Quark deconfinement at finite temperature in the bag model

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A thermodynamical description of a single hadron at finite internal temperature is given within the framework of the MIT bag model. The quark deconfinement transition is characterized by the divergence of the bag radius $R(T)$. The radius behaves like $(T_c{}^4-T^4)^{-1/4}$ as the temperature T increases toward the critical temperature T_c. A possible consequence from this result combined with a percolation picture is that the normalized critical baryon-number density may behave as $\rho_c(T)/\rho_c(0)\simeq(1-T^4/T_c{}^4)^{3/4}$ when T approaches T_c from below.

I. INTRODUCTION

Quantum chromodynamics indicates the existence of a phase transition (or phase transitions) which is related to quark deconfinement and chiral-symmetry restoration at finite temperature and/or finite baryon-number density.[1] It is expected that investigation of this subject gives new insight into the fundamental problems of the quark confinement and of the spontaneous breaking of the chiral symmetry. It is also opening up a new exciting field in high-energy hadron nuclear physics.

There are various models or approaches to study the properties of the phase transition from hadronic matter to quark-gluon plasma at high-temperature and/or high baryon-number density. They may be classified into three categories.

(i) Monte Carlo calculation of lattice quantum chromodynamics.[1] At present, this is the most powerful method to treat all the phases within a single theory. The existence and the nature of the phase transition can be predicted from basic principles (the QCD Lagrangian). However, there are various problems at present. The physical interpretation and hence the modeling of results from the computer experiments are not so easy in general. Ambiguities due to the finite-size effect and/or uncertain scaling behavior are often significant. The computer time required for calculations that include dynamical quarks is tremendous. These situations render the models that belong to category (ii) or (iii) complementary to this approach.

(ii) The two phase models. In this case, the existence of the two phases is assumed from the beginning and both phases are described by different models. The critical point is then determined by imposing the Gibbs criteria, by comparing the free energies of both phases or so on.[2]

(iii) Single models that describe a hadron or hadron gas at finite temperature from zero up to some critical temperature. Or they describe quark-gluon gas at high temperature down to some critical point. In many cases, the models can be easily extended to the whole temperature region. The confinement mechanism is incorporated in an appropriate way, e.g., by using the bag or the string picture, and then the existence and the nature of the phase transition can be predicted. A typical example is the statistical quark-gluon bag model developed by Gorenstein *et al.*[3,4] Another example is the string model for quark matter proposed by Miyazawa.[5] The third example is the flux-tube model proposed by Patel.[6] The SU(2) version of the flux-tube model has been utilized by the present author to calculate the temperature dependence of the effective string tension.[7]

In this paper, we propose another model that belongs to category (iii). It is based on the MIT bag model[8] and provides a smooth interpolation between the ground-state bag at zero temperature and the highly excited bag in the thermodynamical limit at the limiting temperature both of which are described in the original MIT paper.[8] Our model is presented and the temperature dependence of the bag radius is derived from a pressure balance condition in Sec. II. In Sec. III this result is used to evaluate the temperature dependence of the critical baryon-number density. Equivalence between the pressure balance condition and the maximum entropy condition is shown in Sec. IV. Discussions and conclusions are given in Sec. V.

II. THERMODYNAMICS OF A SINGLE BAG

The fundamental assumption of our model is that an excited hadron bag can be described in a statistical-thermodynamical language even when its size is finite. We then consider thermal equilibrium of a single bag at rest instead of a gas of moving bags. Such a single-body approach has already met with a remarkable success in the case of the flux-tube model.[7] The mass, i.e., the energy of a spherical bag (either mesonic or baryonic or even multibaryonic) at rest with the radius R, is thus given as

$$E=\frac{C}{R}+BV+3KT^4V\ , \tag{2.1}$$

where V is the volume of the bag, i.e.,

$$V=\tfrac{4}{3}\pi R^3\ , \tag{2.2}$$

$C=C_i$ is a positive constant that depends on the species (the internal quantum numbers) of the bag (the subscript i refers to the species and will often be suppressed), B is the bag constant, and K is a positive proportionality constant that characterizes the Stefan-Boltzmann law for the quark-gluon gas confined in the bag. For the relativistic free quark-gluon gas, $K=(7N_f/60+8/45)\pi^2$, where N_f is the number of the quark flavors. The first term on the right-hand side of (2.1) represents the energy due to the quantum zero-point oscillations of quarks and gluons. The second term is the volume energy. The third term is the thermal energy of the quark-gluon gas. Keeping only the first and second terms gives the usual description of the ground-state bag at vanishing temperature. Keeping only the second and third terms with

$$T=(B/K)^{1/4}\equiv T_c \tag{2.3}$$

and very large V or even the limit $V\to\infty$ leads to the thermodynamical description of a highly excited bag already given in Ref. 8. Equation (2.1) provides a smooth and natural interpolation between these two limits.

A hadron bag with a definite radius is realized when an appropriate condition about the pressure balance is satisfied. There seems to be two pictures about the pressure balance arising from the literature.

(a) The constant external pressure B balances with the internal pressure which is due to the zero-point oscillation and the thermal excitation; (b) the pressure of the vacuum outside the bag is vanishing independent of the internal temperature, and hence the internal pressure p must be vanishing also. We found that (b) is more appropriate than (a). Hence, one has

$$p=p_0-B+p_r=0\ , \tag{2.4}$$

where p_0 is the pressure due to the zero-point oscillation, i.e.,

$$p_0=-\frac{d(C/R)}{dV}=\tfrac{1}{3}C\left[\frac{4\pi}{3}\right]^{1/3}V^{-4/3}\ , \tag{2.5}$$

$-B=-d(BV)/dV$ is the constant negative pressure due to the volume energy and p_r is the pressure due to the thermal radiation of the quark-gluon gas, i.e.,

$$p_r=KT^4\ , \tag{2.6}$$

where we have used the equation of state that relates p_r to the radiation energy density ϵ_r, i.e., $\epsilon_r=3KT^4=3p_r$. From (2.1) and (2.4), one obtains

$$E=4BV \quad \text{for } 0\le T\le T_c\ , \tag{2.7}$$

where

$$V=\tfrac{4}{3}\pi R^3(T)\equiv V(T) \tag{2.8}$$

and

$$R(T)=C^{1/4}(4\pi K)^{-1/4}(T_c{}^4-T^4)^{-1/4}\ . \tag{2.9}$$

Equation (2.7) is the virial theorem to be satisfied by the solution.[8] Equation (2.9) is the main result of this paper. It implies that the bag radius diverges as T increases toward the "limiting temperature" T_c, indicating a transition into the quark-gluon phase. Therefore, T_c given by (2.3) is identified with the critical temperature for the quark deconfinement transition. This conclusion is consistent with the argument of Kagiyama, Nakamura, and Minaka.[9] Incidentally, $T_c=134$ MeV if one takes $B=190$ MeV and $N_f=2$ with the approximation of free quark-gluon gas. It is remarkable that the same divergent behavior of the effective radius is obtained in the SU(2) flux-tube model.[7]

It may be worthwhile to note that we are considering a statistical ensemble of a single hadron bag at finite internal temperature. The total energy E of the system has a nonvanishing uncertainty ΔE in order to define a microcanonical ensemble. Such a treatment may be justified if the level density is sufficiently high. It would become only a crude approximation at very low temperature where the level density is low. The radius $R(T)$ given by (2.9) is thus the ensemble average, and is not directly related to the radius of a particular hadron, say, the ρ meson. (Note that the index i in the constant C_i could refer only to the flavor quantum numbers but not to the spin and the parity.) Pisarski[10] defines the temperature-dependent bag constant $B(T)\propto T_c{}^4-T^4$ and argues that the ρ meson mass, for example, vanishes like $m_\rho(T)/m_\rho(0)\propto(T_c-T)^{1/4}$ as $T\to T_c$. If one assumes the same behavior for the energy given by (2.7) and uses $B(T)$ in place of the temperature-independent B, one obtains the behavior $V(T)\propto(T_c-T)^{-3/4}$ in apparent agreement with (2.9). However, this does not mean that our model is equivalent to Pisarski's. First of all, in our model, the dependence on B and T cannot be described in terms of a single function $B(T)$ because B and T appear in different combinations in (2.1) and (2.4). Accordingly, the energy E of a bag given by (2.7) with (2.8) and (2.9) does not possess the scaling form $E\propto[B(T)]^{1/4}$ suggested by Pisarski. Second, the virial theorem (2.7) with the temperature-independent B was derived unambiguously in our model. It implies that the energy density inside a hadron bag is constant independent of T for $0\le T\le T_c$ being consistent with the result of Ref. 8 for $T=T_c$ and also with the notion of the mass-proportional cluster volume used by Hagedorn and Rafelski.[2] On the other hand, there is neither derivation nor justification of the corresponding virial theorem with the temperature-dependent bag constant in the paper by Pisarski. However, our model deals with the dominant configurations at a given T (Ref. 3) while Pisarski is concerned with the T dependence of a particular state such as ρ meson. Therefore, it requires further careful study to judge whether the apparent difference between the two models is truly fundamental or not.

III. TEMPERATURE DEPENDENCE OF CRITICAL BARYON-NUMBER DENSITY

The result of Sec. II may be used to evaluate the temperature dependence of the critical baryon-number density $\rho_{Bc}(T)$ at which the quark deconfinement transition takes place. Consider a hadron gas of a finite baryon-number density ρ_B at the gas temperature T and suppose that it consists of many baryonic bags and mesonic bags. Here we have assumed that the internal temperature of individual bags is equal to the "external" one, the gas temperature.[4] For simplicity, the Lorentz contraction effect is neglected in the following argument. Such a nonrelativistic approximation should be valid at such high temperature where $4BV(T)\gg 3T/2$. The fraction of the volume of the whole system occupied by baryons is $\rho_B V_B(T)$, where $V_B(T)$ is the volume of a baryon bag at temperature T. The meson-number density ρ_M will in general depend on both the temperature and ρ_B:

$$\rho_M = \rho_M(T,\rho_B) \ . \tag{3.1}$$

The volume fraction due to the meson component is therefore $\rho_M(T,\rho_B)V_M(T)$, where $V_M(T)$ is the volume of a meson bag at temperature T. The hadron gas may percolate[11] and undergo a phase transition into a quark-gluon matter when the net volume fraction reaches a critical constant ω $(0<\omega\leq 1)$:

$$\rho_B V_B(T) + \rho_M(T,\rho_B)V_M(T) = \omega \ . \tag{3.2}$$

Note that ρ_B is a T-independent constant because of the baryon-number conservation. The effect of baryon-antibaryon pair creation is neglected. The solution of (3.2) for ρ_B gives the critical density $\rho_{Bc}(T)$ as a function of T. In the simplest case where $\rho_M(T,\rho_B) \ll \rho_B$, one obtains $\rho_{Bc}(T) \simeq \omega / V_B(T)$, and, hence,

$$\rho_{Bc}(T)/\rho_{Bc}(0) \simeq V_B(0)/V_B(T) = (1-T^4/T_c{}^4)^{3/4} \ . \tag{3.3}$$

This result is depicted in Fig. 1. The critical exponent is equal to $\frac{3}{4}$. On the other hand, most results from the models that belong to category (ii) indicate that the exponent is unity, or in some cases there is no vanishing point of $\rho_{Bc}(T)$ at all up to $T=\infty$.[12]

More rigorous treatment of (3.2) requires a detailed study of $\rho_M(T,\rho_B)$. Here we give a simple model as an example. It is well known that the excluded volume effect due to the finite hadron volume is crucial to determine the behavior of hadron gas near the critical temperature.[3,4,13–15] We therefore assume that (i) the meson number density is proportional to the fraction of the available volume, i.e., the volume not occupied by either meson bags or baryon bags, (ii) the proportionality constant is a function of the temperature, and (iii) there is no spacial overlap between different bags. One then has

$$\rho_M(T,\rho_B) = [1-\rho_B V_B(T) - \rho_M(T,\rho_B)V_M(T)]f(T) \ ,$$

i.e.,

$$\rho_M(T,\rho_B) = [1-\rho_B V_B(T)]/[1+V_M(T)f(T)] \ . \tag{3.4}$$

Equation (3.2) with (3.4) yields

$$\rho_{Bc}(T) = [\omega - (1-\omega)V_M(T)f(T)]/V_B(T) \ . \tag{3.5}$$

Equation (3.3) is recovered if $\omega=1$ (the close packing without empty space). However, $\rho_{Bc}(T)$ will vanish at a temperature lower than T_c if $\omega<1$. In this sense, the critical temperature given by (2.3) may be regarded as an upper bound for the true critical temperature.

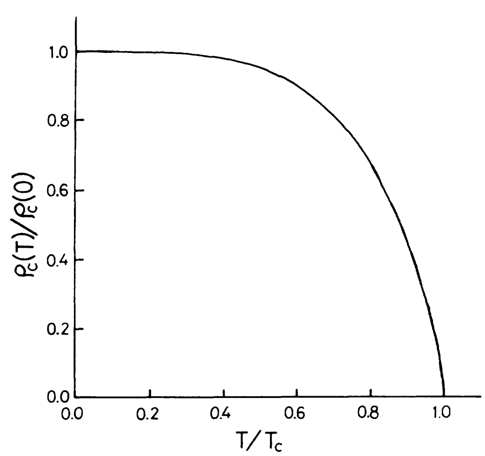

FIG. 1. Temperature dependence of the critical baryon-number density normalized at $T=0$. The curve represents (3.3) in the text.

IV. EQUIVALENCE BETWEEN PRESSURE BALANCE AND MAXIMUM ENTROPY

It may be instructive to justify the pressure balance condition (2.4) in terms of the principle of maximum entropy. The entropy of a single bag at rest is entirely given by that of the thermal radiation of the quark-gluon gas inside the bag because both the zero-point oscillation and the volume energy do not carry the entropy. Suppose that the volume of a bag is virtually fixed and the quark-gluon gas inside the bag is brought into contact with a heat reservoir at some temperature through the bag boundary. The contact with the heat reservior is turned off after the thermal equilibrium is achieved. Then, let the system be an isolated free system with a fixed energy E. Here, the energy E has an uncertainty ΔE so as to define the corresponding microcanonical ensemble. Both the adiabatic change of the volume and the interchange of energy between the gas and the volume through the boundary are allowed. The system will then rearrange itself so that the entropy given by

$$S = 4KT^3V \tag{4.1}$$

becomes maximum. It should be noted here that T and V are not independent of each other. Since E is fixed, only one of T or V is an independent variable available for entropy maximization. Thus, the condition of maximum entropy is

$$\left.\frac{\partial S}{\partial V}\right|_{E \text{ fixed}} = 0 \ . \tag{4.2}$$

Since (2.1) satisfies a general relation

$$\left.\frac{\partial S}{\partial V}\right|_{E \text{ fixed}} = \frac{p}{T} \ , \tag{4.3}$$

the condition (4.2) is equivalent to the pressure balance condition (2.4).

V. DISCUSSIONS AND CONCLUSIONS

The peculiarity of our single bag approach lies in the point that no reference is made to the gas of bags and hence to the thermodynamical limit except for the $T=T_c$ case. Taking that limit is indeed the main point of the hadron gas model developed by Gorenstein *et al.*[3,4] Thermodynamics of a gas of bags was also investigated by Baacke.[13] Our result may hopefully provide useful information which is complementary to those works. It may indeed be interesting to use our result for single bags as an input to the hadron gas models. Comparison with the improved statistical bootstrap model[14] may also be interesting.

To summarize, we have constructed a thermodynamical model for a single MIT bag at finite temperature from zero up to the critical temperature T_c. The quark deconfinement transition is characterized by the divergence of the bag radius $R(T)$ at $T=T_c$. It actually behaves like $(T_c-T)^{-1/4}$ as T increases toward T_c. Utilizing this result, we have also evaluated the behavior of the critical baryon-number density as a function of the temperature. In our model, the system of a single bag at $T>T_c$ is nothing but the quark-gluon gas filling the whole space where the effect of the nonperturbative volume energy is taken into account in terms of the bag constant. In this case, our model is a single theory that can describe the both phases simultaneously. Our approach is complementary to models where the critical temperature is approached from the above and the transition point is identified with the point where the pressure vanishes.[9,16] Such pictures are indeed consistent with our zero-pressure condition (2.4). Boal, Schachter, and Woloshyn proposed a different but quite interesting model where the critical temperature is reached again from the high-temperature side.[17] In their approach, the critical point is identified with the point where the nonvanishing solution to the equation for the quark-gluon number density ceases to exist. Comparison (or unification if possible) of such various models will deepen our understanding of the quark deconfinement transition and accordingly the confinement mechanism itself. Anyway, our model alone cannot be a complete model to describe the hadron phase. It must be supplemented by an appropriate model for hadron gas.

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