Electric dipole transitions of charmonium D states

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We calculate electric dipole transitions of charmonium D states for a scalar confining potential.

Energy levels, radiative decay rates, and leptonic widths provide a basis for testing quarkonium potential models through comparison of theory and experiment. In charmonium, calculations of $E1$ rates for $2\,^3S_1 \to 1\,^3P_J$ and $1\,^3P_J \to 1\,^3S_1$ have been carried out, and now that relativistic corrections have been included, the agreement between theory and experiment is much improved.[1–3] The purpose of the present work is to extend these calculations to include the transitions $1\,^3D_{J'} \to 1\,^3P_J$. Attempts to measure these transitions will be carried out in the near future, and thus a comparison of theory and experiment may be possible.[4]

Following Ref. 3 we find that the decay rate is given by the formula

$$W^{E1}_{1^3D_{J'}\to 1^3P_J} = \Gamma_{\rm NR}(1 + r_1^{JJ'} + r_2^{JJ'} + r_3) , \quad (1)$$

where $\Gamma_{\rm NR}$ is the nonrelativistic rate, $r_1^{JJ'}$ is the relativistic correction arising from relativistic corrections to the wave function, $r_2^{JJ'}$ results from possible quark anomalous moments (this will be ignored in this work), and r_3 derives from relativistic corrections to the $E1$ transition operator.

We find

$$\Gamma_{\rm NR} = \tfrac{8}{27} k_0{}^2 k e_q{}^2 (2j+1) G_1{}^2 , \quad (2)$$

where k is the magnitude of the photon wave vector and $\hbar k_0 = \omega_0$ is the energy separation between the initial and final charmonium states. The quantities $r_1^{JJ'}$ and r_3 are given by

$$r_1^{JJ'} = 2\left[\frac{G_2^{J'} + G_3^J}{G_1}\right] + \left[\frac{G_2^{J'} + G_3^J}{G_1}\right]^2 \quad (3)$$

and

$$r_3 = -\frac{1}{10}\frac{k_0{}^2 G_4}{G_1} + \frac{1}{8}\frac{\omega_0}{mc^2}\left[\frac{G_5 - G_6}{G_1}\right] , \quad (4)$$

where

$$G_1 = \int_0^\infty R^{(0)}_{nP} R^{(0)}_{n'D} r^3 dr, \quad G_2^{J'} = \int_0^\infty R^{(0)}_{nP} R^{(1)}_{n'D_{J'}} r^3 dr ,$$
$$G_3^J = \int_0^\infty R^{(1)}_{1P_J} R^{(0)}_{n'D} r^3 dr, \quad G_4 = \int_0^\infty R^{(0)}_{nP} R^{(0)}_{n'D} r^5 dr , \quad (5)$$
$$G_5 = \int_0^\infty R^{(0)}_{nP} R^{(0)'}_{n'D} r^3 dr, \quad G_6 = \int_0^\infty R^{(0)'}_{nP} R^{(0)}_{nD} r^3 dr ,$$

with

$$R^{(0)'} \equiv 2r\, dR^{(0)}/dr + R^{(0)} ,$$

and $R^{(1)}$ the relativistic correction to the radial wave function. To carry out the calculations we need a model which gives reasonable spectroscopic results. We have used a potential with the same form as that of Gupta, Repko, and Radford[5] but with some modifications. The potential used is

$$V = -\frac{4}{3}\frac{\alpha_s}{r}\left[1 - \frac{3\alpha_s}{2\pi} + \frac{\alpha_s(33-2n_f)}{6\pi}[\ln(\mu r) + \gamma_E]\right] + \frac{r}{A^2} + C_s + C_0 \quad \text{for } r \ge 0.178 \text{ GeV}^{-1}$$
$$= -\frac{8\pi}{25}\frac{1}{r\ln(r\Lambda e^{\gamma_E})^{-1}} + C_0 \quad \text{for } r < 0.178 \text{ GeV}^{-1} . \quad (6)$$

In the region $r < 0.178$ GeV^{-1} the potential is that obtained from asymptotic freedom[6] while in the region $r \ge 0.178$ GeV^{-1} terms to order $\alpha_s{}^2$ are calculated perturbatively and are supplemented by the confining potential $r/A^2 + C_s$. The value $r = 0.178$ GeV^{-1} for scalar confinement is determined by demanding that the potential in one region should match smoothly with that in the second region.

The parameters of the potential are chosen to have the

TABLE I. $c\bar{c}$ spectrum (scalar confinement).

State	Predicted value (GeV)	Experimental value (Ref. 8) (GeV)
$1\,^3S_1$	3.097	3.0969 ± 0.0001
$2\,^3S_1$	3.679	3.6860 ± 0.0001
$3\,^3S_1$	4.072	4.030 ± 0.005
$4\,^3S_1$	4.373	4.415 ± 0.006
$1\,^3D_1$	3.768	3.7699 ± 0.0024
$1\,^3D_2$	3.777	
$1\,^3D_3$	3.778	
$2\,^3D_1$	4.126	4.159 ± 0.020
$1\,^3P_0$	3.423	3.4149 ± 0.0011
$1\,^3P_1$	3.478	3.5107 ± 0.0005
$1\,^3P_2$	3.509	3.5563 ± 0.0004

TABLE II. Decay rates for $1\,{}^3D_{J'} \to 1\,{}^3P_J + \gamma$ (scalar confinement).

Decay (k in GeV)	Nonrelativistic rate Γ_{NR} (keV)	Correction relativistic modification of wave function $r_1^{JJ'}$	Finite-size correction r_3	Predicted rate $\Gamma = \Gamma_{NR}(1 + r_3^{JJ'} + r_3)$ (keV)
$1\,{}^3D_1 \to 1\,{}^3P_0 + \gamma$ $k=0.339$	336	-0.292	-0.102	204
$1\,{}^3D_1 \to 1\,{}^3P_1 + \gamma$ $k=0.251$	401	-0.166	-0.026	324
$1\,{}^3D_1 \to 1\,{}^3P_2 + \gamma$ $k=0.208$	376	-0.060	0.0026	354
$1\,{}^3D_2 \to 1\,{}^3P_0 + \gamma$ $k=0.350$	372	-0.305	-0.114	216
$1\,{}^3D_2 \to 1\,{}^3P_1 + \gamma$ $k=0.263$	461	-0.181	-0.035	362
$1\,{}^3D_2 \to 1\,{}^3P_2 + \gamma$ $k=0.220$	445	-0.076	-0.005	409
$1\,{}^3D_3 \to 1\,{}^3P_2 + \gamma$ $k=0.221$	454	-0.081	-0.005	415

values

$$C_s = -0.614 \text{ GeV}, \quad C_0 = -0.006 \text{ GeV} ,$$

$$\alpha_s = 0.3575, \quad A = 2.1 \text{ GeV}^{-1} ,$$

$$\mu = 1.98 \text{ GeV}, \quad n_f = 4 ,$$

$$\Lambda = \mu e^{-1/2b_0\alpha_s}, \quad b_0 = \frac{33 - 2n_f}{12\pi} ,$$

$$\gamma_E = 0.5772, \ldots \quad \text{(Euler's constant)} .$$

The c-quark mass m_c is 1.75 GeV. We treat the confining potential $r/A^2 + C_s$ as a scalar interaction and the remaining parts as a vector interaction. C_0 is an additive constant which shifts all levels equally and is chosen to set the $1\,{}^3S_1$ mass equal to the experimental value. The Hamiltonian we use has the form given in Ref. 3. With this choice of parameters the charmonium energy levels and their comparison with experiment are shown in Table I.

In Table II we tabulate the results of our calculation of the $E1$ decay widths for the D states. From Eq. (2) we see that the nonrelativistic rate depends on the magnitude of the photon momentum. For the case of ${}^3D_1$ decays to P states the energies of initial and final-states are known and hence we take k_0 and k from experiment. In contrast with this, the corresponding energies for the ${}^3D_2$ and ${}^3D_3$ are much less certain and more model dependent. We obtain these by using the fine-structure separation between the theoretically calculated values of ${}^3D_1$, ${}^3D_2$, ${}^3D_3$ as well as the experimental values of ${}^3D_1$ to obtain the most reliable numbers for use in the decay rate formula. It should be noted from Table II that the relativistic corrections reduce the decay rate and that these corrections are quite large.

For the sake of completeness we also include in Table III the $E1$ decay rates for ψ' to χ_J and χ_J to ψ for the scalar confinement model. These calculations are also quite

TABLE III. Decay rates for $\psi' \to \chi_J + \gamma$, $\chi_J \to \psi + \gamma$ (scalar confinement).

Decay (k in GeV)	Nonrelativistic rate Γ_{NR} (keV)	Correction relativistic modification of wave function $r_1^{JJ'}$	Finite-size correction r_3	Predicted rate $\Gamma = \Gamma_{NR}(1 + r_1^{JJ'} + r_3)$ (keV)
$\psi' \to \chi_0 + \gamma$ $k=0.261$	44	-0.450	-0.031	23
$\psi' \to \chi_1 + \gamma$ $k=0.172$	37	-0.264	0.038	28
$\psi' \to \chi_2 + \gamma$ $k=0.128$	25	-0.099	0.061	24
$\chi_0 \to \psi + \gamma$ $k=0.305$	155	-0.049	0.017	150
$\chi_1 \to \psi + \gamma$ $k=0.391$	335	-0.070	-0.034	300
$\chi_2 \to \psi + \gamma$ $k=0.432$	456	-0.087	-0.064	387

model dependent.

In conclusion, the relativistic corrections to the D-state $E1$ decay rates are quite large. The correction depends on the form and the parameters of the potential. Since the nature of the confining potential has not been determined theoretically, it could be a scalar, a vector, or some combination. Although we have not included the results here we generally find that the vector confining potential increases D-state $E1$ rates relative to the scalar confining potential. However, a pure vector model gives spin splittings of energy levels[7] which are too large and hence unacceptable. It should be noted from Eq. (5) that $G_2^{J'}$ represents relativistic corrections to the D-state wave function which result from admixtures of other states. There is an additional admixture correction, which has not been analyzed here, which is due to coupled-channel mixing. This will be studied in future work on the effect of coupled-channel mixing in quarkonia.

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