# Further remarks on particlelike solutions in spinor-connection theory

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In a recent work the author presented a numerical solution for a neutral pionlike particle in spinor-connection theory. Here, using a more analytical approach and an improved numerical method, that solution is studied in greater and refining detail. A new solution for an electrically charged pionlike particle is also given. The results obtained suggest that the electromagnetic and strong-field coupling constants are closely equal.

## I. INTRODUCTION

The field equations of spinor-connection theory have been given previously[1,2] and an exact solution has been found in the cosmological case.[2] A numerical solution has been given for an electrically neutral, spinless particle which exhibits a short-ranged "strong" field, i.e., a neutral pionlike particle.[1] However, that numerical treatment masked several interesting physical properties of the theory which are set out here.

For ease of reference we shall refer to that earlier paper[1] as EP. The symbols and notations used here will be the same as those in EP. Thus the field equations of the theory are EP(3.3)—EP(3.6).

As a preliminary remark, an essential requirement of a true particle solution is that the rest mass of the particle should be uniquely specified by the solution. In spinor-connection theory there is a fundamental mass $M_0$, which via Eq. EP(3.11), sets the scale of length of the geometry. Accordingly the rest mass $m_0$ of any particle admitted by the theory should have the form

$$m_0 = \theta M_0 \,, \tag{1.1}$$

where $\theta$ is a definite dimensionless number.

Now spinor-connection theory is not, in any conventional sense, a quantum theory. For example the normalization condition EP(4.43) imposed on the spinor-tetrad field $\psi$ in EP, namely,

$$2\pi \int_0^\infty \frac{f_1}{g_1} B_1 dy = 1 \,, \tag{1.2}$$

is not given any quantum-mechanical probabilistic interpretation. Rather, this condition expresses just the classical Maxwell-type requirement that the source of the short-ranged $J_1$ field is a "strong" charge $q_{(1)}$. Likewise, the value of the mass factor $\theta$ in (1.1) apparently cannot be determined by a quantum-type energy-eigenvalue problem.

In EP, attention was concentrated on the energy-conservation requirement of general relativity.[3] This requirement states that there exits a conserved mass $\lambda M_0$ say, of the form

$$\lambda M_0 = \frac{c^2}{8\pi G_n} \int \left[ \frac{\partial U}{\partial g^{4\mu}{}_{,i}} g^{4\mu} \right]_{,i} d^3x \,, \tag{1.3}$$

where $\lambda$ is a constant and $U$ is the pseudoscalar energy density of EP(4.55). Using the field equations, the three-space volume integral (1.3) may be expressed as an integral of the interior and exterior metric, spinor, and "strong" fields of the particle. This integral is

$$\lambda = \int_0^\infty D\, dy \,, \tag{1.4}$$

where $y$ is the radial marker EP(4.25) and the linear energy density $D$ is by EP(4.57),

$$D = 8\pi f_1 g_1 \frac{J_1{}^2}{y^2} + \frac{\pi f_1{}^3}{g_1} \left[ \frac{3A_1{}^2}{32y^2} - K_1 B_1 \right] + 2\pi f_1 (1 - f_1{}^2) \frac{P_1 Q_1}{y} \,. \tag{1.5}$$

We expect that an acceptable solution should give a finite integral in (1.4), which places severe restrictions on the functional form of the fields appearing in (1.5) as we approach the deep interior ($y \to 0$) region of the particle. But we can give no solution at all without specifying initial values for the fields in the far exterior ($y \to \infty$) region. With regard to the metric field functions $f_1$ and $g_1$ in (1.5), there seems to be no reasonable alternative to imposing the exterior Schwarzschild solution $g_1{}^2 = f_1{}^{-2} = 1 - \mu(8\pi y)^{-1}$, for a Schwarzschild mass $\mu M_0$, as was done in EP. With regard to the spinor, torsion, and "strong"-field functions $P_1$, $Q_1$, $B_1$, $A_1$, $J_1$, and $K_1$ in (1.5), these must tend to zero in the exterior region. However, the manner in which these functions tend to zero depends on how the field equations are scaled with respect to the expectedly very large constant $T$ of EP(3.16). The scaling chosen in EP, namely, that given by EP(4.25)—EP(4.28), appears to be the most obvious and simple scaling and it is retained here.

Having now chosen the Schwarzschild solution for a mass $\mu M_0$ as the exterior metric field solution, Eqs. (1.3) and (1.4) raise a question which was not given due consideration in the numerical solution of EP. The pseudoscaler density $U$ of (1.3) is expressible in terms of the metric tensor $g_{\mu\nu}$ alone by EP(4.55). Because of this the integral (1.4) takes the simple form $\lambda = X(\infty) - X(0)$, where

$$X(y) = 8\pi \frac{y g_1}{f_1} (f_1{}^2 - 1) \,, \quad \frac{dX}{dy} = D \,. \tag{1.6}$$

 

Evaluating now $X(\infty)$ with the use of the Schwarzschild solution, we get the identity

$$\mu - X(0) = \lambda . \tag{1.7}$$

The equivalence principle might seem to suggest the equality of the Schwarzschild mass $\mu M_0$ and the conserved mass $\lambda M_0$ of (1.3), or equivalently to require that the function $X(y)$ of (1.6) vanish at the origin $y=0$ due to (1.7). This circumstance is true, for example, in the case of the interior Schwarzschild solution[3] for a neutral sphere of perfect fluid. However, the charged-particle solution in EP does not have $\lambda=\mu$ with $X(0)=0$. The equality $\lambda=\mu$ was tacitly assumed at EP(4.56) and subsequently used to numerically determine the supposedly unique result $\mu=1.3$ for a solution satisfying the normalization condition (1.2). But, from EP(4.59), we find that although $X(0)$ is small, of order $10^{-2}$, it is not zero. Thus the stated result in EP does not express a true particle property. Instead it gives a measure, by way of (1.7), of the global inaccuracy of the numerical computations which were carried out.

The purpose of the present paper is to study EP-type solutions in analytical detail. It is shown that $X(0)$ must be nonzero and further that the normalization condition (1.2) cannot be satisfied unless the Schwarzschild mass $\mu M_0$ is such that $\mu \le \mu_{\max}$, where the value of $\mu_{\max}$ depends on the magnitude of the "strong" coupling constant $q_{(1)}^2(4\pi\hbar c)^{-1}$. Similar results are found for an electrically charged particle, for which the electromagnetic coupling constant is known to be very close to $\frac{1}{137}$.

Concerning the three mass descriptor $\theta$, $\mu$, and $\lambda$, presumably the rest mass $\theta M_0$ of (1.1) is the inertial mass, while $\mu M_0$ is the gravitational mass. Accordingly, we take $\theta=\mu$. Regarding the mass $\lambda M_0$, it is known[3] that its defining integral (1.3) presents quite formidable problems of interpretation. The function $U$, besides possessing a pseudoscalar character, admits no unique expression. Thus the function $X(y)$ of (1.6) likewise has no unique expression. For this reason, in this work no intrinsic physical significance will be ascribed to the value of $\lambda$ other than that it should be finite. However, the precision to which the identity (1.7) is satisfied provides a useful monitor of the accuracy of numerical computations.

In order to fix a particle mass we make a hypothesis which seems to be reasonable from a classical viewpoint. This hypothesis is that for a given charge, the mass of the particle is such that the charge to mass ratio has a minimum value, i.e., the mass is fixed by the cutoff value $\mu_{\max}$ for a normalized solution.

For the electrically charged particle we find, using the known electromagnetic coupling constant, that $\mu_{\max}\simeq 2.18$. Identifying this particle as a charged pion we obtain $T\simeq 7\times 10^{38}$, which is about three times the value estimated in EP. Similarly, the fundamental mass $M_0$ turns out to be roughly half the $\mu$-meson mass, down by a factor of about 2 on the value in EP. Identifying the neutral particle as a neutral pion, the cutoff $\mu_{\max}$ for this solution should be about 97% of that for the electrically charged solution. This is possible if the electromagnetic and strong field coupling constants are closely equal, rather than differing by a factor of 137 as was assumed in EP.

## II. NEUTRAL PIONLIKE PARTICLE

The equations to be solved are, from EP(4.31) to EP(4.36), with the prime denoting $d/dy$,

$$K_1' = -\frac{f_1 g_1}{\gamma}\frac{J_1}{y^2} , \tag{2.1}$$

$$J_1' = \frac{1}{16\gamma}\frac{f_1}{g_1}B_1 , \tag{2.2}$$

$$P_1' = \frac{f_1 P_1}{y} + \frac{1}{2}\frac{f_1}{g_1}\left[K_1 - \frac{3}{16}\frac{B_1}{y^2}\right]Q_1 , \tag{2.3}$$

$$Q_1' = -\frac{f_1 Q_1}{y} - \frac{1}{2}\frac{f_1}{g_1}\left[K_1 - \frac{3}{16}\frac{B_1}{y^2}\right]P_1 , \tag{2.4}$$

$$f_1' = \frac{f_1(1-f_1^2)}{2y} + \frac{f_1^3 J_1^2}{2y^3} + \frac{f_1^3}{g_1^2}\frac{y}{16}\left[\frac{3A_1^2}{32y^4} - \frac{K_1 B_1}{y^2}\right] , \tag{2.5}$$

$$g_1' = -\frac{g_1(1-f_1^2)}{2y} - \frac{g_1 f_1^2 J_1^2}{2y^3} + \frac{f_1^2}{g_1}\frac{y}{16}\left[\frac{3A_1^2}{32y^4} - \frac{K_1 B_1}{y^2}\right] - \frac{f_1^2 P_1 Q_1}{4y^2} , \tag{2.6}$$

where the torsion and spinor intensity functions $A_1$ and $B_1$ are, by EP(4.37),

$$A_1 = B_1 = P_1^2 + Q_1^2 \tag{2.7}$$

and by EP(4.49), the constant $\gamma$ is given by

$$\gamma = -\frac{1}{8\sqrt{\pi\alpha}} , \tag{2.8}$$

where

$$\alpha = \frac{q_{(1)}^2}{4\pi\hbar c} \tag{2.9}$$

is the coupling constant for the "strong" charge $q_{(1)}$. We do not assume, as was done in EP, that $\alpha$ is unity.

For large $y$ the system (2.1)–(2.6) has the solution

$$g_1^2 = f_1^{-2} = 1 - \frac{\mu}{8\pi y} , \tag{2.10}$$

$$K_1 = \frac{-a^2}{32\gamma^2 y^2}\left[1 + \frac{2}{3}\frac{\mu}{8\pi y} + \frac{23}{48}\left[\frac{\mu}{8\pi y}\right]^2 + \cdots\right] , \tag{2.11}$$

$$J_1 = \frac{-a^2}{16\gamma y}\left[1 + \frac{\mu}{8\pi y} + \frac{23}{24}\left[\frac{\mu}{8\pi y}\right]^2 + \cdots\right] , \tag{2.12}$$

$$P_1 = \frac{a^3}{192\gamma^2 y^2}\left[1 + \frac{3}{2}\frac{\mu}{8\pi y} + \cdots\right] , \tag{2.13}$$

$$Q_1 = \frac{a}{y}\left[1 + \frac{1}{2}\frac{\mu}{8\pi y} + \frac{5}{16}\left[\frac{\mu}{8\pi y}\right]^2 + \cdots\right] , \tag{2.14}$$

where $a$ is a constant. On the other hand, as $y \to 0$, we have , as in EP, that

$$\lim_{y\to 0} J_1 = J_1(0) = \frac{-1}{32\pi\gamma} . \tag{2.15}$$

We can obtain a semianalytic global solution provided that the constants $a$ and $a\gamma^{-1}$ are small enough to ensure that the "exterior" solution (2.10)–(2.14) holds good until we approach close to the Schwarzschild radius at $y=\eta$, where

$$\eta=\frac{\mu}{8\pi} \ . \tag{2.16}$$

To investigate the solution as $y \to \eta$ we set

$$y-\eta=\epsilon>0 \tag{2.17}$$

so that, for small $\epsilon$, Eq. (2.10) requires

$$g_1 \simeq f_1^{-1} \simeq \left[\frac{\epsilon}{\eta}\right]^{1/2} \ . \tag{2.18}$$

Equations (2.13) and (2.14) show that $Q_1 \gg P_1$ for large $y$. Evidently this same inequality holds good as $y$ approaches $\eta$ provided

$$a^2(192\gamma^2\eta)^{-1} \ll 1 \ . \tag{2.19}$$

With this condition, combining (2.3) and (2.4), the exact equation

$$P_1P_1'+Q_1Q_1'=-\frac{f_1}{y}(Q_1^2-P_1^2) \tag{2.20}$$

becomes, because of (2.7) and (2.18), $\frac{1}{2}B_1' \simeq -B_1(\eta\epsilon)^{-1/2}$. Therefore

$$B_1 \simeq b_0 \exp[-\tfrac{1}{4}(\epsilon/\eta)^{1/2}] \ ,$$

so that, for small $\epsilon$,

$$B_1 \simeq b_0 \ . \tag{2.21}$$

As an indication of the value of $b_0$, Eq. (2.14) gives

$$b_0 \simeq 3(a/\eta)^2 \ . \tag{2.22}$$

For the behavior of $J_1$ as $\epsilon \to 0$, Eqs. (2.2), (2.18), and (2.21) give $J_1' \simeq \eta b_0(16\gamma\epsilon)^{-1}$, so that $J_1$ varies as $\ln\epsilon$. Substitution of this $J_1$ behavior in (2.1) shows that $K_1'$ also varies as $\ln\epsilon$. Consequently, for small $\epsilon$,

$$K_1 \simeq k_0 \ , \tag{2.23}$$

where as an estimate for this negative constant $k_0$, Eqs. (2.8) and (2.11) give

$$k_0 \simeq -4\pi a^2 \alpha \eta^{-2} \ . \tag{2.24}$$

Focusing attention now on Eq. (2.5) for the metric function $f_1$, this equation can be integrated when the torsion ($A_1^2$) term dominates over the $J_1^2$ and $K_1B_1$ terms. For this purpose we set

$$\frac{1}{t_0^2}=\frac{3b_0^2}{256\eta^2} \tag{2.25}$$

with

$$\frac{1}{t_0^2} \gg -\frac{k_0b_0}{8} \tag{2.26}$$

and

$$\frac{1}{t_0^2} \gg \frac{g_1^2J_1^2}{\eta^2} \ . \tag{2.27}$$

Under these circumstances Eq. (2.5) becomes

$$f_1' \simeq -\frac{f_1^3}{2y}\left[1-\frac{1}{t_0^2g_1^2}\right] , \tag{2.28}$$

while (2.6) becomes, neglecting the $P_1Q_1$ term,

$$g_1' \simeq \frac{g_1f_1^2}{2y}\left[1+\frac{1}{t_0^2g_1^2}\right] . \tag{2.29}$$

The previous two equations give

$$\frac{df_1}{dg_1} \simeq -\frac{f_1}{g_1}\left[\frac{t_0^2g_1^2-1}{t_0^2g_1^2+1}\right] \tag{2.30}$$

which has a solution

$$\frac{f_1}{g_1} \simeq \frac{t_0^2}{g_1^2t_0^2+1} \ . \tag{2.31}$$

Returning now to the $g_1'$ Eq. (2.29), we obtain, with (2.31),

$$g_1' \simeq \frac{g_1t_0^2}{2y(t_0^2g_1^2+1)} \tag{2.32}$$

which has a solution

$$t_0^2g_1^2-1+\ln(g_1^2t_0^2) \simeq t_0^2\ln(y/\eta) \simeq t_0^2\frac{\epsilon}{\eta} \ . \tag{2.33}$$

Equation (2.33) shows that, for small $\epsilon$,

$$g_1 \simeq \frac{1}{t_0} \tag{2.34}$$

and hence, from (2.31),

$$f_1 \simeq \frac{t_0}{2} \ . \tag{2.35}$$

Most importantly, Eq. (2.35) fixes the maximum value of $f_1$ since, according to (2.28), $f_1'$ is zero. Moreover, as this maximum value must exceed unity, Eq. (2.35) establishes an inequality which is also most important in the sequel. This inequality is

$$t_0>2 \ . \tag{2.36}$$

Proceeding now inside the Schwarzschild radius with $y \lesssim \eta$, $g_1$ drops down quite sharply, with $g_1'/g_1$ exceeding $y^{-1}$ according to (2.32). Neglecting $g_1^2t_0^2$ in comparison with unity, Eq. (2.31) then gives

$$\frac{f_1}{g_1} \simeq t_0^2 \tag{2.37}$$

as a reasonable approximation at $y \lesssim \eta$. To find the functional form for $f_1$ in this vicinity we use (2.5) with $f_1^3g_1^{-2} \simeq f_1t_0^4$ according to (2.37). Evaluating the torsion term by (2.25), Eq. (2.5) then becomes

$$\frac{f_1'}{f_1}+\frac{1}{2y}(f_1{}^2-1)\simeq\frac{t_0{}^2}{2y} \tag{2.38}$$

which can be solved by partial fractions to give, for $y \lesssim \eta$,

$$f_1{}^2\simeq\frac{(t_0{}^2+1)y^{t_0{}^2+1}}{y^{t_0{}^2+1}+(3+4t_0{}^{-2})\eta^{t_0{}^2+1}}\ , \tag{2.39}$$

where the constant of integration in this solution has been chosen so that Eq. (2.35) holds at $y=\eta$.

Considering now the deeper and central regions $y\to 0$, Eq. (2.39) indicates

$$f_1\simeq F\left[\frac{y}{\eta}\right]^{(t_0{}^2+1)/2}\ , \tag{2.40}$$

where $F$ is a positive constant. Thus, from (2.36), $f_1$ tends to zero faster than $y^{5/2}$. To determine the behavior of $g_1$, Eqs. (2.5) and (2.6) combine to give

$$\left[\frac{yg_1}{f_1}\right]'=f_1g_1\left[1-\frac{J_1{}^2}{y^2}\right]-\frac{f_1P_1Q_1}{4y}\ . \tag{2.41}$$

Equation (2.41) allows the solution, as $y\to 0$,

$$\frac{yg_1}{f_1}\simeq\text{const}\simeq\frac{\eta}{t_0{}^2} \tag{2.42}$$

since in this circumstance $g_1$ tends to zero faster than $y^{3/2}$ and, as we will shortly see, the $J_1{}^2$ and $P_1Q_1$ terms on the right-hand side of (2.41) are negligible. The value of the constant in (2.42) is chosen so that (2.37) holds at $y\lesssim\eta$. It follows now from (2.42) and (1.6) that the function $X(y)$ tends to the nonzero limit

$$X(0)\simeq-\frac{8\pi\eta}{t_0{}^2} \tag{2.43}$$

and hence $\mu\neq\lambda$ in Eq. (1.7).

Looking now at the spinor field equations (2.3) and (2.4) for $P_1$ and $Q_1$, the only significant term on the right-hand side of these equations is the torsion ($B_1=A_1$) term, with $B_1$ itself being a constant as $y\to 0$ by (2.20) and (2.40). Taking $B_1\simeq b_0$ and noting (2.42), the solution is

$$P_1\simeq b_0{}^{1/2}\cos(\sigma_0\ln y+\phi_0)\ , \tag{2.44}$$

$$Q_1\simeq b_0{}^{1/2}\sin(\sigma_0\ln y+\phi_0)\ , \tag{2.45}$$

$$\sigma_0\simeq\frac{3b_0t_0{}^2}{32\eta}\ , \tag{2.46}$$

with $\phi_0$ being a constant. The remaining field equations (2.2) and (2.1) for $J_1$ and $K_1$ are now solved by

$$J_1\simeq\frac{-1}{32\pi\gamma}\left[1-\frac{\pi t_0{}^2b_0y^2}{\eta}\right]\ , \tag{2.47}$$

$$K_1\simeq k_0+O(y^{t_0{}^2-1})\ , \tag{2.48}$$

where the constant term in (2.47) is chosen to meet the requirement (2.15) for $J_1(0)$.

The solution (2.40)–(2.48) has the same form as the numerical solution for $y\to 0$ given in EP. However, here the constants which appear in the solution are subject to new and physically very significant constraints which originate in the now know behavior of the fields in the vicinity of the Schwarzschild radius at $y=\eta$. To see this we next investigate the normalization condition (1.2). Neglecting the small contribution made to the normalizing integral in the exterior region $y>\eta$, we have, from (1.2),

$$2\pi\int_0^\infty\frac{f_1}{g_1}B_1dy\simeq 2\pi\int_0^\eta\frac{t_0{}^2}{\eta}b_0y\,dy\simeq 1 \tag{2.49}$$

by use of (2.42) and (2.21). By application of (2.25), Eq. (2.49) requires

$$\frac{16\pi}{\sqrt{3}}\eta^2t_0\simeq 1\ . \tag{2.50}$$

Recalling now the fundamental inequality $t_0>2$ of (2.36), Eq. (2.50) gives $\eta<0.13$. Hence, by (2.16)

$$\mu<3.3\ . \tag{2.51}$$

The inequality (2.51) does not imply that the maximum mass consistent with the normalization condition is independent of the value of the "strong" field coupling constant $\alpha$ of (2.9). This is because our solution is not valid unless both of the conditions (2.26) and (2.27) are satisfied for $y\gtrsim\eta$. Now the condition (2.27) is already satisfied by virtue of the normalization requirement (2.50) together with (2.47) and (2.34). However the condition (2.26) imposes a further new restriction on $\mu$ and $\alpha$. To obtain this, notice that from (2.22) and (2.24)

$$\frac{-k_0b_0}{8}\simeq\frac{\pi\alpha}{6}b_0{}^2\ . \tag{2.52}$$

Hence, from (2.25), the inequality (2.26) becomes $256\pi\alpha\eta^2\ll 18$, so that with use of (2.16) we have

$$\alpha\mu^2\ll 15\ . \tag{2.53}$$

The numbers on the right-hand side of the inequalities (2.51) and (2.53) are of course only approximations, since the interior and exterior fields have not been exactly matched in the region $y\simeq\eta$. However the existence of such inequalities does show that, for given $\alpha$, there is a maximum mass $\mu_{\max}M_0$ for which the solution is normalizable. Such an extremal solution has the property that for the given charge, the charge to mass ratio is minimal. In classical terms, the self-repulsion inherent in the charge is opposed by the gravitational and torsional effects of the largest mass consistent with that charge. We adopt the hypothesis that the particles of matter are represented by such extremal solutions.

The particle considered so far has been called "neutral pionlike" on the grounds that it possesses zero angular momentum, it is electrically neutral, and it exhibits a short-ranged "strong" field whose strength was estimated in EP. Of course the particle does not exhibit the temporal instability which is characteristic of the empirical $\pi^0$. Any possibility of this has been precluded by imposing an absence of fields due to external sources. Nevertheless we shall assume that the extremal solution given here does represent an isolated $\pi^0$ whose mass depends on the value of the strong coupling constant $\alpha$. As a guide to an appropriate choice for $\alpha$, we next consider an electrically charged pion, for which the electromagnetic coupling constant is very well known.

## III. ELECTRICALLY CHARGED PIONLIKE PARTICLE

In EP we used a nomenclature where $q_{(1)}$ and $q_{(2)}$ referred to the strong and electric charges, respectively. In the present application it is convenient to make an obvious change in that mere nomenclature and now regard $q_{(1)}$ as the electric charge. The equations to be solved are thus again (2.1) to (2.6), where now $J_1$ refers to the electric field of a charge $\pm q_{(1)}$. Equations (2.7)—(2.9) also hold good, except that $\alpha$, now the electromagnetic coupling constant, should have the value $\frac{1}{137}$.

We want $J_1$ to give rise to the Coulomb electric field $E=\pm q_{(1)}(4\pi r^2)^{-1}$ at large distances $r$ from the particle. Following the same line of argument as in EP Sec. IV, we easily find[4] that such is the case when

$$\lim_{y\to\infty} J_1(y)=J_1(\infty)=+\frac{1}{32\pi\gamma} . \tag{3.1}$$

Also with the normalization condition (1.2), the field equation (2.2) will reduce to the usual Maxwell-Gauss equation $\int \nabla\cdot\mathbf{E}\,dV=\pm q_{(1)}$ in weak fields if we impose the boundary condition

$$\lim_{y\to 0} J_1(y)=J_1(0)=0 . \tag{3.2}$$

At large $y$, where the spinor field functions $P_1$ and $Q_1$ must be negligible, the metric field equations (2.5) and (2.6) give, using (3.1) and (2.8),

$$g_1{}^2=f_1{}^{-2}=1-\frac{\mu}{8\pi y}+\frac{\alpha}{16\pi y^2} . \tag{3.3}$$

Equation (3.3) has the same form as the Reissner-Nordström solution[3] in that it has the same functional dependence on the radial coordinate. Nevertheless it is not the Reissner-Nordström solution. When converted to Heaviside-Lorentz units,[5] the coefficient of the radial $r^{-2}$ term in (3.3) differs dramatically from the corresponding term of the Reissner-Nordström solution. This difference results from the scaling used here, which is that given by Eqs. EP(4.25)—EP(4.28).[6] For our present purpose this disparity in the coefficients need cause no great concern since, to the author's knowledge, the value of the pertinent coefficient in the Reissner-Nordström solution has no experimental support. However it is essential that the scaling used here should allow the Lorentz equation of motion for the electric charge immersed in an external electromagnetic field. It seems that this is the case.[7,8]

The usual Reissner-Nordström solution does not allow the possibility that a charge "elementary" particle should admit a Schwarzschild-type singular sphere in its exterior region.[3] Such is not the case with the exterior solution (3.3). According to this latter solution, the equation $g_1{}^2=0$ will have two real roots provided that (with use of $\alpha=\frac{1}{137}$)

$$\mu>(16\pi\alpha)^{1/2}=0.606 . \tag{3.4}$$

The two roots are a larger one, at $y=\xi$, and a smaller one, at $y=\xi-\Delta$, where

$$\xi=\frac{\mu}{16\pi}\left[1+\left[1-\frac{16\pi\alpha}{\mu^2}\right]^{1/2}\right] , \tag{3.5}$$

$$\Delta=\frac{\mu}{8\pi}\left[1-\frac{16\pi\alpha}{\mu^2}\right]^{1/2} . \tag{3.6}$$

Using (3.1) and (3.3), the field equations (2.1) to (2.4) can now be solved at large $y$ to give

$$K_1=2\alpha y^{-1}+k_1y^{-2\beta}+k_2y^{-2\beta-1}+k_3y^{-4\beta+1}+\cdots , \tag{3.7}$$

$$J_1=-\tfrac{1}{4}(\alpha/\pi)^{1/2}+j_1y^{-2\beta+1}+j_2y^{-2\beta}+j_3y^{-4\beta+2}+\cdots , \tag{3.8}$$

$$P_1=ay^{-\beta}+p_1y^{-\beta-1}+p_2y^{-3\beta+1}+\cdots , \tag{3.9}$$

$$Q_1=-(1+\beta)\alpha^{-1}ay^{-\beta}+q_1y^{-\beta-1}+q_2y^{-3\beta+1}+\cdots , \tag{3.10}$$

where

$$\beta=(1-\alpha^2)^{1/2}\simeq 1 , \tag{3.11}$$

and $a$ is a constant. The coefficients $k_1, j_1, \ldots$ are quite complicated combinations of $a$, $\alpha$, $\mu$, and $\beta$ which are not critically relevant here.

As in Sec. II earlier, we can obtain a semianalytic global solution provided that (3.4) is satisfied and that the constant $a$ in (3.9) and (3.10) is small enough to ensure that the metric field (3.3) holds good until we approach close to the Schwarzschild-type radius at the larger root $y=\xi$ of (3.5). The singular sphere is averted by the full field equations, so that the solution obtained is very similar to the neutral solution. Setting

$$y-\xi=\epsilon>0 \tag{3.12}$$

we have, by (3.3), for small $\epsilon$,

$$g_1\simeq f_1{}^{-1}\simeq \xi^{-1}(\epsilon\Delta)^{1/2} \tag{3.13}$$

so that the metric field has the same functional dependence on $\epsilon$ as was the case earlier in (2.18). Also, from (3.9) and (3.10), $Q_1{}^2 \gg P_1{}^2$ since $(1+\beta)^2\alpha^{-2}\simeq 75\,000$. Hence $B_1\simeq Q_1{}^2$ so that (2.21) also holds good. If, as before, the torsion term dominates on the right-hand side of (2.5) and (2.6), then both the inequality $t_0>2$ of (2.36) and the normalization condition (2.50) will remain true, except, of course, that we must replace $\eta$ of (2.16) by $\xi$ of (3.5). Hence $\xi<0.13$, so that, with use of (3.5), the inequality (2.51) is replaced by

$$\mu\left[1+\left[1-\frac{16\pi\alpha}{\mu^2}\right]^{1/2}\right]<6.6 . \tag{3.14}$$

Similarly, noting (3.2), the solution (2.47) for $J_1$ now becomes

$$J_1\simeq\frac{1}{32\gamma}\frac{t_0{}^2b_0y^2}{\xi} \tag{3.15}$$

in the interior region. Setting $y=\xi$ in (3.15) and using the

$\xi$ form of (2.25) and (2.50), we obtain $J_1 \simeq -\frac{1}{4}(\alpha/\pi)^{1/2}$ as is required by (3.8).

As previously, the inequality (3.14) cannot hold true for arbitrary $\alpha$, since inequalities corresponding to (2.26) and (2.27) must also be satisfied. From (2.27) and (2.34) we need, at $y \gtrsim \xi$, the inequality $\xi^2 \gg J_1^2$. Using (3.8) this last inequality gives $16\pi\xi^2 \gg \alpha$. The inequality corresponding to (2.26) is a little more restrictive. Here we need $3A_1^2/32\xi^2 \gg K_1 B_1$, which gives, by (2.25) and (3.7), $t_0^{-1} \gg 4\alpha/3^{1/2}$. On the one hand, using $t_0 > 2$, this gives the interesting requirement $3^{1/2}8^{-1} \gg \alpha$, which is true for $\alpha = \frac{1}{137}$. Alternatively, using the normalization condition (2.50), we get

$$4\pi\xi^2 \gg \alpha \ . \qquad (3.16)$$

## IV. NUMERICAL RESULTS AND CONCLUSIONS

Actual values of $\mu_{\max}$ for extremal solutions have been numerically estimated using a fifth-order Runge-Kutta method. For the electrically charge particle, with $\alpha = \frac{1}{137}$, the result found was

$$\mu_{\max} \simeq 2.18 \qquad (4.1)$$

with an expected accuracy of no worse than $\pm 0.05$. Using (4.1) and (3.5) we obtain $\xi \simeq 0.0850$, so that the inequalities (3.4) and (3.14) are satisfied. Also, since $4\pi\xi^2\alpha^{-1} \simeq 12.4$, the condition (3.16) is acceptably met. The value of the constant $a$ in (3.9) and (3.10) was found to be $a \simeq 8.52 \times 10^{-5}$. If this solution represents a charged pion with mass $2.18M_0$, then $M_0 \simeq 125 m_e$, where $m_e$ is the electron's rest mass. Hence, by EP(3.16), the constant $T$ of the theory is very large as expected, with $T \simeq 7 \times 10^{38}$.

As $\mu$ is decreased below the extremal value 2.18, the normalized solutions have progressively increasing values of $t_0$, together with increasing values for the maximum attained by the metric function $f_1$. At $\mu = 2.18$ the maximum value of $f_1$ is about 2, as against a maximum value of about 550 at $\mu = 0.61$. On the other hand, the torsion term $t_0^2$ of (2.25) decreases dramatically as $\mu$ decreases. For $\mu > 2.18$, the derivative of $J_1$ in (2.2) is unable to attain a value which is sufficient to pull $J_1$ to zero at the center as is necessary for a normalized solution.

Looking now at the electrically neutral solution, if this is to represent a neutral pion then we need to find a value for $\alpha$ which will lead to $\mu_{\max} \simeq 2.11$, i.e., about 97% of value in (4.1). Two values for $\alpha$ have been considered, namely, $\alpha = 1$ and $\alpha = 10^{-2}$. For $\alpha = 1$ we find $\mu_{\max} \simeq 2.50$, so that the inequality (2.53) is only very marginally met, with $\mu^2\alpha \simeq 6$. On the other hand, for $\alpha = 10^{-2}$ the result found is $\mu_{\max} \simeq 2.17$. Now both (2.51) and (2.53) hold, with $\mu^2\alpha \simeq 0.05 \ll 15$ as is required. The value of the constant $a$ in (2.11) to (2.14) was found to be $a \simeq 1.84 \times 10^{-2}$. Also, using the argument of Sec. V in EP, the ratio of the strengths of the strong and gravitational forces near the "surface" of the particle, at $y \gtrsim \eta$, is

$$S_g \simeq T J_1 (2\mu\gamma y^2 g_1 g_1')^{-1} \simeq 10^{38}$$

as previously.

The above results suggest that the known neutral to charged-pion mass ratio will result if the electromagnetic and strong coupling constants are closely equal. Further, the charged and neutral pions have almost the same "radius" in the sense the $\eta \simeq \xi$ from (2.16) and (3.5). This radius is extremely small. In ordinary units of length it is $\eta\Lambda T^{-1}$, where $\Lambda$ is the Compton length for the mass $M_0$ as in EP(3.11).

Finally, although the significance of the mass factor $\lambda$ remains obscure, Eqs. (1.7), (2.43), (2.16), and (2.36) lead to the constraint $\mu < \lambda < 1.25\mu$. The numerical results satisfied this condition.

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[1]J. T. Lynch, Phys. Rev. D 31, 1287 (1985).

[2]J. T. Lynch, Class. Quantum Gravit. 3, 103 (1986).

[3]R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw-Hill, New York, 1965).

[4]If $r$ is the radial coordinate in cm, then the electric field $E(r)$ in Heaviside-Lorentz units is $E(r) = \hbar c \eta J_1(r)(2q_{(1)}\gamma r^2)^{-1}$. Here $\eta = \pm 1$ refers to the signature $\eta$ of the spinor-tetrad ideals $\psi^{(\eta,\epsilon)}$ in the ansatz EP(4.3). The $r$ and $y$ coordinates are related by $r = \Lambda T^{-1} y$, with $\Lambda$ and $T$ being given by EP(3.11) and EP(3.16), respectively.

[5]In Heaviside-Lorentz units, the coefficient of the $r^{-2}$ term in the Reissner-Nordström solution for an electric charge $q_{(1)}$ is $\phi = G_n q_{(1)}^2 (4\pi c^4)^{-1}$, where $G_n$ is Newton's gravitational constant and $c$ is the speed of light. In the solution (3.3) the corresponding coefficient has the very much smaller value $\phi T^{-1}$. On the other hand, the coefficients of the $r^{-1}$ term in the respective solutions coincide

[6]We can obtain the Reissner-Nordström solutions as the "exterior solution" using the procedure which was mentioned in passing in Sec. III of EP. This is to normalize the spinor field and choose the constant $b$ as

$$\int i\sqrt{\det(g_{\alpha\beta})}\langle \psi \Lambda^4 \psi \rangle d^3x = 1 \ , \quad b = (64\pi\alpha)^{-1} \ .$$

However, no well-behaved "interior solution" has been found in this case. The normalization used in this present work is that used in Sec. IV of EP. Effectively both the above Reissner-Nordström normalization and choice of $b$ pick up a factor of $T^{-1}$ on the right-hand side to become

$$\int i\sqrt{\det(g_{\alpha\beta})}\langle \psi \Gamma^4 \psi \rangle d^3x = T^{-1} \ , \quad b = (64\pi\alpha T)^{-1} \ .$$

[7]We do not obtain the Lorentz equations of motion following the method suggested by Chase (Ref. 8). In that method the electromagnetic field is taken as a free field and the source terms in the energy-momentum tensor $E_{\alpha\beta}$, say, are disregarded. Following Chase's method then gives an unwanted factor of $T^{-1}$ in the equations of motion due to our present choice of $b$. However a much more natural approach (Ref. 3) is to retain the source terms in $E_{\alpha\beta}$ and seek the equations of motion in the standard way from the condition

$$\int i\sqrt{\det(g_{\alpha\beta})} E^{\nu\mu}{}_{;\mu} d^4x = 0 \ .$$

In principle, no obstacle is anticipated with this approach.

[8]D. M. Chase, Phys. Rev. 95, 243 (1954).