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Internal gravity

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The dynamics of the internal gravitons in higher-dimensional unified theories is discussed in a general setting. A generalized nonlinear σ model describes the dynamics of the internal metric. It is shown that when the vacuum of the internal space has a nonvanishing curvature the internal gravitons acquire masses comparable to the Planck mass. This implies that, unlike the five-dimensional case, the internal gravitons will not alter Newton's gravity (or the low-energy physics, in general) in a significant way.

Motivated by the recent discussions[1] on whether Newton's law of gravitation should be modified by a "fifth force," Bars and Visser[2] made a very interesting suggestion that the internal gravitons in a higher-dimensional Kaluza-Klein unification, in the presence of a non-Abelian gauge field, could mediate a fifth force which can compete with Newton's gravity. Their suggestion was based on the five-dimensional result in which the Kaluza-Klein gauge field generates an "antigravity" effect. So it would be very interesting to see whether this antigravity survives in a realistic non-Abelian generalization. In fact, independent of this question, it should be important for us to understand the dynamics of the internal gravitons, especially its low-energy effects, if there are any, to see if the idea of the higher-dimensional unification is indeed a desirable one. This is so because the existence of the internal gravitons[3] is an unavoidable fact in any higher-dimensional unified theory. The purpose of this Rapid Communication is to discuss the dynamics of the internal gravitons and the possibility of a fifth force in a general setting.

A central issue in any (supersymmetric or not) higher-dimensional unified theory is how to reduce it to a four-dimensional theory. A popular method to achieve the dimensional reduction is to make a "zero-mode" approximation[4] of the harmonic expansion, after a spontaneous compactification[5] of the internal space. The justification of this approximation is of course based on the belief that when the compactification scale remains small all the higher modes can safely be neglected in the low-energy physics. Unfortunately, the matter is more complicated,[6] and the zero-mode approximation is plagued with a logical ambiguity due to the possibility of a spontaneous symmetry breaking,[7] and the consistency problems both at the classical and the quantum levels.[6,8] A simpler and unambiguous method is dimensional reduction by isometry.[7,9] Here the reduction is achieved simply by requiring the *right invariance to all the fields,* including the fermions. In this case the zero-mode ansatz is replaced by the right invariance, which allows only a finite number of modes whose internal-space dependence is completely fixed. Thus there is no need for a spontaneous compactification and harmonic expansion. Since the right invariance gives us a unique dimensional reduction which involves no approximation, we will adopt this method in the following.

Let the metric g_{AB} of the $(4+n)$-dimensional unified space P admit an isometry K consisting of n linearly independent Killing fields ξ_a $(a=1,2,\ldots,n)$,

$$\pounds_{\xi_a} g_{AB}=0, \quad [\xi_a,\xi_b]=\frac{1}{\kappa}f_{ab}^c\xi_c , \tag{1}$$

where κ is a scale parameter. The isometry makes the unified space P a principal fiber bundle $P(M,K)$ with the four-dimensional space-time M as the base manifold and K as the structure group. In a local direct-product basis made of a coordinate basis $\partial_\mu (\mu=1,2,3,4)$ of M and the Killing basis $\partial_a (=\xi_a)$ of K, the metric can be written as

$$g_{AB}=\begin{pmatrix} g_{\mu\nu}+e^2\kappa^2\phi_{ab}B_\mu^a B_\nu^b & e\kappa\phi_{ab}B_\nu^b \\ e\kappa B_\mu^a\phi_{ab} & \phi_{ab}\end{pmatrix} ,$$

where e is the coupling constant, and B_μ^a is the gauge potential of K. Now the isometry (1) requires the metric to be right invariant,[9] and determines the internal-space dependence of g_{AB} uniquely:

$$\partial_a g_{\mu\nu}=0 ,$$
$$\partial_a B_\mu^c = -\frac{1}{\kappa}f_{ab}^c B_\mu^b , \tag{2}$$
$$\partial_a\phi_{bc}=\frac{1}{\kappa}f_{ab}^d\phi_{dc}+\frac{1}{\kappa}f_{ac}^d\phi_{bd} .$$

Furthermore, when K is unimodular, the $(4+n)$-dimensional Lagrangian of the Einstein-Hilbert action on $P(M,K)$ becomes *explicitly independent* of internal coordinates. So the dimensional reduction is obtained automatically and one is left with the four-dimensional Lagrangian

$$\mathcal{L}_0 = -\frac{1}{16\pi G}\sqrt{g}\sqrt{\phi}(R_P+\Lambda) \ , \tag{3}$$

where $g=|\det g_{\mu\nu}|$, $\phi=|\det\phi_{ab}|$, R_P is the scalar curvature of $P(M,K)$, and Λ is a cosmological constant. Notice that, up to a total divergence, R_P is given by

$$R_P = R_M + R_K + \frac{e^2\kappa^2}{4}\phi_{ab}G^a_{\mu\nu}G^b_{\mu\nu} + \tfrac{1}{4}\phi^{ab}\phi^{cd}[(D_\mu\phi_{ac})(D_\mu\phi_{bd})-(D_\mu\phi_{ab})(D_\mu\phi_{cd})] \ , \tag{4}$$

where $G^a_{\mu\nu}$ is the field strength of B^a_μ. So with $e^2\kappa^2=16\pi G$ one obtains the desired unification.[3,7]

For the matter fields let us for simplicity introduce a gauge field with symmetry group G minimally coupled to fermions on $P(M,K)$. This would make the bosonic part (and thus the vacuum and the symmetry-breaking pattern) of the theory very similar to ten-dimensional $N=1$ supersymmetric Yang-Mills supergravity,[10] which is the low-energy limit of the type-I superstring. Writing the gauge field of G as A^k_M and its field strength F^k_{MN} $(M,N=1,2,\ldots,4+n)$ one has

$$\mathcal{L}_1 = \sqrt{g}\sqrt{\phi}[\bar\psi(i\Gamma^{\hat A}\tilde\nabla_{\hat A})\psi - \tfrac{1}{4}F^k_{MN}F^k_{MN}] \ , \tag{5}$$

where $\tilde\nabla_A$ is the $(4+n)$-dimensional generally covariant and G-gauge covariant derivative. To determine the internal-space dependence of the matter fields, notice first that A^k_M is made of the four-dimensional potential A^k_μ and the scalars A^k_a which we write as ϕ^k_a. Then one can easily prove that the right invariance requires

$$\partial_a A^k_\mu = 0, \quad \partial_a\phi^k_b = \frac{1}{\kappa}f^c_{ab}\phi^k_c, \quad \partial_a\psi = 0 \ . \tag{6}$$

Again it is emphasized that this is not a zero-mode ansatz, but a simple consequence of the right invariance. Now, notice that since the horizontal basis[3] $D_\mu=\partial_\mu - e\kappa B^a_\mu\partial_a$ is orthogonal to the Killing basis ∂_a on $P(M,K)$, the G-gauge covariant derivative $\tilde D_M$ of the fermions is given by

$$\tilde D_\mu\psi = [\partial_\mu + ig(A^i_\mu - e\kappa B^a_\mu\phi^i_a)t_i]\psi = (\partial_\mu + ig\tilde A^i_\mu t_i)\psi \ ,$$

$$\tilde D_a\psi = (\partial_a + ig\phi^i_a t_i)\psi = ig\phi^i_a t_i\psi \ ,$$

where g and t_i are the coupling constant and generators of G. Also notice that

$$(F^k_{MN})^2 = (\tilde F^k_{\mu\nu})^2 + 2(\tilde D_\mu\phi^k_a)^2 + (F^k_{ab})^2 \ ,$$

where $\tilde F^k_{\mu\nu}$ is the field strength of $\tilde A^k_\mu = A^k_\mu - e\kappa B^a_\mu\phi^k_a$, and

$$\tilde D_\mu\phi^k_a = D_\mu\phi^k_a + f^k_{ij}\tilde A^i_\mu\phi^j_a = \partial_\mu\phi^k_a - ef^b_{ca}B^c_\mu\phi^k_b + gf^k_{ij}\tilde A^i_\mu\phi^j_a \ ,$$

$$F^k_{ab} = \frac{1}{\kappa}f^c_{ab}\phi^k_c + gf^k_{ij}\phi^i_a\phi^j_b \ .$$

This implies that ϕ^i_a could play a crucial role in spontaneous symmetry breaking.

There is one more step to go before we discuss the physics. Notice that with $\phi_{ab}=\phi^{1/n}\rho_{ab}$ $(\det\rho_{ab}=1)$, R_P can be written as

$$R_P = R_M + R_K + 4\pi G\phi^{1/n}\rho_{ab}G^a_{\mu\nu}G^b_{\mu\nu} - \frac{n-1}{4n}\frac{(\partial_\mu\phi)^2}{\phi^2} + \tfrac{1}{4}\rho^{ab}\rho^{cd}(D_\mu\rho_{ac})(D_\mu\rho_{bd}) + \lambda(\det\rho_{ab}-1) \ ,$$

where λ is a Lagrange multiplier. So the Lagrangian (3) appears to be *unstable* because the ϕ field has a negative kinetic term. To remove this defect, one has to make the conformal transformation

$$g_{\mu\nu} \to \sqrt{\phi}g_{\mu\nu} \ , \tag{7}$$

after which $\mathcal{L}_0$ is written, in terms of the new metric, as

$$\mathcal{L}_0 = -\frac{1}{16\pi G}\sqrt{g}\left\{R_M + \exp\left[-\left(\frac{n+2}{n}\right)^{1/2}\sigma\right]\hat R_K + \exp\left[-\left(\frac{n}{n+2}\right)^{1/2}\sigma\right]\Lambda + 4\pi G\exp\left[\left(\frac{n+2}{n}\right)^{1/2}\sigma\right]\rho_{ab}G^a_{\mu\nu}G^b_{\mu\nu} + \tfrac{1}{2}(\partial_\mu\sigma)^2 + \tfrac{1}{4}\rho^{ab}\rho^{cd}(D_\mu\rho_{ac})(D_\mu\rho_{bd}) + \lambda(\det\rho_{ab}-1)\right\} \ , \tag{8}$$

up to a total divergence, where $\hat R_K = R_K(\rho_{ab})$ and we have introduced the dilaton field σ by

$$\sigma = \frac{1}{2}\left(\frac{n+2}{n}\right)^{1/2}\ln\phi \ .$$

The result suggests that *one should treat the new metric $\sqrt{\phi}g_{\mu\nu}$, but not $g_{\mu\nu}$, as the physical space-time metric.* So from now on we will always use the new metric, but express it simply by $g_{\mu\nu}$. With this convention $\mathcal{L}_1$ becomes

$$\begin{aligned}\mathcal{L}_1 = \sqrt{g}\Bigg\{&\exp\left[-\frac{1}{2}\left(\frac{n}{n+2}\right)^{1/2}\sigma\right]\bar\psi i\gamma^{\hat\mu}\tilde\nabla_{\hat\mu}\psi - \exp\left(-\frac{n+1}{[n(n+2)]^{1/2}}\sigma\right)\bar\psi\gamma^{\hat a}\tilde\nabla_{\hat a}\psi \\ &+\frac{i}{4}\left(\frac{n}{n+2}\right)^{1/2}\exp\left[-\frac{1}{2}\left(\frac{n}{n+2}\right)^{1/2}\sigma\right](\partial_{\hat\mu}\sigma)\bar\psi\gamma^{\hat\mu}\psi \\ &-\frac{e\kappa}{4}\exp\left(\frac{1}{[n(n+2)]^{1/2}}\sigma\right)G^{\hat a}_{\hat\mu\hat\nu}\bar\psi(s^{\hat\mu\hat\nu}\otimes\gamma_{\hat a})\psi - \tfrac{1}{4}\exp\left[\left(\frac{n}{n+2}\right)^{1/2}\sigma\right]\tilde F^i_{\mu\nu}\tilde F^i_{\mu\nu} \\ &-\tfrac{1}{2}\exp\left(-\frac{2}{[n(n+2)]^{1/2}}\sigma\right)\rho^{ab}(\tilde D_\mu\phi^i_a)(\tilde D_\mu\phi^i_b) - \tfrac{1}{4}\exp\left(-\frac{n+4}{[n(n+2)]^{1/2}}\sigma\right)\rho^{ab}\rho^{cd}F^i_{ac}F^i_{bd}\Bigg\} \ ,\end{aligned} \tag{9}$$

where we have made the $e^{i(\pi/4)\gamma_5}$ rotation for the fermions[11] to make the mass matrix Hermitian. Then the Lagrangian

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_1 \tag{10}$$

should describe the dynamics of the internal gravitons σ, ρ_{ab}, and B_μ^a with the matter fields. Notice that even though the fermions are neutral under K (i.e., $\partial_a\psi=0$) they do couple to B_μ^a, first through $\tilde{A}_\mu^i$ and then through the $(4+n)$-dimensional general covariance. In fact, the Pauli-type term in (9) is precisely the coupling dictated by the general covariance. So the coupling constant e assumes the role of the "magnetic moment" of K for the fermions. Another interesting aspect of the Lagrangian (10) is that the dynamics of the internal metric ρ_{ab} is described by that of a generalized nonlinear σ model, with the minimal gauge coupling to B_μ^a and the self-interaction potential $\hat{R}_K$.

To discuss the vacuum let us choose $\sigma=0$ (by fixing the scale κ) and flat M. Then one may find $\rho_{ab}=\delta_{ab}$ with vanishing A_μ^i and B_μ^a as the vacuum of the theory under reasonable circumstances. For simplicity we will consider only two cases: (i) when $K=\mathrm{U}(1)$, and (ii) when K is compact semisimple and $K\subset G$. First, when $K=\mathrm{U}(1)$ one can easily show that $\Lambda=0$ with an *arbitrary* ϕ^i becomes the vacuum solution. In this case B_μ remains massless but A_μ^i has the mass matrix

$$m_{ij}=g^2 f_{ik}{}^m f_{jl}{}^m \phi^k\phi^l \ .$$

Thus G is broken, but the scale of the symmetry breaking is left completely arbitrary. For instance, when $G=\mathrm{SU}(2)$ the mass eigenvalues become $(0,g^2\phi^2,g^2\phi^2)$, so that SU(2) is broken down to U(1). The scale invariance of course implies the existence of a massless Higgs mode. Now, when K is compact semisimple and $K\subset G$, one may find

$$e^2=8g^2 \ ,$$
$$\phi_a^i=-\frac{1}{2g\kappa}\delta_a^i \ , \tag{11}$$
$$\Lambda=-\tfrac{1}{2}\hat{R}_K(0) \ ,$$

as the vacuum, where $\hat{R}_K(0)$ is the vacuum curvature of K. So in this case a nonvanishing Λ plays a crucial role for the existence of the vacuum. Another interesting aspect of the above solution is that the two coupling constants e and g are *not* independent even though the gauge symmetry is $K\times G$. The reason for this is that the vacuum value of ϕ_a^i depends on g, which in turn is fixed by the vacuum value of the internal curvature which depends on e. As for the symmetry breaking, notice that the mass matrix of A_μ^i and B_μ^a is given by

$$m_{ij}=\frac{1}{4\kappa^2}f_{ia}{}^k f_{jb}{}^l\delta_{ab}\delta_{kl} \ ,$$
$$m_{ab}=-\frac{1}{2\kappa^2}f_{ac}{}^d f_{bd}{}^c \ .$$

This means that K is completely broken while G is at least partially broken, with all the masses of the order of the Planck mass. For instance, for $K=\mathrm{SU}(2)$ and $G=\mathrm{SU}(3)$, G is broken down to U(1). With the above vacuum the fermions also acquire mass, with the mass matrix

$$m=-igI_4\otimes\gamma^a\phi_a^i t_i+\frac{1}{4\kappa}f_{ab}{}^c I_4\otimes s^{ab}\gamma_c \ .$$

Notice that the second term is induced by the nonvanishing curvature of the internal space, which is absent when $K=\mathrm{U}(1)$.

To discuss the dynamics further, let us first excite the dilaton from the vacuum. In this case one finds

$$\mathcal{L}_\sigma=-\frac{1}{16\pi G}[\tfrac{1}{2}(\partial_\mu\sigma)^2+V(\sigma)] \ ,$$
$$V(\sigma)=-\hat{R}_K(0)\exp\left[-\left(\frac{n+2}{n}\right)^{1/2}\sigma\right]\left[\tfrac{1}{2}\exp\left(\frac{2}{[n(n+2)]^{1/2}}\sigma\right)+\tfrac{1}{2}\exp\left(-\frac{2}{[n(n+2)]^{1/2}}\sigma\right)-1\right] \ . \tag{12}$$

So the dilaton acquires a mass

$$m_\sigma{}^2=V''(0)=-\frac{4}{n(n+2)}\hat{R}_K(0) \ .$$

Next, we excite the internal metric ρ_{ab} and find

$$\mathcal{L}_\rho=-\frac{1}{16\pi G}[\tfrac{1}{4}\rho^{ab}\rho^{cd}(\partial_\mu\rho_{ac})(\partial_\mu\rho_{bd})+V(\rho_{ab})+\lambda(\det\rho_{ab}-1)] \ , \tag{13}$$

where

$$V(\rho_{ab})=\hat{R}_K(\rho_{ab})+4\pi G\rho^{ab}\rho^{cd}F_{ac}^iF_{bd}^i+\Lambda$$
$$=\frac{1}{2\kappa^2}(f_{ac}{}^d f_{bd}{}^c\rho^{ab}+\tfrac{1}{2}f_{ac}{}^e f_{bd}{}^f\rho^{ab}\rho^{cd}\rho_{ef})+\frac{1}{8\kappa^2}f_{ac}{}^e f_{bd}{}^f\rho^{ab}\rho^{cd}\delta_{ef}+\Lambda \ .$$

So it must become clear that at least some of ρ_{ab} should also become massive when K becomes non-Abelian. In fact, when K is compact semisimple *all* the internal gravitons σ, ρ_{ab}, and B_μ^a acquire masses of the order of the Planck mass.

Now we are ready to discuss the possibility of a fifth force. When $K=\mathrm{U}(1)$, $\hat{R}_K$ must vanish so that both σ and B_μ remain massless. Furthermore, since B_μ couples to the fermion mass in the nonrelativistic limit,[2] B_μ generates an antigravity effect. However, as soon as K becomes compact semisimple all the internal gravitons acquire huge masses, which should make them totally irrelevant in any low-energy phenomenology. On the face of this one might try to keep at least some of the internal gravitons massless, which one can certainly do by assuming $K=\mathrm{U}(1)\times K'$, or by judiciously choosing a Ricci-flat vacuum for K. For instance, by choosing $K=\mathrm{E}_2$ (or $\mathrm{E}_2\times\mathrm{E}_2$) one could keep K Ricci flat.[7] In this case the dilaton and one (or two for

$K = E_2 \times E_2$) of B_μ^a remains massless. This would allow the dilaton to modify Newton's gravity. However, from (9) it must become clear that the massless B_μ^a will no longer couple to the fermion mass term when the dimension of the internal space becomes larger than one. So their antigravity effect will completely disappear. Actually even in the five-dimensional case our analysis shows that B_μ couples to the mass of the "heavy" fermions, because the desired coupling arises from the "first step" symmetry breaking where all the masses involved are of the order of the Planck mass. Thus it is not clear whether the antigravity coupling could also apply to the "light" fermions which are supposed to acquire mass in the "second step" symmetry breaking.

In conclusion, my analysis suggests that, for a very limited class of K, a non-Abelian generalization of the Kaluza-Klein unification could contain massless particles and generate a fifth force which could alter Newton's gravity. But it is highly unlikely that one could obtain an antigravity effect from the Kaluza-Klein gauge bosons. In fact, an antigravity effect could more likely come from a massless σ. Because of the unique coupling of σ with the matter fields shown in (9), a massless σ could generate an antigravity effect under certain circumstances.[12,13] As for the low-energy physics in general, our result could be interpreted as an encouraging aspect of the higher-dimensional unification, because it virtually guarantees that the low-energy phenomenology will not be altered significantly by the unification. However, two things should be kept in mind here. First, for a very limited class of K, some of the internal gravitons could remain massless (i.e., light) and thus become relevant to the low-energy physics. The other point is that the presence of the internal gravitons (especially the dilaton) will have a deep impact on cosmology.[12]

Finally, my analysis demonstrates how difficult it is to try to obtain the dimensional reduction by the "zero-mode" approximation. Notice that one might have liked to regard the conditions (2) and (6) as a zero-mode ansatz. However, as soon as the scale invariance is broken and the internal space is compactified by a Planck scale, it becomes very difficult to avoid a spontaneous symmetry breaking which will force some of these zero modes to become extremely heavy. Once such a symmetry breaking does occur, of course, one loses the whole justification of the zero-mode approximation.

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[11]Our notation is

$$g_{AB} = (-,+,+,\ldots,+),\ \{\Gamma_A,\Gamma_B\} = -2g_{AB}\ ,$$

$$\Gamma_\mu = \gamma_\mu \otimes I_k\ ,$$

$$\Gamma_a = \gamma_5 \otimes \gamma_a,\ \{\gamma_a,\gamma_b\} = -2\delta_{ab}\ ,$$

and

$$s_{ab} = \tfrac{1}{4}[\gamma_a,\gamma_b]\ .$$

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