Heterotic strings from the bosonic string in 26 dimensions

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We construct a tachyonless string model from the purely bosonic 26-dimensional string model, without using any projection which excludes the unwanted states. The spectrum of this model is equivalent to that of the heterotic string model proposed by Gross *et al.*, and we show that the one-loop vacuum amplitude vanishes. 16 dimensions have to be compactified leaving 8 uncompactified transverse dimensions. Since the 8 fermion coordinates in the superstring sector are identified with only 4 string coordinates in the compactified 16 dimensions, the residual 12 string coordinates should decouple for consistency. We propose a mechanism for this decoupling to occur: the signature of the metric of the 16-dimensional space should change. Unitarity is, however, restored by imposing some state condition. We show that this decoupling really occurs in the one-loop amplitude.

I. INTRODUCTION

Recently Freund has suggested that the superstring theories with gauge groups SO(32) or $E_8 \times E_8$ might arise as "soliton-type" solutions of 26-dimensional bosonic string theory.[1] Casher, Englert, Nicolai, and Taormina further proposed that all the closed-superstring theories are contained in the bosonic theory.[2] A more detailed analysis is given in Ref. 3. In their scenario, the 16 dimensions among 24 transverse dimensions of bosonic theory have to be compactified, leaving 8 uncompactified dimensions, in such a way that the internal-symmetry group G resulting from the compactification contains an internal group $\widetilde{SO}(8)$, the covering group of SO(8). Furthermore, the rotational group SO(8) in 8 uncompactified transverse dimensions must be mapped onto the $\widetilde{SO}(8)$ so that the diagonal subgroup $SO(8)_{diag}$ of $SO(8)\otimes\widetilde{SO}(8)$ becomes identified with a new transverse subgroup of the 10-dimensional Lorentz group. In this way, the spinor representation of $\widetilde{SO}(8)$ describes fermionic states because a rotation in space induces a half-angle rotation on these states.

However, there exist several problems to be solved. First, in order that we regard $SO(8)_{diag}$ as the subgroup of the 10-dimensional Lorentz group, a mechanism is needed by which $SO(8)\otimes\widetilde{SO}(8)$ breaks down to $SO(8)_{diag}$ since $\widetilde{SO}(8)$ has originally nothing to do with the 10-dimensional Lorentz group. In the case of superstring field theory in the light-cone gauge, the free part of the action has an SO(8) symmetry in the bosonic coordinate sector and another independent SO(8) symmetry in the fermionic coordinate sector. However, the interaction vertex breaks this $SO(8)\otimes SO(8)$ to its diagonal subgroup $SO(8)_{diag}$ and we can regard these fermionic coordinates as spinors in 10-dimensional space-time.[4–6] Therefore we suspect that the interaction vertex in the bosonic string field theory, if it generates superstrings, must be modified to induce the breaking of $SO(8)\otimes\widetilde{SO}(8)$ symmetry. This mechanism may be described by the condensation of string fields.

Englert, Nicolai, and Shellekens[3] have been mainly concerned with string compactification on the even self-dual Euclidean lattice by requiring modular invariance of the closed-bosonic-string one-loop amplitude. We should, however, remark that the modular invariance of the bosonic string has no relation with that of the induced superstring. It is because the trace over the states corresponding to the odd-fermion-number sector in the superstring, which should be space-time fermions, never has the relative minus sign in the usual one-loop calculation of the bosonic string. This minus sign is of course necessary for the vanishing of the cosmological term or supersymmetry.

There is another important problem. Since we need only four bosonic operators to generate $\widetilde{SO}(8)$, the role of the residual 12 string coordinates in the compactified dimensions is not obvious. Casher, Englert, Nicolai, and Taormina have expected that these extra coordinates might be understood to be the superghost and the unphysical Majorana fermions on the basis of the counting argument of the conformal anomaly.[2] If this scenario were true, these coordinates must decouple from the physical transverse state and the norm of the states containing the excitations of those coordinate modes should cancel.[7] The cancellation of their norm is necessary for consistent decoupling at the loop level.[8] In the previous paper,[9] we proposed a mechanism how these coordinates decouple. We expect that the signature of the metric changes and antiperiodic negative-norm bosons appear. The appearance of negative-norm particles is potentially dangerous since they may break unitarity. We have shown, however, that some state condition restores the unitarity.

The purpose of this paper is to construct, based on the previous paper, a purely bosonic string model from which the heterotic string model[10,11] is generated. The model does not contain a tachyon mode due to the condition that there is no distinguished point on a closed string and the spectrum is equivalent to that of the usual heterotic string. It should be noted that we need not use any projection as was done in Ref. 3 to obtain a tachyonless theory. We further calculate the one-loop vacuum amplitude of the model and we find that it vanishes. Furthermore we show that the 12 unphysical string coordinates really decouple in the one-loop amplitude with vertices which are properly defined.

The rest of this paper is organized as follows. In the next section we describe the correspondence between bosons and fermions based on the work by Eguchi and Higashijima[12] and after that we discuss the negative-norm bosons. In Sec. III we review a mechanism proposed in the previous paper,[9] explain how 12 string coordinates decouple, and discuss the condition to restore the unitarity. In Sec. IV a tachyonless bosonic string model is proposed. The closure of 10-dimensional Lorentz algebra is easily shown. In Sec. V the one-loop amplitude is calculated. The last section is devoted to the discussion of further problems including higher-loop calculation.

II. THE CORRESPONDENCE BETWEEN BOSONS AND FERMIONS

The Bose field $\phi(\theta)$ obeying the periodic boundary condition $\phi(\theta+2\pi)=\phi(\theta)$ is expanded into oscillator modes as

$$\phi(\theta)=q+p\theta+i\sum_{n\neq 0}\frac{a_n}{n}e^{-in\theta}\,. \tag{2.1}$$

The commutation relations of the mode variables are given by

$$[q,p]=i,\quad [a_m,a_n]=m\delta_{m,-n}\,. \tag{2.2}$$

The correspondence between bosons and fermions can be understood by examining the partition functions $Z=\mathrm{tr}e^{-\beta H}$, with H being the Hamiltonian:

$$H=\tfrac{1}{2}p^2+\sum_{n=1}^{\infty}a_{-n}a_n\,. \tag{2.3}$$

The partition functions are given, in the case $p=$integer, by

$$\begin{aligned} Z_I&=\frac{1}{\prod_{n=1}^{\infty}(1-q^{2n})}\sum_{n=-\infty}^{\infty}q^{n^2}\\ &=\prod_{n=1}^{\infty}(1+q^{2n-1})^2\,, \end{aligned} \tag{2.4}$$

and in the case $p=$half-integer, by

$$\begin{aligned} Z_H&=\frac{1}{\prod_{n=1}^{\infty}(1-q^{2n})}\sum_{n=-\infty}^{\infty}q^{(n-1/2)^2}\\ &=2q^{1/4}\prod_{n=1}^{\infty}(1+q^{2n})^2\,. \end{aligned} \tag{2.5}$$

Here $q^2=e^{-\beta}$ and we have used the identities

$$\begin{aligned} \sum_{n=-\infty}^{\infty}q^{n^2}&=\prod_{n=1}^{\infty}(1-q^{2n})\prod_{n=1}^{\infty}(1+q^{2n-1})^2\,,\\ \sum_{n=-\infty}^{\infty}q^{(n-1/2)^2}&=2q^{1/4}\prod_{n=1}^{\infty}(1-q^{2n})\prod_{n=1}^{\infty}(1+q^{2n})^2\,. \end{aligned} \tag{2.6}$$

The last expression in Eq. (2.4) [Eq. (2.5)] just coincides with the partition function of two antiperiodic [periodic] fermions. These relations mean the equivalence of the boson system to the fermion system. We obtain two real Fermi fields ψ^1 and ψ^2 from a Bose field ϕ via

$$\psi^1+i\psi^2=\sqrt{2}{:}e^{i\phi}{:}\,. \tag{2.7}$$

Here : : means normal ordering of the boson mode variables.

When the Bose field obeys the antiperiodic boundary condition $\phi(\theta+2\pi)=-\phi(\theta)$, on the other hand, the mode expansion is given by

$$\phi(\theta)=i\sum_{n=\otimes\text{-integer}}\frac{a_n}{n}e^{-in\theta}\,, \tag{2.8}$$

and the Hamiltonian reads

$$H=\sum_{n=1/2}^{\infty}a_{-n}a_n\,. \tag{2.9}$$

In this case the partition function becomes

$$\begin{aligned} Z_A&=\frac{1}{\prod_{n=1}^{\infty}(1-q^{2n-1})}\\ &=\prod_{n=1}^{\infty}(1+q^{2n})\prod_{m=1}^{\infty}(1+q^{2m-1})\,, \end{aligned} \tag{2.10}$$

which tells us that this system is equivalent to that consisting of one periodic fermion and one antiperiodic fermion.

We now turn to discuss the *negative-norm* bosons. The Hamiltonian is given by

$$H=-\tfrac{1}{2}p^2-\sum_{n=1}^{\infty}a_{-n}a_n\,, \tag{2.11}$$

in the period case $\phi(\theta+2\pi)=\phi(\theta)$ and

$$H=-\sum_{n=1/2}^{\infty}a_{-n}a_n \tag{2.12}$$

in the antiperiodic case $\phi(\theta+2\pi)=-\phi(\theta)$. The mode variables satisfy the commutation relations

$$[q,p]=-i,\quad [a_m,a_n]=-m\delta_{m,-n}\,. \tag{2.13}$$

From Eq. (2.11), however, we recognize that periodic negative-norm bosons are ill defined since the Hamiltonian is unbounded below owing to the contribution of the zero mode p (Ref. 13). Therefore we consider only antiperiodic bosons henceforth. The partition function of a negative-norm boson with an antiperiodic boundary condition is given by

$$\begin{aligned} Z_{A-}&=\frac{1}{\prod_{n=1}^{\infty}(1+q^{2n-1})}\\ &=\prod_{n=1}^{\infty}(1+q^{2n})\prod_{m=1}^{\infty}(1-q^{2m-1})\,. \end{aligned} \tag{2.14}$$

Note that the states corresponding to the excitation of an odd number of modes have a negative norm. From Eq. (2.14) we see that this system is equivalent to the combination of one periodic positive-norm fermion ψ_{P+} and one antiperiodic negative-norm fermion ψ_{A-}. We can express these Fermi fields in terms of Bose fields:

$$\begin{aligned}\psi_{P+}&=\frac{1}{\sqrt{2}}(:e^{\phi}:+:e^{-\phi}:)\ ,\\ \psi_{A-}&=\frac{1}{\sqrt{2}}(:e^{\phi}:-:e^{-\phi}:)\ .\end{aligned}\tag{2.15}$$

III. A MECHANISM OF THE DECOUPLING

In this section we explain how the extra 12 coordinates can decouple from the physical state. (The definition of physical state will be given later.) The norm of the states corresponding to the excitation of the modes of these coordinates must cancel for the consistent decoupling. Since the cancellation of the norm means that the product of partition functions is trivial, we examine Eqs. (2.4), (2.5), (2.10), and (2.14) and look for a combination for which such a cancellation occurs. We find that a combination of one integer-p positive-norm boson and two antiperiodic negative-norm bosons gives such an example:

$$\begin{aligned}Z_I(Z_{A-})^2&=\prod_{n=1}^{\infty}(1+q^{2n-1})^2\left[\frac{1}{\prod_{m=1}^{\infty}(1+q^{2m-1})}\right]^2\\ &=1\ .\end{aligned}\tag{3.1}$$

Therefore if we identify the 12 string coordinates with 4 sets of this combination so as to realize their decoupling, the metric of 8 dimensions out of 12 dimensions should be taken negative. The antiperiodicity may be realized by putting the string on a fixed point in some orbifold, e.g., $K3\otimes K3$ (Refs. 14—16).

The appearance of negative-norm particles is potentially dangerous since they may break unitarity. Some physical state condition should be imposed to restore unitarity as in case of gauge theories.[17]

Since we find from Eq. (3.1) that the product of the partition functions of one antiperiodic fermion which originates from the integer-p boson and of one antiperiodic negative-norm boson is unity, we consider an equivalent system whose Lagrangian is given by

$$\mathscr{L}=-\partial_+A\partial_-A+i\psi_L\partial_+\psi_L+i\psi_R\partial_-\psi_R\ .\tag{3.2}$$

Here A is identified with the antiperiodic negative-norm boson coordinate and ψ_L (ψ_R) is a left- (right-) moving antiperiodic fermion which is bosonized into the left- (right-) moving integer-p coordinate in Eq. (4.7) of the next section. This system is invariant under the supersymmetry transformations,

$$\begin{aligned}\delta A&=-i\epsilon_L\psi_L-i\epsilon_R\psi_R\ ,\\ \delta\psi_L&=-\epsilon_L\partial_-A\ ,\\ \delta\psi_R&=-\epsilon_R\partial_+A\ ,\end{aligned}\tag{3.3}$$

and the supercharges are given by

$$\begin{aligned}Q_L&=i\int d\sigma\,\psi_L\partial_-A\ ,\\ Q_R&=i\int d\sigma\,\psi_R\partial_+A\ .\end{aligned}\tag{3.4}$$

Q_L (Q_R) is composed of only a left- (right-) moving part.

We demand that the physical states be annihilated by these superchanges. Since the ground state is not degenerate due to the antiperiodic boundary condition, the unique state annihilated by both of Q_L and Q_R is the vacuum and this condition excludes the states corresponding to the excitation of these unphysical modes.

IV. CONSTRUCTION OF A MODEL WITHOUT TACHYON

We start with the light-cone gauge action

$$\begin{aligned}S=\frac{1}{4\pi\alpha'}\int d\tau\int_0^{\pi}d\sigma(&\partial_\alpha X^i\partial^\alpha X^i+\eta^R_{IJ}\partial_\alpha X^I_R\partial^\alpha X^J_R\\ &+\eta^L_{IJ}\partial_\alpha X^I_L\partial^\alpha X^J_L)\ ,\\ \alpha=1,2,\ \ i=1,\dots,8,&\ \ I,J=1,\dots,16\ .\end{aligned}\tag{4.1}$$

Here η^R_{IJ} (η^L_{IJ}) is a metric tensor in the right- (left-) moving 16-dimensional space. We impose a constraint restricting X^I_R (X^I_L) to consist of the right- (left-) moving modes alone. This can be achieved by demanding that

$$\begin{aligned}(\partial_\tau+\partial_\sigma)X^I_R&=0\ ,\\ (\partial_\tau-\partial_\sigma)X^I_L&=0\ .\end{aligned}\tag{4.2}$$

These constraints (4.2) assure the complete separation of a left-moving and a right-moving part in the string coordinate. The left-right separation given by Eqs. (4.1) and (4.2) might be equivalent to the usual one which is realized by introducing a "winding vector" in the torus compactified string coordinates,[10,3,18] but should be more comprehensible when the left and right sectors are compactified on the different manifold.

The canonical commutation relations are given by

$$\frac{1}{2\pi\alpha'}[X^i(\sigma,\tau),\partial_\tau X^j(\sigma',\tau)]=i\delta(\sigma-\sigma')\delta^{ij}\tag{4.3}$$

and

$$\begin{aligned}\frac{1}{2\pi\alpha'}[X^I_R(\tau-\sigma),\partial_\tau X^J_R(\tau-\sigma')]&=\frac{i}{2}\delta(\sigma-\sigma')\eta^{IJ}_R\ ,\\ \frac{1}{2\pi\alpha'}[X^I_L(\tau+\sigma),\partial_\tau X^J_L(\tau+\sigma')]&=\frac{i}{2}\delta(\sigma-\sigma')\eta^{IJ}_L\ .\end{aligned}\tag{4.4}$$

The factor of $\frac{1}{2}$ on the right-hand side (RHS) of Eq. (4.4) arises due to the constraint (4.2) (Ref. 10).

X^i can be expanded as

$$\sqrt{\alpha'}X^i = x^i + p^i\tau + \frac{i}{2}\sum_{n\neq 0}\left[\frac{\alpha^i_{Rn}}{n}e^{-2in(\tau-\sigma)} + \frac{\alpha^i_{Ln}}{n}e^{-2in(\tau+\sigma)}\right] \qquad (4.5)$$

and we obtain from Eq. (4.3) the commutation relations for the mode variables:

$$[x^i,p^j]=i\delta^{ij},$$
$$[\alpha^i_{Rn},\alpha^j_{Rm}]=[\alpha^i_{Ln},\alpha^j_{Lm}]=n\delta_{n+m,0}\delta^{ij}, \qquad (4.6)$$
$$\text{others}=0 .$$

X^I_R and X^I_L have the expansion

$$\sqrt{\alpha'}X^I_R = \tfrac{1}{2}x^I_R + p^I_R(\tau-\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{\alpha^I_{Rn}}{n}e^{-2in(\tau-\sigma)},$$
$$\sqrt{\alpha'}X^I_L = \tfrac{1}{2}x^I_L + p^I_L(\tau+\sigma) + \frac{i}{2}\sum_{n\neq 0}\frac{\alpha^I_{Ln}}{n}e^{-2in(\tau+\sigma)}, \qquad (4.7)$$

under the periodic boundary condition $X^I(\sigma+\pi)=X^I(\sigma)$ and

$$X^I_R = \tfrac{1}{2}\sum_{n=\text{half-integer}}\frac{\alpha^I_{Rn}}{n}e^{-2in(\tau-\sigma)},$$
$$X^I_L = \tfrac{1}{2}\sum_{n=\text{half-integer}}\frac{\alpha^I_{Ln}}{n}e^{-2in(\tau+\sigma)}, \qquad (4.8)$$

under the antiperiodic boundary condition $X^I(\sigma+\pi) = -X^I(\sigma)$. Here mode variables satisfy the following commutation relations:

$$[\alpha^I_{Rn},\alpha^J_{Rm}]=n\delta_{n+m,0}\eta^{IJ}_R,\quad [\alpha^I_{Ln},\alpha^J_{Lm}]=n\delta_{n+m,0}\eta^{IJ}_L, \qquad (4.9)$$
$$[x^I_R,p^J_R]=i\eta^{IJ}_R,\quad [x^I_L,p^J_L]=i\eta^{IJ}_L .$$

In order to generate heterotic superstring, 16 dimensions have to be compactified, leaving 8 uncompactified dimensions. The left-moving string coordinates X^I_L are compactified on the even self-dual lattice, which corresponds to usual heterotic string theory.[10,11] 12 right-moving string coordinates decouple by the mechanism proposed in the previous section. The remaining 4 right-moving string coordinates are identified with the bosonized 8 fermion coordinates via

$$\psi^{2I-1}+i\psi^{2I}=\sqrt{2}C^I{:}e^{i\tilde{X}^I_R}{:}\quad (I=1,\ldots,4) . \qquad (4.10)$$

Here $\tilde{X}^I_R$ is defined as

$$\tilde{X}^I_R = 2\sqrt{\alpha'}X^I_R \qquad (4.11)$$

and it corresponds to the Bose field ϕ in Eq. (2.1) with $\theta=2(\tau-\sigma)$. C^I is called a twist factor which is introduced to let fermions with different indices $I\neq J$ anticommute.[12] It is given by

$$C^I = \exp\left[\frac{i\pi}{2}\left[\sum_{K<I}-\sum_{K>I}\right]p^K_R\right] . \qquad (4.12)$$

We use a formulation analogous to the old superstring[19] in order to generate a right-moving superstring sector. We need two string fields: one corresponding to the Ramond[20] sector and another corresponding to the Neveu-Schwarz[21] sector.

The states in the Ramond sector in the superstring belong to the SO(8) spinor conjugation class[3,12] and their weight vector w_s is given by

$$w_s = \sum_{i=1}^{4} n_i s_i,\quad \sum_{i=1}^{4} n_i = \text{odd} . \qquad (4.13)$$

Here

$$s_1=(\tfrac12\ \tfrac12\ \tfrac12\ \tfrac12),$$
$$s_2=(\tfrac12\ \tfrac12\ -\tfrac12\ -\tfrac12),$$
$$s_3=(\tfrac12\ -\tfrac12\ \tfrac12\ -\tfrac12),$$
$$s_4=(\tfrac12\ -\tfrac12\ -\tfrac12\ \tfrac12) . \qquad (4.14)$$

On the other hand, the states in the Neveu-Schwarz sector belong to the SO(8) vector conjugation class[3,12] and their weight vector w_v is given by

$$w_v = \sum_{i=1}^{4} n_i v_i,\quad \sum_{i=1}^{4} n_i = \text{odd} , \qquad (4.15)$$

where

$$v_1=(1\ 0\ 0\ 0),$$
$$v_2=(0\ 1\ 0\ 0),$$
$$v_3=(0\ 0\ 1\ 0),$$
$$v_4=(0\ 0\ 0\ 1) . \qquad (4.16)$$

In order to generate the superstring, 4 right-moving string coordinates in one string field are compactified on a weight lattice which is given by

$$w_R = \sum_{i=1}^{4} n_i s_i \qquad (4.17)$$

and 4 coordinates in another string field are compactified on another weight lattice,

$$w_{NS} = \sum_{i=1}^{4} n_i v_i . \qquad (4.18)$$

Since we do not impose any restriction like $\sum_{i=1}^4 n_i=\text{odd}$ in Eq. (4.13) or (4.15), the compactification on the weight lattice in (4.17) or (4.18) is realized by usual torus compactification.

In the following we explain how only the $\sum_{i=1}^4 n_i=\text{odd}$ sector is realized in the "physical" space defined in the previous section.

Since there is no distinguished point on a closed string, we are free to shift the origin of the σ coordinate by an arbitrary amount. This gives a constraint

$$M_L - 1 = M_R + \overline{M}_R - \tfrac{1}{2} . \qquad (4.19)$$

Here M_L, $\overline{M}_R$, and M_R are the normal-ordered mass operators for the left movers, the decoupled 12 right movers, and the remaining right movers:

$$M_L=\sum_{n=1}^{\infty}\left[\sum_{i=1}^{8}\alpha^i_{L-n}\alpha^i_{Ln}+\sum_{I=1}^{16}\alpha^I_{L-n}\alpha^I_{Ln}\right]+\frac{1}{2}\sum_{I=1}^{16}(p^I_L)^2\,, \tag{4.20}$$

$$\overline{M}_R=\sum_{n=1}^{\infty}\sum_{I=5}^{8}\alpha^I_{R-n}\alpha^I_{Rn}+\frac{1}{2}\sum_{I=5}^{8}(p^I_R)^2-\sum_{n=1/2}^{\infty}\sum_{I=9}^{16}\alpha^I_{R-n}\alpha^I_{Rn}\,, \tag{4.21}$$

$$M_R=\sum_{n=1}^{\infty}\left[\sum_{i=1}^{8}\alpha^i_{R-n}\alpha^i_{Rn}+\sum_{I=1}^{4}\alpha^I_{R-n}\alpha^I_{Rn}\right]+\frac{1}{2}\sum_{I=1}^{4}(p^I_R)^2\,. \tag{4.22}$$

Note that $\overline{M}_R$ vanishes in the "physical" space which is annihilated by Q_R of Eq. (3.4). The zero intercept -1 ($-\frac{1}{2}$) in the LHS (RHS) of Eq. (4.19) is due to the normal ordering of M_L (M_R and $\overline{M}_R$). Note that the contribution of one boson to the zero intercept is $-\frac{1}{24}$ if the boundary condition is periodic, and $\frac{1}{48}$ if antiperiodic.[14,22] Since we have 24 periodic string coordinates in the left-moving sector and 16 periodic string coordinates and 8 antiperiodic ones in the right-moving sector, the zero intercept in the left-moving sector is given by

$$24\times(-\tfrac{1}{24})=-1 \tag{4.23}$$

and that in the right-moving sector is given by

$$16\times(-\tfrac{1}{24})+8\times\tfrac{1}{48}=-\tfrac{1}{2}\,. \tag{4.24}$$

Since $\sum_{I=1}^{16}(p^I_L)^2=\text{even}$ on the even lattice, the LHS of Eq. (4.19) has an integer eigenvalue and hence the RHS of Eq. (4.19) must also have an integer eigenvalue. From this fact and the vanishing of $\overline{M}_R$ in the "physical" space, we see that $\sum_{I=1}^{4}(p^I_R)^2$ in Eq. (4.22) should be an odd integer. Since $p^I_R=w_{\rm R}$ or $w_{\rm NS}$ in Eq. (4.17) or (4.18) and

$$\sum_{i=1}^{N}{n_i}^2=\left[\sum_{i=1}^{N}n_i\right]^2-2\sum_{i>j}^{N}n_in_j\,, \tag{4.25}$$

the condition

$$\sum_{I=1}^{4}(p^I_R)^2=\sum_{i=1}^{4}(n_i)^2=\text{odd}$$

gives a constraint $\sum_{i=1}^{4}n_i=\text{odd}$ in Eq. (4.13) or (4.15). Therefore $w_{\rm R}$ ($w_{\rm NS}$) is restricted to the weight lattice of the SO(8) spinor (vector) conjugation class and the spectrum of Ramond (Neveu-Schwarz) sector is reproduced.

The total mass operator $m^2=2p_+p_--\sum_{i=1}^{8}(p^i)^2$ is given by

$$m^2=M_L-1+M_R+\overline{M}_R-\tfrac{1}{2}\,. \tag{4.26}$$

Since Eq. (4.19) is satisfied only in case that both of the RHS and the LHS of Eq. (4.19) is positive semidefinite, m^2 is also positive semidefinite. Therefore we have obtained a tachyonless purely bosonic string theory.

When we consider the zero intercept, we obtain the following partition functions:

$$Z(q)=(Z_0)^8(q^{-1/12}Z_I)^8(q^{1/24}Z_{A-})^8 \\ =q^{-1}\prod_{n=1}^{\infty}\left[\frac{1+q^{2n-1}}{1-q^{2n}}\right]^8 \tag{4.27}$$

in the right-moving parts corresponding to the Ramond or NS sector. Here Z_0 is the partition function of the oscillators in one string coordinate in the uncompactified transverse dimensions:

$$Z_0=q^{-1/12}\frac{I}{\prod_{n=1}^{\infty}(1-q^{2n})}\,,$$

and Z_I and Z_{A-} are defined in Eqs. (2.4) and (2.14), respectively.

Since only the even powers of q can survive due to the constraint Eq. (4.19), we symmetrize Z with respect to q and obtain

$$\overline{Z}(q)=\tfrac{1}{2}[Z(q)+Z(-q)] \\ =8\prod_{n=1}\left[\frac{1+q^{2n}}{1-q^{2n}}\right]^8\,. \tag{4.28}$$

Here we have used the Jacobi identity for the θ functions:

$$q^{-1}\prod_{n=1}^{\infty}(1+q^{2n-1})^8-q^{-1}\prod_{n=1}^{\infty}(1-q^{2n-1})^8 \\ =16\prod_{n=1}^{\infty}(1+q^{2n})^8\,. \tag{4.29}$$

The partition function in Eq. (4.28) correctly reproduces those of the corresponding Ramond (Neveu-Schwarz) sector in superstring theories.[19]

The fermion operators defined in Eq. (4.10) are explicitly given in terms of oscillators as

$$\psi^{2I-1}+i\psi^{2I}=\sqrt{2}C^I\exp\left[-i\sum_{n=1}^{\infty}\frac{\alpha^I_{R-n}}{n}e^{in\theta}\right] \\ \times\exp\left[i\sum_{n=1}^{\infty}\frac{\alpha^I_{Rn}}{n}e^{-in\theta}\right] \\ \times\exp(ix^I_R)\exp[i(\tfrac{1}{2}+p^I_R)\theta]\,, \\ \theta=2(\tau-\sigma)\,. \tag{4.30}$$

Since the oscillating (non-zero-) mode part in Eq. (4.30) is a periodic function of θ and hence of σ, the periodicity of fermions is determined by the last factor $\exp[i(\frac{1}{2}+p^I_R)\theta]$. Fermions are periodic functions in the physical space corresponding to the Ramond sector since p^I_R is half-integer-valued due to Eq. (4.13) and are antiperiodic functions in the sector corresponding to the Neveu-Schwarz sector since p^I_R is integer valued due to Eq. (4.15). This periodicity reproduces the properties of the fermions in the corresponding sectors.

We now discuss the 10-dimensional Lorentz algebra.

The generators are obtained by replacing the fermion coordinates of the generators in the corresponding sectors in the superstring, with the bosonized fermion coordinates in Eq. (4.10). The closure of the algebra is trivial since these generators are nothing but the boson representation of the generators in the corresponding theories. The commutativity of the generators with Virasoro operators L_n in the right mover also holds of course. Note that L_n has the following form:

$$L_n = L_n^{(1)} + L_n^{(2)} . \tag{4.31}$$

Here $L_n^{(1)}$ is the boson representation of the Virasoro operators in the corresponding sectors and it commutes with the Lorentz generators. $L_n^{(2)}$ is composed of the mode operators in the decoupled dimensions and it also commutes with Lorentz generators, which do not contain these mode operators.

Finally, we note that the 4 string coordinates which generate fermion coordinates by Eq. (4.10) are not compactified on the different manifolds between the Ramond and Neveu-Schwarz sectors but the same manifold. Both s_i in Eq. (4.14) and v_i in Eq. (4.16) are orthogonal unit vectors in 4-dimensional space and they are connected with each other by the rotation. The difference comes from the choice of coordinate system. Then why do the weight vectors in the Ramond sector belong to spinor conjugation class and those in the Neveu-Schwarz sector belong to the vector conjugation class? It is because of the mapping of the rotational group SO(8) in 8 uncompactified transverse dimensions onto the internal group $\widetilde{\mathrm{SO}}(8)$ which results from the compactification are different in the Ramond and Neveu-Schwarz sectors. The mapping is accompanied with the breaking of the symmetry $\mathrm{SO}(8)\otimes\widetilde{\mathrm{SO}}(8)$ down to the diagonal subgroup $\mathrm{SO}(8)_{\mathrm{diag}}$ and the angle of the breaking may be different for a two-string field in general.

V. ONE-LOOP CALCULATION

Let us start by defining the propagator Δ:

$$\begin{aligned}\Delta &\equiv \frac{1}{H}\delta(M_L - M_R - \overline{M}_R - \tfrac{1}{2}) \\ &= \int_0^{+\infty} dt \int_{-\infty}^{+\infty} ds \exp[-tH + is(M_L - M_R - \overline{M}_R - \tfrac{1}{2})] \\ &= \frac{1}{2i}\int_{\text{upper half-plane}} d\tau\, d\bar{\tau} \exp(i\tau L - i\bar{\tau}\bar{L}) .\end{aligned} \tag{5.1}$$

Here

$$\begin{aligned} H &= \tfrac{1}{2}\sum_{i=1}^{10}(p^i)^2 + M_L + M_R + \overline{M}_R - \tfrac{3}{2} , \\ L &= \tfrac{1}{4}\sum_{i=1}^{10}(p^i)^2 + M_L - 1 , \\ \bar{L} &= \tfrac{1}{4}\sum_{i=1}^{10}(p^i)^2 + M_R + \overline{M}_R - \tfrac{1}{2} ,\end{aligned} \tag{5.2}$$

and

$$\tau = s + it . \tag{5.3}$$

The region of τ integration is consistent with the path-integral formulation[23–25] and will be restricted to the fundamental one by modular invariance.

The one-loop free energy in the sector corresponding to the Neveu-Schwarz one is given by[26]

$$= \int \frac{d\tau\, d\bar{\tau}}{\mathrm{Im}\tau}\left[\frac{2}{\mathrm{Im}\tau}\right]^5 \frac{x^{-1}\prod_{n=1}^{\infty}(1+x^{2n-l})^8}{\prod_{n=1}^{\infty}(1-x^{2n})^8}\, \frac{\bar{x}^{2}\left[1+\sum_{m=1}^{\infty}480\sigma_7(m)\bar{x}^{2m}\right]}{\prod_{m=1}^{\infty}(1-\bar{x}^{2m})^{24}} , \quad x \equiv e^{(i/2)\tau},\ \ \bar{x} \equiv e^{-(i/2)\bar{\tau}} . \tag{5.4}$$

Here tr for an operator θ is defined by

$$\mathrm{tr}\theta = \sum_i \langle i \,|\, \theta \,|\, i \rangle \tag{5.5}$$

and i runs all the states in each sector. We have assumed that the left mover is compactified on the $\mathrm{E}_8\times\mathrm{E}_8$ weight lattice and $\sigma_7(m)$ is the sum of the seventh powers of the divisors of m (Refs. 10 and 11). Since the integrand of Eq. (5.4) contains only even powers of $\bar{x}$, the odd powers of x vanish after integrating over σ. Using Eq. (4.29) we obtain the final expression:

$$F_{\mathrm{NS}} = \int \frac{d\tau\, d\bar{\tau}}{\mathrm{Im}\tau}\left[\frac{2}{\mathrm{Im}\tau}\right]^5 \frac{8\prod_{n=1}^{\infty}(1+x^{2n})^8}{\prod_{n=1}^{\infty}(1-x^{2n})^8} \times \frac{\left[1+\sum_{m=1}^{\infty}480\sigma_7(m)\bar{x}^{2m}\right]}{\bar{x}^{-2}\prod_{m=1}^{\infty}(1-\bar{x}^{2m})^{24}} . \tag{5.6}$$

Calculating one-loop free energy in the sector corresponding to the Ramond sector, we obtain the same expression as Eq. (5.6) with the relative minus sign. This sign difference comes from the fact that the Ramond sector corresponds to fermions. Combining the contributions from both sectors, the total one-loop free energy vanishes, which signals supersymmetry.

The emission vertices are obtained by replacing the fermion coordinates of the vertices in the heterotic string theory with the bosonized fermion coordinates of Eq. (4.10). These vertices V are physical since

$$[V,Q_R]=0\ . \tag{5.7}$$

Here Q_R is defined by Eq. (3.4). Equation (5.7) results from the fact that these vertices do not contain mode operators of 12 unphysical right-moving coordinates.

Now we show the decoupling of 12 unphysical right-moving string coordinates in the one-loop amplitude with vertices which satisfy Eq. (5.7). Let us consider the N-point amplitude

$$\begin{aligned} A_N &= \mathrm{tr}(\Delta V_1 \Delta V_2 \cdots \Delta V_N) \\ &= \int \prod_{j=1}^{N} d\tau_j d\bar{\tau}_j \mathrm{tr}(e^{i\tau_1 L - i\bar{\tau}_1 \bar{L}} V_1 e^{i\tau_2 L - i\bar{\tau}_2 \bar{L}} V_2 \cdots e^{i\tau_N L - i\bar{\tau}_N \bar{L}} V_N)\ . \end{aligned} \tag{5.8}$$

Since the vertices do not contain the unphysical mode operators due to Eq. (5.7), the integrand of Eq. (5.8) is rewritten into the factorized form:

$$A_N = \int \prod_{j=1}^{N} d\tau_j d\bar{\tau}_j \mathrm{tr}_P(e^{i\tau_1 L - i\bar{\tau}_1 \bar{L}_P} V_1 e^{i\tau_2 L - i\bar{\tau}_2 \bar{L}_P} V_2 \cdots e^{i\tau_N L - i\bar{\tau}_N \bar{L}_P} V_N) \mathrm{tr}_U \left[\exp\left[-i \sum_{j=1}^{N} \bar{\tau}_j \bar{M}_R \right] \right]\ . \tag{5.9}$$

Here

$$\bar{L}_P = \tfrac{1}{4} \sum_{i=1}^{8} (p^i)^2 + M_R - \tfrac{1}{2}\ , \tag{5.10}$$

tr_U and tr_P are the trace over all the states corresponding to the excitation of unphysical modes and that over all the physical states. Since

$$\mathrm{tr}_U \left[\exp\left[-i \sum_{j=1}^{N} \bar{\tau}_j \bar{M}_R \right] \right] = 1\ , \tag{5.11}$$

due to Eq. (3.1), we obtain

$$A_N = \int \prod_{j=1}^{N} d\tau_j d\bar{\tau}_j \mathrm{tr}_P(e^{i\tau_1 L - i\bar{\tau}_1 \bar{L}_P} V_1 e^{i\tau_2 L - i\bar{\tau}_2 \bar{L}_P} V_2 \times \cdots e^{i\tau_N L - i\bar{\tau}_N \bar{L}_P} V_N)\ . \tag{5.12}$$

The expression of Eq. (5.12) means the complete decoupling of unphysical 12 right-moving string coordinates.

It is straightforward to show that the N-point amplitude of the vector or graviton vanishes for $N=1,2,3$.

VI. SUMMARY AND DISCUSSION

In this paper a tachyonless string model has been constructed from the purely bosonic 26-dimensional string model. The 16 dimensions have to be compactified leaving 8 uncompactified transverse dimensions. Since we need only 4 string coordinates to obtain the 8 bosonized fermion coordinates in the superstring sector, the residual 12 string coordinates should decouple for the consistency. We have proposed a mechanism for this decoupling and shown that it really occurs in calculating one-loop amplitude.

The spectrum of the model proposed in this paper is equivalent to that of the usual heterotic string model and we have shown that one-loop vacuum amplitude vanishes. We do not need any projection which kills spurious states.

However there remain some points to be clarified. In this paper we have shown that the heterotic string theory can be reformulated in terms of the bosonic theory at least at the one-loop level. We should extend the argument to higher-loop levels. This could be done by string field theory or by path integration.[22–25] In the former the action of the superstring field theory should be rewritten by the action of the bosonic string one. In the latter the correspondence between fermions and bosons on the Riemann surface with arbitrary genus[27] will be a clue.

Recently Hata *et al.*[28] have shown that the usual closed bosonic string is equivalent to the theory whose action contains no kinetic term and consists soley of cubic terms in the string field. The kinetic term can be generated through the condensation of an infinitesimally small string which is a solution of the equation of motion. The solution would be, however, unstable because the usual bosonic string theory contains tachyons. It is not obvious if indeed there exists any stable solution at all. If it exists, however, there might be a solution describing superstrings and the mechanism proposed in this paper would be realized.

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$$\sum_{n=1}^{\infty} \tfrac{1}{2}n = -\tfrac{1}{24}, \quad \sum_{n=1}^{\infty} \tfrac{1}{2}(n-\tfrac{1}{2}) = \tfrac{1}{48} \; .$$

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$$\ln A = -\int_0^\infty \frac{dt}{t} e^{-At} + C \; .$$

Here C is an infinite constant which can be absorbed into the string tension. This equation can be easily derived by differentiating both hand sides with respect to A.

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