PHYSICAL REVIEW D VOLUME 35, NUMBER 7 1 APRIL 1987

Effects of core motion on the nucleon electric form factors

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(Received 8 December 1986)

When the nucleon is described as the self-consistent nontopological soliton ground state of a translation-invariant model Hamiltonian for a nonstatic baglike core interacting with a pion field, the motion of the core is shown to have a significant effect on the proton and neutron electric form factors, as compared to previous cloudy-bag-model (CBM) calculations that neglect the core motion and treat the baglike core as a static source of pion field. In the translation-invariant model, the charge density of the nucleon has contributions from the core and from the pion cloud, as in the usual static CBM; in addition, and in contrast to the usual static CBM, there are effects due to the spreading of the core charge density as a result of the self-consistent motion of the core within the pion field that it generates. The spreading of the core density tends to weaken the core-pion interaction for a fixed bag radius so that the binding of the core-pion system is less than in the static cloudy bag. Hence, the motion of the core softens the electric form factors as compared to static CBM calculations.

I. INTRODUCTION

For a decade or so models motivated by quantum chromodynamics have provided a useful framework for trying to understand the structure of the nucleon.[1–10] Two important physical features of these models are (a) quark substructure at short distances and (b) meson-field structure at larger distances. Some models of this type require the quarks to interact self-consistently with mean meson fields,[9,10] while others treat the QCD core as a unit whose properties are calculated independently of the surrounding meson field.[2–5] The present work is in the context of this second group of phenomenological models of nucleon structure, which can appropriately be called "cloudy bag models" (CBM's).[5] In a CBM the core is taken to have a sharp spherical boundary of radius R within which the physics is assumed to be like the MIT bag model;[1] for example, the charge distribution of the core is taken to be that of the MIT bag with three massless quarks in the lowest S state in the bag. The core has a Yukawa interaction with the pion field that bears some relation to the idea of chiral current conservation; in the present work, the form of the Yukawa interaction is taken to be the interaction that is derived in chiral bag models,[5] namely,

$$\frac{f}{m}\frac{3j_1(kR)}{kR}i\tau_\lambda\boldsymbol{\sigma}\cdot\mathbf{k}\ ,$$

where R is the radius of the bag. The Yukawa interaction of the core with the pion field generates a cloud of virtual pions around the core. Both the quarks in the bag and the virtual pions around the bag contribute to the total charge of the nucleon as well as to other static properties such as the nucleon charge radius and magnetic moment. Nonstatic quantities such as the nucleon electric form factor are also the sum of contributions from the intrinsic quark charge density and the pion charge density.

In the past, the core in CBM's was treated in a static approximation, in which the core remains fixed and acts as a static source of pion field.[5] Generally speaking, a CBM Hamiltonian that is invariant under translations must include core motion, that is, it must include a kinetic energy term for the core and the Yukawa interaction of the static core with the pion field must be generalized to an interaction of the dynamic core with the pion field that is invariant under translations. In a recent paper[11] it was shown that when the static CBM Hamiltonian is modified so as to include core motion in these ways, the ground state of the nucleon can be a pionic nontopological soliton state of the core with its cloud of virtual pions. Under the assumption that meson fields other than the pion field could be neglected, the pionic soliton state was used to compute charge radii and magnetic moments of the nucleon, with the result that the motion of the core was shown to alter the values of these static properties significantly for a range of values of the bag radius R. In addition to contributions from the core and pion field to nucleon electromagnetic properties, there is now a third contribution due to the self-consistent motion of the core in the pion field it generates. If the core has an intrinsic size or bag radius R and the core motion is of extent S, it is clear that the apparent size of the core will be of order $(R^2+S^2)^{1/2}$; that is, the intrinsic bag radius can be substantially less than the apparent size. In order to give a quantitative aspect to this rather intuitive argument, Ref. 11 showed how it applies to the nucleon magnetic moments and charge radii.

In this paper, we calculate the effect of core motion on the electric form factor of the nucleon for the particular form of core motion that was used in the computations of Ref. 11. The translation-invariant Hamiltonian used in Ref. 11 to represent the system consisting of the core interacting with a pion field is

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$$
\begin{aligned}
H &= T_c + T_\pi + H_{\text{Yukawa}} + H^\dagger_{\text{Yukawa}} , \\
T_c &= \int \tilde{\Psi}^\dagger(\mathbf{p}) \frac{p^2}{2M} \tilde{\Psi}(\mathbf{p}) d\mathbf{p} , \\
T_\pi &= \int \omega(k) a^\dagger_\lambda(\mathbf{k}) a_\lambda(\mathbf{k}) d\mathbf{k} , \\
H_{\text{Yukawa}} &= -\int \frac{a_\lambda(\mathbf{k})}{[16\pi^3\omega(k)]^{1/2}} \delta(\mathbf{p}-\mathbf{q}-\mathbf{k}) \\
&\quad \times \tilde{\Psi}^\dagger(\mathbf{p}) \tau_\lambda \sigma \cdot \mathbf{J}_\pi \left[\mathbf{k}, \frac{\mathbf{p}+\mathbf{q}}{2} \right] \tilde{\Psi}(\mathbf{q}) d\mathbf{p}\, d\mathbf{q}\, d\mathbf{k} ,
\end{aligned} \tag{1}
$$

where the summation convention is used for the pion isospin index λ. For the specific case of the cloudy bag, the pion-nucleon current operator $\mathbf{J}$ is independent of its second argument:

$$
\mathbf{J}(\mathbf{k},\mathbf{K}) = if \frac{\mathbf{k}}{m} \frac{3j_1(kR)}{kR} = i \frac{\mathbf{k}}{m} c(k) , \tag{2}
$$

where m is taken to be charged pion mass.

References 11 and 12 describe the methods used to find an approximate ground state of the Hamiltonian of (1) with the pion-nucleon current operator of (2). With the renormalized πNN coupling constant required to have the value 0.08 and the mass M of the core taken to be the nucleon mass (see Sec. IV below for further discussion), the only free parameter is the bag radius R. An essential feature of the approximation method described in Refs. 11 and 12 is that the core is assumed to occupy just the four isospin-spin substates of a single S state; this is a single-wave-function or "single-mode" approximation for the core field operator. Similarly, the virtual pions are restricted to a single mode, the P state $\sqrt{3}\hat{\mathbf{k}}\phi(k)$ (with nine orthonormal isospin-spin substates). Thus, the effective field operators $\Psi(\mathbf{r})$ for the core and $a_\lambda(\mathbf{k})$ for the pion field are

$$
\begin{aligned}
\Psi(\mathbf{r}) &\simeq f(r) \sum_{i=1}^{4} \alpha_i B_i , \\
a_\lambda(\mathbf{k}) &\simeq \sum_{i=1}^{3} \frac{k_i\sqrt{3}}{k} \phi(k) A_{\lambda i} ,
\end{aligned} \tag{3}
$$

where B_i annihilates the core in the isospin-spin substrate α_i of the S state with radial wave function $f(r)$ and $A_{\lambda i}$ annihilates a pion in the substrate λi of the P state with i momentum-space component $(k_i\sqrt{3}/k)\phi(k)$. The mode functions are normalized,

$$
\int |f(r)|^2 d\mathbf{r} = \int |\phi(k)|^2 d\mathbf{k} = 1 , \tag{4}
$$

and $\Psi(\mathbf{r})$ is the Fourier transform of the field operator $\tilde{\Psi}(\mathbf{p})$ that appears in Eq. (1). The choice of the pion-field mode function ϕ that minimizes the energy is[13]

$$
\phi(k) = \frac{ik}{Gm} \frac{c(k)\tilde{\rho}(k)}{[48\pi^3\omega^3(k)]^{1/2}} , \tag{5}
$$

with $c(k)$ defined by Eq. (2), $\tilde{\rho}(k)$ the Fourier transform of the probability density of the core S state,

$$
\tilde{\rho}(k) = \int e^{i\mathbf{k}\cdot\mathbf{r}} |f(r)|^2 d\mathbf{r} , \tag{6}
$$

and G the normalization constant for $\phi(k)$. The dependence in Eq. (5) of the pion-field mode function $\phi(k)$ on the core probability density $\tilde{\rho}(k)$ expresses the self-consistent relationship between the motion of the core and the pion field it generates.

When the single-mode approximations of Eq. (3) are substituted into the Hamiltonian of Eq. (1) an effective Hamiltonian results. References 11 and 12 describe the variational method used to find approximate eigenvectors and eigenvalues of this effective Hamiltonian. The variational state described in Ref. 11 and used here to compute the nucleon electric form factor is a localized state that breaks translation invariance because of the single-mode approximations for the core- and pion-field operators. This state minimizes the expectation value of the Hamiltonian over the single-mode subspace and satisfies the constraint that the expectation value of the total momentum of the state be zero. Within the single-mode approximation, the variational states used are allowed to contain large numbers of virtual pions by the use of coherent-state techniques; this is important for bag radii less than about 1 fm, where the pion-nucleon coupling is no longer weak. Finally, the variational states are constructed to be eigenstates of the total isospin and total spin of the system; projection methods are not used.

II. CHARGE DENSITY

In the Hamiltonian of (1) and in the approximate ground-state vector described in Ref. 11, the core is treated as a point object moving in the pion field it generates. Of course, the bag of quarks is not a point object, so that the various core current operators, such as the core charge-density operator, must include effects associated with the bag's extended nature. In order to avoid confusion, the fictitious phenomenological point core whose field annihilation operator is $\Psi(\mathbf{r})$ will be called the "pcore" (for "point core"). As in Ref. 11, it is assumed that the core charge-density operator is a convolution of the (c-number) quark charge density within the bag $K_{\text{bag}}(r)$ and the charge density of the pcore wave function:

$$
\begin{aligned}
\hat{\rho}_{\text{core}}(\mathbf{r}) &= \int K_{\text{bag}}(\mathbf{r}-\mathbf{r}') \hat{\rho}_{\text{pcore}}(\mathbf{r}') d\mathbf{r}' , \\
\hat{\rho}_{\text{pcore}}(\mathbf{r}) &= \Psi^\dagger(\mathbf{r}) \frac{1+\tau_3}{2} \Psi(\mathbf{r}) .
\end{aligned} \tag{7}
$$

The bag charge density is[1,14]

$$
\begin{aligned}
K_{\text{bag}}(r) &= \frac{N^2\theta(R-r)}{8\pi} \left[j_0{}^2\left[\frac{\omega r}{R}\right] + j_1{}^2\left[\frac{\omega r}{R}\right] \right] , \\
N^2 &= \frac{\omega^3}{2R^3(\omega-1)\sin^2\omega} ,
\end{aligned} \tag{8}
$$

with $\omega \simeq 2.04$. For the case that the pcore isoscalar and isovector probability densities are delta functions, this form for the core charge density reduces to the standard static-bag charge density $K_{\text{bag}}(r)(1+\tau_3)/2$. The use of a convolution here is analogous to the way that the charge density of a nucleon is used in computing the charge density of a nucleus that consists of phenomenological point nucleons.

The combination $\sum_{ij} B_i^\dagger \alpha_i^\dagger \hat{\beta} \alpha_j B_j$ that appears when $\hat{\rho}_{\text{pcore}}$ is expanded in terms of the S state of (3) is just the operator $\hat{\beta}$ for the pcore in the S state with radial function $f(r)$, where $\hat{\beta}$ is any of τ, σ, or $\tau\sigma$. It follows, as would be expected, that

$$
\hat{\rho}_{\text{pcore}}(\mathbf{r}) = |f(r)|^2 \frac{1+\tau_3}{2} \tag{9}
$$

when the pcore is restricted to be in the single S state with radial wave function $f(r)$.

The pion electric charge density operator should probably also be taken of the form

$$\hat{\rho}_{\text{pion}}(\mathbf{r})=\int K_\pi(\mathbf{r}-\mathbf{r}')\hat{\rho}_{\text{ppion}}(\mathbf{r}')d\mathbf{r}' ,$$
$$\hat{\rho}_{\text{ppion}}(\mathbf{r})=e\epsilon_{3\mu\nu}\Phi_\mu(\mathbf{r})\dot{\Phi}_\nu(\mathbf{r}) , \tag{10}$$

where $\hat{\rho}_{\text{ppion}}(\mathbf{r})$ is the usual Noether electric charge-density operator for the pion field.[15] The distribution $K_\pi(\mathbf{r})$ can be used to take into account the charge distribution of the quarks that make up the pion or to incorporate the hypothesis of vector-meson dominance. In this latter case, the form of $K_\pi(\mathbf{r})$ would be

$$K_\pi^{\text{VMD}}(\mathbf{r})=\frac{m_V{}^2}{4\pi}\frac{e^{-m_Vr}}{r} . \tag{11}$$

In the present work, however, the size of the pion has been taken to be zero, that is, $K_\pi(\mathbf{r})$ has been taken to be $\delta(\mathbf{r})$ and

$$\hat{\rho}_{\text{pion}}(\mathbf{r})=\hat{\rho}_{\text{ppion}}(\mathbf{r})=e\epsilon_{3\mu\nu}\Phi_\mu(\mathbf{r})\dot{\Phi}_\nu(\mathbf{r}) . \tag{12}$$

When this is evaluated in terms of the modes of (3), the effective pion charge-density operator in the ϕ-mode subspace is

$$\hat{\rho}^{\text{eff}}_{\text{pion}}(\mathbf{r})=\frac{d\chi(r)}{dr}\frac{d\psi(r)}{dr}P_3^\pi ,$$
$$P_\lambda^\pi=-3i\epsilon_{\lambda\mu\nu}\hat{r}_i\hat{r}_jA^\dagger_{\mu i}A_{\nu j} \tag{13}$$

with the functions $\chi(r)$ and $\psi(r)$ given by

$$\chi(r)=\frac{f}{\sqrt{6}Gm}\int e^{i\mathbf{k}\cdot\mathbf{r}}\frac{c(k)\tilde{\rho}(k)}{\omega^2(k)}\frac{d\mathbf{k}}{(2\pi)^3} ,$$
$$\psi(r)=\frac{f}{\sqrt{6}Gm}\int e^{i\mathbf{k}\cdot\mathbf{r}}\frac{c(k)\tilde{\rho}(k)}{\omega(k)}\frac{d\mathbf{k}}{(2\pi)^3} . \tag{14}$$

From these forms it follows that

$$\int \chi'(r)\psi'(r)d\mathbf{r}=1 . \tag{15}$$

Now it is easy to see that

$$P_\lambda^\pi=I_\lambda^\pi+3i\epsilon_{\lambda\mu\nu}(\hat{r}_i\hat{r}_j-\tfrac{1}{3}\delta_{ij})(A^\dagger_{\mu i}A_{\nu j}-\tfrac{1}{3}A^\dagger_{\mu k}A_{\nu k}\delta_{ij}) , \tag{16}$$

where I_λ^π is the pion-field total isospin operator:

$$I_\lambda^\pi=-i\epsilon_{\lambda\mu\nu}A^\dagger_{\mu i}A_{\nu i} . \tag{17}$$

Since the last term in (16) has angular momentum 2 in the pcore variable and the pcore is in a state with $j=\frac{1}{2}$, only the I_λ^π term contributes to the pion charge density in the nucleon state.

Thus, finally, the total charge-density operator in the nucleon state is

$$\hat{\rho}_{\text{nucleon}}(r)=\frac{1+\tau_3}{2}\int K_{\text{bag}}(\mathbf{r}-\mathbf{r}')\,|f(r')|^2d\mathbf{r}' +\chi'(r)\psi'(r)I_3^\pi . \tag{18}$$

The nucleon electric form factor is the Fourier transform of the expectation value of this operator in the variational state of Ref. 11; the isoscalar part is

$$F_{\text{el}}^{\text{isoscalar}}(q)=\tfrac{1}{2}\tilde{K}_{\text{bag}}(q)\tilde{\rho}(q) , \tag{19}$$

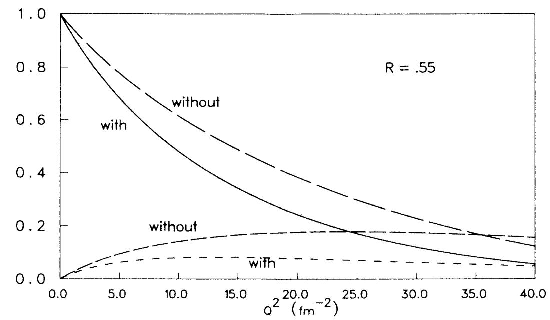

FIG. 1. Proton (above) and neutron (below) electric form factors with and without taking core motion into account for a bag radius of 0.55 fm.

where $\tilde{K}_{\text{bag}}(q)$ is the Fourier transform of $K_{\text{bag}}(r)$ of Eq. (8); the isovector part is

$$F_{\text{el}}^{\text{isovector}}(q)=\tfrac{1}{2}\langle\tau_3\rangle\tilde{K}_{\text{bag}}(q)\tilde{\rho}(q) +\langle I_3^\pi\rangle\int e^{-i\mathbf{q}\cdot\mathbf{r}}\chi'(r)\psi'(r)d\mathbf{r} , \tag{20}$$

where the expectation values $\langle\tau_3\rangle$ and $\langle I_3^\pi\rangle$ are evaluated in the nucleon-pionic soliton state.

III. COMPUTATIONS

Figure 1 shows the proton and neutron electric form factors computed as described above for a bag radius of 0.55 fm, both with and without core motion. The effect of the core motion is to increase the magnitude of the slope of the proton form factor at $Q^2=0$, that is, to increase the rms charge radius; the smaller values at large Q^2 are also consistent with a softer or more extended core. The neutron form factor involves core and pion contributions of opposite sign; Ref. 11 discusses how the results for the mean-square charge radius of the neutron show the effects of the more extended core. Figure 2 shows a plot of the proton electric form factor for various values of the bag radius along with the usual dipole fit to the data and shows that as the bag radius gets larger the bag-pion system is more loosely bound and therefore the electric form factor becomes softer.

For reference, Fig. 3 shows the sizes of the various contributions to the proton electric form factor. It was

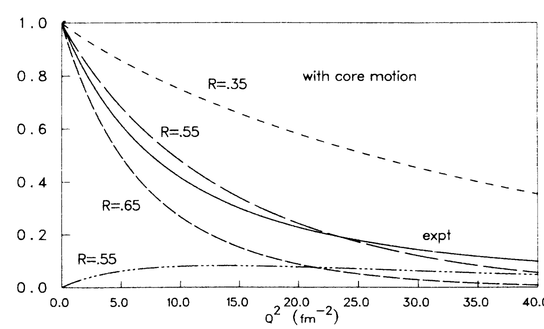

FIG. 2. Proton (above) and neutron (below) electric form factors with core motion taken into account for bag radii varying from 0.35 to 0.65 fm. The dipole fit to the experimental proton form factor is also shown, and the neutron form factor is shown for a bag radius of 0.55 fm.

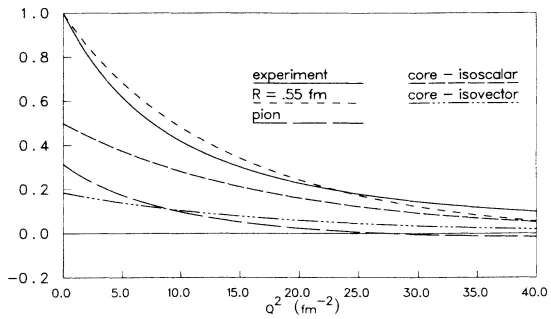

FIG. 3. A breakdown of the contributions to the proton electric form factor.

shown in Ref. 11 that the point-core wave function is very nearly an exponential, and therefore the point-core contribution $\tilde{\rho}(q)$ to the electric form factor, which is characteristic of the pionic-soliton description of the nucleon, has a dipole form. At a bag radius of 0.55 fm the core contribution is more than half of the form factor, so that it may be possible to attribute the dipole form largely to the self-consistent wave function of the core if the present calculations at a bag radius of about 0.55 fm are close to the physical nucleon. However, there are some physical effects that have been omitted, and the approximations used in treating the Hamiltonian are not completely satisfactory, so that such a conclusion may well be premature.

IV. REMARKS

There are effects that have not yet been incorporated into these cloudy-bag-model computations. The smearing of the pion isovector part of the form factor that would arise from a pion form factor of the type that appears in Eq. (10) seems likely (cf. Fig. 3) to produce a minor alteration of the form factors. Perhaps the most important physics that needs to be studied is the influence of translation invariance on the form factors. An improved state, compared to the one used here, would be an eigenstate of linear momentum that minimizes the total energy of the system. Such a state would also give a more reliable value for the effective mass of the ground state and, hence, a better value for the (bare) mass of the core; the core wave function and, hence, the form factor will be affected by this change in the core mass.

V. SUMMARY

In a model in which the nucleon consists of a nonstatic baglike core and its pion field in a self-consistent nontopological soliton state, the motion of the core has been shown to have a significant effect on the proton and neutron electric form factors. The charge density of the nucleon has contributions from the core and from the pion cloud, as in the usual static cloudy bag model; in addition, and in contrast to the usual CBM, there are effects due to the spreading of the core charge density that is a consequence of the self-consistent motion of the core within the pion field that it generates. The trend of these effects indicates that core motion tends to soften the electric form factors as compared to static CBM calculations. At a more speculative level, the calculations also indicate that the physical basis of the dipole fit to the proton electric form-factor data may, at least for small momentum transfers, be just this charge distribution of the bag or core as it moves in the pion field it generates. Within the context of the Hamiltonian and the approximations used in our nonstatic CBM calculations, the self-consistent wave function of the bag or core is an exponential, which translates into a dipole form factor that multiples the more slowly varying intrinsic core charge form factor.

ACKNOWLEDGMENTS

This work was performed under the auspices of the U.S. Department of Energy. The work of J.A.P. was supported in part by the Department of Energy, Division of Nuclear Physics, under Grant No. DE-FG05-86ER40252.

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