Effects of heavy subdominantly coupled neutrino mixing on l^+ polarization in $K_L^0 \to \pi^- l^+ \nu_l$ decays

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The effects of heavy subdominantly coupled (HSC) neutrino mixing, without and with the inclusion of right-handed currents (RHC's), on l^+ polarization in K_{l3}^0 decays are investigated. The changes in average polarization components due to the HSC neutrino mass (≤ 125 MeV) and the RHC factor (≤ 0.2) are less than one percent for μ^+ and about 0.01% for e^+. A large uncertainty in the experimental value of ξ renders it difficult to discern HSC neutrino mixing effects or RHC contributions in μ^+ polarization measurements. Electron polarization, if measured, could be decisive in ascertaining the RHC admixture even if a few percent of HSC neutrino mixing is present.

In the past few years there has been a good deal of activity investigating the effects of neutrino masses and mixing in the decays of leptons[1] and mesons,[2,3] among several other processes.[4] In this paper we investigate the effects of heavy subdominantly coupled[5] (HSC) neutrino mixing on the l^+ ($l=\mu,e$) polarization in $K_L^0 \to \pi^- l^+ \nu_l$ decays[6] with a view to ascertain whether these effects could be discernible in present-day experiments.[7] In particular, we obtain the expression for the Cabibbo-Maksymowicz polarization vector[8] with the inclusion of neutrino mass mixing[1-3] and right-handed currents[3,9] (RHC's), and estimate the effects of HSC neutrino mixing on longitudinal, perpendicular, and normal components of the l^+ polarization without and with the admixture of RHC's.

The matrix element for the decay with the inclusion of neutrino mixing and RHC is given by[3]

$$\mathcal{M}=(G/2^{1/2})V_{us}^L \sum_i U_{li}^{L\dagger}[(p_K+p_\pi)^\lambda f_+(q^2)+(p_K-p_\pi)^\lambda f_-(q^2)]\,[\bar{\nu}_i\gamma_\lambda(1-\gamma_5)l+C_R\bar{\nu}_i\gamma_\lambda(1+\gamma_5)l]\ , \tag{1}$$

where

$$G=2^{1/2}g_L{}^2/8M_L{}^2\ ,$$

$$C_R=g_R{}^2M_L{}^2V_{us}^R U_{li}^{R\dagger}/g_L{}^2M_R{}^2V_{us}^L U_{li}^{L\dagger}\ ,$$

g_L and g_R denote, respectively, the field strengths for left-handed (LH) and right-handed (RH) gauge fields in the non-left-right-symmetric $SU(2)_L\times SU(2)_R\times U(1)$ model; M_L and M_R are the masses of W_L and W_R gauge bosons; V_{us}^L and V_{us}^R are the Kobayashi-Maskawa mixings for LH and RH quarks;[10] U_{li}^L and U_{li}^R are the LH and RH neutrino mixing elements and C_R denotes the RHC-contribution parameter.[11]

The expression for the Cabibbo-Maksymowicz polarization vector $\mathbf{P}=\mathbf{A}/|\mathbf{A}|$, with the inclusion of neutrino mixing and RHC's, is derived by using the matrix element (1). The vector $\mathbf{A}$, in the rest frame of the kaon, is given by

$$\mathbf{A}=\sum_i |U_{li}^L|^2\left[(1-C_R{}^2)\left\{\left[a_1(\xi)+\frac{m_i{}^2 m_K\,\mathrm{Re}b(q^2)}{m_l}\right]\mathbf{p}_l -[a_2(\xi)-m_i{}^2|b(q^2)|^2]\left[\mathbf{p}_l\left[\frac{m_K-E_\pi}{m_l}+\frac{(\mathbf{p}_\pi\cdot\mathbf{p}_l)(E_l-m_l)}{m_l|\mathbf{p}_l|^2}\right]+\mathbf{p}_\pi\right]\right\} +m_K\,\mathrm{Im}\xi(q^2)\mathbf{p}_\pi\times\mathbf{p}_l\left[1+C_R{}^2-2C_R\frac{m_i}{m_l}\right]\right]\ , \tag{2}$$

where

$$a_1(\xi)=2m_K{}^2[E_{\nu_i}+\mathrm{Re}b(q^2)(W_0-E_\pi)]/m_l\ ,$$

$$a_2(\xi)=m_K{}^2+2\,\mathrm{Re}b(q^2)m_K E_l+|b(q^2)|^2 m_l{}^2\ ,$$

$$b(q^2)=\tfrac{1}{2}[\xi(q^2)-1],\quad W_0=(m_K{}^2+m_\pi{}^2-m_l{}^2)/2m_K\ ,$$

$$\xi(q^2)=f_-(q^2)/f_+(q^2),\quad m_i=m(\nu_i)\ .$$

The form factors $f_\pm(q^2)$ specify the behavior of the hadronic current. Time-reversal invariance requires f_+ and f_- to be relatively real, i.e., $\mathrm{Im}\xi=0$. Consequently the last term in Eq. (2) representing the l^+ polarization component normal to the decay plane vanishes. In general, to allow for the possibility of breakdown of time-reversal invariance, f_+ and f_- are taken to be complex (see, e.g., Ref. 12, pp. 474—476). At present, measurements of the normal component of muon polarization give an average value $\mathrm{Im}\xi=-0.020\pm0.022$ (see Ref. 13). On retaining the term containing $\mathrm{Im}\xi$, Eq. (2) reduces to the usual expression of Cabibbo and Maksymowicz[8] for the case of zero neutrino masses, no mixing and zero RHC admixture.

The longitudinal, perpendicular, and normal components of the l^+ polarization are defined parallel to $\hat{\mathbf{p}}_l$, perpendicular to $\hat{\mathbf{p}}_l$ [in the decay plane along the direction $\hat{\mathbf{p}}_l\times(\hat{\mathbf{p}}_\pi\times\hat{\mathbf{p}}_l)$] and normal to the decay plane along the direction $\hat{\mathbf{p}}_\pi\times\hat{\mathbf{p}}_l$, respectively.[14] The expression for the longitudinal polarization (P_L^l) is given by[15]

$$P_L^l(E_l,m_\nu,C_R)=\frac{1-C_R{}^2}{1+C_R{}^2}F_L^l(E_l,m_\nu,C_R)P_L^l(E_l,0,0)\ , \tag{3}$$

where

$$P_L^l(E_l,0,0)\equiv P_L^l(E_l,m_\nu=0,C_R=0)\ ,$$

is the longitudinal polarization[12] for the case of vanishing neutrino masses and zero C_R, and

$$F_L^l(E_l,m_\nu,C_R)=\frac{\sum_i |U_{li}|^2 f_L^l(E_l,m_i)}{\sum_i |U_{li}|^2\{h_1^l(E_l,m_i)-[C_R/(1+C_R{}^2)]h_2^l(E_l,m_i)\}}\ . \tag{4}$$

The complete expressions of the functions $f_L^l(E_l,m_i)$ and $h_{1,2}^l(E_l,m_i)$ are lengthy and are given in the Appendix.

It may be emphasized that the neutrino-mixing contributions are congregated in the function F_L^l which attains a constant value one for massless neutrinos. As such, in order to have an idea of neutrino mass contributions to this function, we display the behavior of its constituents f_L^l ($l=\mu,e$) and h_2^l in Figs. 1 and 2, respectively. The nature (not shown here) of the function h_1^l is almost identical to that of f_L^l except for a slight difference in numerical values.

The expressions for perpendicular (P_P^l) and normal (P_N^l) components of the l^+ polarization, with the inclusion of neutrino mixing and RHC's, are given by[15]

$$P_P^l(E_l,m_\nu,C_R)=[(1-C_R{}^2)/(1+C_R{}^2)]F_P^l(E_l,m_\nu,C_R)P_P^l(E_l,0,0) \tag{5a}$$

and

$$P_N^l(E_l,m_\nu,C_R)=F_N^l(E_l,m_\nu,C_R)P_N^l(E_l,0,0)\ , \tag{5b}$$

where

$$F_{P,N}^l(E_l,m_\nu,C_R)=\frac{\sum_i |U_{li}|^2 f_{P,N}^l(E_l,m_i)}{\sum_i |U_{li}|^2\{h_1^l(E_l,m_i)-[C_R/(1+C_R{}^2)]h_2^l(E_l,m_i)\}}\ .$$

The functions $f_{P,N}^l(E_l,m_i)$ are given in the Appendix. The dependence of f_P^μ, f_N^μ, f_P^e, and f_N^e on lepton energy and neutrino mass is almost identical to that of f_L^μ and f_L^e shown in Fig. 1.

In the present three-generation neutrino world ($i=1,2,3$), the current experimental limits on neutrino masses are $20<m_1\le45$ eV (Ref. 16), $m_2\le250$ keV (Ref. 17), and $m_3\le125$ MeV (Ref. 18). This holds if one assumes[19] that ν_e is principally ν_1, ν_μ is mainly ν_2, and ν_τ is ν_3. In order to have an order-of-magnitude estimate of the effects of neutrino mass mixing, we note that the relative contribution of neutrino mass m_i depends on the ratio $\delta_i=m_i/m_K$. The present mass limits give $\delta_1\le10^{-7}$, $\delta_2\le10^{-4}$, and $\delta_3\le0.25$. As such, we may take $\delta_1=\delta_2=0$ for estimating the effects of HSC neutrino ν_3 mixing. With the use of the unitarity condition $|U_{l1}|^2+|U_{l2}|^2=1-|U_{l3}|^2$, expression (3) becomes

$$P_L^l(E_l,m_3,C_R)=[(1-C_R{}^2)/(1+C_R{}^2)]F_L^l(E_l,m_3,C_R)P_L^l(E_l,0,0)\ , \tag{6}$$

where

$$F_L^l(E_l,m_3,C_R)=\frac{1-|U_{l3}|^2[1-f_L^l(E_l,m_3)]}{1-|U_{l3}|^2\{1-h_1^l(E_l,m_3)+[C_R/(1+C_R{}^2)]h_2^l(E_l,m_3)\}}\ .$$

We plot in Fig. 3 the function $F_L^\mu(E_\mu,m_3,C_R)$ for $m_3=75$ and 125 MeV, $C_R=0$ and 0.2, for two different values of neutrino mixing element.[20] We note that the HSC neutrino mixing contributions to this function are small and change slightly for finite[21] $C_R(=0.2)$. At the kinematic cutoff of E_μ, F_L^μ approaches unity. The nature (not shown here) of the other two functions F_P^μ and F_N^μ that would occur in the corresponding expressions of P_P^μ and

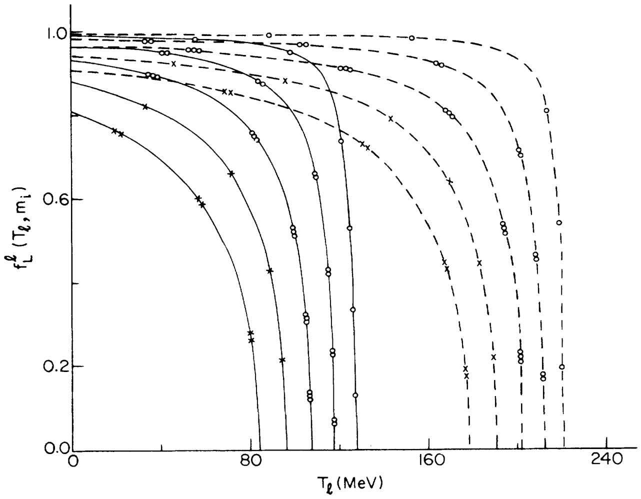

FIG. 1. The variation of $f_L^l(T_l,m_i)$ with lepton kinetic energy T_l ($l=\mu^+,e^+$). The solid curves are for $f_L^\mu(T_\mu,m_i)$ and the dashed curves are for $f_L^e(T_e,m_i)$. The single circle, double circle, triple circle, single cross, and double cross on the curves represent $m_i=25$, 50, 75, 100, and 125 MeV, respectively.

P_N^μ in the ν_3 dominance limit is similar to that of F_L^μ. We give below the estimates of HSC neutrino mixing effects to the three components of μ^+ polarization for the case of $m_3\le 125$ MeV, $|U_{\mu 3}|^2\le 0.059$ (Ref. 22), without and with the inclusion of finite C_R.

We define

$$\Delta P_\beta^l(E_l,m_3,0)\equiv |P_\beta^l(E_l,m_3,0)-P_\beta^l(E_l,0,0)| ,$$

where $\beta=L$, P, or N. Then, we find that (i) for $C_R=0$,

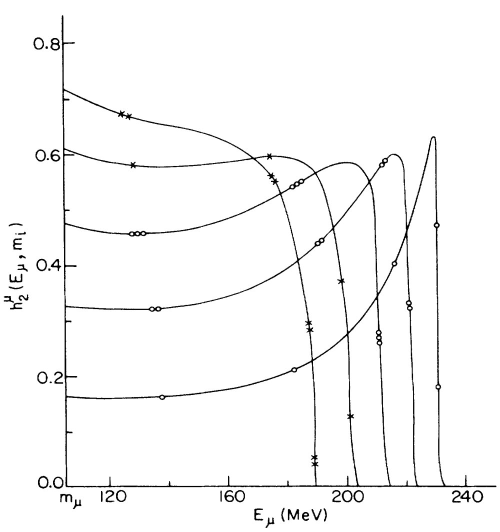

FIG. 2. The nature of the function $h_2^\mu(E_\mu,m_i)$. The description of the curves is identical to that of the solid curves shown in Fig. 1. The function $h_2^e(E_e,m_i)$ (not shown here) is approximately 2 orders of magnitude (m_e/m_μ) smaller than h_2^μ and has an identical behavior.

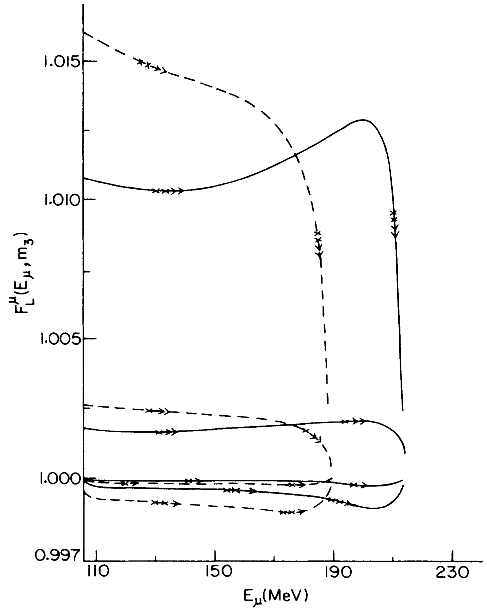

FIG. 3. The variation of $F_L^\mu(E_\mu,m_3)$ with E_μ. The solid and dashed curves are for $m_3=75$ and 125 MeV, respectively. The single and double arrow on the curves are for $C_R=0$ and 0.2, and the single and double cross on the curves are for $|U_{\mu 3}|^2=0.01$ and 0.059, respectively.

$$\frac{\Delta P_L^\mu(E_\mu,m_3,0)}{P_L^\mu(E_\mu,0,0)}<0.12\% ,\qquad \frac{\Delta P_{P,N}^\mu(E_\mu,m_3,0)}{P_{P,N}^\mu(E_\mu,0,0)}<1.2\% , \tag{7}$$

(ii) for $C_R=0.2$ (Ref. 21),

$$\frac{\Delta P_{L,P}^\mu(E_\mu,m_3,0.2)}{P_{L,P}^\mu(E,0,0.2)}<1.6\% ,\qquad \frac{\Delta P_N^\mu(E_\mu,m_3,0.2)}{P_N^\mu(E,0,0.2)}<0.4\% , \tag{8}$$

where

$$P_{L,P}^\mu(E_\mu,0,C_R)=[(1-C_R{}^2)/(1+C_R{}^2)]P_{L,P}^\mu(E_\mu,0,0) ,$$

and

$$P_N^\mu(E_\mu,0,C_R)=P_N^\mu(E_\mu,0,0) .$$

The corresponding contributions to e^+ polarization, for $m_3\le 125$ MeV, $|U_{e3}|^2\le 0.044$ (Ref. 23), and $C_R\le 0.2$, are[24]

$$\frac{\Delta P_L^e(E_e,m_3,0)}{P_L^e(E_e,0,0)}<0.002\% ,\qquad \frac{\Delta P_L^e(E_e,m_3,0.2)}{P_L^e(E_e,0,0.2)}<0.1\% . \tag{9}$$

The experiments on lepton polarization measurements[25] report average values of polarization components. The relevant estimates for average values of $P_{L,P,N}^l$ with the inclusion of HSC neutrino ν_3 mixing and finite C_R are obtained by averaging Eqs. (3) and (5) over the entire Dalitz plot. We express average polarization components as

$$\langle P^l_{L,P}(m_3,C_R)\rangle=[(1-C_R{}^2)/(1+C_R{}^2)]\times G^l_{L,P}(m_3,C_R)\langle P^l_{L,P}(0,0)\rangle \quad (10)$$

and

$$\langle P^l_N(m_3,C_R)\rangle=G^l_N(m_3,C_R)\langle P^l_N(0,0)\rangle\ , \quad (11)$$

where the variation of the functions $G^\mu_{L,P}(m_3,C_R)$ with m_3, for $K^0_{\mu 3}$, is shown in Fig. 4. For $m_3\le 125$ MeV, $|U_{\mu 3}|^2\le 0.059$ (Ref. 22), and $C_R\le 0.2$, we obtain

$$\Delta\langle P^\mu_{L,P}(m_3,C_R)\rangle/\langle P^\mu_{L,P}(0,C_R)\rangle<0.5\% \quad (12)$$

and

$$\Delta\langle P^\mu_N(m_3,C_R)\rangle/\langle P^\mu_N(0,C_R)\rangle<1.0\%\ . \quad (13)$$

In the case of K^0_{e3} decay the nature of the function $G^e_L(m_3,C_R)$ is similar to that of $G^\mu_L(m_3,C_R)$ shown in Fig. 4. It is found that[24] for $m_3\le 125$ MeV, $C_R\le 0.2$, and $|U_{e3}|^2\le 0.044$ (Ref. 23), $G^e_L(m_3,C_R)=1$ within 0.01%. Therefore, for K^0_{e3} decay, Eq. (10) becomes

$$\langle P^e_L(m_3,C_R)\rangle\simeq[(1-C_R{}^2)/(1+C_R{}^2)]\langle P^e_L(0,0)\rangle\ . \quad (14)$$

This shows that any significant decrease in $\langle P^e_L\rangle$ may be attributed to RHC contributions even though large HSC neutrino mixing ($U_{e3}=0.21$) may be present.

It is well known that the value of μ^+ polarization depends significantly on the value of ξ. The uncertainty in the presently known best-fit value[13] $\xi(0)=-0.11\pm0.09$ gives

$$\frac{\langle P_L(\xi=-0.2)\rangle-\langle P_L(\xi=-0.02)\rangle}{\langle P_L(\xi=-0.11)\rangle}=4.5\%\ ,$$

$\Delta\langle P_P(\xi)\rangle/\langle P_P(\xi)\rangle=4.6\%$ and $\Delta\langle P_N(\xi)\rangle/\langle P_N(\xi)\rangle=3.2\%$. As such, the uncertainty in ξ makes it almost impossible to discern the effects of HSC neutrino mixing or the RHC admixture from the present μ^+ polarization measurements.[25] The situation could improve if ξ is known with a much better precession.

The numerical results given here refer to $K^0_L\to\pi^-l^+\nu$ decays but the conclusions apply equally well to $K^+\to\pi^0l^+\nu$. In Ref. 3, for K^+ decays, the longitudinal component $P^l_L(E_l,m_3,C_R)$ of charged-lepton polarization has been calculated. It is shown that, in principle, (i) a study of the variation of muon polarization P^μ_L with E_μ could be a useful place to ascertain the RHC contributions and ν_3 mixing effects, (ii) the existence of electron polarization $P^e_L<1$ would be indicative of the RHC admixture, and (iii) any variation of P^e_L with electron energy E_e would be indicative of contributions of finite m_3. Here, we find that the presently known uncertainty in ξ [$\xi(0)=-0.35\pm0.15$ for $K^+_{\mu 3}$ decay[13]] would render it almost impossible to discern the effects of HSC neutrino mixing or the RHC admixture in $K^+_{\mu 3}$ decay also. Further, due to ν_3 mixing, the change expected in P^e_L, for any value of kinematically allowed E_e, is less than about 0.1% for $C_R\le 0.2$, $m_3\le 125$ MeV and as such difficult to detect. However, the average longitudinal polarization of e^+ is not sensitive to the value of ξ and the HSC neutrino mixing contributions to $\langle P^e_L\rangle$ are also insignificant (less than 0.01%). Therefore, this measurement, if carried out,[26] and giving $\langle P^e_L\rangle$ value different from unity[27] could be a positive signature of the RHC admixture even if a few percent of HSC neutrino mixing exists.[28]

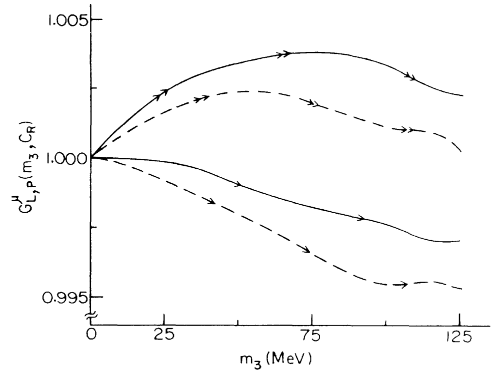

FIG. 4. The variation of $G^\mu_{L,P}(m_3,C_R)$ with the HSC neutrino mass m_3 (for $|U_{\mu 3}|^2=0.059$). The solid curves are for G^μ_L and the dashed curves are for G^μ_P. The single and double arrow on the curves represent $C_R=0$ and 0.2, respectively. The nature of the function $G^\mu_N(m_3,C_R)$ (not shown here) is similar to that of the G^μ_P except for difference in numerical values.

APPENDIX

For brevity we use the notation $f^l_{L,P,N}$ to represent one of the three functions f^l_L, f^l_P, or f^l_N. All relations are expressed in the kaon rest frame:

$$f^l_{L,P,N}(E_l,m_i)=\int_{\alpha_i}^{\beta_i}X_{L,P,N}(E_l,E_\pi,m_i)dE_\pi\Big/\int_a^b X_{L,P,N}(E_l,E_\pi,m_i=0)dE_\pi\ ,$$

$$h^l_{1,2}(E_l,m_i)=\int_{\alpha_i}^{\beta_i}Y_{1,2}(E_l,E_\pi,m_i)dE_\pi\Big/\int_a^b Y_1(E_l,E_\pi,m_i=0)dE_\pi\ ,$$

$$P^l_{L,P,N}(E_l,0)=\int_a^b X_{L,P,N}(E_l,E_\pi,m_i=0)dE_\pi\Big/\int_a^b Y_1(E_l,E_\pi,m_i=0)dE_\pi\ ,$$

where

$$X_L(E_l,E_\pi,m_i)=m_l\bar{\lambda}\{[a_1(\xi)+\mathrm{Re}\,b(q^2)m_i{}^2m_K/m_l]\,|\mathbf{p}_l|-[a_2(\xi)-|b(q^2)|^2m_i{}^2][\,|\mathbf{p}_l|(m_K-E_\pi)+|\mathbf{p}_\pi|E_l\cos\theta_{\pi l}]/m_l\}\ ,$$

$$X_P(E_l,E_\pi,m_i)=-m_l\bar{\lambda}[a_2(\xi)-|b(q^2)|^2m_i{}^2]\,|\mathbf{p}_\pi|\sin\theta_{\pi l}\ ,$$

$$X_N(E_l,E_\pi,m_i)=-m_l m_K\bar{\lambda}[1-2C_R m_i/(1+C_R{}^2)m_l]\,|\,\mathbf{p}_l\,|\,\mathrm{Im}\xi(q^2)\,|\,\mathbf{p}_\pi\,|\,\sin\theta_{\pi l}\ ,$$

$$Y_1(E_l,E_\pi,m_i)=m_K\bar{\lambda}[A+B\,\mathrm{Re}\xi(q^2)+C\,|\,\xi(q^2)\,|^2+m_i{}^2D-|\,b(q^2)\,|^2 m_i{}^4/2m_K]\ ,$$

$$Y_2(E_l,E_\pi,m_i)=\tfrac{1}{2}m_i m_l\bar{\lambda}\{m_K{}^2\,|\,1+\xi(q^2)\,|^2+m_\pi{}^2\,|\,1-\xi(q^2)\,|^2+2m_K E_\pi[1-|\,\xi(q^2)\,|^2]\}\ ,$$

$$A=m_K[2E_lE_{\nu_i}-m_K(W_0-E_\pi)]+m_l{}^2(W_0-E_\pi)/4-m_l{}^2E_{\nu_i}\ ,$$

$$B=m_l{}^2[E_{\nu_i}-(W_0-E_\pi)/2],\quad C=m_l{}^2(W_0-E_\pi)/4\ ,$$

$$D=m_K/2+2E_l\,\mathrm{Re}b(q^2)+(W_0-E_\pi+3m_l{}^2/2m_K)\,|\,b(q^2)\,|^2\ ,$$

$$\bar{\lambda}=(1+\lambda_+q^2/m_\pi{}^2)\ .$$

The integration limits $\alpha_i=E^i_{\pi,\min}$ and $\beta_i=E^i_{\pi,\max}$ are obtained from the following kinematic limits on E_π:

$$E^i_{\pi,\max(\min)}=\{[(m_K-E_l)^2-|\,\mathbf{p}_l\,|^2+m_\pi{}^2-m_i{}^2](m_K-E_l)(\pm)\,|\,\mathbf{p}_l\,|\,[(m_K-E_l)^2-|\,\mathbf{p}_l\,|^2-(m_\pi-m_i)^2]^{1/2}$$
$$\times[(m_K-E_l)^2-|\,\mathbf{p}_l\,|^2-(m_\pi+m_i)^2]^{1/2}\}\{2[(m_K-E_l)^2-|\,\mathbf{p}_l\,|^2]\}^{-1}\ . \qquad \text{(A1)}$$

The limits $a=E^0_{\pi,\min}$ and $b=E^0_{\pi,\max}$ are obtained by substituting $m_i=0$ in Eq. (A1).

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[26]We are not aware of any measurement of e^+ polarization in $K^0_L\to\pi^-e^+\nu_e$ decay.

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