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Gravitational particle creation and inflation

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Particle creation due to the changing spacetime metric at the end of an inflationary era in the early Universe is discussed. The Universe is assumed to make a transition from de Sitter space to either a radiation-dominated or matter-dominated universe. A perturbation approach is used to calculate the number density and energy density of massless, nonconformally coupled particles created by this transition. It is found that their energy density is typically of the order of $\rho_v{}^2/\rho_{\rm Pl}$, where ρ_v is the value of the cosmological constant in the de Sitter phase and $\rho_{\rm Pl}$ is the Planck energy density. This is approximately the energy density of a thermal bath at the Gibbons-Hawking temperature of de Sitter space. The possible applications of this effect to inflationary models is discussed. It is shown that gravitational particle creation is capable of reheating the Universe after inflation and of being the source of the matter in the Universe. This effect makes it possible to avoid the difficulty with reheating which inflationary models with weakly coupled scalar fields otherwise encounter.

I. INTRODUCTION

Since the original proposal of the inflationary model by Guth,[1] several variations of inflation have been proposed.[2] All versions of the inflationary model contain certain essential features. These include a de Sitter phase in which the Universe expands by a factor of at least e^{60} and a mechanism for ending the de Sitter phase and reheating the Universe. The reheating is typically assumed to occur through the damping of the coherent oscillations of a Higgs scalar field by coupling to other fields. However, the change in the spacetime metric at the end of inflation will itself create particles due to their coupling to the spacetime curvature. This type of particle creation was first discussed by Parker.[3]

The purpose of this paper is to investigate this gravitational particle creation in spacetimes which make a transition from de Sitter space to another Robertson-Walker universe, and to discuss its possible role as a reheating mechanism in inflation.

II. PERTURBATION CALCULATION OF THE PARTICLE CREATION

We take the metric to be that of a spatially flat Robertson-Walker universe:

$$ds^2=dt^2-a^2(t)d\mathbf{x}^2=a^2(\eta)(d\eta^2-d\mathbf{x}^2)\ , \tag{1}$$

where $dt=a\,d\eta$. In such a conformally flat spacetime, quanta of conformally invariant fields are not created.[3] However, massive particles and particles which are not conformally coupled to the background gravitational field are produced. An example of the latter is a scalar field which satisfies the wave equation

$$\Box\phi+\xi R\phi=0\ , \tag{2}$$

where R is the scalar curvature and ξ is an arbitrary constant. If $\xi\neq\frac{1}{6}$, this field is not conformally invariant and particle production occurs.

In general, the solution of Eq. (2) in a given metric is a nontrivial task, and there are few scale factors for which the particle creation rate may be calculated exactly. However, if one performs a perturbation calculation treating $|\xi-\frac{1}{6}|$ as a small parameter, it is possible to obtain simple expressions for the number density and energy density of created particles. This was done by Zel'dovich and Starobinsky[4] and by Birrell and Davies.[5] The essential results will be quoted below. Let

$$V(\eta)=(\tfrac{1}{6}-\xi)C(\eta)R(\eta)\ , \tag{3}$$

where $C=a^2$. The scalar curvature R may be expressed as

$$R=3C^{-1}(\dot{D}+\tfrac{1}{2}D^2)\ , \tag{4}$$

where $D=\dot{C}/C$ and an overdot denotes differentiation with respect to η. If

$$\beta_\omega=\frac{i}{2\omega}\int_{-\infty}^{\infty}e^{-2i\omega\eta}V(\eta)d\eta\ , \tag{5}$$

the number density of created particles is

$$n=(2\pi^2a^3)^{-1}\int_0^\infty|\beta_\omega|^2\omega^2d\omega \tag{6}$$

and their energy density is

$$\rho=(2\pi^2a^4)^{-1}\int_0^\infty\omega^3|\beta_\omega|^2d\omega\ . \tag{7}$$

These quantities may also be expressed as coordinate-space integrals:

$$n=(16\pi a^3)^{-1}\int_{-\infty}^{\infty}V^2(\eta)d\eta \tag{8}$$

and

$$\rho=-(32\pi^2a^4)^{-1}\int_{-\infty}^{\infty}d\eta_1\int_{-\infty}^{\infty}d\eta_2\ln(|\eta_1-\eta_2|\mu)\times\dot{V}(\eta_1)\dot{V}(\eta_2)\ . \tag{9}$$

Here μ is an arbitrary mass. These expressions for n and

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ρ are of order $(\xi-\frac{1}{6})^2$. The contributions which have been ignored will in general be $O(|\xi-\frac{1}{6}|^3)$. Gravitons in a particular gauge are equivalent to massless scalars with $\xi=0$ (Ref. 6). Thus Eqs. (8) and (9) with $\xi=0$ and an extra factor of 2 to account for the two polarization degrees of freedom yield a reasonable estimate of the number and energy densities of gravitons. This estimate is at least qualitatively correct and is presumably accurate to about 20%.

We require that $V(\eta)\to 0$ as $\eta\to\pm\infty$ sufficiently rapidly that the above integrals converge at both limits. Consequently, ρ is independent of μ because

$$\int_{-\infty}^{\infty}\dot{V}(\eta)d\eta=0\ . \tag{10}$$

The requirement that V vanish in the past and in the future is linked to the need for asymptotic in and out regions where particle number is well defined. This would certainly be satisfied if spacetime were to be asymptotically flat in both the past and future. More generally, it will be satisfied if one can define adiabatic vacuum states in either region. This is the case in our present problem, where the in region is de Sitter space, in which the de Sitter-invariant vacuum state is an adiabatic vacuum state, and the out region is a Robertson-Walker universe which is asymptotically flat in the future. We will assume that $\xi>0$ so that the scalar field is both classically stable in de Sitter space and possesses a de Sitter-invariant vacuum state.[7,8] The in-vacuum state for the particle creation calculation is taken to be the de Sitter-invariant vacuum, and the out vacuum is the usual Minkowski vacuum. The latter is the appropriate choice because the spacetime is asymptotically flat in the future. It may be regarded as effectively flat after the curvature has fallen to some small fraction of its value in the de Sitter phase. Particle number is less well defined in the in region where the curvature is nonzero. Fortunately this does not introduce any significant ambiguity into our calculation. After several e-folding times of inflation, essentially any choice of in state is indistinguishable from the de Sitter-invariant state and hence the choice of in state does not affect the results of a particle creation calculation.

Because the perturbation approach is not well suited for massive particles if the scale factor does not approach the same constant value in both the past and the future,[5] we restrict our attention to the massless, nonconformally coupled scalar field. Although in de Sitter space, where R is constant, both fields satisfy the same wave equation, they cease to be equivalent after the end of the de Sitter phase.

The factors of a^{-3} and a^{-4} in n and ρ, respectively, reflect the dilution of the particles by the expansion after their creation. We can regard the particle production as having effectively ceased after V and $\dot{V}$ have become small compared to their maximum values. From that time onward, the particles behave as a photon gas.

Let us first consider an abrupt transition from de Sitter space to a radiation-dominated universe. The scale factor may be taken to be

$$a(\eta)=\begin{cases}(H|\eta|)^{-1}, & \eta<\eta_0<0\ ,\\ H[\eta-\eta_0+(H^2|\eta_0|^{-1})], & \eta>\eta_0\ .\end{cases} \tag{11}$$

The scalar curvature is

$$R=\begin{cases}12H^2, & \eta<\eta_0\ ,\\ 0, & \eta>\eta_0\ .\end{cases} \tag{12}$$

Hence $V=0$ for $\eta>\eta_0$, and the number density of created particles just after the transition is

$$\begin{aligned} n&=[16\pi a^3(\eta_0)]^{-1}\int_{-\infty}^{\eta_0}V^2(\eta)d\eta\\ &=(12\pi)^{-1}(1-6\xi)^2H^3\ . \end{aligned} \tag{13}$$

Note that this density is independent of the time η_0 at which the transition occurs.

We may also investigate the spectrum and the energy density associated with this transition. From Eq. (5) we find that β_ω is expressible as an incomplete Γ function:

$$\beta_\omega=2(1-6\xi)\Gamma(-1,2i\omega\eta_0)\ . \tag{14}$$

The frequency spectrum of particles falls off as ω^{-4} at high frequencies:

$$|\beta_\omega|^2\sim\tfrac{1}{4}(1-6\xi)^2(\eta_0\omega)^{-4},\quad \omega\to\infty\ . \tag{15}$$

Thus the integral in Eq. (7) for ρ is logarithmically divergent at the upper range of integration. This reflects the fact that this abrupt change in the metric produces too many particles in the higher-frequency modes. As will be shown below, a smoother transition leads to finite results for both n and ρ.

The basic difficulty with the abrupt transition is that the scalar curvature changes discontinuously at $\eta=\eta_0$ so V acquires a δ-function term. If the scale factor and its first two derivatives are continuous, then ρ will be finite. Let us consider the following choice of scale factor:

$$C(\eta)=a^2(\eta)=f(H\eta)\ , \tag{16}$$

where

$$f(x)=\begin{cases}1/x^2, & x<-1\ ,\\ a_0+a_1x+a_2x^2+a_3x^3, & -1<x<x_0-1\ ,\\ b_0(x+b_1)^2, & x>x_0-1\ .\end{cases} \tag{17}$$

This is a universe which makes a transition from de Sitter space to a radiation-dominated universe on a time $\Delta\eta=H^{-1}x_0$. We require that $f(x)$, $f'(x)$, and $f''(x)$ be continuous at $x=-1$ and at $x=x_0-1$. These six conditions then uniquely determine the coefficients a_0,a_1,a_2,a_3,b_0,b_1 to be

$$\begin{aligned} a_0&=6-S_1/(x_0S_2)\ ,\\ a_1&=8-3S_1/(x_0S_2)\ ,\\ a_2&=a_1-5,\quad a_3=a_0-6\ ,\\ b_0&=3S_2^{-1}(9x_0^2+12x_0+4)\ ,\\ b_1&=S_1[6(3x_0+2)]^{-1}\ , \end{aligned} \tag{18}$$

where

$$\begin{aligned} S_1&=2\sqrt{3}(3x_0^4+6x_0^3+7x_0^2+6x_0+3)^{1/2}\\ &\quad-2(3x_0^2+2x_0-1) \end{aligned} \tag{19}$$

and

$$S_2=S_1+9x_0^2+10x_0+1\ . \tag{20}$$

The energy density may be expressed as

$$\rho=\frac{H^4}{128\pi^2a^4}(1-6\xi)^2I\ , \tag{21}$$

where

$$I=-\int_{-\infty}^{x_0-1}dx_1\int_{-\infty}^{x_0-1}dx_2\ln|x_1-x_2|\,\tilde{V}'(x_1)\tilde{V}'(x_2)\ , \tag{22}$$

and

$$\tilde{V}(x)=f^{-2}[f''f-\tfrac{1}{2}(f')^2]\ . \tag{23}$$

For $x<-1$, $\tilde{V}(x)=4/x^2$ and for $x>x_0-1$, $\tilde{V}=0$. Its behavior in the interval $-1<x<x_0-1$ can be obtained from Eqs. (17) and (18). However, in the limit that $x_0\ll 1$, it is approximately a linear function of slope $-4/x_0$. If we let

$$\tilde{V}'=\begin{cases}-\dfrac{8}{x^2}, & x<-1\ ,\\[2mm] -\dfrac{4}{x_0}, & -1<x_1<x_0-1\ ,\end{cases} \tag{24}$$

then I may be explicitly evaluated to be

$$I=-16\ln x_0+32(x_0-1)^{-1}x_0\ln x_0+4\ ,$$

so

$$I\sim-16\ln x_0 \quad \text{as } x_0\to 0\ . \tag{25}$$

Because $a=1$ at the time of the transition, $\eta=-H^{-1}$, we have that $\Delta t\simeq\Delta\eta$ if $x_0=|H\Delta\eta|\ll 1$. Thus if the transition occurs on a comoving time scale Δt small compared to H^{-1}, we can approximate the energy density of created particles as

$$\rho=\frac{(1-6\xi)^2H^4}{8\pi^2a^4}\ln(1/H\Delta t)\ . \tag{26}$$

Thus ρ diverges logarithmically as $\Delta t\to 0$. As our previous calculation has shown, the number density n is finite in this limit and is given by Eq. (13).

For arbitrary values of x_0, both ρ and η may be evaluated numerically. The results of such a calculation are shown in Fig. 1. We see that as $x_0\to 0$, n approaches the finite limit of Eq. (13) while ρ is asymptotically given by Eq. (26).

Let us now consider a universe which makes a smooth transition from de Sitter space to a matter-dominated universe. Let $V(\eta)$ be given by

$$V(\eta)=2(1-6\xi)(\eta^2+\eta_0^2)^{-1}\ . \tag{27}$$

The scale factor which corresponds to this form of V may be found by solving the equation

$$\dot{D}+\tfrac{1}{2}D^2=(\tfrac{1}{2}-3\xi)V=4(\eta^2+\eta_0^2)^{-1}\ . \tag{28}$$

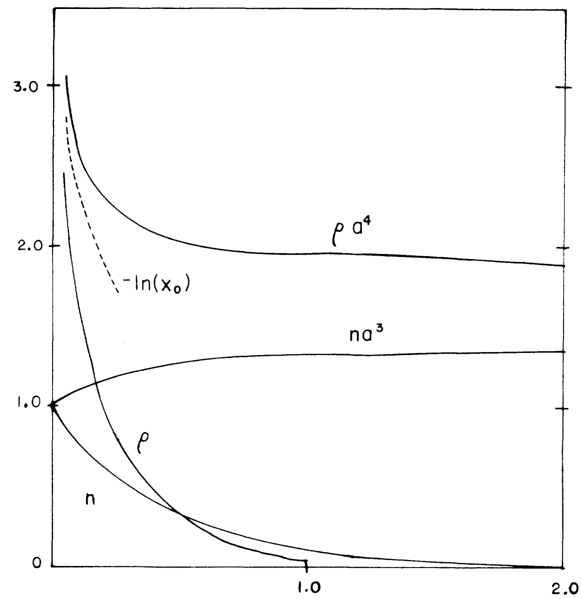

FIG. 1. The energy density ρ and number density n of created particles are shown for the case of a transition to a radiation-dominated universe. Here $x_0=H|\Delta\eta|$ where $\Delta\eta$ is the transition interval in conformal time. For $x_0\ll 1$, n approaches a constant and $\rho\propto a^{-4}\ln(1/x_0)$, where a is the scale factor at the end of the transition. For $x_0\gg 1$, $\rho\propto a^{-4}$ and $n\propto a^{-3}$.

First consider the limits $\eta\to\pm\infty$. Here the solutions are $D\sim-2/\eta$ and $D\sim 4/\eta$, which lead to $C\sim C_0\eta^{-2}$ and $C\sim C_1\eta^4$, which are the de Sitter space and the matter-dominated universe, respectively. We must still show that there is a solution which is de Sitter space in the past ($\eta\to-\infty$) and the matter-dominated universe in the future ($\eta\to+\infty$). This may be done by numerically solving Eq. (28). The resulting solutions for $a(\eta)$ are shown in Fig. 2. In all of the cases investigated, a solution which is asymptotically de Sitter space in the past becomes a matter-dominated universe in the future (rather than returning to de Sitter space) and makes the transition on a time scale of $\Delta\eta\approx\eta_0$.

From Eqs. (5) and (27), the coefficients β_ω are found to be

$$\beta_\omega=-\frac{\pi i}{\omega\eta_0}(1-6\xi)e^{-2\eta_0\omega} \tag{29}$$

and, from Eqs. (6) and (7), the number density and energy density are found to be

$$n=\frac{(1-6\xi)^2}{2\eta_0^3a^3}\ , \tag{30}$$

and

$$\rho=\frac{(1-6\xi)^2}{32\eta_0^4a^4}\ . \tag{31}$$

We note that the spectrum of created particles decays ex-

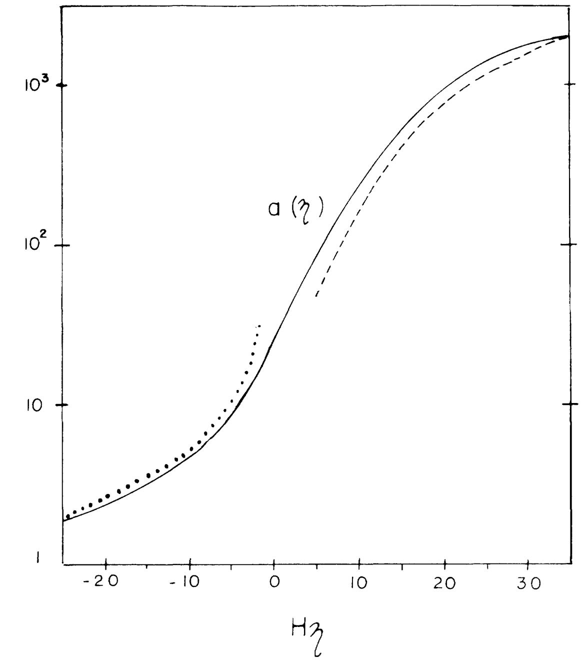

FIG. 2. The scale factor $a(\eta)$ obtained by numerically integrating Eq. (28) with $\eta_0 = 5.0H^{-1}$ is shown. The dotted line is the function $49.9/|\eta|$ and the dashed line is the function $1.90\eta^2$. Because $a(\eta)$ makes a smooth transition from the former to the latter, this represents a universe which makes a transition from de Sitter space to the matter-dominated universe on a time of order of η_0.

ponentially at high frequencies.

Until $\eta \approx -\eta_0$, the scale factor is approximately that of de Sitter space: $a = (H|\eta|)^{-1}$. Thus the value of a at the beginning of the transition is $a \approx (H\eta_0)^{-1}$. In the limit $\eta_0 \to 0$, the product $\eta_0 a$ remains finite and approaches H^{-1}. Thus both η and ρ are finite in the limit $\eta_0 \to 0$. The reason for this is presumably that the $V(\eta)$ given in Eq. (27) is continuous for all η_0. That is, although $V(\eta)$ is large at $\eta = 0$ when η_0 is small, it does not develop a discontinuity at the origin for small η. The effect of the large increase in V in the interval $-\eta_0 \le \eta \le 0$ is canceled by a corresponding decrease in the interval $0 \le \eta \le \eta_0$. If one were to have a sudden transition from de Sitter space to the matter-dominated universe in which a and $\dot{a}$ are continuous but R is discontinuous, then V would be discontinuous and ρ will diverge. If the transition is governed by the dynamics of scalar field whose energy density ρ_v is continuous, this will be the case. In de Sitter space, the trace of the Einstein equation yields $R = 2\pi\rho_v$, whereas in the matter-dominated phase the corresponding relation is $R = 8\pi\rho_v$. Thus a transition to a matter-dominated universe which occurs on a time scale $\Delta t \ll H^{-1}$ will lead to a rapid increase in R and hence V. The expressions for the energy density ρ of created particles should then be of the form of Eq. (31) but with an extra factor of $\ln(1/H\Delta t)$, as in the case of the transition to a radiation-dominated universe, Eq. (26).

III. DISCUSSION AND APPLICATIONS TO INFLATIONARY COSMOLOGY

We have seen that a universe which makes a transition from de Sitter space to a matter- or radiation-filled universe results in the creation of an energy density of the order of

$$\rho \simeq (1-6\xi)^2 10^{-2} H^4 a^{-4} \tag{32}$$

as long as the transition is not too abrupt. This energy density is of the order of that associated with a thermal bath at the Gibbons-Hawking temperature[9] $T = H/2\pi$. One is tempted to ask whether these particles are created by the transition, or whether one can regard them as already being present in de Sitter space and then "dumped" at the end of inflation. This question does not have an unambiguous answer because particle number is only clearly defined after the end of the de Sitter phase. If one chooses the latter interpretation, then particles must be continuously created in de Sitter space to compensate for the effects of the expansion and maintain a constant particle density. In the case of quick transition which changes the scalar curvature, at least the high-frequency part of the spectrum is created by the transition. In any case, the present results show no evidence of an instability of de Sitter space due to particle creation, in contradiction to the recent claim of Mottola.[10] Such an instability would require particles to be created faster than they are redshifted by the expansion in order that the energy density grow in de Sitter space and produce an instability. In the present calculation this would require that the energy density increase as the length of the inflationary phase increases, which is not the case.

Let us now consider the possible relevance of particle production by the gravitational field to inflationary cosmology. Suppose that the inflation is produced by a vacuum energy (cosmological constant) ρ_v. If inflation were to occur at the grand-unified-theory (GUT) scale, then $\rho_v \simeq (10^{15}\ \text{GeV})^4$. Einstein's equations relate ρ_v to the parameter H:

$$H^2 = \frac{8\pi}{3}\rho_v \rho_{\text{Pl}}^{-1/2}, \tag{33}$$

where $\rho_{\text{Pl}} \simeq (10^{19}\ \text{GeV})^4$ is the Planck density. If we then make a transition to a radiation-dominated universe, the energy density of created particle is (in Planck units)

$$\rho = (1-6\xi)^2 (\rho_v^2/\rho_{\text{Pl}}) a^{-4} \ln\left[\frac{1}{H\Delta t}\right]. \tag{34}$$

If $\Delta t \simeq H^{-1}$, $\rho \simeq (1-6\xi)^2(\rho_v^2/\rho_{\text{Pl}})$ just after the transition. Thus if inflation were to occur near the Planck scale, we would have $\rho \simeq \rho_{\text{Pl}}$. On the other hand, if $\rho_v \simeq (10^{15}\ \text{GeV})^4$, then $\rho \simeq (1-6\xi)^2(10^{11}\ \text{GeV})^4$ (Ref. 11).

Gravitational particle creation will reheat the Universe to a temperature of the order of $(\rho_v^2/\rho_{\text{Pl}})^{1/4}$. If this is greater than the mass of the gauge bosons, one can generate the observed baryon asymmetry by *CP*-violating decays of these bosons.[12,13] Baryon asymmetry can also be generated by decay of Higgs bosons if the reheating temperature is at least 10^{11} GeV (Ref. 14). It is also possible for gravitational particle creation to generate a baryon asymmetry directly if CP-violating interactions are present.[15]

It is usually assumed that the end of inflation is governed by the dynamics of a scalar field Φ. In the "new inflation" model of Linde[16] and Albrecht and Steinhardt,[17] this field evolves classically from an unstable maximum to a minimum. If the oscillations about this minimum are damped, then particles are produced and the Universe reheats.[13,18] In Linde's "chaotic inflation" model,[19] the field evolves from a nonmaximum value, but reheating is again assumed to occur through damping of oscillations. If these oscillations are damped efficiently, then the vacuum energy ρ_v of the Φ field is essentially all converted into thermal radiation. Unless inflation occurs near the Planck scale so $\rho \approx \rho_v \approx \rho_{\rm Pl}$, the effect of gravitational particle creation will then be small. However, if the damping of the oscillations of Φ is weak, it is possible for gravitational particle creation to be the dominant mechanism for reheating the Universe.[20] The constraints on inflationary models that adequate inflation occur and that density fluctuations not be too large in fact require Φ to be very weakly coupled to other fields. This requirement makes it difficult to achieve adequate reheating by damping of Φ oscillations alone. Gravitational particle creation offers a possible solution to this difficulty.

The change in the metric from de Sitter space to that of another Robertson-Walker model will create a matter energy density $\rho \approx (\rho^2/\rho_{\rm Pl})$. However, the total energy density is still dominated by the Φ field, if $\rho_v < \rho_{\rm Pl}$. In order that most of the mass of the Universe at later epochs be that arising from particle creation, it is necessary that the scalar field Φ have a large positive pressure so that its energy density red-shifts more rapidly than does ρ. Let the equation of state of the Φ field be

$$p_\Phi = \alpha \rho_\Phi \ . \tag{35}$$

Then the energy density red-shifts as

$$\rho_\Phi = \rho_v \left[\frac{a_0}{a} \right]^{3(1+\alpha)} , \tag{36}$$

where a_0 is the scale factor at the end of inflation. As long as ρ_Φ dominates the expansion, the scale factor is given by

$$a(t) \propto t^{2/[3(1+\alpha)]} , \tag{37}$$

and hence, because $\rho \propto a^{-4}$,

$$\left[\frac{\rho_\Phi}{\rho} \right] = \left[\frac{\rho_v}{\rho_0} \right] \left[\frac{t_0}{t} \right]^{\gamma}, \quad \gamma = \frac{2(3\alpha - 1)}{3(\alpha+1)} \ . \tag{38}$$

Here ρ_0 is the initial energy density of created particles at t_0, the time at which inflation ends. If $\alpha > \frac{1}{3}$, the scalar field's energy density will red-shift faster than that of the created particles and eventually become negligible.

The value of α depends upon the details of the potential $V(\Phi)$ which governs the evolution of Φ. If the potential is approximately quadratic, then the coherent, harmonic oscillations of Φ behave as a pressureless fluid. However, more generally there will be a pressure. Consider the potential

$$V(\Phi) = \lambda \Phi^{2n} , \tag{39}$$

for which the equation of motion is

$$\Box \Phi + 2n\lambda \Phi^{2n-1} = 0 , \tag{40}$$

and the energy-momentum tensor is

$$T_{\mu\nu} = \Phi_{,\mu}\Phi_{,\nu} - \tfrac{1}{2} g_{\mu\nu} \Phi_{,\rho}\Phi^{,\rho} + g_{\mu\nu} V(\Phi) \ . \tag{41}$$

If $\Phi = \Phi(t)$, then the energy density and pressure are, respectively,

$$\rho_\Phi = \tfrac{1}{2}\dot{\Phi}^2 + \lambda \Phi^{2n} \tag{42}$$

and

$$p_\Phi = \tfrac{1}{2}\dot{\Phi}^2 - \lambda \Phi^{2n} \ . \tag{43}$$

If the period of oscillation is short compared to the expansion time, then $\Box \Phi \approx \ddot{\Phi}$, and

$$\ddot{\Phi} + 2\lambda n \Phi^{2n-1} = 0 \tag{44}$$

or

$$\tfrac{1}{2}\dot{\Phi}^2 + \lambda \Phi^{2n} = \lambda {\Phi_0}^{2n} , \tag{45}$$

where Φ_0 is the amplitude of the oscillations. Thus

$$\rho_\Phi = \lambda {\Phi_0}^{2n}, \quad p_\Phi = \lambda({\Phi_0}^{2n} - 2\Phi^{2n}) \ . \tag{46}$$

To find the average pressure, we must average Φ^{2n} over a period of oscillation. Note that

$$\frac{d}{dt}(\Phi\dot{\Phi}) = \dot{\Phi}^2 + \Phi\ddot{\Phi} = \dot{\Phi}^2 - 2\lambda n \Phi^{2n} \ . \tag{47}$$

Average this equation over a period to find

$$\langle \dot{\Phi}^2 \rangle = 2n\lambda \langle \Phi^{2n} \rangle , \tag{48}$$

and hence from Eq. (45), that

$$\langle \Phi^{2n} \rangle = (n+1)^{-1} {\Phi_0}^{2n} \ . \tag{49}$$

Thus we have an equation of state of the form $\langle p_\Phi \rangle = \alpha \rho_\Phi$, where[21]

$$\alpha = \frac{n-1}{n+1} \ . \tag{50}$$

Anharmonic oscillations produce a positive pressure. For $n=2$, $\alpha = \frac{1}{3}$; the oscillations in the conformally invariant Φ^4 model behave like radiation. All values of $n > 2$ yield $\alpha > \frac{1}{3}$, so the energy in the oscillations red-shifts more rapidly than that in radiation. If the potential does not have a minimum but rather approaches a constant value as $\Phi \to \infty$, there will be no oscillations but Φ will grow without bound. The possibility of potentials of this type in supersymmetric theories has been discussed by Witten.[22] Here we have asymptotically that $\rho_\Phi = p_\Phi = \frac{1}{2}\dot{\Phi}^2$; so $\alpha = 1$. For a generic potential in which oscillations about a minimum occur, large amplitude oscillations can be expected to be anharmonic. Just after the end of inflation such large amplitude oscillations are expected, so in general there will be a nonzero pressure. If the potential is not of the power-law form, Eq. (39), the effective value of α may decrease as the amplitude of the oscillations decreases. If $\alpha > \frac{1}{3}$, the scalar field energy density will eventually become smaller than that of the created particles. In order that the scalar field energy density not alter the standard-model nucleosynthesis predictions, we should have $\rho_\Phi < \rho$ at $t \approx 1$ sec. If $(\rho_v / \rho_0) \approx 10^{16}$ at $t_0 = 10^{-35}$ sec, which are values associated with inflation at a scale of 10^{15} GeV, then $\rho_\Phi < \rho$ at $t = 1$ sec if $\alpha \gtrsim 0.7$. This can be satisfied in several ways: (1) by the potentials without a minimum and $\alpha = 1$ discussed above; (2) a power-law potential, Eq. (39), with $n \gtrsim 5$; and (3) by more general flat-bottomed potentials for which the effective value of α remains $\gtrsim 0.7$. It is thus possible to have models in which the energy density of particles is eventually dominant, and most of the matter in the Universe arose from gravitational particle creation. This avoids the problem of inadequate reheating from which inflationary models with weakly coupled scalar fields otherwise suffer.

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[1]A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[2]For a recent review, see A. D. Linde, Rep. Prog. Phys. **47**, 925 (1984).

[3]L. Parker, Phys. Rev. **183**, 1057 (1969).

[4]Ya. B. Zel'dovich and A. A. Starobinsky, Zh. Eksp. Teor. Fiz. **61**, 2161 (1971) [Sov. Phys. JETP **34**, 1159 (1972)]; Pis'ma Zh. Eksp. Teor. Fiz. **26**, 373 (1977) [JETP Lett. **26**, 252 (1977)].

[5]N. D. Birrell and P. C. W. Davies, J. Phys. A **13**, 2109 (1980); *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1982), Sect. 5.6.

[6]L. P. Grishchuk, Zh. Eksp. Teor. Fiz. **67**, 825 (1974) [Sov. Phys. JETP **40**, 409 (1975)]; L. H. Ford and L. Parker, Phys. Rev. D **16**, 1601 (1977).

[7]A. Vilenkin and L. H. Ford, Phys. Rev. D **26**, 1231 (1982).

[8]B. Allen, Phys. Rev. D **32**, 3136 (1985).

[9]G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).

[10]E. Mottola, Phys. Rev. D **31**, 754 (1985).

[11]Krauss *et al.* [L. M. Krauss, A. H. Guth, D. N. Spergel, G. B. Field, and W. H. Press, Nature (London) **319**, 748 (1986)] have recently discussed an inflationary model based upon $E_8 \times E_8$ superstring theory in which inflation and reheating occur in one E_8 sector and the second sector is reheated entirely by gravitational interactions of particles. The ratio of the energy densities in the two sectors after reheating is found to be $\rho_2/\rho_1 \approx (T_R/M_{Pl})^4$. Hence the contribution of gravitational particle creation is suppressed by a factor of T_R/M_{Pl} compared to the mechanism discussed by Krauss *et al.*

[12]A. D. Dolgov and A. D. Linde, Phys. Lett. **116B**, 329 (1982).

[13]L. F. Abbott, E. Farhi, and N. Wise, Phys. Lett. **117B**, 29 (1982).

[14]N. Sato, H. Kodama, and K. Sato, Prog. Theor. Phys. **74**, 405 (1986).

[15]N. J. Papastamatiou and L. Parker, Phys. Rev. D **19**, 2283 (1979).

[16]A. D. Linde, Phys. Lett. **108B**, 431 (1982).

[17]A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).

[18]A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, Phys. Rev. Lett. **48**, 1437 (1982).

[19]A. D. Linde, Phys. Lett. **129B**, 177 (1983).

[20]We are here treating the Φ field as a classical field and ignoring creation of its own quanta. More generally, scalar particles will be created as a result of self-coupling of the scalar field to itself. The magnitude of this effect presumably depends upon the details of this self-coupling. However, the effect of gravitational particle creation will dominate if there are a sufficiently large number of fields whose quanta are created by coupling to the spacetime curvature. The results for ρ given by either Eq. (32) or (33) are multiplied by N, the number of nonconformal fields present, if $N > 1$.

[21]This result was also obtained by Turner [M. S. Turner, Phys. Rev. D **28**, 1243 (1983)].

[22]E. Witten, Phys. Lett. **105B**, 267 (1981).