Constraints on bag formation from the scalar sector

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Previously it was shown that a bag would form automatically when the Skyrme model was minimally modified to satisfy the scale anomaly equation of QCD. Here, we first calculate the nucleons' static properties for the two characteristic fits corresponding to deep- and shallow-bag types. Second we further embellish the model in order for the required scalar sector to contain particles in a realistic mass range with physically narrow widths. This requirement favors the shallow-bag type of solution.

I. INTRODUCTION AND SUMMARY

At present there are two different general approaches to models of the nucleons' properties at low energies. On one hand, there is the approach in which three quarks are confined by fiat to a bag.[1] This bag is generally pictured at a more fundamental level as corresponding to a bubble of higher energy density in a nonperturbative sea of condensed gluons. In the other approach the nucleon is treated as a soliton in an effective chiral Lagrangian of mesons. A theoretical justification for this procedure in terms of QCD has been given[2] on the basis of the "$1/N_c$" expansion, but the model itself was proposed a long time ago by Skyrme.[3,4]

It seems fair to say that both approaches contain a certain amount of truth, and furthermore both require modification from their original forms to achieve an accurate description of the nucleon. Hence it is interesting to try to combine the two in some way. There is actually a very large amount of work[5] in which the bag model is modified to obtain chiral symmetry by adding a pionic tail. This tail has the identical structure as the "Skyrmion" which thus may perhaps be considered as an approximation to this more complicated type of model.

It also seems interesting to see how the combining of the two approaches can be accomplished from the other direction—by starting from a chiral-invariant theory of mesons and obtaining the nucleon as a soliton which is located within a region from which gluon condensation has been suppressed. In an earlier paper[6] it was shown that this could be accomplished by introducing an order parameter field H proportional to the square of the gluon field strength tensor and minimally modifying the Skyrme model so that it obeys the scale anomaly equation of QCD. This type of model may be conveniently used to ascertain how large an effect the bubble confinement mechanism has on the soliton. Of course, for a very accurate description of the nucleon, vector and other mesons should be included[7] since they are important in low-energy meson phenomenology. The more complete low-energy effective Lagrangian thus obtained is fairly complicated. Hence it appears reasonable to first study in detail a Lagrangian in which only the confining fields are present in addition to the pions. The minimal "bag formation" chiral Lagrangian of this type was seen[6] to accommodate two distinct fits. The first is of the "deep bag" type and has the phenomenological advantage that it permits one to use the experimental value of the pion decay constant F_π. The second is of the shallow-bag type and gives a numerical fit similar to the original Skyrme model which features a value of F_π substantially less than the experimental value.

In this paper we first (in Sec. II) briefly review the minimal chiral model which satisfies the trace anomaly and calculate moments of the vector and axial-vector currents for comparison with experiment. The effect of nonzero pion mass (introduced in accordance with the trace-anomaly equation) is considered and found to be relatively small, though not negligible.

Since our model permits the formation of both deep and shallow bags, it is interesting to compare the physics of the two situations. As far as the comparison of F_π and nucleon moments with experiment is concerned, the work in Sec. II shows the deep-bag fit to be slightly favored. However, there are several factors which suggest that the shallow bag may provide a more accurate description of nature. For one thing, it was noted[6] that the model satisfies the $1/N_c$ counting rules and that as $N_c \to \infty$ the deep bag will disappear (although for "physical" values of the parameters this does not necessarily hold). Another difficulty from a phenomenological point of view is that if one does not make the scalar glueball state so massive that it is "frozen out" as an auxiliary field, the model (with both the shallow- and deep-bag fits) predicts its width to be very large (of the order of several GeV). To overcome this difficulty we investigate (in Sec. III) a similar model which possesses a more realistic scalar sector, consisting of both a quarkonium as well as a gluonium scalar. As discussed in Sec. IV, a small modification of this model (which does not significantly change the solution properties) can give physically reasonable decay widths to both scalar particles. In the two-scalar-field model of Sec. III it is found that both the deep- and shallow-bag fits persist but that the very deep bag is inevitably associated with an unphysically low-mass scalar particle, which is rather unlikely. This feature is due to an upper bound on the lightest scalar particle in the model—$m_{\mathrm{scalar}}{}^2 < 8\langle H \rangle / F_\pi{}^2$—which had been observed earlier.[8] The point is that the very-deep-bag fit can only be obtained for a relatively small value of the gluon condensate $\langle H \rangle$. A plausible case for $\langle H \rangle \approx (0.34\ \text{GeV})^4$ has been made by study of the QCD sum rules.[9] This value is in good agreement with that predicted[8] by a consideration of the U(1) anomaly: $\langle H \rangle = \frac{5}{6} F_\pi{}^2 m_{\eta'}{}^2$.

Thus at the present stage the shallow bag is somewhat favored.[10] Strictly speaking, the bag is shallow for the gluon condensate field but somewhat more enhanced for the quarkonium condensate field. One is left with a fit similar to the usual Skyrme model,[4] which does not give good predictions for F_π and the axial-vector renormalization constant g_A. The introduction of vector mesons and/or explicit quarks[11] seems to help but more work is clearly needed. It will be interesting to investigate the role of the bag in these more sophisticated models. Perhaps the deep-bag fit will then get resuscitated.

II. MINIMAL CHIRAL MODEL WITH BAG FORMATION

We previously showed[6] that a minimal modification of the Skyrme chiral Lagrangian to achieve the correct QCD scale anomaly automatically resulted in the formation of a bag in the presence of a chiral soliton. This seems interesting for several reasons. First, it provides an explicit "QCD ingredient" in the Skyrme model which as it stands is based on spontaneously broken chiral symmetry rather than QCD directly. Thus it may provide a useful clue for a "first principles" derivation of low-energy phenomena. Second, it provides a mechanism for implementing a smooth transition from the bag to the nonbag region. Finally and more pragmatically it provides one with another handle to try to improve the predicted properties of the nucleon. Of course, there are many other effects (such as the presence of vector mesons, etc.) which may also play a role in the nucleons' appearance at low energies.

The effective Lagrangian for this model is

$$\begin{aligned}\mathscr{L} = &-\tfrac{1}{2} b^2 (\partial_\mu \psi)^2 - \tfrac{1}{4}\psi^4 \ln\left[\frac{\psi^4}{\Lambda^4}\right] \\ &- \frac{F_\pi{}^2 \psi^2}{8\langle\psi\rangle^2} \mathrm{Tr}(\partial_\mu U \partial_\mu U^\dagger) \\ &+ \frac{1}{32 e_S{}^2} \mathrm{Tr}([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2)\,, \end{aligned} \tag{2.1}$$

where b is a dimensionless constant,[12] e_S is the dimensionless Skyrme constant, F_π is the pion decay constant which is experimentally 132 MeV, and $\langle\psi\rangle = \Lambda/e^{1/4}$ is a measure of the gluon condensate in QCD. The first two terms represent additions to the usual Skyrme model. The scalar field ψ is taken to be an order-parameter field related to the fundamental QCD field strength as $\psi^4 \equiv H = -[\beta(g)/g]\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu})$. The first term is a minimal kinetic term for this field and the second term reproduces the trace anomaly. An additional change is that the third term contains a factor of ψ^2 to ensure that it is scale invariant and hence does not spoil the trace anomaly. The last term in (2.1) is the original Skyrme term which does not require any modification.

Equation (2.1) has static solitonic solutions for the chiral field U, which are very similar to those in the Skyrme model. At large distances the second term's contribution to the energy is minimized when ψ becomes equal to $\Lambda/e^{1/4}$. This makes the potential energy at large distances more negative than what one would have for the $\psi=0$ or "perturbative" vacuum situation. On the other hand, at small distances the third term provides an effective potential energy for ψ which is minimized for $\psi=0$. Depending on the numerical values of the parameters the latter term thereby generates a more or less pronounced suppression of ψ, i.e., a bag. A more quantitative discussion of this feature is contained in Ref. 6.

Although $\mathscr{L}$ depends on four constants we do have at least a rough idea of the value of each. First the value of the gluon condensate has been roughly estimated by the QCD sum-rule approach[9] giving $\langle\psi\rangle \approx 0.34$ GeV. Since the mass associated with the scalar field excitation (presumably the low-lying scalar glueball) is given by $2\langle\psi\rangle/b$, we might expect $b \approx 0.5$ corresponding to a mass choice of about 1.4 GeV. The Skyrme constant e_S has been roughly estimated by assuming that the Skyrme term arises by "integrating out" the effects of a $\rho(770)$ meson; this yields[13] $e_S \approx m_\rho/F_\pi = 5.83$. Finally the pion decay constant F_π has been very accurately measured to be 132 MeV. In a modern discussion of the nucleons' properties in the Skyrme model Adkins, Nappi, and Witten[4] (ANW) found it necessary to choose (with our normalization) $F_\pi = 91$ MeV rather than 132 MeV. An obvious question of interest is to what extent this may be improved in the present model with the retention of reasonable choices for the other less-well-determined parameters.

In Ref. 6 it was found that a fit to the nucleon and Δ masses could be obtained with the true F_π if $\langle\psi\rangle$ was taken to be less than about 0.19 GeV rather than about 0.34 GeV. This could be achieved for a large range of glueball masses. A typical solution of this type is shown in Fig. 1. It features a deep bag and a chiral function $F(r)$, defined from the Skyrme ansatz $U = \exp[i\hat{\mathbf{x}}\cdot\boldsymbol{\tau}F(r)]$, which is slightly enhanced within the bag. On the other hand, if we take $\langle\psi\rangle$ around 0.34 GeV, a fit requires F_π closer to the original value[4] of about 91 MeV. This again holds for a large range of glueball masses and is also illustrated in Fig. 1. The possibility of fitting F_π to its experimental value requires a nontrivial behavior of the function $\psi(r)$. If $\langle\psi\rangle$ is larger than about 0.19 GeV we cannot achieve this and will obtain a rather shallow bag. As pointed out in Ref. 6 the shallow bag is not inconsistent with the difference of inside and outside vacuum energy densities obtained in the MIT bag model.[14]

To further understand this model we would like to compare its predictions for some of the basic static properties of the nucleon with those of the original Skyrme model. The formulas for these quantities are very straightforward modifications of those given by ANW. Specifically, the proton and neutron charge densities are still given by

$$\begin{aligned}\rho_p(r) &= \tfrac{1}{2}[\lambda(r)/\lambda + B(r)]\,, \\ \rho_n(r) &= \tfrac{1}{2}[-\lambda(r)/\lambda + B(r)]\,,\end{aligned} \tag{2.2}$$

where $B(r)=-(2/\pi)F'\sin^2 F$ is a "topological" isoscalar density, $F' \equiv \partial F/\partial r$, and the isovector density $\lambda(r)$ is now

$$\lambda(r)=\frac{4\pi r^2}{3}\sin^2 F\left[8K+\frac{2}{e_S{}^2}\left[F'^2+\frac{\sin^2 F}{r^2}\right]\right], \quad (2.3)$$

where $-K$ is the coefficient of $\mathrm{Tr}(\partial_\mu U \partial_\mu U^\dagger)$ in (2.1). Note that the moment of inertia λ is given by $\int_0^\infty \lambda(r)dr$. We then have the isoscalar and isovector mean-square charge densities

$$\langle r^2\rangle_{I=0}=\int_0^\infty dr\, r^2 B(r)\,,$$
$$\langle r^2\rangle_{I=1}=\int_0^\infty dr\, r^2\lambda(r)/\lambda\,. \quad (2.4)$$

The isoscalar and isovector mean-square magnetic radii are given by

$$\langle r^2\rangle_{M,I=0}=\int_0^\infty dr\, r^4 B(r)/\langle r^2\rangle_{I=0}\,,$$
$$\langle r^2\rangle_{M,I=1}=\langle r^2\rangle_{I=1}\,. \quad (2.5)$$

We shall not discuss separately the $I=0$ and $I=1$ nucleon g factors since they are not independent of other quantities in the model; ANW show that $g_0=(\mu_p+\mu_n)/2=\frac{4}{9}N(\Delta-N)\langle r^2\rangle_{I=0}$ and $g_1=(\mu_p-\mu_n)/2=2N/(\Delta-N)$ where N and Δ stand for the nucleon and Δ masses:

$$N=M+3/(8\lambda),\quad \Delta=M+15/(8\lambda)\,. \quad (2.6)$$

In this formula M is the soliton mass given in Eq. (7) of Ref. 6. Finally the axial-vector renormalization constant[15] in neutron decay g_A is

$$g_A=\begin{Bmatrix}\frac{3}{2}\\ 1\end{Bmatrix}\times\left[\frac{-4\pi}{9}\right]\int\left[8K\left[F'+\frac{\sin 2F}{r}\right]+\frac{2}{e_S{}^2}\left[\frac{\sin 2F}{r}F'^2+\frac{2\sin^2 F}{r^2}F'+\frac{\sin^2 F\sin 2F}{r^3}\right]\right]r^2 dr\,, \quad (2.7)$$

where the factor of $\frac{3}{2}$ is used in the case when the pion mass is zero and the factor 1 is used in the case when the pion mass is nonzero.

It is interesting to consider the effects of a nonzero pion mass term in the model. We would like to include it in such a way[16] that it has the right chiral transformation properties and also mocks up the trace-anomaly equation for massive quarks:

$$\theta_{\mu\mu}=-[\beta(g)/g]\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu})-[1+\gamma(g)]\sum_{i=1}^{3} m_i\bar{q}_i q_i\,. \quad (2.8)$$

Here we have included for generality the *three* light quarks; the quantity $\gamma(g)$ is the anomalous dimension of the $\bar{q}q$ operator. The operator in the last term of (2.8) transforms with scale dimension $(3-\gamma)$ so a suitable mass term in a chiral SU(3)×SU(3) effective Lagrangian would be

$$\mathscr{L}_m=\frac{F_\pi}{2}\left[\frac{\psi}{\langle\psi\rangle}\right]^{3-\gamma}\mathrm{Tr}[A(U+U^\dagger)]\,, \quad (2.9)$$

where the diagonal matrix A has elements $A_1=A_2=F_\pi m_\pi{}^2/4$ and $A_3=(F_\pi/2)(m_k{}^2-m_\pi{}^2/2)$. [For the SU(2) case of present interest one should set A_3 to zero.] Equation (2.9) should be added to (2.1); note that we shall measure the static energy M of the soliton solution by subtracting from the Hamiltonian the vacuum value of the effective potential at $r=\infty$. For the two-flavor case with $\gamma=0$ the asymptotic value of ψ needed to obtain this subtraction constant is obtained as the solution of

$$\langle\psi\rangle=(\Lambda/e^{1/4})\exp\left[\frac{3}{8}\frac{F_\pi{}^2 m_\pi{}^2}{\langle\psi\rangle^4}\right]. \quad (2.10)$$

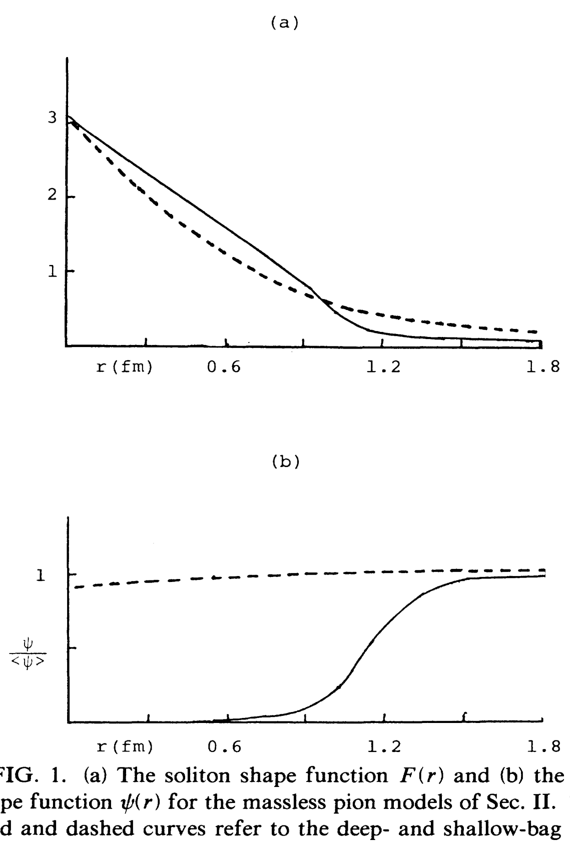

FIG. 1. (a) The soliton shape function $F(r)$ and (b) the bag shape function $\psi(r)$ for the massless pion models of Sec. II. The solid and dashed curves refer to the deep- and shallow-bag fits, respectively.

Now we shall briefly discuss the results of our computation of the nucleons' static properties. In Fig. 2, the charge densities of the proton and neutron [computed from (2.2)] are displayed for both the shallow-bag and deep-bag cases illustrated in Fig. 1. These pictures correspond to the zero-pion-mass case; the effect of the mass term (2.9) on them turns out to be unimportant. It is ob-

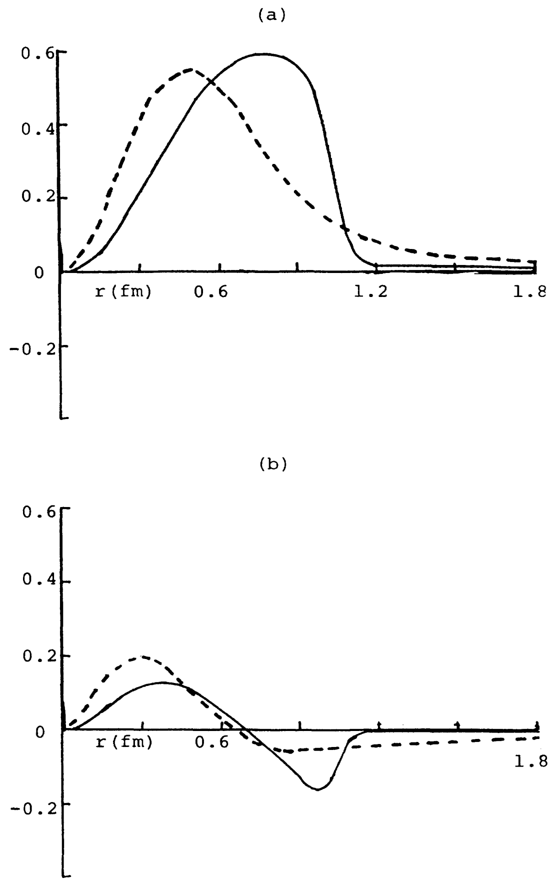

FIG. 2. (a) The proton and (b) the neutron charge densities [from Eq. (2.2)] for the massless pion models of Sec. II. The solid and dashed curves refer to the deep- and shallow-bag fits, respectively.

vious from Fig. 2 that the effect of the deep bag is to significantly decrease the size of the tail. In Table I we list the numerical values of characteristic static properties for the two massless cases illustrated in Fig. 1 as well as for shallow- and deep-bag fits with nonzero pion masses. In the latter cases the anomalous dimension γ in (2.9) was set to zero; we found that the results were not very sensitive to reasonable variations of this parameter. Our Lagrangian contains four independent parameters but it is probably best to think of the four input parameters as the mass of the nucleon, the mass of the Δ, the glueball mass, and the vacuum condensate value $\langle\psi\rangle$. The predictions are then F_π, the Skyrme constant e_S, the three charge radii $\langle r^2\rangle_{I=0}$, $\langle r^2\rangle_{M,I=0}$, $\langle r^2\rangle_{I=1}=\langle r^2\rangle_{M,I=1}$, and the β decay constant g_A. We should first remark that the predictions in the shallow-bag case are essentially the same as those of ANW (for the massless case) and Adkins and Nappi[17] for the massive case. In the massless case, first, we see that the deep bag gives better predictions for F_π, $\langle r^2\rangle_{I=0}$, and $\langle r^2\rangle_{M,I=0}$ than does the shallow bag. It is somewhat worse for g_A but for this quantity the model of ANW is so far off that one suspects that a new mechanism is needed. One possibility,[5,11] which has been extensively discussed in the literature, is that the Skyrme model represents in some way the very substantial "tail" of the nucleon. If quarks were present in the "core" they would also contribute to g_A. In such an eventuality it might be desirable to have a smaller g_A as in the deep bag. Another possible solution to this problem involves the inclusion of vector and other mesons. Finally the effect of the mass term does not change the predictions much for the deep-bag case but improves some and worsens others for the shallow bag. In all our fits we kept the glueball mass in the 1.5-GeV region.

III. MODEL WITH QUARKONIUM SCALAR

The model of the previous section introduces a new degree of freedom which, assuming[18] that $b\neq 0$, corresponds to a rather wide $I=0$ scalar particle of typical hadronic mass. It would be nice to try to compare its properties with experiment. Unfortunately, both the experimental and theoretical situations in the $I=0$ scalar channel are far from certain. A summary from the present point of view is given in Ref. 8. Most physicists expect not only a glueball scalar somewhere very roughly around 1.5 GeV but also a quarkonium scalar very roughly around 1 GeV to exist. These may well mix with each other. Hence it seems desirable to give a modification of our model in

TABLE I. Inputs and predictions for various fits in the model of Sec. II. F_π, $\langle\psi\rangle$, and e_S were chosen to fit the N and Δ masses and to give a glueball mass around 1.5 GeV.

	Model				
Quantity	Shallow bag	Shallow bag, $m_\pi\neq 0$	Deep bag	Deep bag, $m_\pi\neq 0$	Expt.
F_π (GeV)	0.092	0.078	0.132	0.132	0.132
$\langle\psi\rangle$ (GeV)	0.34	0.34	0.156	0.14	
e_S	5.40	4.84	4.53	4.44	
Glueball mass (GeV)	1.70	1.70	1.42	1.75	
$\psi(0)/\langle\psi\rangle$	0.91	0.94	0	0	
$\langle r^2\rangle_{I=0}{}^{1/2}$ (fm)	0.59	0.68	0.67	0.65	0.72
$\langle r\rangle_{M,I=0}{}^{1/2}$ (fm)	0.88	0.95	0.80	0.78	0.81
$\langle r^2\rangle_{I=1}{}^{1/2}$ (fm)		0.99		0.78	0.88
$\langle r^2\rangle_{M,I=1}{}^{1/2}$ (fm)		0.99		0.78	0.80
g_A	0.60	0.68	0.45	0.33	1.23

which a quarkonium scalar is also present.

Let us for generality first consider the three-flavor Lagrangian[8] with quarkonium scalars as well as quarkonium pseudoscalars. This is a linear σ model with a 3×3 matrix field M which behaves as $M\rightarrow U_L M U_R^\dagger$ under chiral transformations. If the chiral symmetry is spontaneously broken to $U_V(3)$ it is convenient to decompose

$$M=SU,\quad S^\dagger=S,\quad U^\dagger=U^{-1},\quad \langle S\rangle=\frac{F_\pi}{2}1\ . \tag{3.1}$$

We note the identity for a kinetic term

$$-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu M\partial_\mu M^\dagger)=-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu S\partial_\mu S)-\tfrac{1}{2}\mathrm{Tr}(\partial_\mu U\partial_\mu U^\dagger S^2)\ . \tag{3.2}$$

The second term in (3.2) is similar in structure to the third term of (2.1) except that the standard nonlinear σ model is multiplied by quarkonium fields rather than by a gluonium field. Actually the quarkonium and gluonium fields will mix because of the need to construct a scale-invariant potential term which results in a nonzero vacuum value for S. The simplest term of this type is

$$-c\,[2\,\mathrm{Tr}(MM^\dagger)-Rb^2\psi^2]^2=-c\,[2\,\mathrm{Tr}(S^2)-Rb^2\psi^2]^2\ , \tag{3.3}$$

where c and R are new dimensionless positive parameters (chosen in this particular way to agree with the notation of Ref. 8). Now let us specialize to the usual SU(2) σ model by setting $S=(\sigma/\sqrt{2})1$. Our modified model then consists of (3.2) plus the simplest[19] (which seem sufficient for our present purposes) potential term (3.3) as well as the pure ψ terms and Skyrme term:

$$\mathcal{L}=-\tfrac{1}{2}b^2(\partial_\mu\psi)^2-\tfrac{1}{4}\psi^4\ln\frac{\psi^4}{\Lambda^4}-\tfrac{1}{2}(\partial_\mu\sigma)^2-\frac{\sigma^2}{4}\mathrm{Tr}(\partial_\mu U\partial_\mu U^\dagger)+\frac{1}{32e_S^{\,2}}\mathrm{Tr}([\partial_\mu UU^\dagger,\partial_\nu UU^\dagger]^2)-c\,(\sigma^2-Rb^2\psi^2)^2\ . \tag{3.4}$$

In this case it might be reasonable to modify the mass term in (2.9) to

$$\mathcal{L}_m=\left[\frac{\psi}{\langle\psi\rangle}\right]^{2-\gamma}\mathrm{Tr}[A(M+M^\dagger)]=+\frac{m_\pi^{\,2}F_\pi}{4\sqrt{2}}\sigma\left[\frac{\psi}{\langle\psi\rangle}\right]^{2-\gamma}\mathrm{Tr}(U+U^\dagger)\ , \tag{3.5}$$

since it is plausible[20] that $M+M^\dagger$ is related to the $\bar{q}q$ term in the fundamental QCD Lagrangian. We notice that $\mathcal{L}$ in (3.4) will lead to bag formation in a similar way as (2.1). The key feature was the suppression of ψ at small distances due to its coupling to $\mathrm{Tr}(\partial_\mu U\partial_\mu U^\dagger)$. Now this coupling is indirect; the last term in (3.4) links ψ to σ which in turn is coupled to $\mathrm{Tr}(\partial_\mu U\partial_\mu U^\dagger)$.

Before discussing the soliton excitations of (3.4) let us investigate the two scalar mesons. Actually we may just carry over some previous work[8,21] here. The 2×2 squared mass matrix in the space of the properly normalized fields σ and $h\equiv b\psi$ is

$$M^2=\begin{bmatrix}\mathcal{A} & -\mathcal{B}\\ -\mathcal{B} & \mathcal{C}+\dfrac{\mathcal{B}^2}{\mathcal{A}}\end{bmatrix},\qquad \mathcal{A}=4cF_\pi^{\,2},\quad \mathcal{B}=\frac{4cF_\pi^{\,3}}{\sqrt{2}\langle\psi\rangle b},\quad \mathcal{C}=\frac{4\langle\psi\rangle^2}{b^2}\ . \tag{3.6}$$

Diagonalization of (3.6) yields the physical fields σ_p and h_p:

$$\begin{bmatrix}\sigma\\ h\end{bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\sigma_p\\ h_p\end{bmatrix}, \tag{3.7}$$

where the mixing angle is given by

$$\tan 2\theta=\frac{2\mathcal{B}}{\mathcal{A}-\mathcal{C}-\mathcal{B}^2/\mathcal{A}} \tag{3.8}$$

and the squared masses are

$$m^2(\sigma_p,h_p)=\tfrac{1}{2}(\mathcal{A}+\mathcal{C}+\mathcal{B}^2/\mathcal{A}\pm 2\mathcal{B}/\sin 2\theta)\ . \tag{3.9}$$

Since it is well known that the usual nonlinear σ model can be obtained as a certain limit of the linear one,[22] it is interesting to explicitly see how the model of the previous section emerges as a limiting case of the present one. For this purpose we should take c in (3.4) to infinity. Then σ and ψ will no longer be independent since the equations of motion will require $\sigma^2=Rb^2\psi^2$ to hold. It is important to note that

$$R=\frac{F_\pi^{\,2}}{2b^2\langle\psi\rangle^2}\ , \tag{3.10}$$

must always be positive. For (3.6), (3.8), and (3.9) we learn that, as $c\rightarrow\infty$,

$$m^2(\sigma_p)\rightarrow\infty\ ,\qquad m^2(h_p)\rightarrow\frac{4\langle\psi\rangle^2}{b^2(1+R)}\ ,\qquad \tan\theta\rightarrow R^{1/2}\ . \tag{3.11}$$

If the scale-anomaly term were absent we would have $\langle\psi\rangle=0$ and h_p would be a "dilaton." In the $c\rightarrow\infty$ limit the field σ_p is frozen out and we should replace σ_p by its vacuum value $\langle\sigma_p\rangle$ which is easily seen to be zero using (3.7) and (3.10) with $\tan\theta=R^{1/2}$. The Lagrangian (3.4) takes the form

$$-\tfrac{1}{2}(\partial_\mu h_p)^2-\tfrac{1}{2}(\partial_\mu\sigma_p)^2-\tfrac{1}{4}\psi^4\ln\frac{\psi^4}{\Lambda^4}-\frac{\sigma^2}{4}\mathrm{Tr}(\partial_\mu U\partial_\mu U^\dagger)+\cdots\ .$$

We ~~delete~~ the σ_p kinetic term and substitute $h_p=b\sqrt{1+R}\,\psi$ and $\sigma=bR^{1/2}\psi$ into the above expression to achieve exactly (2.1) wherein b has been replaced by b' satisfying

$$b'^2=b^2(1+R)\ . \tag{3.12}$$

Eliminating b between (3.10) and (3.12) gives

$$R=\left[\frac{2\langle\psi\rangle^2 b'^2}{F_\pi^2}-1\right]^{-1}. \tag{3.13}$$

Since R must be positive we have a restriction on the parameter b' which must hold if the model of Sec. II is considered as a limit of the Lagrangian in (3.4) [of course, one might accept (2.1) on its own without deriving it from (3.4); then there is no restriction]:

$$b'^2>\frac{F_\pi^2}{2\langle\psi\rangle^2}. \tag{3.14}$$

Since the scalar-meson mass in the model (2.1) is given by $m^2=4\langle\psi\rangle^2/b'^2$ [this agrees with (3.11) and (3.12)] Eq. (3.14) amounts to an upper bound

$$m^2<\frac{8\langle\psi\rangle^4}{F_\pi^2}. \tag{3.15}$$

This agrees with the upper bound derived in Ref. 8 for the lightest particle in the scalar channel assuming a Lagrangian of the type (3.4) but with the most general potential compatible with the scale-anomaly equation. Note that (3.15) holds for two rather than three quark flavors.

It is also interesting to observe that with typical values $F_\pi=0.132$ GeV, $\langle\psi\rangle=0.34$ GeV, $m\approx1.5$ GeV, $R^{1/2}$ from (3.13) will be about $\frac{3}{4}$. This corresponds to a fairly substantial gluonium-quarkonium mixing angle. Thus for the model of (2.1), one may consider the field ψ to contain a sizable quarkonium fraction. This would affect the calculations of electromagnetic properties such as two-photon decay.[8,23]

Now let us discuss the general features of the physical situation described by the two-scalar field Lagrangian (3.4). For simplicity we shall not include the pion mass term (3.5) since we have seen that its effect was small for our previous model in Sec. II. Compared to the old model, the new one of (3.4) contains only one new parameter—the dimensionless quantity c. [Note that R which appears in (3.4) is related to the other quantities by (3.10).] Equation (3.6) shows that $2F_\pi c^{1/2}$ may be interpreted as a "bare" quarkonium scalar mass. It is sent to infinity in the ordinary nonlinear σ model but realistically may be expected to be around 1.2 GeV (corresponding to $c=20$). Even though the present model contains an extra parameter it actually is a more restrictive one. This extra restriction is due to the mass bound in (3.15) which forces one of the scalar masses to be rather light when a fit with physical F_π is attempted. The origin of the bound may be traced to the fact that the potential must be such that the theory chooses to break chiral symmetry spontaneously (i.e., $R>0$). In contrast, the model of (2.1) is, by construction, already in the spontaneously broken phase. In the present model the quarkonium field σ is also suppressed for small values of coordinate r. Actually since σ^2 is the coefficient of the meson kinetic term in (3.4) this suppression may be considered as the more direct one and turns out, for typical fits, to be fairly pronounced even when one has a more shallow suppression in ψ.

For orientation, first consider the limit when c becomes very large, say $c\approx1000$. As discussed above, the model in this case effectively reduces to the previous one in which two different characteristic fits can be made for $F_\pi=0.132$ GeV and $F_\pi\approx0.091$ GeV. The new feature is that the $F_\pi=0.132$ GeV fit now requires the scalar mass to be rather low. In the model of Sec. II, the scalar mass could be raised more or less at will simply by lowering b. Now however the analog of b [see (3.14)] must be greater than $F_\pi/(\sqrt{2}\langle\psi\rangle)$ which is approximately 0.8 when one puts $\langle\psi\rangle=0.12$ (in order to fit the nucleon and Δ masses). This should be contrasted with the value of $b=0.22$ for the deep-bag fit in Table I. In this model an $F_\pi=0.132$ GeV fit requires [see (3.15)] the scalar mass to be less than 310 MeV which seems rather unphysical. It is interesting to remark that the value $\langle\psi\rangle\approx0.34$ GeV obtained by the "sum rule" approach[9] gives a physically reasonable bound of about 2.5 GeV in (3.15). The fit for $F_\pi=0.091$ GeV, which gives a reasonable mass for the lightest scalar, leads to a shallow depression in both ψ and σ. When we lower c to a more physical value a shallow bag in ψ always is accompanied by a more substantial suppression in σ since the linkage between the two [last term of (3.4)] is weakened.

We do not encounter any surprises when the physical choice $c=20$ is made. Solutions with $F_\pi=0.132$ GeV require a low-mass scalar meson. A typical fit of the shallow-bag type with $F_\pi=0.097$ GeV can be achieved with $e_S=4.93$, $b=0.49$, and $\langle\psi\rangle=0.34$ GeV. Note that we are able to make a fit with a slightly better value of F_π than the earlier shallow-bag model[6] and the original Skyrme model.[4] The low-energy properties of the nucleon are very similar to the shallow-bag solution in Table I; specifically $\langle r^2\rangle_{I=0}{}^{1/2}=0.61f$, $\langle r^2\rangle_{M,I=0}{}^{1/2}=0.86f$, and $g_A=0.60$. The scalar masses [see (3.7)–(3.9)] are $\sigma_p=0.83$ GeV and $h_p=1.47$ GeV with a mixing angle $\theta=-0.22$ rad. Figure 3 shows σ, ψ, and F as functions of radius. Although the fit obtained here is similar to the shallow-bag fit in the minimal model of Sec. II, the present model seems to give a more reasonable description of the scalar-meson sector. The scalar widths are still extremely large (order of several GeV) but can, as we will show in the next section, be reduced by a simple modification of the kinetic term, without changing other results.

It is interesting to observe that the difference of the inside energy density (approximated by its value at the ori-

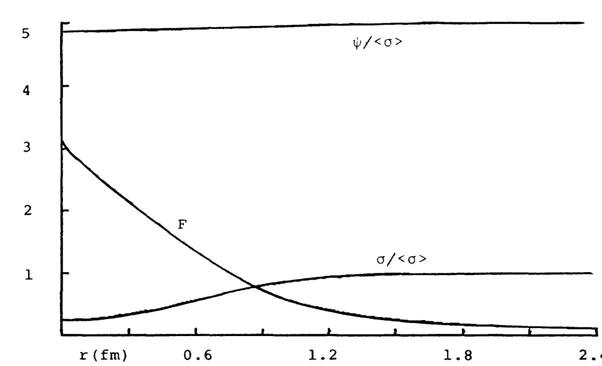

FIG. 3. $F(r)$, $\sigma(r)/\langle\sigma\rangle$, and $\psi(r)/\langle\sigma\rangle$ for the shallow-bag fit of Sec. III.

gin) and the outside energy density is in agreement with that of the MIT bag model[14] for the above fit. Explicitly $\epsilon_{\text{inside}}-\epsilon_{\text{outside}}\approx V(0)-V(\infty)=(141\ \text{MeV})^4$, where the potential $V=\psi^4\ln(\psi/\Lambda)+c(\sigma^2-\langle\sigma\rangle^2\psi^2/\langle\psi\rangle^2)^2$.

IV. SCALAR-MESON WIDTHS

A traditional feature of the old linear σ model and its descendents is an extremely large decay width for the $I=0$ scalar to go into two pions. One possibility is that this is in fact a true description of nature and that the relatively narrow states listed in the tables of the Particle Data Group are exotic objects such as[24] "$K\bar{K}$ molecules." A more orthodox interpretation of the scalars, on the other hand, would require us to find a way of suppressing their widths. This does not seem possible for the minimal model of Sec. II, whose form is tightly constrained by requiring it to be chiral symmetric and to obey the trace anomaly (see footnote 15 of Ref. 6 for a discussion of the scalar width in this model). As we pointed out in Ref. 8 the widths for *both* scalars can be reduced to the order of several hundred MeV rather than several GeV in the two-scalar model of Sec. III, when the meson kinetic term is slightly modified. Here we would like to give a more transparent explanation of this effect as well as to point out that improving the scalar-meson widths does not significantly change the properties of the nucleon.

Let us adopt the following simple modification of the pseudoscalar kinetic term in (3.4):

$$\frac{-\sigma^2}{4}\text{Tr}(\partial_\mu U\partial_\mu U^\dagger)\rightarrow\frac{-F_\pi{}^2}{8}\left[\frac{\sigma}{\langle\sigma\rangle}\right]^2\left[(1-t)+t\left[\frac{\sigma\langle\psi\rangle}{\langle\sigma\rangle\psi}\right]^2\right]\text{Tr}(\partial_\mu U\partial_\mu U^\dagger)\ , \tag{4.1}$$

where t is a new dimensionless parameter. Equation (4.1) is both chiral and scale invariant, though not unique. It is clear that the possibility of adding the additional parameter t is due to the fact that we have two scalar fields rather than one present. With the coupling constants for $\sigma_p\rightarrow\pi\pi$ and $h_p\rightarrow\pi\pi$ defined by

$$\Gamma\left[\begin{bmatrix}\sigma_p\\ h_p\end{bmatrix}\rightarrow\pi\pi\right]=\frac{3}{32\pi}m^{-1}\begin{bmatrix}\sigma_p\\ h_p\end{bmatrix}g^2\begin{bmatrix}\sigma_p\\ h_p\end{bmatrix}\ , \tag{4.2}$$

we find, from (4.1) and (3.7),

$$\begin{aligned} g(\sigma_p)&=-m^2(\sigma_p)\left[\frac{\sqrt{2}(1+t)\cos\theta}{F_\pi}+\frac{t\sin\theta}{\langle\psi\rangle}\right]\ ,\\ g(h_p)&=m^2(h_p)\left[\frac{-\sqrt{2}(1+t)\sin\theta}{F_\pi}+\frac{t\cos\theta}{\langle\psi\rangle}\right]\ . \end{aligned} \tag{4.3}$$

To understand how *both* widths may be suppressed we take $t\approx-1$. Then the first terms in (4.3) will both be suppressed. The second terms have a characteristic denominator $\langle\psi\rangle$ instead of $F_\pi/\sqrt{2}$. Thus the characteristic widths will be smaller by a factor of about $(F_\pi/\sqrt{2}\langle\psi\rangle)^2\approx\frac{1}{10}$ compared to a usual linear σ model. For this mechanism it is important that $\langle\psi\rangle$ be at least as large as the value obtained from the sum-rule approach.[9]

As an illustration consider the case when $t=-0.7$ and the other parameters are similar to those of the shallow-bag fit of the previous section: $c=20$, $b=0.59$, $e_S=4.94$, $\langle\psi\rangle=0.34$ GeV. This yields $F_\pi=0.092$ GeV and similar parameters for the nucleon. The scalars of mass 0.79 and 1.2 GeV have the relatively narrow widths 0.42 and 0.23 GeV, respectively. These can, without changing other predictions, be modified by varying t and can be reduced further by increasing $\langle\psi\rangle$.

ACKNOWLEDGMENTS

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[1]For references and discussion, see T. D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood Academic, New York, 1981).

[2]E. Witten, Nucl. Phys. **B160**, 57 (1979).

[3]T. Skyrme, Proc. R. Soc. London **A260**, 127 (1961); Nucl. Phys. **31**, 556 (1962); J. Math. Phys. **12**, 1735 (1971); J. G. Williams, *ibid.* **11**, 2611 (1970); N. K. Pak and H. C. Tze, Ann. Phys. (N.Y.) **117**, 164 (1979); A. P. Balachandran, V. P. Nair, S. G. Rajeev, and A. Stern, Phys. Rev. D **27**, 1153 (1983); E. Witten, Nucl. Phys. **B223**, 422 (1983); **B223**, 433 (1983).

[4]G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983). See also E. Braaten, S-M. Tse, and C. Willcox, Phys. Rev. D **34**, 1482 (1986).

[5]A. Chodos and C. B. Thorn, Phys. Rev. D **12**, 2733 (1975). For more recent references see, for example, P. J. Mulders, *ibid.* **30**, 1073 (1984). See also V. P. Nair and V. G. J. Rodgers, Institute for Advanced Study report, 1984 (unpublished).

[6]H. Gomm, P. Jain, R. Johnson, and J. Schechter, Phys. Rev. D **33**, 3476 (1986).

[7]The effect of vector mesons in the Skyrme model has been investigated by, among others, G. S. Adkins and C. R. Nappi,

Phys. Lett. **137B**, 251 (1985); I. J. R. Aitchison, C. M. Fraser, and P. J. Miron, Phys. Rev. D **33**, 1994 (1986); V. G. Meissner and I. Zahed, Phys. Rev. Lett. **56**, 1035 (1986); M. Lacombe, B. Loiseau, R. Vinh Mau, and W. N. Cottingham, *ibid.* **57**, 170 (1986); Y. Igarishi *et al.*, Nucl. Phys. **B259**, 721 (1985); U-G. Meissner, N. Kaiser, A. Wirzba, and W. Weise, Phys. Rev. Lett. **57**, 1676 (1986); M. Mashaal, T. N. Pham, and T. N. Truong, *ibid.* **56**, 436 (1986).

[8]H. Gomm, P. Jain, R. Johnson, and J. Schechter, Phys. Rev. D **33**, 801 (1986). Further references on the scale-anomaly and dilaton models are contained here; one we missed, pointed out by A. Zee, is N. V. Krashikov and A. A. Pivovarov, in *Quantum Gravity,* proceedings of the Third Seminar, Moscow, 1984, edited by M. A. Markov, V. A. Berezin, and V. P. Frolov (World Scientific, Singapore, 1985).

[9]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).

[10]This is in agreement with the point of view in Ref. 9 and also seems to be favored by work in related, but somewhat different, "color dielectric" models by I. Duck, Phys. Rev. D **34**, 1493 (1986) and W. Broniowski, M. K. Banerjee, and T. D. Cohen, University of Maryland Report No. 87-035, 1986 (unpublished).

[11]S. Kahana, G. Ripka, and V. Soni, Nucl. Phys. **A415**, 351 (1984); M. C. Birse and M. K. Banerjee, Phys. Lett. **136B**, 284 (1984). See also R. Mackenzie, F. Wilczek, and A. Zee, Phys. Rev. Lett. **53**, 2203 (1984).

[12]To compare with the notation of Ref. 6, note that $b^2=16a$.

[13]T. N. Pham and T. N. Truong, Phys. Rev. D **31**, 3027 (1985).

[14]J. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2066 (1975).

[15]Simpler formulas for g_A in both the massless and massive pion cases may be obtained from the identity

$$\int d^3x\, J_i^5 = \int d\mathbf{S}\cdot\mathbf{J}^5 x_i - \int d^3x\, x_i \nabla\cdot\mathbf{J}^5 ,$$

where J_μ^5 is the axial-vector current. ANW have already noted that only the first term contributes for massless pions which leads to a simple alternative formula for g_A in terms of the asymptotic form of $F(r)$. In the massive case $F(r)\sim e^{-m_\pi r}/r$ which makes the *first* term above vanish. The second term may be simply evaluated using the mass term $\mathscr{L}_m$ in (2.9). This yields

$$g_A=(4\pi/9)m_\pi{}^2F_\pi{}^2\int_0^\infty dr\, r^3\left[\frac{\psi}{\langle\psi\rangle}\right]^3 \sin F .$$

[16]See Ref. 8 for relevant references.

[17]G. S. Adkins and C. R. Nappi, Nucl. Phys. **B223**, 109 (1984).

[18]For $b=0$ or b very small the scalar particle mass becomes very large so it may be more accurately regarded as an effective auxiliary field.

[19]This potential term is slightly more general when considered in the context of (3.4). Other possible terms of the type $c_1\sigma^4+c_2\psi^4$ can be absorbed in the form of (3.4) by suitable redefinitions of the parameters Λ, c, and R.

[20]We do not have a strong motivation to make a similar modification of the Skyrme term since it is apparently not a fundamental one.

[21]H. Gomm and J. Schechter, Phys. Lett. **158B**, 499 (1985).

[22]One can also consider the limit of this model (roughly $b\to\infty$) when ψ is simply set to its vacuum value in (3.4) and one is left with the usual linear model which does not obey the trace-anomaly condition. In this case the value of the σ field is still suppressed from its vacuum value inside the soliton. There are now three parameters e_S, F_π, and c with which to fit the two baryon masses and the σ mass. Using the physical F_π we find an unreasonably small value of the σ mass around 100 MeV. Choosing $F_\pi=0.091$ GeV gives a fit with the σ mass 1.5 GeV.

[23]J. Ellis and J. Lanik, Phys. Lett. **175B**, 83 (1986).

[24]J. Weinstein and N. Isgur, Phys. Rev. D **27**, 588 (1983).