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Study of correlations in fermionic matter

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We study dense quark matter at zero temperature to explore effects induced by short-range forces on fermionic correlations. Some results known earlier about Fermi liquids are generalized and extended.

INTRODUCTION

The study of Fermi liquids for fermion correlations induced by short-range interactions led to the discovery of collective states[1] that were observed subsequently.[2] Similar phenomenon in quark liquids were investigated, assuming Yukawa-type interactions, in the early 1970s (Ref. 3). The possibility of a phonon-type mode in such fluids has also been suggested.[4]

The impact of long-range forces in a plasma of quark and glue, studied in the weak-coupling, perturbative domain, suggests screening of "electrostatic" forces.[5] The glue self-interactions provide a further "magnetic" mass. The higher-loop effects, however, contribute to the same order as the lower ones, making perturbative results unreliable.[6,7]

As a result, the nature of the "deconfined" phase remains perturbatively inaccessible; the mass gap in the effective three-dimensional theory remains uncomputed.[8] One may assume when densities of quarks reach a few times their densities in nucleons, the wave functions overlap sufficiently for them to move about in this "deconfined" region.[9]

Nucleons, both mesons and baryons, have sufficient quark densities and overlaps that short-range effects, such as weak interactions, are important. This overlap leads many of them to decay in very short times. When densities a few times the nucleonic densities are reached, these short-range effects are likely to become even more significant. We study fermion correlations induced by weak-interaction effects in such Fermi liquids. As the liquid cools, the Pauli principle greatly limits possible collisions; modes that depend on too many collisions cease, and coherent states, like the ones we discuss, predominate. A generalization of earlier results[1] indicate possible modes that in the long-wavelength limit have dispersions that are phononlike. Other modes, structurally similar to the ones known before, yet different and new, exist in this liquid. The chemical potential in all these cases is sufficiently small compared to typical gauge-boson masses of W and Z.

FERMION CORRELATIONS

We compute fermion correlations, induced by weak interactions, in a plasma of quarks, at zero temperature, consisting of the up and the down quarks, both maintained at chemical potential μ. No significant antiparticle density is assumed.

The propagator for the fermions, up or down, is given by[10]

$$S_F^{u,d}=\frac{\not p}{2E_{\mathbf{p}}}\left[\frac{1-\theta(\mu-E_{\mathbf{p}})}{p_0-E_{\mathbf{p}}+i\epsilon}+\frac{\theta(\mu-E_{\mathbf{p}})}{p_0-E_{\mathbf{p}}-i\epsilon}-\frac{1}{p_0+E_{\mathbf{p}}-i\epsilon}\right],$$
$$E_{\mathbf{p}}{}^2=\mathbf{p}^2+m^2\ . \tag{1}$$

The superscripts u and d denote flavor. The three terms in large parentheses are going to be denoted by P, H, and A, respectively, for reasons described below.

Particles have momenta lying above the Fermi surface, which at zero temperature is characterized by the chemical potential μ; holes lie below it. The first P, the second H, and the third A terms of Eq. (1) denote propagation of particles, holes, and antiparticles, respectively; numerators characterize the zero-temperature limit of the usual Fermi-Dirac distribution function. Note that the ground state has all the levels up to the Fermi surface filled with particles.

To study fermionic correlations induced by the weak vector boson W, we study the self-energy diagram given in Fig. 1; self-energy is denoted Π. The diagram corresponds to evaluating an integral of the type

$$\int S_F^u(q)S_F^d(p+q)d^4q\ . \tag{2}$$

Terms of the type PP, HH, or AA go to zero; poles lie on one side of the q_0 axis, and one may choose to close the q_0 contour excluding term. The usual zero-density limit is obtained when PA-type terms are considered. At density characterized by μ, the kinematical region of our inves-

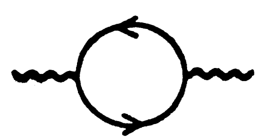

FIG. 1. The self-energy of the W boson due to matter fluctuations. The wavy line corresponds to the spin-one boson, and fermions of different flavor propagate in the loop.

tigation is confined to almost zero external energy and momentum. In this $\mu \gg p_0, |\mathbf{p}|$ limit, the PH and HA are dominant, and the particle hole [PH], and also [HA] correlations are evaluated explicitly.

Before proceeding further, it is convenient to rewrite the propagator, using

$$\frac{1}{\omega \pm i\eta} = \mathscr{P}\left[\frac{1}{\omega}\right] \mp i\pi\delta(\omega) , \tag{3}$$

as

$$S_F^{u,d} = \frac{\not p}{2E_{\mathbf{p}}}\left[\frac{1}{p_0 - E_{\mathbf{p}} + i\epsilon} + 2\pi i \delta(p_0 - E_{\mathbf{p}})\theta(\mu - E_{\mathbf{p}}) - \frac{1}{p_0 + E_{\mathbf{p}} - i\epsilon}\right] . \tag{4}$$

We continue to denote the three terms as P, H, and A, even though there is now a lack of precision in this usage. Collecting HA terms we get, for the self-energy Π,

$$\Pi_{\alpha\nu}^{HA} = -\frac{g^2}{32}\int \frac{d^4q}{(2\pi)^3}\frac{\mathrm{Tr}[\gamma_\alpha(1+\gamma_5)(\not p + \not q)\gamma_\nu(1+\gamma_5)\not q]\delta(p_0+q_0-E_{\mathbf{p+q}})\theta(\mu - E_{\mathbf{p+q}})}{E_{\mathbf{p+q}}E_{\mathbf{q}}(q_0+E_{\mathbf{q}})}$$
$$-\frac{g^2}{32}\int \frac{d^4q}{(2\pi)^3}\frac{\mathrm{Tr}[\gamma_\alpha(1+\gamma_5)(\not p + \not q)\gamma_\nu(1+\gamma_5)\not q]\delta(q_0-E_{\mathbf{q}})\theta(\mu - E_{\mathbf{q}})}{E_{\mathbf{p+q}}E_{\mathbf{q}}(p_0+q_0+E_{\mathbf{p+q}})} . \tag{5}$$

Note that the masses of the up and the down quarks have been set equal to zero. We do not know how good this approximation is; we choose to adopt it purely for convenience. We know, however, that inclusion of mass does not alter our conclusions qualitatively.

The two terms above, in Eq. (5), are denoted (I) and (II), respectively. Carrying out the q_0 integration in (II) and evaluating the trace, we get

$$(\mathrm{II})_{00} = -\frac{g^2}{16}\int \frac{d^3q}{(2\pi)^3}\frac{(8\mathbf{q}^2 + 4p_0|\mathbf{q}| + 4|\mathbf{p}||\mathbf{q}|\cos\theta)\theta(\mu - E_{\mathbf{q}})}{E_{\mathbf{p+q}}E_{\mathbf{q}}(p_0 + |\mathbf{q}| + E_{\mathbf{p+q}})} . \tag{6}$$

As far as (I) is concerned, if we change variables

$$q \to -q - p , \tag{7}$$

carry out the q_0 integration, and evaluate the trace, we get

$$(\mathrm{I})_{00} = -\frac{g^2}{16}\int \frac{d^3q}{(2\pi)^3}\frac{(8\mathbf{q}^2 - 4p_0|\mathbf{q}| + 4|\mathbf{p}||\mathbf{q}|\cos\theta)(\mu - E_{\mathbf{q}})}{E_{\mathbf{p+q}}E_{\mathbf{q}}(-p_0 + |\mathbf{q}| + E_{\mathbf{p+q}})} . \tag{8}$$

Thus, (I) + (II) is symmetric with respect to p_0, and thus only positive p_0 need to be studied. The rest of the integration may be done analytically. Similar results hold true for particle-hole (PH) correlation; it is a symmetric function of p_0, and the integrations are straightforward. This symmetry in p_0 is destroyed if the down- and up-quark chemical potentials are different.

The whole kinematical region is not of interest; by experience with similar computations in the past,[1] we specialize to the following domain: the limit $p_0 \to 0$, $|\mathbf{p}| \to 0$, such that $p_0/|\mathbf{p}|$ is held to some value X. Thus, the chemical potential is much larger than both p_0 and $|\mathbf{p}|$; the correlation is dominated by PH and HA terms. In this limit Π_{00} becomes

$$\Pi_{00} = \frac{g^2\mu^2}{8\pi^2}X\ln\left|\frac{p_0+|\mathbf{p}|}{p_0-|\mathbf{p}|}\right| - \frac{g^2\mu^2}{6\pi^2} . \tag{9}$$

The procedure repeats for the other components of $\Pi_{\alpha\nu}$, and it is sufficient to consider only one more, say Π_{zz}, which becomes

$$\Pi_{zz} = \frac{g^2\mu^2}{8\pi^2}X^3\ln\left|\frac{p_0+|\mathbf{p}|}{p_0-|\mathbf{p}|}\right| - \frac{g^2\mu^2}{6\pi^2}X^2 . \tag{10}$$

The propagator of the W boson, denoted by $D_{\alpha\nu}$ in the 't Hooft-Feynman gauge, in the "chain" approximation is

$$D_{\alpha\nu} = D_{\alpha\nu}^0 + D_{\alpha\beta}^0\Pi_{\beta\delta}D_{\delta\nu} \tag{11}$$

with

$$D_{\alpha\nu}^0 = -\frac{g_{\alpha\nu}}{k^2 - M_W{}^2} . \tag{12}$$

Thus, from Eq. (11) we get

$$D_{00} = -\frac{1}{k^2 - M_W{}^2 + \Pi_{00}} \tag{13}$$

and

$$D_{zz} = \frac{1}{k^2 - M_W{}^2 - \Pi_{zz}} . \tag{14}$$

It is sufficient that p_0 and $|\mathbf{p}|$ be small compared to μ and not get close to the mass shell. The relevant dispersion relations are obtained when the denominator of the above two equations are set to zero, and solved. Thus, from Eq. (13), we get

$$k^2 - M_W{}^2 + \Pi_{00} = 0 . \tag{15}$$

Note we are interested in the case when the chemical potential is significantly smaller than the mass of the W boson. The other case, when μ is comparable or greater, will lead to symmetry restoration, and has been dealt with by others.[5] It is also important to note that only $X>1$ leads to stable modes, as the imaginary part of Π goes like

$$X\theta(1-|X|)\ . \tag{16}$$

A solution to Eq. (15), under the conditions listed above, in the kinematical region of interest, is found by assuming a long-wavelength dispersion of the type $p_0=C\,|\mathbf{p}|$. We get

$$p_0=\left\{1+2\exp\left[-\left[\frac{8\pi^2 M_W{}^2}{\mu^2 g^2}+\frac{4}{3}\right]\right]\right\}|\mathbf{p}| \tag{17}$$

which are phononlike modes. What is interesting is that the other components of Π yield similar results. Considering Eq. (14), when μ is small compared to M_W, we get the following dispersion:

$$p_0=\frac{\sqrt{6}\pi M_W}{g\mu}|\mathbf{p}|\ . \tag{18}$$

Note that as M_W goes to zero, as would happen in the symmetric phase, this mode will cease to exist. A mode of this type in the symmetric phase, with a somewhat different dispersion, has been obtained earlier.[11] In this sense, these modes, which in the long-wavelength limit are phononlike (different slope than the longitudinal modes), are interesting, and different from previous results.

It is worth noting that these modes of paired quarks and holes of different flavor are going to carry quantum numbers such as isospin, etc.

It is important to understand what effect the strong interactions—the dominant electron—will have on these modes. The effect of exchange of gluons by quarks and holes proceeds as in the case of long-range correlations,[5] and just as in that case are unlikely to destabilize these states. The strong interaction of the quark-hole color interaction is attractive and overwhelms the repulsive nature of the electromagnetic interactions in the u-quark, d-hole channel. Experience with similar computations in the past indicate, when higher loops and exchange are considered, that[12,7] (i) some of our results are likely to become gauge dependent and (ii) the dispersion relations are likely to be altered.

As far as the dispersion relations are concerned, the impact of strong-interaction corrections, in analogy with previous calculations, are likely to change the coefficient C.

DISCUSSIONS

We have presented a study of Fermi liquids in the presence of a short-range interaction, mediated by a spin-one particle. While we have chosen to present the computation in the setting of quark liquids, the results are in no way confined to this system alone. One could equally well apply these results to, for example, neutrino matter in galactic halos interacting via exchange of the Z boson. While the longitudinal mode is a generalization of earlier work,[1] the other modes have not been dealt with before. Some changes are likely to come about in finite-density computations because of their presence.

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[1] L. D. Landau, Zh. Eksp. Teor. Fiz. **32**, 59 (1957) [Sov. Phys. JETP **5**, 101 (1957)].

[2] W. R. Abel, A. C. Anderson, and J. C. Wheatley, Phys. Rev. Lett. **17**, 74 (1966); B. E. Keen, P. W. Matthews, and J. Wilks, Proc. R. Soc. London **A284**, 125 (1965).

[3] P. Carruthers, Collect. Phenom. **1**, 147 (1973).

[4] P. Carruthers, Phys. Rev. Lett. **50**, 1179 (1983).

[5] D. Pines, *Many-Body Problems* (Benjamin, New York, 1962); E. Fradkin, *Quantum Field Theory and Hydrodynamics* (Consultants Bureau, New York, 1967); M. B. Kislinger and P. D. Morley, Phys. Rev. D **13**, 2765 (1976); E. Shuryak, Phys. Rep. **61**, 71 (1980); D. J. Gross, R. D. Pisarski, and L. G. Yaffe, Rev. Mod. Phys. **53**, 43 (1981). See the above two papers for further references.

[6] A. Linde, Rep. Prog. Phys. **42**, 389 (1979). Gross, Piarski, and Yaffe (Ref. 5).

[7] S. Nadkarni, Phys. Rev. D **27**, 917 (1983); K. Kajantie and J. Kapusta, Ann. Phys. (N.Y.) **160**, 477 (1985); T. Toimela, Z. Phys. C **27**, 289 (1985); L. D. McLerran and B. Svetitsky, Phys. Rev. D **24**, 450 (1981); T. Appelquist and R. D. Pisarski, *ibid.* **23**, 2305 (1981); R. Anishetty, J. Phys. G **10**, 423 (1984); **10**, 439 (1984); J. Polonyi and H. W. Wyld, University of Illinois Report No. ILL-(TH)-85-23, 1985 (unpublished).

[8] A. Billoire, G. Lazarides, and Q. Shafi, Phys. Lett. **103B**, 450 (1981); T. A. DeGrand and D. Toussaint, Phys. Rev. D **25**, 526 (1982); G. Lazarides and S. Sarantakos, *ibid.* **31**, 389 (1985).

[9] J. Engels, F. Karsch, H. Satz, and I. Montvay, Nucl. Phys. **B205**, 545 (1982); L. McLerran and B. Svetitsky, Phys. Lett. **98B**, 195 (1981); J. Kogut, M. Stone, H. Wyld, W. Gibbs, J. Shigemitsu, S. Shenker, and D. Sinclair, Phys. Rev. Lett. **50**, 393 (1983).

[10] R. L. Bowers and R. L. Zimmerman, Phys. Rev. D **7**, 296 (1973).

[11] Shuryak (Ref. 5).

[12] A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).