$K_L \to \pi^0 e^+ e^-$ as a probe of CP violation

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We point out that the decay mode $K_L \to \pi^0 e^+ e^-$ is predominantly CP violating in the standard model and is expected to occur with a branching ratio of several parts in 10^{12}. The decay should be sensitive to direct $\Delta S = 1$ CP violation. Analysis of the Dalitz plot may enable the CP-violating nature of the process to be confirmed.

I. INTRODUCTION

The "standard model" of weak and electromagnetic interactions[1] has enjoyed remarkable success in allowing the understanding of a wide range of phenomenology, from low ($\lesssim 1$ GeV) to high (~ 100 GeV) energies, in terms of the exchange of massless virtual photons and heavy gauge bosons. Nevertheless several mysteries remain, one of which is whether the phenomena of CP violation[2] is simply a natural consequence of this model, as a result of the CP-nonconserving phase δ permitted by a three-generation unitary weak mixing matrix,[3] or whether some sort of "new" physics must be invoked.

One of the problems in answering this important question is the lack of experimental data in this area. Despite two decades of intense effort, the only definite experimental confirmation of CP violation lies in the mixing between the K^0 and its antiparticle $\bar{K}^0$ which has been observed via detection of the decays $K_L \to \pi^+\pi^-, \pi^0\pi^0$ and of the asymmetry in the semileptonic modes $K_L \to \pi^\pm \mu^\mp \nu_\mu$. Using the conventional definitions

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv \varepsilon + \varepsilon' ,$$
$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv \varepsilon - 2\varepsilon' , \tag{1}$$
$$\delta = \frac{\Gamma(K_L \to \pi^-\mu^+\nu_\mu) - \Gamma(K_L \to \pi^+\mu^-\bar{\nu}_\mu)}{\Gamma(K_L \to \pi^-\mu^+\nu_\mu) + \Gamma(K_L \to \pi^+\mu^-\bar{\nu}_\mu)} ,$$

the present experimental situation[4] can be summarized by the single parameter ε, with

$$|\varepsilon| = (2.27 \pm 0.03) \times 10^{-3} ,$$
$$\mathrm{Arg}\varepsilon = (44.6 \pm 12.0)^\circ , \tag{2}$$

and by the result that ε' is very much smaller:

$$\frac{\varepsilon'}{\varepsilon} = -0.004 \pm 0.005(\pm 0.004) . \tag{3}$$

In fact, since ε' corresponds to direct CP violation within the $\Delta S = 1$ $K \to 2\pi$ transition while ε denotes CP-violating contributions buried within the $\Delta S = 2$ "mass matrix" which measures mixing between K^0 and $\bar{K}^0$, the specific origin of the effect becomes even more difficult to pinpoint.

Although improved experiments on these and other systems are underway, the question of the genesis of CP violation is so fundamental that it is important to explore all possible avenues to its answer. With this in mind we analyze here one promising possibility for probing this effect—the decay $K_L \to \pi^0 e^+ e^-$. With present techniques, it should be possible to detect this mode at the $\sim 10^{-12}$ level and, as we shall show, this should be sufficient to find a CP-violating signal, provided the standard Kobayashi-Maskawa (KM) scenario is responsible for CP nonconservation. In fact, we shall demonstrate that the CP-violating piece of the amplitude is actually the *dominant* one.

In Secs. II and III we analyze the CP-nonconserving, and -conserving amplitudes, respectively, for this process while in Sec. IV we discuss the promise for detection.

II. $K_L \to \pi^0 e^+ e^-$: CP VIOLATING

In the standard model, the effective weak Hamiltonian responsible for $K_L \to \pi^0 e^+ e^-$ has been derived by Gilman and Wise for the case of three quark generations.[5] One begins with the full $\Delta S = 1$ weak Hamiltonian. By successively considering the W boson and the t, b, and c quarks, respectively, as heavy compared to the mass scale involved, these field operators are removed from the problem. The resulting effective Hamiltonian contains only u-, d-, and s-quark fields in the combination

$$H_{\mathrm{eff}} \sim \sum_{i=1}^{6} C_i Q_i , \tag{4}$$

where C_i is a Wilson coefficient and Q_i is a local four-quark operator, plus an additional term

$$\alpha_{\mathrm{EM}} C_7 Q_7 \equiv C_7 \alpha_{\mathrm{EM}} \bar{s}\gamma_\mu(1+\gamma_5) d \bar{e}\gamma^\mu e \tag{5}$$

which is the one responsible for the $K_L \to \pi^0 e^+ e^-$ mode, in which we are interested. Note that if CP were a good symmetry, then in single-photon-exchange approximation we would have

$$\langle \pi_p^0 e_{s_1}^+ e_{s_2}^- | H_w | K_{Lk} \rangle \sim \alpha_{\mathrm{EM}} \bar{u}(s_2) \gamma^\mu v(s_1) \frac{1}{(k-p)^2} A_\mu(\mathbf{k},\mathbf{p}) , \tag{6}$$

where

$$A_\mu(\mathbf{k},\mathbf{p})=\int d^4x\, e^{i(k-p)\cdot x}\langle\pi^0_{\mathbf{p}}|T(V_\mu^{\rm EM}(x)H_w(0))|K_{-\mathbf{k}}\rangle \tag{7}$$

and

$$|K_-\rangle=\frac{1}{\sqrt{2}}(|K^0\rangle-|\bar{K}^0\rangle) \tag{8}$$

is the CP-odd admixture of K^0, $\bar{K}^0$ states. However, under CP we find

$$A_\mu(\mathbf{k},\mathbf{p})\to -A^\mu(-\mathbf{k},-\mathbf{p})=-A_\mu(\mathbf{k},\mathbf{p}) \tag{9}$$

so that this diagram is forbidden if CP were to be conserved.

In the real world, of course, this is not the case. Writing

$$|K_L\rangle\cong|K_-\rangle+\bar{\varepsilon}|K_+\rangle\ , \tag{10}$$

where $|K^0\rangle$, $|\bar{K}^0\rangle$ states are chosen to have the quark content $\bar{s}d$, $\bar{d}s$, respectively, so that the $K^0\to\pi\pi$ $(I=2)$ amplitude is real, we find

$$A_\mu(\mathbf{k},\mathbf{p})=A_\mu^-(\mathbf{k},\mathbf{p})+\bar{\varepsilon}A_\mu^+(\mathbf{k},\mathbf{p})\ , \tag{11}$$

where

$$A_\mu^\pm(\mathbf{k},\mathbf{p})=\int d^4x\, e^{i(k-p)\cdot x}\langle\pi^0_{\mathbf{p}}|T(V_\mu^{\rm EM}(x)H_w(0))|K_{\pm k}\rangle\ . \tag{12}$$

Here both A_μ^+, A_μ^- are nonvanishing, the former since we are now dealing with the CP-*even* eigenstate K_+, the latter because the weak Hamiltonian itself now contains a CP-violating component.

In the absence of strong-interaction (gluonic) corrections, the effective Hamiltonian responsible for the decay $K_L\to\pi^0e^+e^-$ arises from the charged quark loop diagram shown in Fig. 1—the so-called "electromagnetic penguin"—and is found to be[5]

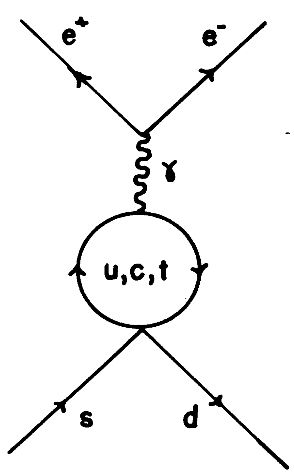

FIG. 1. The electromagnetic penguin diagram responsible for the CP-violating part of the decay $K_L\to\pi^0e^+e^-$.

$$H_{\rm eff}=-\frac{G_F}{\sqrt{2}}\alpha_{\rm EM}\frac{2}{9\pi}\left[s_1c_2(c_1c_2c_3-s_2s_3e^{-i\delta})\ln\frac{m_c{}^2}{\mu^2}+s_1s_2(c_1s_2c_3+c_2s_3e^{-i\delta})\ln\frac{m_t{}^2}{\mu^2}\right]\times\bar{s}\gamma_\mu(1+\gamma_5)d\bar{e}\gamma^\mu e\ , \tag{13}$$

where $c_i,s_i\equiv\cos\theta_i,\sin\theta_i$ and $\theta_1,\theta_2,\theta_3,\delta$ are the usual KM angles. In terms of the quantity

$$\tau\equiv s_2{}^2+s_2s_3e^{-i\delta} \tag{14}$$

we find then

$$H_{\rm eff}\approx-\frac{G_F}{\sqrt{2}}s_1\alpha_{\rm EM}\frac{2}{9\pi}\left[\ln\frac{m_c{}^2}{\mu^2}(1+\tau)+\tau\ln\frac{m_t{}^2}{m_c{}^2}\right]Q_7\ . \tag{15}$$

In the presence of gluonic corrections, mixing between the various operators occurs. Nevertheless the problem may still be solved perturbatively[5] and the interaction is found to be

$$\begin{aligned}H_{\rm eff}&\sim-\frac{G_F}{\sqrt{2}}s_1\alpha_{\rm EM}(-0.1+0.2\tau)Q_7\\&\equiv-\frac{G_F}{\sqrt{2}}s_1\alpha_{\rm EM}C_7Q_7\ .\end{aligned} \tag{16}$$

The sign change between the constant and τ term in Eqs. (15) and (16) is an interesting feature of the strong-interaction renormalization. The specific numerical values are calculated by Gilman and Wise for $m_t\sim40$ GeV and a scale parameter $\Lambda^2\sim0.01$ GeV^2. However, values of C_7 for other parameters are not dissimilar.

We now proceed to apply this interaction to the case at hand. Defining

$$\begin{aligned}&\langle\pi^0_{\mathbf{p}}|\bar{s}\gamma_\mu(1+\gamma_5)d|K^0_k\rangle\\&\quad=\langle\pi^0_p|\bar{s}\gamma_\mu d|K^0_k\rangle\\&\quad\equiv f_+((k-p)^2)(k+p)_\mu+f_-((k-p)^2)(k-p)_\mu\end{aligned} \tag{17}$$

we can relate these form factors via isospin rotation to corresponding quantities measured in K_{l3} decay. Thus we find, assuming K^* dominance for the slope of the form factor,

$$f_+(q^2)\approx\frac{1}{1-q^2/m_{K^*}{}^2}\left[\frac{1}{2}\right]^{1/2}$$

while $f_-(q^2)$ is not needed since

$$(k-p)_\mu\bar{u}(s_2)\gamma^\mu v(s_1)=(s_1+s_2)_\mu\bar{u}(s_2)\gamma^\mu v(s_1)=0\ . \tag{18}$$

The $K_L\to\pi^0e^+e^-$ transition amplitude is influenced by other aspects of the kaon system. Let us work in the natural phase convention for the kaon field such that the $K\to2\pi$ amplitudes of the $\Delta I=\frac{3}{2}$ weak Hamiltonian are real. (The final results are independent of this convention.) With standard definitions we have

$$|K_S^0\rangle = \frac{1}{\sqrt{2}}[(1+\bar{\varepsilon})|K^0\rangle + (1-\bar{\varepsilon})|\bar{K}^0\rangle] ,$$
$$|K_L^0\rangle = \frac{1}{\sqrt{2}}[(1+\bar{\varepsilon})|K^0\rangle - (1-\bar{\varepsilon})|\bar{K}^0\rangle] , \tag{19}$$

and

$$\langle \pi^+\pi^- | H_w | K^0\rangle = |A_0| e^{i\xi} e^{i\delta_0} + \frac{1}{\sqrt{2}} A_2 e^{i\delta_2} ,$$
$$\langle \pi^0\pi^0 | H_w | K^0\rangle = |A_0| e^{i\xi} e^{i\delta_0} - \sqrt{2} A_2 e^{i\delta_2} , \tag{20}$$

with δ_I being the final-state phase shift for isospin $I=0,2$. This allows us to identify ε and ε' via

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | H_w | K_L^0\rangle}{\langle \pi^+\pi^- | H_w | K_S\rangle} = \varepsilon + \varepsilon' = \bar{\varepsilon} + i\xi - \frac{i\xi A_2}{\sqrt{2} A_0} e^{i(\delta_2 - \delta_0)} ,$$
$$\eta_{00} = \frac{\langle \pi^0\pi^0 | H_w | K_L^0\rangle}{\langle \pi^0\pi^0 | H_w | K_S^0\rangle} = \varepsilon - 2\varepsilon' = \bar{\varepsilon} + i\xi + i\sqrt{2}\frac{\xi A_2}{A_0} e^{i(\delta_2 - \delta_0)} , \tag{21}$$

to first order in the small quantities $\bar{\varepsilon}$, ξ, and A_2/A_0. These yield

$$\varepsilon = \bar{\varepsilon} + i\xi , \quad \varepsilon' = -\frac{i\xi}{\sqrt{2}} \frac{\text{Re}A_2}{\text{Re}A_0} e^{i(\delta_2 - \delta_0)} (1-\Omega) , \tag{22}$$

where $|\Omega| \lesssim \frac{1}{3}$ accounts for electromagnetic and mixing corrections to ε' (Ref. 6). Experimentally

$$\varepsilon = 2.27 \times 10^{-3} e^{i\pi/4} ,$$
$$\left| \frac{\varepsilon'}{\varepsilon} \right| < 0.01 , \tag{23}$$
$$\frac{\text{Re}A_2}{\text{Re}A_0} = 0.045 = \frac{1}{22} ,$$

from which one finds

$$\left| \frac{\xi}{\varepsilon} \right| < 0.5 . \tag{24}$$

Turning to the transition $K_L \to \pi^0 e^+ e^-$, we find

$$A(K_L \to \pi^0 e^+ e^-) = \frac{G}{\sqrt{2}} s_1 \alpha_{\text{EM}} f_+((k-p)^2) 2\sqrt{2}\, k_\mu \bar{u}(s_2)\gamma^\mu v(s_1) \text{Re}C_7 \left[i\frac{\text{Im}C_7}{\text{Re}C_7} + (\varepsilon - i\xi) \right] , \tag{25}$$

where the factor in large parentheses can also be written

$$[\;\;] = \left[\varepsilon + i\frac{\text{Im}(e^{-i\xi} C_7)}{\text{Re}(C_7 e^{-i\xi})} \right] . \tag{26}$$

The factor of ε is obviously to be expected, while the second term measures the direct $\Delta S = 1$ *CP* violation. In order to estimate its magnitude we note that theoretically ε arises dominantly from the box diagram, and has a magnitude

$$\varepsilon \sim s_2 s_3 s_\delta e^{i\pi/4} \left[\frac{B}{0.33} \right] , \tag{27}$$

where B is the $K^0\bar{K}^0$ hadronic matrix element. For the electromagnetic penguin, Eq. (16) yields then

$$\frac{\text{Im}\,C_7}{\text{Re}C_7} = -2\,\text{Im}\tau = +2 s_2 s_3 s_\delta . \tag{28}$$

In addition ξ is expected to be negative (in units of $s_2 s_3 s_\delta$) so that all terms add constructively yielding

$$[\;\;] = |\varepsilon| \frac{1}{\sqrt{2}} \left\{ 1 + i\left[1 + \left[\frac{0.33}{B} \right] \left[2.8 + \left| \frac{\xi}{\varepsilon} \right| \right] \right] \right\} . \tag{29}$$

We see that direct $\Delta S = 1$ *CP* violation is expected to be as large as or dominant over the usual mass-matrix effect ε.

In addition to the electromagnetic penguin diagram, there is also a long-distance contribution from pole diagrams which can contribute due to the nonzero K^0 charge radius. These have the form

$$A(K_L \to \pi^0 e^+ e^-) = e^2 \varepsilon \frac{\langle K_+ | J_\mu^{\text{EM}} | K_- \rangle \langle K_- | H_w | \pi^0 \rangle}{m_\pi^2 - m_K^2} \frac{1}{q^2} \bar{u}\gamma^\mu v .$$

In the case of charged kaon decay via $K^\pm \to \pi^\pm e^+ e^-$, similar pole diagrams are very important. However here, due to the small neutral-kaon charge radius[4]

$$\langle K_+ | J_\mu^{\text{em}} | K_- \rangle = [(k+p)_\mu q^2 - q_\mu (k+p)\cdot q]$$

and the fact that the weak amplitude is evaluated at $q^2 = m_\pi^2$,

$$\langle K_-(q^2 = m_\pi^2) | H_w | \pi^0(q^2 = m_\pi^2) \rangle = \sqrt{2} F_\pi (A_0 - \sqrt{2} A_2) m_\pi^2 / m_K^2 ,$$

the contribution to $K_L \to \pi^0 e^+ e^-$ is less than one tenth of the magnitude of the electromagnetic penguin.

Finally we can calculate the branching ratio due to this *CP* violation, outlined in the Appendix, yielding (using $B = 0.33$, $\xi/\varepsilon = 0$)

$$R = 0.5\times 10^{-12}\left|\frac{\varepsilon + i\,\mathrm{Im}(C_7 e^{-i\xi})/\mathrm{Re}(C_7 e^{-i\xi})}{\varepsilon}\right|^2$$

$$\sim 3.7\times 10^{-12}\ . \tag{30}$$

Thus the *CP*-violating signal alone is at a level where it may be detectable. Also we see that the size of the effect will be further enhanced by as much as 30% if $\xi \neq 0$. However, it is also necessary to consider the *CP*-conserving amplitude, to which we now turn.

III. $K_L \to \pi^0 e^+ e^-$: *CP* CONSERVING

A *CP*-conserving contribution to the decay $K_L \to \pi^0 e^+ e^-$ can arise from a two-photon intermediate state[7]

$$K_L \to \pi^0\gamma\gamma \to \pi^0 e^+ e^-\ .$$

In the previous section we considered the one-photon-exchange (*CP*-violating) contribution to $K_L \to \pi^0 e^+ e^-$. This contribution must be of order

$$\mathrm{Amp}(1\gamma) \sim G_F m_K{}^2 \frac{\alpha}{\pi}\varepsilon \sim 10^{-6} G_F m_K{}^2\ . \tag{31}$$

One would expect then the two-photon-exchange (*CP*-conserving) amplitude to be of magnitude

$$\mathrm{Amp}(2\gamma) \sim G_F m_K{}^2\left[\frac{\alpha}{\pi}\right]^2 \sim 10^{-6} G_F m_K{}^2 \tag{32}$$

and thus comparable to its *CP*-violating counterpart. Actually, this amplitude is somewhat smaller, for reasons which we shall point out, so that the *CP*-violating amplitude is expected to be dominant.

Although a reliable result for the amplitude $K_L \to \pi^0\gamma\gamma$ is not yet available, we should be able to obtain a reasonable estimate by use of current-algebra—PCAC (partial conservation of axial-vector current) techniques

$$\langle \pi_p^0 \gamma_{q_1}\gamma_{q_2} | H_w | K_{Lk}\rangle \underset{p\to 0}{\to} -\frac{i}{F_\pi}\langle \gamma_{q_1}\gamma_{q_2} | [F_{\pi^0}{}^5, H_w] | K_{Sk}\rangle$$
$$= -\frac{i}{2F_\pi}\langle \gamma_{q_1}\gamma_{q_2} | H_w | K_{Sk}\rangle\ . \tag{33}$$

The $K_S \to \gamma\gamma$ amplitude has recently been calculated[8] in chiral-symmetric models, where it arises in one-loop order and is *finite* since no counterterm with the correct chiral-transformation properties is available. The result is

$$\langle \gamma_{q_1}\gamma_{q_2} | H_w | K_{Sk}\rangle = \lambda \frac{G_F}{\sqrt{2}} s_1 \times \frac{\alpha_{\mathrm{EM}}}{\pi}(\varepsilon_1\cdot\varepsilon_2 q_1\cdot q_2 - \varepsilon_1\cdot q_2 \varepsilon_2\cdot q_1)\ , \tag{34}$$

where

$$\lambda \cong 10.6 F_\pi \frac{m_\pi{}^2 - m_K{}^2}{m_K{}^2}\left\{1 - \frac{m_\pi{}^2}{m_K{}^2}\left[\pi^2 - \ln^2\left[\frac{x_+}{x_-}\right] - 2i\pi\ln\left[\frac{x_+}{x_-}\right]\right]\right\}$$

and

$$x_\pm = \frac{1}{2}\left[1 \pm \left[1 - \frac{4m_\pi{}^2}{m_K{}^2}\right]^{1/2}\right]\ .$$

The imaginary component of this amplitude agrees precisely with previous calculations using unitarity with the 2π intermediate state, while the real part is comparable in size:

$$\frac{\mathrm{Re}\lambda}{\mathrm{Im}\lambda} \sim -0.6\ . \tag{35}$$

The predicted branching ratio

$$\frac{\Gamma(K_S \to \gamma\gamma)}{\Gamma(K_S \to \mathrm{all})} = 3.5\times 10^{-6} \tag{36}$$

is still 2 orders of magnitude below the present experimental bound.

We can then use this result in a corresponding unitarity calculation (cf. Fig. 2) to yield the imaginary piece of the $K_L \to \pi^0 e^+ e^-$ amplitude. Thus

$$2\mathrm{Abs}\langle e_{s_1}^+ e_{s_2}^- \pi_p^0 | H_w | K_{Lk}\rangle = e^2 \int \frac{d^3q_1 d^3q_2}{(2\pi)^3 2q_{10}(2\pi)^3 2q_{20}}(2\pi)^4\delta^4(k - q_1 - q_2 - p)$$
$$\times \bar{u}(s_2)\gamma_\mu \frac{1}{-\not q_1 + \not s_2 - m_e}\gamma_\nu v(s_1)\left[\frac{\lambda}{2F_\pi}\frac{G_F}{\sqrt{2}}s_1\frac{\alpha_{\mathrm{EM}}}{\pi}\right](g^{\mu\nu}q_1\cdot q_2 - q_1^\nu q_2^\mu)$$
$$\cong \frac{\lambda}{F_\pi}\frac{G_F}{\sqrt{2}}s_1\left[\frac{\alpha_{\mathrm{EM}}{}^2}{\pi}\right]\frac{1}{4}m_e\frac{1}{\left[1 - \dfrac{4m_e{}^2}{s}\right]^{1/2}}\ln\left[\frac{1 + (1 - 4m_e{}^2/s)^{1/2}}{1 - (1 - 4m_e{}^2/s)^{1/2}}\right]\bar{u}(s_2)v(s_1)\ , \tag{37}$$

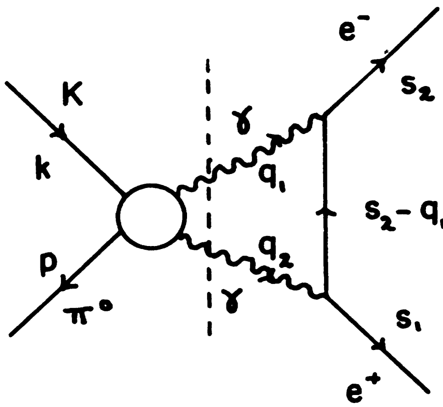

FIG. 2. The unitarity diagram for the decay $K_L \to \pi^0 e^+ e^-$.

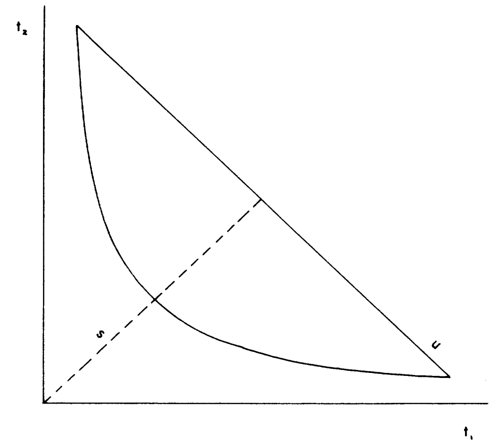

FIG. 3. The Dalitz plot for the decay $K_L \to \pi^0 e^+ e^-$.

where

$$s = (k-p)^2 = m_K{}^2 + m_\pi{}^2 - 2m_K E_\pi ,$$
$$t_1 = (k - s_1)^2 , \quad t_2 = (k - s_2)^2 .$$

If the unitarity contribution were the only one, the decay rate would be

$$\Gamma(K_L \to \pi^0 e^+ e^-) = \int dt_1 dt_2 \frac{m_e{}^2}{32\pi^3 m_K{}^3} \left[\frac{\lambda G_F}{32 F_\pi} s_1 \frac{\alpha_{\text{EM}}{}^2}{\pi} \right]^2 \times s \ln^2 \frac{\sqrt{s} - \sqrt{s - 4m_e{}^2}}{\sqrt{s} + \sqrt{s - 4m_e{}^2}} = 3 \times 10^{-27} \text{ MeV} \tag{38}$$

which corresponds to a branching ratio of

$$\frac{\Gamma(K_L \to \pi^0 e^+ e^-)}{\Gamma(K_L \to \text{all})} = 2.3 \times 10^{-13} . \tag{39}$$

Even if the dispersive component of the $K_L \to \pi^0 e^+ e^-$ amplitude is a factor of 2 larger than its absorptive counterpart (and its actual magnitude is expected to be much less) the predicted branching ratio of the *CP*-conserving component is still considerably lower than that of the *CP*-violating piece. The reason that our previous estimate

$$\frac{\text{Amp}(2\gamma)}{\text{Amp}(1\gamma)} \sim \left[\frac{\alpha}{\pi \varepsilon} \right] \sim 1 \tag{40}$$

is faulty is that there is an additional suppression factor of $\pi^2 m_e / F_\pi$ built into the actual result. Thus a more appropriate estimate is given by

$$\frac{\text{Amp}(2\gamma)}{\text{Amp}(1\gamma)} \sim \pi \frac{\alpha m_e}{\varepsilon F_\pi} \sim 10\% \tag{41}$$

which explains the dominance of the *CP*-violating amplitude.

We emphasize that this same electron-mass dependence also serves to suppress the interference between *CP*-conserving and -violating amplitudes. Thus, writing generically

$$\text{Amp(tot)} \sim \text{Amp}(2\gamma) \bar{u}(s_2) v(s_1) + \text{Amp}(1\gamma) k^\mu \bar{u}(s_2) \gamma_\mu v(s_1) \tag{42}$$

we observe that because of the helicity mismatch between $\bar{u}v$ and $\bar{u}\gamma_\mu v$ any such interference term is also suppressed by an amount m_e / E_e and hence has only a small impact on the decay rate.

IV. CONCLUSION

We have seen then that in the standard model wherein *CP* violation arises due to the phase δ in the KM matrix, one expects the decay mode $K_L \to \pi^0 e^+ e^-$ to be predominantly *CP* violating with a branching ratio at the level of 4×10^{-12}. Any *CP*-conserving contaminant should appear suppressed by at least one order of magnitude. Thus, just seeing the existence of the $\pi^0 e^+ e^-$ mode at this level should be very suggestive.

This conclusion depends upon the extra suppression built into the *CP*-conserving amplitude by its $m_e \bar{u} v$ form, as discussed above. We have studied this question in some detail and believe the suppression to be a general feature of the 2γ contribution. Because of the helicity-conserving nature of the photon coupling, the helicity-flip matrix element $\bar{u}v$ must always be accompanied by a factor of the electron mass. In order for the suppression factor of m_e to be absent, the matrix element must have a helicity-conserving form such as $(k+p)_\mu \bar{u} \gamma^\mu v$. However, for the latter to appear in a *CP*-conserving mechanism it must appear with a *CP*-odd coefficient such as

$$M^{CP} \sim (k \cdot s_1 - k \cdot s_2) \bar{u}(s_1)(\not{k} + \not{p}) v(s_2) . \tag{43}$$

We have checked the tree-level diagrams and one-loop effects and find that this matrix element does not occur. However we do not at this point have a rigorous proof to all orders. This point and the form of the $K_L \to \pi^0 \gamma\gamma$ amplitude are under further study.

Fortunately, since this decay has a three-body final state, this uncertainty can be resolved experimentally by a study of the Dalitz plot (Fig. 3). The point is that each of these matrix elements has a distinctive Dalitz-plot signature. For example, the *CP*-conserving process calculated

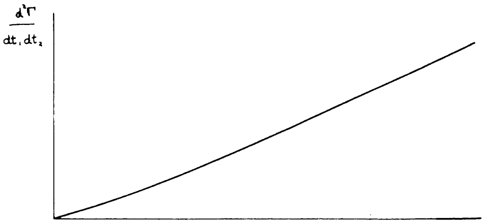

FIG. 4. Shown is the s dependence of the doubly differential decay rate arising from the (CP-conserving) unitarity contribution to $K_L \to \pi^0 e^+ e^-$.

in the text has a form which depends solely on s, i.e.,

$$\frac{d^2\Gamma}{dt_1 dt_2} \sim s \ln^2 \frac{\sqrt{s} + \sqrt{s - 4m_e^2}}{\sqrt{s} - \sqrt{s - 4m_e^2}} \tag{44}$$

which is a monotonically increasing function of s and is shown in Fig. 4. While this specific form is for the absorptive component only, the dependence of a general $\bar{u}v$ matrix element should be similar. On the other hand, for the CP-violating mode we find a very different and distinctive form:

$$\frac{d\Gamma}{dt_1 dt_2} \sim |\bar{u} k\!\!\!/ v|^2$$

$$\sim t_1 t_2 - m_e^2(t_1 + t_2) + m_e^4 - m_K^2 m_\pi^2 \ . \tag{45}$$

The implications of this shape are easiest to see in terms of the variables

$$s = m_K^2 + m_\pi^2 + 2m_e^2 - t_1 - t_2 \ , \quad u = t_1 - t_2 \ , \tag{46}$$

for which we find

$$\frac{d\Gamma}{ds\, du} \sim \tfrac{1}{4}(m_K^2 + m_\pi^2 + 2m_e^2 - s)^2 - \tfrac{1}{4}u^2$$

$$- m_e^2(m_K^2 + m_\pi^2 + m_e^2 - s) - m_K^2 m_\pi^2 \ . \tag{47}$$

Then the dependence upon u is such that the event rate drops as one moves off the s axis, as shown clearly in Fig. 5, as opposed to the constant off-axis behavior expected for the CP-even mode. Likewise the helicity- and CP-conserving matrix element Eq. (43), which would not have the m_e suppression, has a distinctive Dalitz plot,

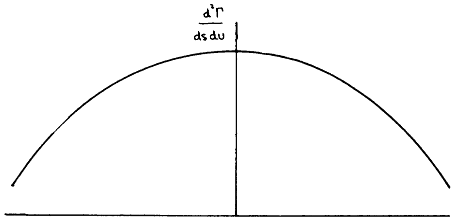

FIG. 5. Shown is the u dependence of the doubly differential decay rate arising from the CP-violating electromagnetic penguin diagram. The shape is evaluated at the midpoint of the allowed range of s.

$$\frac{d\Gamma}{dt_1 dt_2} \sim (t_1 - t_2)^2 (t_1 t_2 - m_K^2 m_\pi^2) \ , \tag{48}$$

with a characteristic vanishing along the center line of the Dalitz plot. Thus, provided the event rate is high enough it should be possible to confirm the CP properties of the decay amplitude by seeking the u dependence of the Dalitz plot.

In addition, it should be possible to determine if the $\Delta S = 1$ direct CP effect is present. For the parameters used in the text the rate is almost an order of magnitude larger than if the decay were due to ε alone. This is an extremely important issue because it would distinguish the KM model from superweak theories in which there are no $\Delta S = 1$ effects. If the rate is as large as Eq. (30) suggests, the distinction should be clear. If the observed rate is somewhat smaller, but still larger than that expected with ε alone, a more careful study of the consequences of the QCD renormalization may be needed in order to firmly conclude that $\Delta S = 1$ effects are present. Alternatively a measurement of $K_S \to \pi^0 e^+ e^-$ could provide the normalization which is needed to distinguish the superweak theory from milliweak ones. In any case, the decay $K_L \to \pi^0 e^+ e^-$ is the only mode in the kaon system where $\Delta S = 1$ effects are expected to stand out so strongly.

We conclude that study of the $K_L \to \pi^0 e^+ e^-$ decay should offer a clear window into the mechanism of CP violation, which is unavailable by the study of only nonleptonic processes.

Note added in proof. Recently there has been a reevaluation of the coefficient of the usual penguin operator, by W. A. Bardeen, A. J. Buras, and J.-M. Gérard [Phys. Lett. **180B**, 133 (1986)]. In this work, the authors criticize the method of Gilman and Wise and find a considerably larger real part of the coefficient. If valid, this could possibly also modify the real part of the electromagnetic penguin diagram, i.e., ReC_7. If ReC_7 turns out to be different than the Gilman-Wise value which we quote, our results can be easily modified to account for the change.

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APPENDIX

The phase space for the decay $K_L \to \pi^0 e^+ e^-$ is given, in the kaon rest frame, as

$$\Phi = \frac{1}{2m_K} \int \frac{d^3p}{(2\pi)^3 2p_0} \frac{m_e}{s_{10}} \frac{d^3 s_1}{(2\pi)^3} \frac{m_e}{s_{20}} \frac{d^3 s_2}{(2\pi)^3}$$

$$\times (2\pi)^4 \delta^4(k - p - s_1 - s_2)$$

$$= \int \frac{1}{64\pi^3 m_K^3} m_e^2 dt_1 dt_2 \equiv \int \frac{d^2\Phi}{dt_2 dt_1} dt_2 dt_1 \ , \tag{A1}$$

where $t_1 = (k - s_1)^2$, $t_2 = (k - s_2)^2$. The region of integration is defined, for a given t_1 by

$$t_2^{\max,\min} = (M_1 + M_2)^2 - [(M_1{}^2 - m_e{}^2)^{1/2} \mp (M_2{}^2 - m_\pi{}^2)^{1/2}]^2 , \quad \text{(A2)}$$

where

$$M_1 = \frac{t_1 + m_e{}^2 - m_\pi{}^2}{2t_1} , \quad M_2 = \frac{m_K{}^2 - t_1 - m_e{}^2}{2t_1} , \quad \text{(A3)}$$

yielding the Dalitz plot as shown in Fig. 3. A numerical integration over this area yields

$$\Phi = 2.03 \times 10^{-2} \text{ MeV}^{-1} . \quad \text{(A4)}$$

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