag{21a}$$

$$\phi_0(r)\to\eta/\sqrt{2},\ \ r\gg\eta^{-1}\ , \tag{22b}$$

$$A^{26}{}_\theta=A_+(r)\ , \tag{21c}$$

$$A^a{}_\mu=0,\ \ \mu\neq\theta \ \text{ or } a\neq 26\ . \tag{21d}$$

Since $u_{16}\times v_{16}=(\,|1\rangle+|-1\rangle)/2+|0\rangle\sqrt{2}$, where $|q\rangle$ is an eigenstate of q^{26} with eigenvalue q, the boundary conditions on $\phi_+(r)$ [there is no significance in the choice of $+$ rather than $-$ to label $\phi_+(r)$ and $A_+(r)$] and $\phi_0(r)$ at large r, corresponding to Eq. (16a) are

$$\phi_+(r)\to\eta/2,\ \ r\gg\eta^{-1}\ , \tag{22a}$$

$$\phi_0(r)\to\eta/\sqrt{2},\ \ r\gg\eta^{-1}\ , \tag{22b}$$

while the boundary condition on $A_+(r)$ at large r is

$$A_+(r)\to 1/(er),\ \ r\gg\eta^{-1}\ . \tag{22c}$$

The boundary conditions for ϕ_+ and A_+ at the origin are given by Eq. (15). Since $\phi_0(r)$ is invariant under rotations generated by τ^{26}, it is not forced to vanish at $r=0$, so that it satisfies the boundary condition

$$\phi_0(r)\to c_0,\ \ r\to 0\ . \tag{23}$$

As in the case of Eq. (14c), one can verify that Eq. (21d) is consistent in that the corresponding currents $j^a{}_u$ given by Eq. (10) vanish.

Thus, in the case of the model in (3), two different kinds of strings are possible. In one of them the rotations of the vacuum state in going around the string are generated by τ^{25}, i.e., $g(\theta)$ is given by Eq. (18), and the magnetic flux is in what we have called the 25-direction, while in the second the rotations are generated by τ^{26}, and the magnetic flux is in the 26-direction. We shall henceforth refer to these as τ^{25} and τ^{26} strings. There are other possible choices besides τ^{26} for the generator of the second kind of strings. However, we shall see that the structure of the lowest-mass strings can be determined by studying τ^{25} and τ^{26} strings, which we proceed to do. Note that τ^{25} and τ^{26} are not gauge equivalent, as is clear from the fact that they have different sets of eigenvalues in the 16-dimensional representation, and hence cannot be transformed into one another by any unitary transformation. [The group containing all transformations taking one generator of SO(10) into another is the much larger group SO(45).] Thus, there is no reason to expect τ^{25} strings to have the same energy as those due to τ^{26} rotations; it is, in fact, clear from the differences between Eqs. (14)–(16) and Eqs. (21)–(23) that they will not have the same energy. Strings generated by τ^{25} and τ^{26} are topologically equivalent. This follows from the fact that the group manifold of SO(10) is simply connected, so that any two paths, such as 1 and 2 in Fig. 1, connecting the two disconnected pieces of H, can be deformed into one another smoothly. However, the paths corresponding to τ^{25} and τ^{26} strings are dynamically different; the string energy changes as the path followed by $g(\theta)$ in the group manifold as θ varies from 0 to 2π is deformed from that given by Eq. (18) to that of Eq. (19), since the gauge transformation connecting the two paths depends on θ and is thus singular as $r \to 0$; the string energy corresponding to different paths will be the same only if they can be deformed into one another by a global, and thus nonsingular, gauge transformation.

The masses of τ^{25} and τ^{26} strings will differ for two reasons. For τ^{26} strings one sees from Eqs. (20) and (21b) that, since ϕ_0 has no θ dependence, one has, loosely speaking, only half a string. The string mass/unit length $\mu \sim |\langle\phi\rangle|^2=\eta^2$, half of which comes from $|\langle\phi_0\rangle|^2$. The θ independence of ϕ_0 means that is does not contribute to the part of the kinetic energy of the string coming from the nonvanishing θ component of the covariant derivative of ϕ within the string, while the fact that ϕ_0 is not constrained to vanish at $r=0$ lowers the string potential energy coming from the departure of $\langle\phi\rangle$ from its equilibrium value at small r. Thus, roughly one expects Eqs. (21)–(23) to describe a string with $\mu \approx 2|\langle\phi_+\rangle|^2=\eta^2/2$. A second difference between the two kinds of strings may be seen by comparing Eqs. (14b) and (21c), from which one sees that the effective value of the coupling constant in the case of strings generated by τ^{25} rotations is $e_{\rm eff}$, where

$$e_{\rm eff}=eq^{25}=1.58e\ , \tag{24}$$

where we have used Eq. (13). Thus, we must consider how the string mass is expected to vary with the coupling constant. One can argue that, at least for small values of e, the string mass decreases as the coupling constant increases. To see this, note that the gauge-boson mass m_B is of order $e\eta$. The gauge field of the string vanishes on the axis, and will approach its asymptotic value at $r=m_B{}^{-1}$. For r appreciably less than $(e\eta)^{-1}$, $D_\theta\phi\neq 0$, and the string will have a nonzero energy density which falls off as $1/r^2$. For $r \ll m_B{}^{-1}$, a gauge string is similar to a global string. The energy of such a string diverges as $\log(\Lambda\eta)$, where Λ is a cutoff length.[5] For a gauge string with small coupling, the role of the cutoff will be played by $m_B{}^{-1}$, and hence one expects the energy of a gauge string to increase at fixed η with $\log(1/e)$. We shall actually be interested in values of e of order 1 where it is doubtful that this argument is quantitatively reliable, but it does suggest that, in comparing τ^{25} and τ^{26} strings, the difference in the coupling constant will affect the relative mass in the opposite sense from the effect of the θ independence of ϕ_0. Thus, a reliable determination of which is the lighter, and thus presumably stable, structure for strings arising in the symmetry-breaking pattern (3) requires a numerical evaluation of the masses associated with the two string configurations, and we turn to this task in the next section. We shall find that the invariance of ϕ_0 dominates the coupling-constant effect, and hence that τ^{26} strings are lighter than those generated by τ^{25} rotations.

III. NUMERICAL CALCULATIONS

In the case of τ^{25} strings, we must obtain the functions $\phi(r)$ and $A(r)$ of Eqs. (14) by solving Eqs. (8) and (9) subject to the boundary conditions of Eqs. (15) and (16). The resulting pair of coupled equations are exactly those of Nielsen and Olesen[6] for an Abelian gauge string, with the Abelian group in question being that generated by τ^{25}, and with the coupling constant replaced by $e_{\rm eff}=eq^{25}$. We write the potential V as

$$V=c_4(|\phi|^2-\eta^2)^2=c_4|\phi|^4-c_2|\phi|^2+\eta^2 \tag{25}$$

with $\eta=(c_2/2c_4)^{1/2}$. Then the equations are

$$-\partial_r(r\partial_r\phi)/r+[(1/r-e_{\rm eff}A)^2-2c_2+4c_4\phi^2]\phi=0\ , \tag{26}$$

$$-\partial_r[\partial_r(rA)/r]+(e_{\rm eff}{}^2A-e_{\rm eff}/r)\phi^2=0\ . \tag{27}$$

Once these equations are solved, the string mass/unit length μ is obtained from

$$\mu=\int_0^\infty H(r)2\pi r\,dr\ , \tag{28}$$

where $H(r)$, the Hamiltonian density, is given by $H(r)=-L(r)$ since the time derivatives of all fields vanish. Taking L from Eqs. (6) and (25), one obtains explicitly

$$\begin{aligned}H&=(\partial_rA+A/r)^2/2\\&\quad+[(\partial_r\phi)^2+\phi^2/r^2-2e_{\rm eff}A\phi^2/r+e_{\rm eff}{}^2A^2\phi^2]/2\\&\quad+(-c_2\phi^2+c_4\phi^4)=H_{\rm I}+H_{\rm II}+H_{\rm III}\ ,\end{aligned} \tag{29}$$

where $H_{\rm I}$, $H_{\rm II}$, and $H_{\rm III}$ stand for the three terms in brackets, respectively. Physically, $H_I=(B^{25})^2/2$ is the energy density in the magnetic field, $H_{\rm II}$ the "kinetic energy" associated with the covariant derivative of the Higgs field, and including the interaction between the Higgs and gauge fields, and $H_{\rm III}$ the potential energy. In our calculations we take the dimensionless coupling constant c_4 to have the value 0.1, unless otherwise specified.

Equations (26) and (27) were solved by numerical integration using the Runga-Kutta method. Using Eqs. (26) and (27), and requiring ϕ and A to be finite at the origin, one can write the boundary conditions at the origin, Eqs. (15), as

$$\phi(r) \sim k_1 r, \quad A(r) \sim k_2 r, \quad r \to 0 . \tag{15'}$$

We impose (15′) at some very small value of $r = r_1$, typically $r_1 = 0.03\eta^{-1}$, and vary the constants k_1 and k_2 until the solutions obtained by numerical integration satisfy the asymptotic boundary conditions (16). Equations (15′) are, of course, exact only at $r = 0$. While they provided a useful starting point, the asymptotic behavior was often extremely sensitive to the behavior at r_1, and in order to get good asymptotic behavior we frequently had to allow ourselves the freedom to relax Eqs. (15′) and vary $d\phi/dr$ and dA/dr independently of ϕ and A at $r = r_1$. Since one does not have exactly the right solution at $r = r_1$, the asymptotic conditions are never satisfied exactly and the solutions begin to diverge at some point. However, we obtain solutions for which Eqs. (16) are well satisfied, and consequently $H(r) \approx 0$, over a range of r of order a few times η^{-1} before the divergent part of the solution becomes important. If we choose the upper cutoff r_2 on the integral in Eq. (28) to lie in this region, we expect our solutions to give a good approximation to the correct $H(r)$ for $r < r_c$, while the error from setting $H(r) = 0$ at $r > r_c$, where the exact $H(r)$ vanishes exponentially, will be negligible. The range of variation in the values of μ coming from varying the starting values at $r = r_1$ over the ranges which yield reasonable solutions, as well as from varying r_c over the range of r for which the asymptotic conditions, Eqs. (16), are well satisfied indicate that our values of μ should be correct to an accuracy of about $\pm 2\%$.

The renormalization-group equations[7] give a value for the gauge coupling $e^2/4\pi \approx \frac{1}{45}$, which, from Eq. (24), yields $e_{\text{eff}} \approx 0.85$ with an uncertainty of perhaps 10%, reflecting the experimental uncertainty in the low-energy values of the Weinberg angle and the color coupling constant, either of which may be used to determine e. We have obtained values of μ for a range of values $0.5 \le e_{\text{eff}} \le 1.05$, both because we wish to include the case $e_{\text{eff}} = 0.55$ in making a comparison with τ^{26} strings, and also for possible applications to other models with different values of e_{eff}. Figure 2 shows typical solutions obtained for $\phi(r)$ and $A(r)$, in this case for $e_{\text{eff}} = 1.0$. Solutions of comparable quality were obtained for other values of e_{eff} within the range studied. Outside of this range of e_{eff} it became very difficult to obtain solutions, as very small variations, often in the eighth significant figure, in the input parameters at $r = r_1$ tended to have drastic effects on the asymptotic form of the solution. To get a solution, such as that of Fig. 2, for an initial value of e_{eff} is quite time consuming, and requires a lengthy trial and error process in adjusting the input values. Once a solution has been obtained for one value of e_{eff}, it is relatively easy to find a solution for a nearby value, at least within the range of e_{eff} being considered.

In Fig. 3 we show the values of $2\pi r H(r)$ obtained from the solution of Fig. 2, together with the values of the individual terms $2\pi r H_{\text{I}}$, $2\pi r H_{\text{II}}$, and $2\pi r H_{\text{III}}$. The value of μ obtained from the $H(r)$ in Fig. 3, and taking the cutoff value $r_c = 6.3\eta^{-1}$, is $\mu = 3.0\eta^2$. The value of μ changes by only about three parts in 10^5 when r_c varies from $3.3\eta^{-1}$ to 9.3^{-1}. The insensitivity of the result to r_c establishes the fact that $H(r)$ calculated from our solution vanishes rapidly as r becomes greater than η^{-1}. This is a

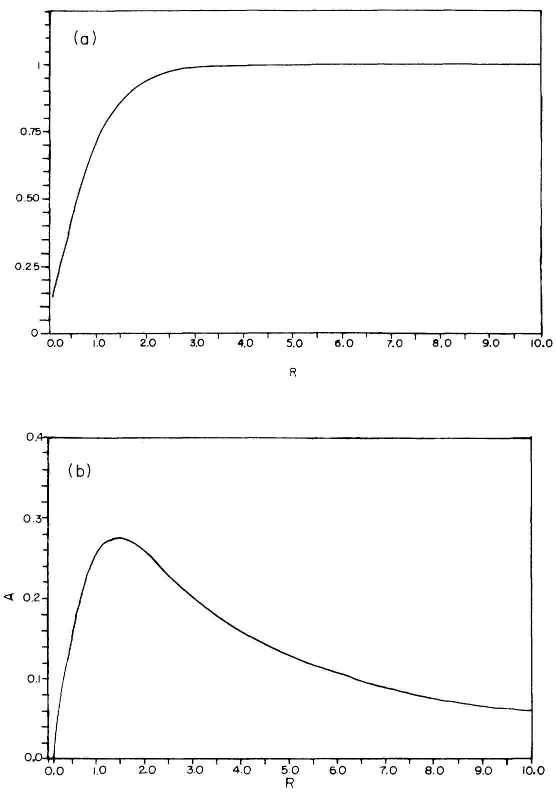

FIG. 2. The functions (a) ϕ and (b) A, defined in Eqs. (14a) and (14b), as functions of r, for the case $e = 1.0$, with ϕ and A in units with $\eta = 1$ and r in units with $\eta^{-1} = 0.633$.

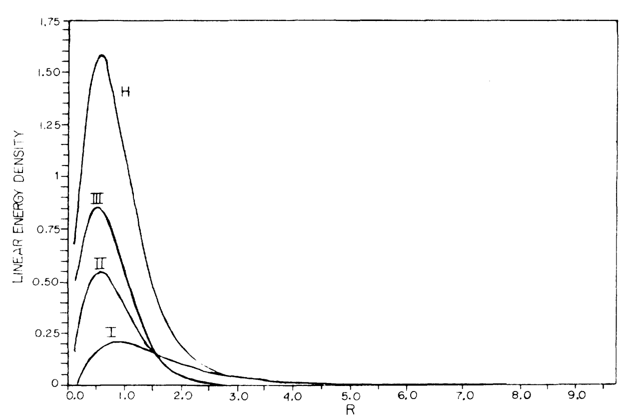

FIG. 3. The curve labeled H gives the radial Hamiltonian density $2\pi r H(r)$, with $H(r)$ given by Eq. (29), as a function of r, in units with $\eta^{-1} = 0.633$, as computed from the solutions in Fig. 2. The curves I, II, and III give the separate contributions from the three terms H_{I}, H_{II}, and H_{III}.

strong test of the validity of our solution, since the largest contribution to μ comes from the piece $\mu_{\rm II}$ arising from the integral of $H_{\rm II}$, the kinetic energy density, and $H_{\rm II}$ contains individual pieces which vanish only as $1/r^2$, and which individually give contributions to the integral in Eq. (28) increasing logarithmically with r_c; the fact that the result is insensitive to the choice of r_c means that the required cancellation among these terms is taking place.

In Table I we present the values of μ, as well as the individual pieces $\mu_{\rm I}$, $\mu_{\rm II}$, and $\mu_{\rm III}$ coming from the integrals of the corresponding terms in H in Eq. (29), for the complete set of values of $e_{\rm eff}$ for which calculations were done. Taking the value from Table I for $e_{\rm eff}=0.85$ we obtain, for μ^{25}, the mass/unit length of a τ^{25} string: $\mu^{25}=3.2\eta^2$. As expected, the string mass increases as the coupling constant is decreased. Note, however, that the value of μ for $e_{\rm eff}=0.55$ is much less than a factor of 2 greater than that for $e_{\rm eff}=0.85$. The qualitative argument made earlier suggests, therefore, that the effect of the change in $e_{\rm eff}$ will not dominate that due to the difference in structure of τ^{25} and τ^{26} strings, and thus that τ^{26} strings are likely to be the energetically stable configuration. We confirm this below.

In Table II we give the values of $\mu_{\rm I}$, $\mu_{\rm II}$, $\mu_{\rm III}$, and μ for $c_4=0.2$ and a range of values of $e_{\rm eff}$, and also for $c_4=0.05$ and $e_{\rm eff}=0.75$ Also shown is the ratio of these values of μ to the values of μ with the same $e_{\rm eff}$ and our standard value of $c_4=0.1$. One sees that the string mass increases with c_4, but that the dependence is rather slow, with μ changing by only about 25% for a factor of 4 change in c_4. Thus, the expected order-of-magnitude relation $\mu \sim \eta^2$ should hold for a wide range of c_4.

We turn now to the calculation of μ^{26}, the mass of τ^{26} strings. Since the fields on the right-hand side of Eq. (20) are orthogonal, we have, recalling Eq. (21a),

$$\begin{aligned}|\phi|^2&=|\phi_+|^2+|\phi_-|^2+|\phi_0|^2\\&=\phi_1(r)^2+\phi_0(r)^2\ ,\end{aligned} \tag{30}$$

where we have defined

$$\phi_1(r)=\sqrt{2}\phi_+(r)\ . \tag{31}$$

We must also consider the appropriate form of potential to use for τ^{26} strings. The full SO(10)-invariant potential is of course quite complicated. However, we are concerned with the case when only ϕ_1 and $\phi_0 \neq 0$. Since we are assuming that the minimum occurs when $|\phi|=\eta$ and when the Higgs field is in the $u_{16}\times v_{16}$ direction, i.e., when $\phi_0=\phi_1$, the part of the full potential involving ϕ_0 and ϕ_1 can be written as

$$V'=c_4({\phi_1}^2+{\phi_0}^2-\eta^2)^2+c_4'({\phi_1}^2-{\phi_0}^2)^2 \tag{25$'$}$$

with c_4' as well as $c_4>0$. Making use of Eqs. (6) and (25′), we find that the Lagrangian and Hamiltonian densities in the case of τ^{26} strings are given by

$$L^{26}=-H^{26}=L_A+D_\mu\phi_1D^\mu\phi_1/2+V'+(\partial_r\phi_0)^2/2\ , \tag{32}$$

where L_A is the free gauge-boson field contribution and the appearance of the field ϕ_1 is due to the summation of identical ϕ_+ and ϕ_- terms. Equation (32) leads to the field equations

$$\begin{aligned}-\partial_r(r\partial_r\phi_1)/r+[\,(1/r-eA_+)^2-2c_2+4(c_4+c_4'){\phi_1}^2&\\+4(c_4-c_4'){\phi_0}^2]\phi_1=0\ ,&\end{aligned} \tag{33}$$

$$\begin{aligned}-\partial_r(r\partial_r\phi_0)/r+[-2c_2+4(c_4+c_4'){\phi_0}^2&\\+4(c_4-c_4'){\phi_1}^2]\phi_0=0\ ,&\end{aligned} \tag{34}$$

$$-\partial_r(r\partial_rA_+)/r+A/r^2+(e^2A_+-e/r){\phi_1}^2=0\ . \tag{35}$$

Equations (33)–(35) can be integrated numerically, using procedures similar to those for Eqs. (26) and (27). The starting conditions near the origin for ϕ_1 and A_+ are the same as in Eq. (15′); for ϕ_0 it follows from (34) that Eq. (23) can be cast in the form

$$\phi_0 \rightarrow k_3+k_3'r^2,\quad r\rightarrow 0 \tag{15$''$}$$

with

$$k_3'=-c_2k_3/2+(c_4+c_4'){k_3}^3\ .$$

There are two interesting special cases where the problem of solving the full set of Eqs. (33)–(35) can be reduced to the problem of solving only a pair of equations,

TABLE I. Values in units of η^2 of the mass/unit length μ and of the individual contributions $\mu_{\rm I}$, $\mu_{\rm II}$, and $\mu_{\rm III}$ coming from the integrals of the corresponding terms in the Hamiltonian density of Eq. (29), for τ^{25} strings as a function of the coupling constants $e_{\rm eff}$, for $c_4=0.1$.

$e_{\rm eff}$	$\mu_{\rm I}$	$\mu_{\rm II}$	$\mu_{\rm III}$	μ
0.50	0.86	2.30	0.86	4.02
0.55	0.82	2.21	0.82	3.85
0.60	0.79	2.14	0.77	3.70
0.65	0.76	2.07	0.76	3.59
0.70	0.73	2.00	0.73	3.46
0.75	0.72	1.97	0.69	3.38
0.80	0.69	1.91	0.67	3.27
0.85	0.67	1.88	0.64	3.19
0.9	0.64	1.84	0.66	3.14
0.95	0.62	1.81	0.64	3.07
1.00	0.63	1.78	0.61	3.02
1.05	0.59	1.78	0.62	2.99

TABLE II. The same as Table I, but for the case $c_4=0.2$, and for the last set of entries $c_4=0.05$.

$e_{\rm eff}$	$\mu_{\rm I}$	$\mu_{\rm II}$	$\mu_{\rm III}$	μ
0.55	0.94	2.56	0.93	4.43
0.65	0.90	2.39	0.90	4.19
0.70	0.84	2.35	0.90	4.09
0.75	0.82	2.22	0.82	3.86
0.80	0.80	2.14	0.77	3.72
0.85	0.78	2.09	0.76	3.63
0.90	0.75	2.07	0.78	3.60
0.95	0.73	2.06	0.78	3.57
1.00	0.72	1.95	0.69	3.36
0.75	0.59	1.79	0.61	2.99

essentially identical to (26) and (27), for ϕ_1 and A_+, with ϕ_0 being obtained by inspection. First consider the case $c_4'=c_4$. In this case ϕ_0 decouples, and in fact ϕ_0 is simply a constant, given by its asymptotic value, $\phi_0=\eta/\sqrt{2}$; one easily finds that the Lagrangian of Eq. (32) for a τ^{26} string reduces to that of a τ^{25} string with equivalent parameters $c_{4eq}=2c_4$, $\eta_{eq}=\eta/\sqrt{2}$ and the same value of $e_{\rm eff}$. Note that $c_2=2c_4\eta^2$ is unchanged by the replacement $c_4\rightarrow c_{4eq}$, $\eta\rightarrow\eta_{eq}$. Since the string mass is proportional to η^2, we conclude that

$$\mu^{26}(e,c_4=c_4')=\mu^{25}(e,2c_4)/2 \ . \tag{36}$$

Note that the factor of 2 on the right-hand side of Eq. (35), i.e., the replacement of η^2 by $\eta^2/2$ for τ^{26} strings, is just the expression of the factor of 2 mentioned in our previous qualitative discussion of τ^{26} strings; it just reflects the fact that only half of the Higgs field ϕ varies with θ and couples to the gauge field. Taking the value of μ^{25} for $c_4=0.2$ and $e_{\rm eff}=0.55$ from Table II and multiplying by $\frac{1}{2}$ gives $\mu^{26}=2.2\eta^2$ for $c_4=c_4'=0.1$.

A similar trick can be used to obtain μ^{26} in the limit $c_4'\gg c_4$. The two terms in the potential of Eq. (31) have opposite effects on the behavior of ϕ_0 for $r\rightarrow 0$. The first term causes ϕ_0 to increase to compensate for the decrease in ϕ_1 and maintain $|\phi|^2$ close to η^2, thus reducing the string potential energy; the second term in V' tends to cause ϕ_0 to follow ϕ_1 and decrease. The string mass thus increases as c_4' increases. For c_4' very large, the second term in V' forces $\phi_0=\phi_1$; from Eq. (34) one has

$$\phi_0{}^2=\phi_1{}^2+O(1/c_4') \tag{37}$$

so that the c_4' term in V' vanishes for $c_4'\rightarrow\infty$. Making use of (35) and (32), one finds that

$$L^{26}(c_4,\eta)=L^{25}(2c_4,\eta/\sqrt{2})+(\partial_r\phi_0)^2/2 \ . \tag{38}$$

where L^{25} is the Lagrangian density for a τ^{25} string with the indicated values of the parameters. The result in Eq. (38) for large c_4' differs from that discussed previously for the case $c_4'=c_4$ by the addition of the last term representing the energy coming from the fact that now $\phi_0=\phi_1$ and varies with r (but not θ). The exact value of μ^{26} for large c_4' can only be obtained by solving the field equations for the Lagrangian of Eq. (38) numerically. However, we can get a useful upper bound on μ^{26} for this case from the results already at hand. Since in our static problem with $H=-L$ the field equations simply represent the condition that μ be a minimum, an upper bound for μ is obtained by evaluating H using arbitrary trial functions for ϕ_1 and A_+ which satisfy the boundary conditions. In particular, we can get μ for large c_4 by using the results for ϕ_1 and A_+ from the case $c_4'=c_4$, for which the mass corresponds to the first term on the right-hand side of Eq. (38). Thus, an upper bound on the value of μ^{26} for large c_4' is given by taking the value of μ^{26} for $c_4'=c_4$ and adding to it the contribution of the last term in Eq. (38) evaluated for $c_4'=c_4$. The latter contribution, obtained from the results used in obtaining Table II, is $\approx 0.5\eta^2$, giving $\mu<2.7\eta^2$ for $c_4'\rightarrow\infty$.

In the opposite limit $c_4'=0$, we obtained μ by solving the full set of Eqs. (33)–(35) for the three fields ϕ_1, ϕ_0, and A_+. As expected, we found that in this case ϕ_0 increased within the string, and in fact, $\phi_0{}^2\approx 0.95\eta^2$ at $r=0$, so that the potential energy density was very small at all r. The quality of the numerical solutions obtained was not as good as those obtained for τ^{25} strings. Because of the small potential energy, the scalar fields vary slowly with r and do not begin to reach their constant asymptotic values until $r\approx 9\eta^{-1}$. On the other hand, even with the most delicate possible adjustment of the initial values, divergent behavior of the fields sets in around 11 or $12\eta^{-1}$, and thus there is only a small range of r for which the fields are reasonable approximated by their asymptotic forms. However, the energy density is small over about twice this range of r and hence the value of μ is again insensitive to reasonable variations in the upper cutoff in the integral of $H(r)$. We obtained a value $\mu^{26}=1.0\eta^2$ for $c_4'=0$. Our three results for μ^{26} are collected in Table III. We see that, for c_4' varying over its physical range $c_4'>0$, $1.0<\mu^{26}/\eta^2<2.7$. Thus, for all values of c_4', $\mu^{26}<\mu^{25}$; for large c_4' the mass difference is relatively small, but appears still to be outside the uncertainty in our calculations.

We next note that the value of μ^{26} given by our analysis may be, in fact, to some extent too high for the following reason. Consider an $SU(2)_R$-singlet member ϕ_s of the multiplet of Higgs fields. Since ϕ_s is invariant under rotations generated by τ^{26}, if $g(\theta)$ is given by Eq. (19) there is no need for ϕ_s to vanish on the string axis, and like ϕ_0 it may take on a nonzero value there in order to minimize the potential energy. Because of its invariance under τ^{26}, ϕ_s will have no angular, but only a radial dependence. Outside the string it will vanish exponentially at the same rate at which ϕ_0 and ϕ_1 approach their asymptotic values. A nonzero ϕ_s will result in the addition of a term V_s to the potential, where

$$V_s=a\,|\phi_s|^4-2c_4\eta^2|\phi_s|^2 + |\phi_s|^2(c_5|\phi_1|^2+c_5'|\phi_0|^2) \ . \tag{39}$$

In order for the potential minimum to occur at $|\phi|=\eta$, $\phi_s=0$, one must have $a>c_4$. It becomes energetically favorable for ϕ_s to develop a nonzero VEV when

$$2c_4\eta^2>c_5\phi_1{}^2+c_5'\phi_0{}^2 \ . \tag{40}$$

In order for the asymptotic vacuum, with $|\phi|=\eta$, to be energetically stable, it follows from (40) that $c_5+c_5'>2c_4$. Since ϕ_1 vanishes at small r, if $c_5'>2c_4\eta^2/\phi_0(0)^2$, where $\phi_0(0)$ is the value of ϕ_0 at $r=0$, ϕ_s will never become nonzero. If the parameter c_4' in Eq. (25') is relatively small, $\phi_0(0)$ may be appreciable, as we recall, and thus in this case one may well find it energetically unfavorable to have any $\phi_s\neq 0$. Even if some $\phi_s=0$, the effect on μ^{26} will be relatively small for small c_4', since then ϕ_0 already varies in such a way that the potential energy density is small, so that little further reduction in energy is possible.

TABLE III. Values of μ^{26}, the mass/unit length of τ^{26} strings, for various values of the potential parameter c_4' defined in Eq. (25').

	$c_4'=0$	$c_4'=c_4$	$c_4'\gg c_4$
μ^{26}	η^2	$2.2\eta^2$	$<2.7\eta^2$

However, for large c_4', ϕ_0 becomes small along with ϕ_1 $r \to 0$, and we will in general expect that condition (40) will be satisfied and $\phi_s \neq 0$ for some range of r near the axis; that range, and hence the effect on μ^{26}, will be small, however, if $c_5 + c_5' >> 2c_4$. Finally, the effect of $\phi_s \neq 0$ on μ^{26} will be small if $a >> c_4$, since the effective scale associated with the potential energy V_s is $\eta(c_4/a)^{1/2}$. In summary, then, we expect that taking account of a nonzero ϕ_s might appreciably reduce our value of μ^{26} for the general range of parameters $c_4' > c_4$, $a \lesssim c_4$, $c_5 + c_5' \lesssim 2c_4$.

A similar phenomenon occurs in the case of τ^{25} strings. For a τ^{25} string all members of the Higgs multiplet have $q^{25} \neq 0$, and hence are forced to vanish on the string axis. However, consider a closely related possibility, raised by Hindmarsh and Kibble,[8] in which τ^{25} is replaced by τ^{28}, the third component of the right-handed weak isospin. The generator τ^{28} can be written as

$$\tau^{28} = \tau^{25}\cos\beta + \tau' \sin\beta \ , \tag{41}$$

where τ' is one of the diagonal generators of the unbroken SU(5). Consider now a string in which $g(\theta)$ is given by

$$g(\theta) = \exp(ip\tau^{28}e) \tag{42a}$$

rather than by Eq. (18). Since the vacuum is invariant under τ', only the τ^{25} piece of τ^{28} is effective in transforming the vacuum, and the single valuedness of the vacuum along spatial curves enclosing the string requires that the coefficients of τ^{25} in the exponents of Eqs. (18) and (42a) be equal, i.e., that

$$p = 1/(10N\cos\beta) \ . \tag{42b}$$

Likewise Eq. (14a) continues to hold, since τ' does not affect the angular dependence of ϕ. However, having τ^{28} rather than τ^{25} in $g(\theta)$ does mean that $SU(2)_R$ singlets are now invariant under $g(\theta)$, and hence again it is possible for an $SU(2)_R$-invariant component ϕ_s to acquire a VEV which is angle independent and nonvanishing on the string axis and vanish exponentially outside the string. The situation in this case is somewhat more complicated when it comes to discussing the gauge fields. Outside the string, where $\phi_s = 0$, the requirement $D_\mu \phi = 0$ asymptotically implies that A_μ^{25} must satisfy Eq. (16b). However, since SU(5) generators, such as τ', give 0 acting on ϕ_{16}, the only asymptotically nonvanishing Higgs component, the corresponding gauge fields, are not determined by the condition $D_\mu \phi = 0$, and are instead determined dynamically through the field equations (9) and (10). For the case of a τ^{25} string, this implies $A^a{}_\mu = 0$, $a = 25$, as we have already remarked, because the corresponding currents vanish. However, with $g(\theta)$ given by Eq. (42a) so that ϕ_s can be nonzero within the string, this is no longer true. Since the charge q^{28} carried by ϕ_s is zero as a result of cancellation between the two terms on the right-hand side of Eq. (41), it follows that the charge q_s', associated with τ', carried by ϕ_s is nonvanishing. From this it follows that the last term on the right-hand side of the current j_θ' given by Eq. (10), proportional to the matrix element of $A_\theta^{25}(\tau'\tau^{25})$, is also nonvanishing, and hence

$$A_\theta' \neq 0 \ . \tag{43}$$

The current j_θ' will vanish exponentially outside the string, since ϕ_s vanishes and ϕ has $q' = 0$. This implies that $A_\theta' \sim 1/r$; locally this is a pure gauge artifact, and hence A_θ' does not contribute any energy density outside the string.

[Note that it is not true in the case of τ^{28} strings, with $g(\theta)$ given by Eq. (42a), that $A^a{}_\mu = 0$, $a \neq 28$, as might have been expected by analogy with Eq. (14c). Let τ'' be the linear combination of τ^{25} and τ' orthogonal to τ^{28}, and A_μ'' the gauge field coupled to τ''. The only constraint on A_μ^{28} and A_μ'' is that

$$\cos\beta A_\mu^{28} + \sin\beta A_\mu'' = A_\mu^{25} \tag{44}$$

with A_μ^{25} being given asymptotically by Eqs. (14b) and (14c). All of the SO(10) generators which commute with τ^{25} and are linearly independent of it are generators of the unbroken SU(5) symmetry and leave the asymptotic Higgs field invariant; τ^{28} does not share this property because of the existence of τ''.]

We now argue that the inclusion of the effects due to $\phi_s \neq 0$ is unlikely to alter the conclusion that τ^{26} strings have smaller mass than τ^{25} (actually τ^{28}) strings. The contribution of ϕ_s to the potential is given by V_s, Eq. (39), with $\phi_1 = \phi_0 = \phi/\sqrt{2}$ since the Higgs field is in the $u_{16} \times v_{16}$ direction for all r in the case of a τ^{28} string. Thus, the form of the ϕ_s contribution to the potential of a τ^{28} string is the same as for a τ^{26} string in the case of large c_4' for which μ^{26} has its largest value. We would thus expect the reductions in the mass of τ^{28} strings and of τ^{26} strings with large c_4' to be roughly of the same magnitude. There is some uncertainty in this statement, because the radial functions $\phi(r)$ for which V_s is evaluated in the two cases differ somewhat, since they are calculated with different values of the parameters η, c_4, and e_{eff}, as discussed previously, but it seems unlikely that the V_s contributions in the τ^{28} case and in the τ^{26} case with large c_4' differ greatly. We have not attempted to study this question numerically, since the difficulty of getting good numerical solutions increases markedly as the number of unknown functions and coupled equations increases.

In addition to the contribution of V_s, in the τ^{28} case there is another extra term in the Hamiltonian density when $\phi_s \neq 0$ which involves the gauge fields; we denote this term by H_s, and it is given by

$$H_s = H'_{\text{mag}} + e^2(q_s' A' + q_s^{25} A^{26})^2 \, | \phi_s |^2 \ , \tag{45}$$

where H'_{mag} is the energy stored in the H' component of the magnetic field and q_s^{25} is the value of q^{25} for the field ϕ_s. Since both terms in H_s are positive definite, the extra energy associated with a nonzero ϕ_s in the case of τ^{28} strings contains a positive gauge field contribution which is not present in the case of τ^{26} strings. Since $A' \sim j' \sim \phi_s{}^2$, and A^{25} vanishes at small r, for small ϕ_s and small r there will always be a negative contribution to the energy density from V_s which will dominate the positive contribution from H_s. Thus, it will always be energetically favorable for some ϕ_s to develop a nonzero VEV at small r, meaning that τ^{28} strings will always have a mass $\mu^{28} < \mu^{25}$. We have seen that $\mu^{26} < \mu^{25}$ before account is taken of effects due to ϕ_s. Since we expect the lowering of the potential energy in going from μ^{25} to μ^{28}

should be comparable to the corresponding lowering of μ^{26} in the case of large c'_4, and since μ^{28} receives a positive contribution from H_s which is not present for τ^{26} strings, it seems very likely that $\mu^{26}<\mu^{28}$ for large c'_4 where μ^{26} has its maximum value, and therefore for all values of c'_4. The result is not totally rigorous, because we do not know precisely the relative contributions of V_s to μ^{26} and μ^{28} or the relative contributions of V_s and H_s to μ^{28}. We cannot totally exclude the possibility that for some range of potential parameters the magnitude of the contribution of V_s to μ^{28} is large enough compared to the magnitude of its contribution to μ^{26} that it overcomes the H_s contribution as well as the difference between μ^{26} and μ^{25} before ϕ_s effects are taken into account, but this seems to us unlikely. In any event it appears clear that for a wide range of potential parameters τ^{26} strings have the lower mass, even for large c'_4. Since μ^{26} decreases with c'_4, this result will hold *a fortiori* for smaller c'_4.

Now let us examine possible choices of the SO(10) generator appearing in $g(\theta)$ besides τ^{26} and τ^{25} (i.e., τ^{28}). Clearly, in place of τ^{26} one could choose any linear combination of τ^{26} and τ^{27} and the resulting strings would be gauge equivalent to τ^{26} strings. We obtain 12 other SO(10) generators by commuting τ^{26} and τ^{27} with the 6 baryon-nonconserving generators of SU(5). These generators are similar to τ^{26} and τ^{27} in having 8 nonzero eigenvalues of equal magnitude in the spinor representation; they have the effect of rotating the left-handed antineutrino into left-handed u or d quarks, and they give rise to strings which are physically equivalent to τ^{26} strings. There are an additional six generators which are the baryon-nonconserving generators of the Pati-Salam SU(4) group.[9] These matrices have 16 nonzero eigenvalues of equal magnitude in the spinor representation, and thus after normalization, the SU(4) charges are smaller by a factor of $\sqrt{2}$ then the SU(2) charges. Since we have seen that the string mass increases as the charge decreases, SU(4) strings will have a larger mass than τ^{26} strings and thus will be unstable. Finally, one might consider the possibility of generators which are linear combinations of τ^{26} and τ^{28}. The lower mass of τ^{26} strings is due, as we have seen, to the presence of the ϕ_0 component, which is left invariant by $g(\theta)$. It is easy to show that the amplitude of the component of the Higgs field, analogous to ϕ_0, which is invariant decreases monotonically as the admixture of τ^{28} in the generator appearing in $g(\theta)$ increases while there is no compensating advantage, in terms of energy, gained by such an admixture. Thus, the string mass will increase monotonically as one varies the string smoothly from a τ^{26} to a τ^{28} configuration. We conclude that the lowest-mass, and therefore stable, strings are τ^{26} strings, and strings which are physically equivalent to them. This statement is certainly true for a wide range, and we believe is probably true for all, values of the potential parameters.

IV. STRING PROPERTIES

We now examine two ways in which some properties of Z_2 strings, besides the mass, depend on the type of generalized magnetic flux carried by the string. This illustrates further the physical significance of the question as to which is the stable configuration for such strings.

We first consider the question of the equivalence of strings and antistrings, where, in an appropriate gauge, an antistring is a configuration having the same vacuum state at $\theta=0$, but in which the Higgs field rotates in the opposite sense along a path enclosing the string; the direction of the magnetic flux carried by an antistring is also opposite to that carried by a string. Since strings and antistrings differ only in the path followed by $g(\theta)$ between the two disconnected pieces of H in Fig. 1, they are topologically equivalent, and may be smoothly deformed into one another. Put another way, if one imagines bringing two strings together, the path followed by $g(\theta)$ along a closed curve in coordinate space enclosing the resulting double string will be the closed curve in the manifold of G formed by one path from H_1 to H_2 in Fig. 1 followed by a second path from H_2 to H_1, the latter corresponding to the inverse of the path from H_1 to H_2 followed by $g(\theta)$ around an antistring. The resulting closed path can be smoothly contracted to a point in the manifold of G; thus two strings can annihilate into the vacuum, and are topologically equivalent to it, showing again that strings and antistrings are topologically equivalent.

In general, however, the set of configurations through which one passes in deforming a string into an antistring are not gauge equivalent (by a nonsingular transformation) to the string and so are not degenerate with it in energy. This will, e.g., clearly be true in the case of symmetry-breaking pattern (4), where the efective symmetry group at the time of the formation of strings is the multiply connected group SU(5)$\times$U(1) with respect to which strings and antistrings are not even topologically equivalent. It is this situation of gauge nonequivalent strings and antistrings which was discussed by Hindmarsh and Kibble in Ref. 8. They point out that in this situation segments of a string may have magnetic flux pointing in opposite directions; these will be separated by beadlike solitons with nonzero mass. In the case (4), U(1) magnetic monopoles, whose flux becomes confined in flux tubes when the U(1) symmetry is broken, act as such solitons.

Now consider the symmetry-breaking pattern (3). Strings and antistrings will be gauge equivalent if there exists a gauge transformation which leaves the vacuum state at $\theta=0$, invariant and thus belongs to the unbroken subgroup H, but which takes $\tau\to-\tau$, where τ is the group generator appearing in $g(\theta)$, thus reversing the magnetic flux and the sense of rotation of ϕ. No such transformation exists for $\tau=\tau^{25}$ or τ^{28}, since they commute with all generators of the unbroken SU(5). However, $\tau^{26}\to-\tau^{26}$ under a 180° gauge rotation generated by the electric charge operator Q; this follows straightforwardly from the fact that the two members of each of the four right-handed isospin doublets in the spinor representation of SO(10) differ by one unit of electric charge. Hence, no gauge-invariant distinction exists between τ^{26} strings and antistrings. Thus, we expect that stable strings formed in the symmetry-breaking pattern (3) are self-conjugate.

However, even for τ^{26} strings one can, in principle, distinguish between the relative direction of the magnetic flux at two different points along a string, or on two different strings. One can imagine deforming the string(s) adiabatically in space so as to bring the two points into coincidence, and measuring the gauge-invariant magnitude of the total magnetic flux at the coincident point. At separated points the flux can always be brought into the same direction by a local gauge transformation, but when this is done the gauge field configuration in the region between the points will depend, although in general only weakly, on the relative direction of the flux at the two points as defined by bringing them into coincidence. In the case of τ^{26} strings, and similar strings in other models where the direction of the flux is not gauge invariant, localized beads of the type discussed in Ref. 7 will not exist between string segments having directions of the flux which differ in the sense just discussed. The point is that since such segments can be connected smoothly through field configurations which are gauge equivalent and hence degenerate in energy, the energy of the transition region involves only the "kinetic energy" associated with the spatial variation of the fields; the energy is thus minimized by making the transition occur as smoothly, over as large a distance, as possible. Eventually, either by soliton-antisoliton annihilation in the case that beads exist, or by gradual smoothing where they do not, a single string will dissipate its excess energy and evolve to a configuration where the flux direction is the same throughout.

A second area in which the properties of τ^{26} and τ^{28} strings differ is in the processes by which they emit photons. Photon emission by strings has been discussed by Vachaspati, Everett, and Vilenkin.[10] They find that the dominant mechanism is two-photon emission resulting from the quartic coupling of the electromagnetic field to the gauge field of the string. (The analysis of Ref. 10 does not apply in the case of superconducting strings.[11]) This quartic diagram is nonzero only if the group generator coupled to the gauge field fails to commute with the generator coupled to the electromagnetic field. This condition is satisfied by τ^{26}. However, τ^{28} commutes with the electric charge operator so that the dominant radiation mechanism of Ref. 9, electromagnetic radiation, occurs through radiative corrections and is much suppressed.

V. CONCLUSIONS

We have seen that for many, and very plausibly for all, of the possible values of the parameters in the effective potential, stable Z_2 strings, which arise in the symmetry-breaking pattern (3), will be τ^{26}-like. As we have seen, this means that they will be self-conjugate, so that the beadlike solitons of Hindmarsh and Kibble will not occur. Also there will be a quartic coupling between the gauge field within the string and the electromagnetic field, so that the string can emit photons through the mechanism of Ref. 10. These conclusions are changed if there is an intermediate phase with an unbroken U(1), as in (4). In that case the Z_2 strings formed will be τ^{28} strings. Such strings are not self-conjugate. Magnetic monopoles will play the role of beads connecting regions of string with opposite directionality, as in Ref. 8. Also the dominant photon radiation mechanism suggested in Ref. 9 will not occur.

While our detailed calculations apply specifically to the model in (3), it would appear that this model has a number of general features which are likely to be true of, and to lead to the same general conclusions in, a wide class of models in which Z_2 strings occur. Namely, in the specific model we have studied, the Higgs field transformed as a direct product, with the discrete symmetry corresponding to the multiplication of each factor in the direct product by -1. This will be true of many models giving Z_2 strings, so let us consider a model in which the Higgs field transforms as $u_1 \times v_1$ where $\{u_i\}$ and $\{v_i\}$ are two sets of objects transforming according to the same representation of the unbroken-symmetry group G. We suppose there is an operator Y in G for which u_1 and v_1 carry nonzero charge y. Then strings will arise in the theory for which $g(\theta)=\exp(iT^{25}p\theta)$, with $T^{25}=Y/N$, N a normalizing constant, and $p=N/2y$; T^{25} is thus the analog of τ^{25} in our specific model. T^{25} strings will be non-self-conjugate; since the Higgs field is an eigenstate of T^{25}, T^{25} cannot be reversed by a transformation which leaves the vacuum at $\theta=0$ invariant. (Moreover, if Y is chosen to give the largest value of y, then in general the set of eigenvalues $\{y_i\}$ of Y will not be invariant under $Y \to -Y$, showing that there is no unitary transformation of any kind which will reverse T^{25}.)

Now let us suppose that u_1 (and v_1) are members of a doublet under a subgroup $SU(2)_1$ of G. Let T^{26} be the operator with submatrix σ_1 within a doublet. Then T^{26} strings, analogous to τ^{26} strings in our specific model, will also exist. Moreover, in exact analogy with our previous discussion, the Higgs field $u_1 \times v_1$ will contain a piece which is invariant under T^{26} rotations. Since that was the basic reason for the relatively lower energy of τ^{26} strings in the model in (3), we expect that T^{26} strings will in general have the lower mass, and thus be the stable string configuration, in the general case. Finally we assume that G contains at least one subgroup $SU(2)_1 \times U(1)_X$, where u_1 and v_1 have nonzero charges for the U(1) generator X. (It can well be that $X=Y$.) Then one can define an operator $Q=X/N'+T^{28}$, where T^{28} is the $SU(2)_1$ generator with submatrix $\sigma_3/2$ and N' is defined so that u_1 and v_1 have charge $q=0$ for the operator Q. Since the eigenvalues of Q for the two members of a doublet differ by 1 (X is proportional to the unit matrix within a doublet), the transformation $T^{26} \to \exp(iQ\pi)T^{26}\exp(-iQ\pi)$ takes $T^{26} \to -T^{26}$, while leaving the vacuum invariant because $q=0$. Thus, under these quite weak assumptions, T^{26} strings, which we expect to be stable, are self-conjugate by the same argument as for τ^{26} strings. Also, if Q represents the electromagnetic charge operator (so that $q=0$ arises from the condition that electromagnetic gauge invariance is not spontaneously broken) then T^{26} and Q fail to commute, so that the dominant photon radiation mechanism found in Ref. 9 is allowed.

As a simple example of the preceding argument, we observe that one can break $SU(N) \to SU(N-1) \times Z_2$ for any $N>2$ with a Higgs field belonging to the symmetric part of the direct product of two fundamental N-dimensional representations. The resulting strings can have either a T^{25} or T^{26} configuration, with the self-conjugate T^{26} configuration expected to be the stable one. Note that the assumptions above fail for SU(2); for $N=2$, no operator q can be constructed for which u_1 and v_1 have $q=0$, and thus the symmetry-breaking pattern $SU(2) \rightarrow Z_2$ leads to non-self-conjugate strings, a point which has been made in Ref. 8. However, for a wide class of symmetry-breaking patterns of type 1, with no intermediate U(1) phase, we expect that the lowest-mass strings which result will be T^{26}-like and self-conjugate.

Note added. Following the completion and submission of this work, we learned that Jacobs and Rebbi[12] had calculated the mass of an Abelian gauge string for a range of gauge and Higgs coupling constants using a variational technique. Their results allow one to obtain values for μ^{25} for a range of parameters having substantial overlap with those we have considered. Our numerical results for μ^{25} given in Tables I and II agree with those obtained from the results of Jacobs and Rebbi where the parameter ranges overlap. Jacobs and Rebbi also prove, by a rescaling argument, the interesting result that the mass of an Abelian gauge string depends on c_4 and e_{eff} only through the combination $(\sqrt{c_4})/e_{\text{eff}}$. We are indebted to Neil Turok and Paul Shellard for drawing our attention to Ref. 12.

[1] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).

[2] For a review, see A. Vilenkin, Phys. Rep. **121**, 265 (1985).

[3] Ya. B. Zel'dovich, Mon. Not. R. Astron. Soc. **192**, 664 (1980); A. Vilenkin, Phys. Rev. Lett. **46**, 1169 (1981); **46**, 1496(E) (1981).

[4] T. W. B. Kibble, G. Lazarides, and Q. Shafi, Phys. Lett. **113B**, 237 (1982).

[5] A. Vilenkin and A. E. Everett, Phys. Rev. Lett. **48**, 1867 (1982).

[6] H. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).

[7] H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[8] M. Hindmarsh and T. W. B. Kibble, Phys. Rev. Lett. **55**, 2398 (1985); **57**, 647 (1986).

[9] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).

[10] T. Vachaspati, A. Everett, and A. Vilenkin, Phys. Rev. D **30**, 2046 (1984).

[11] E. Witten, Nucl. Phys. **B249**, 557 (1985).

[12] L. Jacobs and C. Rebbi, Phys. Rev. B **19**, 4486 (1979).