Higgs-boson mass bound in E_6-based supersymmetric theories

Howard E. Haber and Marc Sher*
Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, California 95064
(Received 13 August 1986; revised manuscript received 10 September 1986)

We consider low-energy supersymmetric models which come from E_6-based grand unified or superstring-inspired theories whose scalar fields come from **27**'s, $\overline{\mathbf{27}}$'s, or 1's of E_6. Two classes of models emerge. In one, an upper bound of 108 GeV to the mass of the lightest Higgs scalar is found, independent of the precise gauge group, soft-supersymmetry-breaking parameters, or superpotential parameters. In the other, the requirement of perturbative unification is sufficient to arrive at a similar bound. Other Higgs-boson mass relations are also discussed. Failure to find a Higgs boson with a mass near or below 108 GeV would therefore rule out nearly all such E_6-based models.

I. INTRODUCTION

One of the most dramatic predictions of supersymmetric theories is the doubling of the particle spectrum. This is also one of the most frustrating predictions since very little theoretical guidance as to the masses of these new particles exists. Short of actually discovering these particles, experimentalists can only push up the lower bounds on their masses. Theoretically, one of the main motivations for supersymmetry is to understand the hierarchy of scales from the Planck scale to the electroweak scale.[1] If supersymmetry is connected with the origin of electroweak symmetry breaking, then supersymmetric particle masses must not be larger than ~ 1 TeV. However, this bound is not precise and, in the absence of firm theoretical upper bounds to these masses, it appears to be impossible to experimentally rule out most supersymmetric theories in the near future. However, an additional feature of all such supersymmetric theories is the highly restrictive nature of the Higgs potential.[2] This, as we will see, does lead to firm (and accessible) upper bounds to the mass of at least one Higgs particle.

For example, consider the minimal supersymmetric model with an $SU(3)_c\times SU(2)_L\times U(1)_Y$ gauge group and two Higgs doublets H_1 and H_2 with hypercharges -1 and $+1$, respectively. Since there can be no gauge-invariant cubic terms in the superpotential, the quartic terms in the potential are generated entirely through gauge interactions and are thus completely determined. The most general potential is then (allowing arbitrary soft-supersymmetry-breaking terms)

$$\begin{aligned}V&=m_1{}^2H_1^\dagger H_1+m_2{}^2H_2^\dagger H_2-m_3{}^2(H_1H_2+\text{H.c.})\\&\quad+\tfrac{1}{2}g^2\sum_i\left|H_1^\dagger\frac{\tau_i}{2}H_1+H_2^\dagger\frac{\tau_i}{2}H_2\right|^2\\&\quad+\tfrac{1}{8}g'^2\,|H_1^\dagger H_1-H_2^\dagger H_2|^2\,,\end{aligned}\tag{1a}$$

where the τ_i are the $SU(2)_L$ generators and g and g' are the $SU(2)_L$ and $U(1)_Y$ coupling constants. This equation can be rewritten as

$$\begin{aligned}V&=m_1{}^2H_1^\dagger H_1+m_2{}^2H_2^\dagger H_2-m_3{}^2(H_1H_2+\text{H.c.})\\&\quad+\tfrac{1}{8}(g^2+g'^2)(H_1^\dagger H_1-H_2^\dagger H_2)^2+\tfrac{1}{2}g^2\,|H_1^\dagger H_2|^2\,.\end{aligned}\tag{1b}$$

Minimizing this potential will give four physical Higgs-boson masses: a charged scalar $H^\pm$, a neutral pseudoscalar H_3^0, and two neutral scalars H_1^0 and H_2^0. Since there are only three arbitrary parameters in V, there are mass relations. It is straightforward to show that the neutral-scalar mass matrix is

$$M^2=\begin{bmatrix}m_Z{}^2\cos^2\beta+m_{H_3^0}{}^2\sin^2\beta & -(m_Z{}^2+m_{H_3^0}{}^2)\sin\beta\cos\beta\\ -(m_Z{}^2+m_{H_3^0}{}^2)\sin\beta\cos\beta & m_Z{}^2\sin^2\beta+m_{H_3^0}{}^2\cos^2\beta\end{bmatrix},\tag{2}$$

where

$$\tan\beta=v_2/v_1\,,$$
$$m_Z{}^2=\tfrac{1}{2}(g^2+g'^2)(v_1{}^2+v_2{}^2)\,,$$
$$m_{H_3^0}{}^2=2m_3{}^2/\sin2\beta\,,$$

and v_1 and v_2 are the (real) vacuum expectation values of the neutral parts of H_1 and H_2, respectively. Since

$$\det|M^2-m_Z{}^2I\,|=-m_Z{}^2m_{H_3^0}{}^2\sin2\beta$$

which is clearly negative, one eigenvalue of this matrix is less than $m_Z{}^2$, and thus there must be a Higgs scalar lighter than the Z. One can also show that the smallest eigenvalue increases as $m_{H_3^0}{}^2$ increases, and in the limit of $m_{H_3^0}{}^2\to\infty$, its value is $m_Z{}^2\cos^22\beta$, which is explicitly less than $m_Z{}^2$.

One can easily show[3] that this bound is independent of the number of Higgs doublets by defining H_1 and H_2 to be the linear combinations (one for each hyperchange) of doublets which point along the ray from the origin to the minimum of the potential. The part of the matrix which involves H_1 and H_2 will then be identical to Eq. (2) and one can diagonalize this part of the matrix. There will then be a number less than $m_Z{}^2$ on the diagonal, and since the smallest eigenvalue of a Hermitian matrix is smaller than any diagonal element, there will still be a Higgs boson lighter than the Z.

One can also derive two other mass relations from the potential of Eq. (1):

$$m_{H_1^0}{}^2+m_{H_2^0}{}^2=m_Z{}^2+m_{H_3^0}{}^2 ,\qquad m_{H_3^0}{}^2+m_W{}^2=m_{H^\pm}{}^2 \tag{3}$$

implying that the charged scalar must be heavier than the W.

The recent flurry of excitement over superstring theory has awakened interest in supersymmetric models with extended gauge groups. When six dimensions of the ten-dimensional $E_8\times E_8$ heterotic superstring theory[4] are compactified on a Ricci-flat manifold of SU(3) holonomy,[5] the resulting four-dimensional gauge symmetry must be a rank-five or -six subgroup of E_6, with matter fields coming from **27**'s and/or $\overline{\mathbf{27}}$'s of E_6. This motivates the study[6] of low-energy supersymmetric models based on gauge groups (which are subgroups of E_6) which have a larger gauge symmetry than $SU(3)_c\times SU(2)_L\times U(1)_Y$.

In this paper we consider the upper bound to the mass of the lightest neutral Higgs scalar in these models. All phenomenologically acceptable subgroups of E_6 will be considered, and it will be assumed that Higgs scalars come from **27**, $\overline{\mathbf{27}}$, or **1** representations of E_6. It will be shown that in spite of the addition of many extra parameters, an upper bound of approximately 110 GeV can be placed on the mass of the lightest Higgs scalar in any of these models.

II. A BOUND FOR THE LIGHTEST HIGGS BOSON IN E_6-BASED MODELS

To fix our notation, the structure of the **27** of E_6 under the decomposition into $SU(3)_c\times SU(3)_L\times SU(3)_R$ is given as

$$[27]=[3,3,1]+[\bar{3},1,\bar{3}]+[1,\bar{3},3]$$
$$=(\tilde{u}\tilde{d}\tilde{g})+\begin{bmatrix}\bar{\tilde{u}}\\ \bar{\tilde{d}}\\ \bar{\tilde{g}}\end{bmatrix}+\begin{bmatrix}(H_2)^+ & (H_2)^0 & \tilde{e}^+\\ (H_1)^0 & (H_1)^- & N_2\\ \tilde{\nu} & \tilde{e}^- & N_1\end{bmatrix} . \tag{4}$$

The neutral Higgs fields are $(H_1)^0$, $(H_2)^0$, N_1, and N_2. It may be possible that the left-handed scalar neutrino $\tilde{\nu}$ could get a vacuum expectation value, but this will only be relevant in one of the models which we will consider.

A. Rank-5 models

The simplest rank-5 subgroup of E_6 is $SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_{Y'}$. For definiteness, we choose the $U(1)_{Y'}$ which can come directly from the compactification (other possible choices will be discussed later). The quantum numbers of H_1, H_2, N_1, and N_2 under this group are $\frac{1}{3}$, $\frac{4}{3}$, $-\frac{5}{3}$, and $-\frac{5}{3}$, respectively. We first consider the minimal case in which there is only one set of H_1, H_2, and N_1. The most general gauge-invariant superpotential is then $W=\lambda H_1H_2N_1$ and thus the most general potential is

$$\begin{aligned}V=&m_1{}^2H_1^\dagger H_1+m_2{}^2H_2^\dagger H_2+m_{N_1}{}^2N_1^\dagger N_1-\lambda A(H_1H_2N_1+\text{H.c.})\\&+\lambda^2(H_1^\dagger H_1H_2^\dagger H_2+H_1^\dagger H_1N_1^\dagger N_1+H_2^\dagger H_2N_1^\dagger N_1)+\tfrac{1}{8}(g^2+g'^2)(H_1^\dagger H_1-H_2^\dagger H_2)^2\\&+\tfrac{1}{72}g_1{}^2(H_1^\dagger H_1+4H_2^\dagger H_2-5N_1^\dagger N_1)^2+(\tfrac{1}{2}g^2-\lambda^2)\,|H_1^\dagger H_2|^2 ,\end{aligned} \tag{5}$$

where g_1 is the $U(1)_{Y'}$ coupling constant. From this potential, one finds the neutral-scalar mass matrix is

$$M^2=\begin{bmatrix}B_1v_1{}^2+\dfrac{\lambda Av_2n_1}{v_1} & B_2v_1v_2-\lambda An_1 & B_3v_1n_1-\lambda Av_2\\ B_2v_1v_2-\lambda An_1 & B_4v_2{}^2+\dfrac{\lambda Av_1n_1}{v_2} & B_5v_2n_1-\lambda Av_1\\ B_3v_1n_1-\lambda Av_2 & B_5v_2n_1-\lambda Av_1 & \frac{25}{18}g_1{}^2n_1{}^2+\dfrac{\lambda Av_1v_2}{n_1}\end{bmatrix} , \tag{6a}$$

where

$$\begin{aligned}B_1&=\tfrac{1}{2}(g^2+g'^2)+\tfrac{1}{18}g_1{}^2 ,\\ B_2&=2\lambda^2+\tfrac{2}{9}g_1{}^2-\tfrac{1}{2}(g^2+g'^2) ,\\ B_3&=2\lambda^2-\tfrac{5}{18}g_1{}^2 ,\\ B_4&=\tfrac{1}{2}(g^2+g'^2)+\tfrac{8}{9}g_1{}^2 ,\\ B_5&=2\lambda^2-\tfrac{10}{9}g_1{}^2 ,\end{aligned} \tag{6b}$$

and n_1 is the vacuum expectation value of N_1. (Note that by phase rotations of the Higgs fields, we can assume, without loss of generality, that λ, A, and the vacuum expectation values are real.) It is of interest to examine the scalar mass matrix M^2 in the limit of large n_1. This limit is physically interesting since it corresponds to the limit where the Z'-boson mass [corresponding to the $U(1)_{Y'}$ gauge group] is large. In this limit the mixing of the Z'

with the ordinary Z is negligible and we may use the formula

$$m_Z{}^2=\tfrac{1}{2}(g^2+g'^2)(v_1{}^2+v_2{}^2)\ .$$

Also, in this limit the pseudoscalar mass is large. To see this, one must compute the neutral-pseudoscalar Higgs-boson mass matrix. After removing the Goldstone bosons, only one physical pseudoscalar survives, with a mass given by

$$m_{\rm PS}{}^2=\frac{\lambda A\,(v_1{}^2n_1{}^2+v_2{}^2n_1{}^2+v_1{}^2v_2{}^2)}{v_1v_2n_1}\ . \tag{7}$$

Thus we see that $m_{\rm PS}$ gets large in the limit of large n_1. In this limit we find the value of the smallest eigenvalue of the matrix is given by

$$m_S{}^2=m_Z{}^2\cos^2 2\beta+\lambda^2(v_1{}^2+v_2{}^2)\left[\sin^2 2\beta+\tfrac{4}{5}(\cos^2\beta+4\sin^2\beta)-\frac{72}{25}\frac{\lambda^2}{g_1{}^2}\right]\ , \tag{8}$$

where $\tan\beta\equiv v_2/v_1$ as before. To see how the value of $m_S{}^2$ varies with n_1, we have diagonalized the scalar mass matrix numerically. The maximum value of its lightest eigenvalue was found to be within 2 GeV of the value obtained in the large-n_1 limit over a large range of parameter space.[7] Thus to obtain a good approximation to the desired bound, we may use the expression for $m_S{}^2$ given by Eq. (8). This mass does depend on an arbitrary parameter λ, but clearly it has a maximum value at

$$\lambda^2=\frac{25g_1{}^2}{144}[\sin^2 2\beta+\tfrac{4}{5}(\cos^2\beta+4\sin^2\beta)]\ ,$$

which corresponds to

$$(m_S{}^2)_{\rm max}=m_Z{}^2\cos^2 2\beta+\frac{25g_1{}^2}{288}(v_1{}^2+v_2{}^2)[\sin^2 2\beta+\tfrac{4}{5}(\cos^2\beta+4\sin^2\beta)]^2\ . \tag{9}$$

This in turn has its maximum value at $\beta=\pi/2$, in which case we conclude that

$$m_S{}^2\le m_Z{}^2+\frac{8g_1{}^2}{9}(v_1{}^2+v_2{}^2)\ . \tag{10}$$

We will take the coupling constant of the extra U(1) to be approximately the same as the coupling of the $\mathrm{U}(1)_Y$ group. In realistic models,[8] they can differ by at most a few percent. By setting $g_1=g'$ in Eq. (10), we obtain $m_S\le 108$ GeV. It must be emphasized that for most of the allowed region of parameter space, m_S will be much smaller.[7]

One can also add fields from the $\overline{27}$. If an $\overline{N_1}$ is used, the sign of the $|N_1|^2$ part of the D term in the potential is changed—this does not affect the bound. If both N_1 and $\overline{N_1}$ are used, then the H_1,H_2,N_1 submatrix is unchanged and the bound still exists. If $\overline{H_1}$ or $\overline{H_2}$ fields are added, then one can have $H_1\overline{H_1}$ or $H_2\overline{H_2}$ terms added to the potential, and our previous bound no longer applies precisely. We have found using numerical techniques that the maximum value of the smallest eigenvalue of the 5×5 mass matrix formed by H_1, H_2, $\overline{H_1}$, $\overline{H_2}$, and N_1, allowing completely arbitrary $H_1\overline{H_1}$ and $H_2\overline{H_2}$ terms is 73 GeV.

The above calculation has been carried out assuming the existence of one set of Higgs fields H_1, H_2, and N_1. Our bound is unaffected by adding N_2 since the superpotential term $H_1H_2N_2$ is forbidden by the E_6 invariance. However, realistic models require more than one generation of 27's, which implies that additional Higgs fields should be considered. First, suppose that we include additional Higgs-doublet fields. Using an argument analogous to the one we previously used for the minimal model in Sec. I, we conclude that the bound on the Higgs-boson mass obtained above cannot be weakened. Second, suppose we include additional Higgs-singlet fields with quantum numbers identical to N_1. There are then two possibilities. Suppose that the superpotential is $W=\lambda H_1H_2N_1+\lambda' H_1'H_2'N_1'$, where the primes refer to a second generation. In this case, the symmetries of the model have been chosen to avoid terms which mix generations. Then, the argument referred to above can be extended and our bound is unchanged. On the other hand, if we do not forbid generation-mixing terms, a new feature arises, and our bound requires modification. To see what happens, consider the simplest example of a minimal rank-5 model with a field N_1' added. The superpotential is taken to be $W=H_1H_2(\lambda N_1+\lambda' N_1')$. The potential (including soft-supersymmetry-breaking terms) previously given by Eq. (5) acquires new terms:

$$\begin{aligned}\Delta V={}&m_{N_1'}{}^2N_1'^\dagger N_1'-m_3{}^2(N_1^\dagger N_1'+\mathrm{H.c.})-\lambda' A'(H_1H_2N_1'+\mathrm{H.c.})\\&+\lambda'^2(H_1^\dagger H_1H_2^\dagger H_2+H_1^\dagger H_1N_1'^\dagger N_1'+H_2^\dagger H_2N_2'^\dagger N_1'-|H_1^\dagger H_2|^2)+\lambda\lambda'(H_1^\dagger H_1+H_2^\dagger H_2)(N_1^\dagger N_1'+\mathrm{H.c.})\\&-\frac{5g_1{}^2}{36}N_1'^\dagger N_1'(H_1^\dagger H_1+4H_2^\dagger H_2-5N_1^\dagger N_1-\tfrac{5}{2}N_1'^\dagger N_1')\ .\end{aligned} \tag{11}$$

Let us denote $\langle N_1'\rangle=n_1'$. The scalar mass matrix in this case is 4×4. Let us evaluate its smallest eigenvalue in the limit of large n_1 and n'_1. The easist way to do this is to define

$$\lambda_1=\frac{\lambda n_1+\lambda' n'_1}{(n_1{}^2+n'_1{}^2)^{1/2}},\quad \lambda_2=\frac{\lambda' n_1-\lambda n'_1}{(n_1{}^2+n'_1{}^2)^{1/2}} . \tag{12a}$$

In addition, we define rotated fields N_a and N_b such that the superpotential is $W=H_1H_2(\lambda_1 N_a+\lambda_2 N_b)$. In this case $\langle N_a\rangle=(n_1{}^2+n'_1{}^2)^{1/2}$ and $\langle N_b\rangle=0$. It is sufficient to examine the 3×3 submatrix (corresponding to H_1,H_2,N_a). The calculation is nearly the same as before with one exception. In the limit of large $\langle N_a\rangle$, the value of the smallest eigenvalue is

$$m_S{}^2=m_Z{}^2\cos^2 2\beta+\lambda_1{}^2(v_1{}^2+v_2{}^2)\left[\sin^2 2\beta+\tfrac{4}{5}(\cos^2\beta+4\sin^2\beta)-\frac{72\lambda_1{}^2}{25g_1{}^2}\right]+\lambda_2{}^2(v_1{}^2+v_2{}^2)\sin^2 2\beta . \tag{12b}$$

Equation (12b) is similar in form to Eq. (8) with one important difference. Since λ_2 is a free parameter, we cannot derive a model-independent bound for $m_S{}^2$ without employing renormalization-group arguments to place an upper bound on λ_2. In fact λ_2 cannot be too large or its value will become inconsistent with perturbative unification. This gives $\lambda_2 \lesssim 0.72$ which implies that the lightest eigenvalue must be less than 125 GeV. Actually, in realistic models one expects this bound to be much smaller. Since H_2 couples to the top quark, one will always have $m_2{}^2<m_1{}^2$, which gives $v_2>v_1$ and

$$\lambda A n_1+\lambda' A' n'_1>\frac{5g_1{}^2}{24}(n_1{}^2+n'_1{}^2)\,|\tan 2\beta| .$$

All models have upper bounds to A and A' which arise from requiring that the vacuum not break electric charge or color (and from requiring that the potential not possess other undesired minima);[9] this translates into an upper bound on $\sin^2 2\beta$. The end result is that the bound on $m_S{}^2$ obtained from Eq. (12b) is lowered by at least 10%, which brings the bound near our previously obtained value of 108 GeV. This bound is unchanged in models with additional N_1 fields. As above, we simply rotate the vacuum expectation values into one of the fields (say, N_a). Then, Eq. (12b) remains valid if we replace $\lambda_2{}^2$ by $\lambda_2{}^2+\lambda_3{}^2+\cdots$, where the latter λ_i correspond to the couplings of the remaining N_1 fields to H_1H_2 in the superpotential. The arguments above bounding λ_2 can be used to obtain an identical bound for $(\lambda_2{}^2+\lambda_3{}^2+\cdots)^{1/2}$.

Finally, let us consider adding E$_6$ singlets to the rank-5 models considered above. In models with no $\overline{27}$'s, the bounds obtained above remain unchanged, since there is no E$_6$-invariant cubic coupling between the singlet field and the 27's. In models with 27's and $\overline{27}$'s, we can have such cubic couplings (e.g., $H_1\overline{H_1}S$). The analysis of such models resembles the analysis of the rank-5 models with more than one N field. Namely, new couplings in the superpotential introduce new λ parameters which appear in the formula for the lightest scalar Higgs-boson mass. Again, we must resort to renormalization-group arguments to bound these λ parameters in order to bound this mass. Based on our numerical analysis of such models, we expect the resulting bound to be even tighter than the ones found above since numerous Higgs fields now contribute to the W and Z masses and thus must "share" their vacuum expectation values.

In our analysis of the rank-5 models, we chose the quantum numbers of the states by assuming that the low-energy gauge group emerged directly from the compactification of the ten-dimensional string theory. Another way to get an extra U(1) is to have a rank-6 subgroup break at some intermediate scale to the rank-five group. This can occur if either N_1, N_2, or a linear combination of the two gets a vacuum expectation value at some intermediate scale (typically of order 10^{11} GeV). Consider the case in which either N_1 or a combination of N_1 and N_2 gets a vacuum expectation value. In that case, an $H_1H_2N_1$ term in the superpotential cannot exist since it would generate intermediate scale vacuum expectation values for H_1 and H_2. But if this term does not exist then the tree-level mass of the pseudoscalar vanishes (the pseudoscalar can still acquire a small mass through gaugino loops). Since the determinant of M^2 in Eq. (2) would then vanish, one of the scalar masses vanishes (at the tree level). As a result, the case in which either N_1 or a combination of N_1 and N_2 acquires an intermediate scale vacuum expectation value results in a scalar mass which is very small. In the case in which N_2 acquires an intermediate scale vacuum expectation value, the quantum numbers of H_1, H_2, and N_1 under the remaining additional U(1) are $3/\sqrt{6}$, $2/\sqrt{6}$, and $-5/\sqrt{6}$, respectively. In this case the factor of $\frac{8}{9}$ in Eq. (10) becomes $\frac{3}{4}$, tightening the bound. Finally, the other rank-5 group $SU(4)_c\times SU(2)_L\times U(1)_Y$ has severe phenomenological problems with proton decay or neutron oscillations (as discussed in Refs. 6 and 8) and will not be discussed further.

We can also consider the generalization of the relations in Eq. (3). It is straightforward to show, independent of the quantum numbers under the extra U(1), that the first relation in Eq. (3) becomes

$$\mathrm{Tr}M_\phi{}^2=\mathrm{Tr}M_Z{}^2+\mathrm{Tr}M_{H_3^0}{}^2 , \tag{13a}$$

where $M_Z{}^2$ is the neutral-vector mass matrix, $M_\phi{}^2$ is the neutral-scalar mass matrix, and $M_{H_3^0}{}^2$ is the pseudoscalar mass matrix. This relation turns out to be far more general. One can easily show that Eq. (13a) holds in any supersymmetric model based on an extended gauge group in which gauge-singlet fields are absent. The second relation of Eq. (3) is slightly more model dependent. In the minimal rank-5 model, we have found that

$$m_{H^\pm}{}^2=m_W{}^2+m_{H_3^0}{}^2-\lambda^2(v_1{}^2+v_2{}^2)+O\left[\frac{1}{n_1{}^2},\frac{1}{n_2{}^2}\right] \tag{13b}$$

as expected from analyses of the minimal model with a singlet added.[10]

B. Rank-6 models

We now turn to the rank-6 subgroups of E_6. The simplest rank-6 subgroup is $SU(3)_c\times SU(2)_L\times U(1)_Y \times U(1)_{Y'}\times U(1)_{Y''}$. Since only one of the U(1)'s (the hypercharge) is specified, there is freedom in defining Y' and Y''. This freedom is manifested by the presence of an arbitrary angle in the quantum numbers under $U(1)_{Y'}$ and $U(1)_{Y''}$. This arbitrary angle thus enters the potential through the D terms. Denoting the couplings of the $U(1)_{Y'}$ and $U(1)_{Y''}$ groups as g_1 and g_2, respectively, and denoting the arbitrary angle as θ, the potential is

$$\begin{aligned}V=&m_1{}^2H_1^\dagger H_1+m_2{}^2H_2^\dagger H_2+m_{N_1}{}^2N_1^\dagger N_1+m_{N_2}{}^2N_2^\dagger N_2-\lambda A(H_1H_2N_1+\text{H.c.})\\&+\lambda^2(H_1^\dagger H_1H_2^\dagger H_2+H_1^\dagger H_1N_1^\dagger N_1+H_2^\dagger H_2N_1^\dagger N_1)+(\tfrac{1}{2}g^2-\lambda^2)\,|H_1^\dagger H_2|^2+\tfrac{1}{8}(g^2+g'^2)(H_1^\dagger H_1-H_2^\dagger H_2)^2\\&+\frac{g_1{}^2}{72}[c_\theta(H_1^\dagger H_1+4H_2^\dagger H_2-5N_1^\dagger N_1-5N_2^\dagger N_2)-\sqrt{15}s_\theta(H_1^\dagger H_1-N_1^\dagger N_1+N_2^\dagger N_2)]^2\\&+\frac{g_2{}^2}{72}[s_\theta(H_1^\dagger H_1+4H_2^\dagger H_2-5N_1^\dagger N_1-5N_2^\dagger N_2)+\sqrt{15}c_\theta(H_1^\dagger H_1-N_1^\dagger N_1+N_2^\dagger N_2)]^2\,,\end{aligned}\tag{14}$$

where c_θ and s_θ are the cosine and sine of the arbitrary angle θ. Note that N_1 and N_2 do not have the same quantum numbers, and both are needed to break the group down to rank 4. The F terms and the cubic soft-supersymmetry-breaking terms are identical to the previous model.

The neutral-scalar mass matrix can be easily calculated from this potential. Again, we find that the smallest eigenvalue of the matrix is very near its maximum value when n_1 and n_2 are large.[7] This value is given by

$$(m_S{}^2)_{\max}=m_Z{}^2\cos^22\beta+\lambda^2(v_1{}^2+v_2{}^2)\left[\sin^22\beta+\tfrac{4}{5}(3\cos^2\beta+2\sin^2\beta)-\tfrac{6}{125}\lambda^2\left[\frac{(5s_\theta-\sqrt{15}c_\theta)^2}{g_1{}^2}+\frac{(5c_\theta+\sqrt{15}s_\theta)^2}{g_2{}^2}\right]\right].\tag{15}$$

As discussed above, in realistic models $g_1\simeq g_2\simeq g'$. Setting them equal, the angle θ drops out. Again, $m_S{}^2$ has a maximum for a particular value of λ and we find

$$m_S{}^2\le m_Z{}^2\cos^22\beta+\frac{25g'^2}{192}[\sin^22\beta+\tfrac{4}{5}(3\cos^2\beta+2\sin^2\beta)]^2(v_1{}^2+v_2{}^2)\,.\tag{16}$$

This is maximized when $\beta=0$, thus

$$m_S{}^2\le m_Z{}^2+\tfrac{3}{4}g'^2(v_1{}^2+v_2{}^2)=107\text{ GeV}\,.$$

It is also straightforward to verify that Eqs. (13a) and (13b) are unchanged. The discussion of models with additional generations, $\overline{27}$'s and/or E_6 singlets, is similar to the one given in the previous section.

The next rank-6 group to consider is $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$. Under $SU(2)_L\times SU(2)_R$, H_1 and H_2 form a (2,2) representation, N_2 is in a (1,2) representation denoted by

$$\eta_2\equiv\begin{bmatrix}\tilde e_R\\ N_2\end{bmatrix}$$

and N_1 is in a singlet representation. The most general potential is (after some simplification)

$$\begin{aligned}V=&m^2(H_1^\dagger H_1+H_2^\dagger H_2)+m_{N_1}{}^2N_1^\dagger N_1+m_{N_2}{}^2\eta_2^\dagger\eta_2-\lambda A(H_1H_2N_1+\text{H.c.})\\&+\lambda^2(H_1^\dagger H_1H_2^\dagger H_2+H_1^\dagger H_1N_1^\dagger N_1+H_2^\dagger H_2N_1^\dagger N_1)+\frac{g_{2L}{}^2}{8}(H_1^\dagger H_1-H_2^\dagger H_2)^2+\tfrac{1}{2}(g_{2L}{}^2+g_{2R}{}^2-\lambda^2)\,|H_1^\dagger H_2|^2\\&+\frac{g_{2R}{}^2}{8}[(H_1^\dagger H_1-H_2^\dagger H_2+\eta_2^\dagger\eta_2)^2+4\tilde e_R^*\tilde e_R(H_2^\dagger H_2-H_1^\dagger H_1)+4(\tilde e_R^*N_2H_1^\dagger H_2+\text{H.c.})]\\&+\frac{5g_{1L}{}^2}{72}(H_1^\dagger H_1+H_2^\dagger H_2-2N_1^\dagger N_1-2\eta_2^\dagger\eta_2)^2+\frac{5g_{1R}{}^2}{72}(H_1^\dagger H_1+H_2^\dagger H_2-2N_1^\dagger N_1+\eta_2^\dagger\eta_2)^2\,.\end{aligned}$$

Going through the same procedure as before, we again see that the smallest eigenvalue is very near its maximum when n_1 and n_2 get large, and is given by

$$(m_S{}^2)_{\max}=m_Z{}^2\cos^22\beta+\lambda^2(v_1{}^2+v_2{}^2)\left[\sin^22\beta+2+\frac{2g_{2R}{}^2(2g_{1L}{}^2-g_{1R}{}^2)\cos2\beta-2\lambda^2(\frac{9}{5}g_{2R}{}^2+4g_{1L}{}^2+g_{1R}{}^2)}{D}\right],\tag{17}$$

where

$$D = g_{2R}{}^2(g_{1L}{}^2 + g_{1R}{}^2) + 5g_{1L}{}^2 g_{1R}{}^2 .$$

Here,

$$m_Z{}^2 = m_W{}^2(1+a)/a ,$$

where

$$a = g_{2L}{}^2 \left[\frac{1}{g_{2R}{}^2} + \frac{1}{5g_{1L}{}^2} + \frac{1}{5g_{1R}{}^2} \right] . \tag{18}$$

Setting $g_{2L} \simeq g_{2R}$ and $g_{1L} \simeq g_{1R}$ as expected in realistic models and using the value of m_Z to fix g_{1L}, we find $m_S \leq 100$ GeV. (Here we have used the fact that $\tan\beta > 1$ in order that the top quark be heavier than the bottom quark. If we relax that restriction, the bound is 117 GeV.) It is also straightforward to show that relations (13a) and (13b) are still valid.

The final model that we need to consider in detail is the group $SU(3)_c \times SU(2)_L \times SU(2)_N \times U(1)_Y \times U(1)_{Y'}$. Here, $SU(2)_N$ is the subgroup of $SU(3)_R$ which commutes with electric charge, and the $U(1)_{Y'}$ is the same U(1) as in the rank-5 group discussed earlier. The $SU(2)_N$ gauge coupling is denoted by g_{2N}. Under the $SU(2)_N$ group, H_2 is a singlet,

$$\mathcal{H} \equiv \begin{bmatrix} H_1^0 \\ \tilde{\nu} \end{bmatrix}$$

and

$$\mathcal{N} \equiv \begin{bmatrix} N_2 \\ N_1 \end{bmatrix}$$

are doublets, and we need a second

$$\mathcal{N}' \equiv \begin{bmatrix} N_2' \\ N_1' \end{bmatrix}$$

doublet in order to break the group down to $SU(3)_c \times U(1)_{em}$. Note that a field with the quantum numbers of a scalar neutrino gets a vacuum expectation value. This will give the associated neutrino a large Majorana mass (through t-channel exchange of a $\tilde{Z}$). However, if $\tilde{\nu}$ comes from either a fourth **27** or from an incomplete multiplet, this may not be a problem.

The mass matrices for the neutral scalars and pseudoscalars are now 7×7 matrices. The former has one zero eigenvalue and the latter has four zero eigenvalues, corresponding to the five Goldstone bosons eaten by the five massive neutral vector bosons [the zero eigenvalue of the scalar mass matrix corresponds to the freedom to perform an $SU(2)_N$ rotation in order to set one of the neutral vacuum expectation values to zero]. We must resort to numerical techniques to find the eigenvalues of the scalar mass matrix (the presence of the required number of zero eigenvalues provides an excellent check on the numerical computation). As another check, we note that the relation in Eq. (11a) is still valid. Interestingly, in this case, the trace of the neutral-vector mass matrix in Eq. (11a) must include the W_N fields, which are the neutral nondiagonal bosons of the $SU(2)_N$ group.

The superpotential can contain two terms involving Higgs fields: $\lambda H_2 \mathcal{H} \mathcal{N}$ and $\lambda' H_2 \mathcal{H} \mathcal{N}'$. The complete potential thus has nine parameters: λ, λ', the coefficients of the two trilinear A terms, the four mass-squared parameters (one for each multiplet), and a term $(-m_3{}^2 \mathcal{N}^\dagger \mathcal{N}' + \text{H.c.})$ which couples $\mathcal{N}$ and $\mathcal{N}'$. There are six physically relevant vacuum expectation values [one of the seven vacuum expectation values can be set to zero by an $SU(2)_N$ rotation], thus three undetermined parameters remain, which we take to be λ, λ', and $m_3{}^2$. For every set of values of λ, λ', and $m_3{}^2$, we then searched numerically for the minimum of the potential (by adjusting the various vacuum expectation values until the eigenvalues of the Higgs-boson mass matrices are positive or zero) and read off the value of the smallest nonzero eigenvalue of the scalar mass matrix. We then varied λ, λ', and $m_3{}^2$ to find the largest possible value of this eigenvalue. We found that the largest value of the smallest nonzero eigenvalue is 33.4 GeV; this value occurs when $\lambda' = 0$, $\lambda = \frac{1}{2} g_{2N}{}^2$, and $m_3 = 1700$ GeV. Thus this model must have a Higgs scalar lighter than 33.4 GeV. An interesting feature of this model is that λ must be greater than $\frac{1}{2} g_{2N}{}^2$ in order for all of the relevant vacuum expectation values to be nonzero. [Although we found this result numerically, we have also found the same feature analytically in a simplified $SU(2)_N$ model with only one $\mathcal{N}$ multiplet.]

The only other rank-6 subgroups of E_6 contain low-energy $SU(3)_L$ groups (which are ruled out by the weak mixing angle), low-energy $SU(4)_c$ or other extended color groups, which are either ruled out by proton decay or neutron oscillations.[8]

C. Rank-4 models

The recent activity concerning superstring-inspired E_6-based models has focused on rank-5 or -6 gauge groups, as these groups can arise naturally from compactification on Calabi-Yau spaces. However, we can extend our arguments to the rank-4 case. Consider a model based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ with a Higgs sector consisting of the fields H_1, H_2, and N_1, as in the minimal rank-5 case, with superpotential $W = \lambda H_1 H_2 N_1$. Note that the superpotential must be composed of E_6-invariant terms if we follow the standard superstring scenario. However, in general the $SU(2)_L \times U(1)_Y$ electroweak gauge symmetry no longer prevents extra soft-supersymmetry-breaking terms proportional to $H_1 H_2$, N_1, $N_1{}^2$, and $N_1{}^3$ from being added to the potential. The general case will be discussed briefly in Sec. III. There are two ways in which such terms can be avoided. First, let us recall that the soft-supersymmetry-breaking terms are determined by the structure of the hidden sector which breaks supergravity.[9] Most authors choose a particularly simple form for the hidden sector, in which case the resulting A and B terms are proportional to terms in the superpotential. (However, we note that such a procedure is not the most general. Following the method of Soni and Weldon,[11] using a more complicated hidden sector can result in *arbitrary* gauge-invariant soft-supersymmetry-breaking terms.) Second, one can assume that the "low-energy" theory just below the compactification scale has rank 5 or 6 which later breaks to $SU(3)_c \times SU(2)_L$

$\times U(1)_Y$. Then the terms proportional to H_1H_2, N_1, $N_1{}^2$, and $N_1{}^3$ are forbidden and the potential is the same as in Eq. (5) with $g_1=0$. However, it would be incorrect to simply put $g_1=0$ in Eq. (9) and claim that $m_S \le m_Z$. The reason is that Eqs. (8) and (9) were derived under the assumption that the 3-3 element of M^2 in Eq. (6a) is of $O(n_1{}^2)$. This is no longer true if we set $g_1=0$. The best one can do without additional assumptions is to diagonalize the upper 2×2 block of M^2. The end result is

$$(m_S{}^2)_{\max}=m_Z{}^2\cos^2 2\beta+\lambda^2(v_1{}^2+v_2{}^2)\sin^2 2\beta \ .$$

As before [see Eq. (12b) and the discussion following], a bound on m_S can be obtained by using renormalization-group arguments to bound λ, with similar results to those previously obtained.[12]

III. GENERALIZATION TO MODELS NOT DERIVABLE FROM E_6

For rank-5 or -6 groups, we only assumed that the gauge group is a phenomenologically acceptable subgroup of E_6. For the rank-four standard-model group, we added the additional assumption that only terms which could arise from E_6-invariant terms are permitted, as would be the case in an E_6 grand unified theory which broke down to $SU(3)_c\times SU(2)_L\times U(1)_Y$. Can this latter assumption be relaxed? Consider a model with an $SU(3)_c \times SU(2)_L\times U(1)_Y$ gauge group and a superpotential given by $W=\lambda H_1H_2S+\lambda' S^3$. Following the same procedure as before, we find that the lightest Higgs scalar must satisfy

$$(m_S{}^2)_{\max}=m_Z{}^2\cos^2 2\beta+\lambda^2(v_1{}^2+v_2{}^2)\sin^2 2\beta -\lambda^2(v_1{}^2+v_2{}^2)\left[\frac{\lambda}{\lambda'}+\sin 2\beta\right]^2 . \tag{19}$$

Since one could choose $\sin 2\beta=1$ and $\lambda'=-\lambda$, one can get a bound of $m_S{}^2\le\lambda^2(v_1{}^2+v_2{}^2)$. As before, if λ is too large, its value will diverge before reaching the unification scale. If we require that λ not get large before reaching the unification scale, we find that $\lambda\lesssim 0.7$, and so $m_S\lesssim 125$ GeV. By imposing the condition $m_2{}^2<m_1{}^2$, as expected in realistic models and as we did in Sec. II A, this bound will be reduced by approximately 10%. [In addition, we note that the general relation given by Eq. (13a) no longer is valid, due to the presence of a gauge singlet.]

In the rank-four model in which the superpotential is composed of E_6-invariant terms, but the most general $SU(3)_c\times SU(2)_L\times U(1)_Y$-invariant soft-supersymmetry-breaking terms are allowed in the potential, we also find a bound similar to Eq. (19) but with the last term replaced by a different (but negative) combination of the parameters. As in the rank-5 model with two N_1 fields, renormalization-group arguments give a bound close to our bound of 108 GeV.

The only way to avoid such a bound completely is to require dimensionful terms in the superpotential (such as an m^2S term or an mH_1H_2 term,[13] where m is an arbitrary mass scale). This, in combination with the S^3 and H_1H_2S terms, allows all such bounds to be evaded. However, there may be severe naturalness problems in putting an ~ 100 GeV parameter in the superpotential of a grand unified or superstring-inspired model. We conclude that a bound similar to our bound of 108 GeV will occur in any model in which only dimensionless parameters are allowed in the superpotential. These arguments will also apply to SU(5) and SO(10) models.

IV. PHENOMENOLOGICAL IMPLICATIONS

We now turn to the phenomenological implications of the bound obtained in this paper. Consider the two lightest Higgs scalars in the model. (Following the notation used in the minimal model, we denote the lightest scalar by H_2^0 and the second lightest scalar by H_1^0. The pseudoscalar is denoted by H_3^0. The corresponding masses will be denoted by $m_i{}^2$, $i=1,2,3$.) We have shown that the value of the lightest scalar mass (m_2) is near its maximum in the limit where $n_1,n_2\to\infty$. In this limit, $m_3\to\infty$ and all additional Z-boson masses (beyond the standard model Z), $m_{Z_i}\to\infty$ ($i=2,3$); with $m_3/m_{Z_i}\to 0$ in the same limit. Remarkably, in this limit the formula for the mass of H_1^0 is the same in all models we have examined (which contain a single λ term):

$$m_1{}^2=m_3{}^2+m_Z{}^2\sin^2 2\beta -\lambda^2(v_1{}^2+v_2{}^2)\sin^2 2\beta+O\left[\frac{1}{n_1{}^2},\frac{1}{n_2{}^2}\right] . \tag{20}$$

Thus if the H_2^0 mass is near the maximum allowed by our bound, all other Higgs bosons are much heavier and therefore phenomenologically irrelevant. On the other hand, it is possible for both H_2^0 and H_3^0 to be light if the bound is far from being saturated.

The best experimental signature for finding the H_2^0 is via the process $e^+e^-\to H_2^0 f\bar f$ through s-channel Z exchange. This process has the advantage that the H_2^0 can be inferred by studying the invariant mass recoiling against the $f\bar f$ system. Hence, this signal is independent of the various Higgs decay channels and depends solely on the strength of the ZZH_2^0 coupling. If the H_2^0 is sufficiently light, the production rate for the decay of a real $Z\to H_2^0 f\bar f$ will be large enough to be seen at the Stanford Linear Collider or CERN LEP. Otherwise, one needs to have the s-channel Z be virtual. In this case, the optimal e^+e^- center-of-mass energy is $\sqrt{s}\simeq m_Z+\sqrt{2}m_{H_2}$ (Ref. 14). Using our bound of $m_{H_2}\lesssim 110$ GeV implies that $\sqrt{s}\lesssim 250$ GeV. Given that LEP II is expected to have $\sqrt{s}=200$ GeV, we see that most of the allowed parameter space will be explored at LEP II. As discussed earlier, the bound is only saturated for very particular (and unrealistic) values of the parameters and any realistic model will have a much lower bound. We thus expect that the small window remaining after LEP II will be insignificant.

However, even if the mass of H_2^0 is light enough to be observed, we must check that the rate for $e^+e^-\to H_2^0 f\bar f$ is not unduly suppressed by a small ZZH_2^0 coupling. This coupling has been worked out in the case of the minimal supersymmetric (SUSY) model.[15] If we compare this coupling $g(ZZH_2^0)$ to its value in the standard (nonsupersymmetric) model (SM), we find

$$\frac{g(ZZH_2^0)_{\text{minimal SUSY}}}{g(ZZH_2^0)_{\text{SM}}}=\frac{m_1{}^2(m_1{}^2-m_Z{}^2)}{(m_1{}^2-m_2{}^2)(m_1{}^2+m_2{}^2-m_Z{}^2)} . \tag{21}$$

Note that in the limit where m_2 attains its maximal value of m_Z, the above ratio approaches one. Also, in the limit in which the second lightest eigenvalue m_1 gets large the ratio is also one. The only way for the ratio to be significantly suppressed is in the region where $m_1 \to m_Z$. However, if $g(ZZH_2^0)$ is suppressed, then $g(ZZH_1^0)$ is nearly full strength, and it would be H_1^0 which would be detected. These considerations can be extended to all the extended gauge models we have studied. It is not hard to see that the conclusions in these models are similar to those of the minimal supersymmetric model. We conclude that the lightest Higgs boson of E_6-based supersymmetric models will be produced in e^+e^- collisions at a rate roughly equal to the rate of a SM Higgs boson. Detection of such a Higgs boson is then assured (in principle) by the standard missing-mass techniques.

V. SUMMARY

We have analyzed supersymmetric models in which the gauge group is a subgroup of E_6 and in which scalar fields come from **27**, $\overline{\mathbf{27}}$, or **1** representations of E_6. In models in which the relevant Higgs fields come from one generation of **27** and/or $\overline{\mathbf{27}}$, we find that the lightest Higgs scalar in the model is lighter than 108 GeV. This is nearly independent of the numerous parameters of the models and the precise gauge group. In more general models, one must resort to renormalization-group analysis and the assumption of perturbative unification to limit the λ parameter (which appears in the superpotential). This in turn puts an upper bound on the lightest Higgs scalar mass which is at most 120 GeV. We also find a new generalized Higgs-boson mass relation Eq. (13a) which is valid in all extended gauge models without gauge singlets. Thus failure to find a Higgs scalar lighter near or below 108 GeV in future e^+e^- collider experiments can rule out all low-energy E_6-based superstring-inspired models proposed to date.

Note added in proof. In our derivation of Eq. (8), we considered the limit of $n_1 \to \infty$ with λ and λA fixed. After this work was submitted, M. Drees [Phys. Rev. D (to be published)] pointed out that if A were of order n, then one could fine-tune A to cancel the λ^4 term in Eq. (8). By choosing $\sin^2 2\beta \approx 1$, and λ large, our bound of 108 GeV could be evaded. Note that this requires A to be large (of order n) and to be find-tuned [to an accuracy of $O(v^2/n^2)$].

A potential problem with such a large value of A is the possibility that the potential may possess unacceptable minima (see, e.g., Ibanez and Mas[6]). In fact, for $\sin^2 2\beta = 1$, it is straightforward to show that the value of A required to cancel the λ^4 term in Eq. (8) does not satisfy the conditions necessary to avoid unacceptable vacua. However, as shown more recently by J. F. Gunion, H. E. Haber, and L. Roszkowski [Report No. UCD-86-41, 1986 (unpublished)], a very narrow region of parameter space still remains, for A of $O(n)$ and fine-tuned appropriately, λ large and $\sin^2 2\beta \approx 1$, which satisfy the necessary conditions. (Since these conditions are necessary but not sufficient, it is still possible that this region of parameter space will be disallowed.)

Even though a narrow range of parameter space may exist for which our bound is not valid, we may recover the Higgs mass bound by imposing the constraint of perturbative unification, as discussed in this paper. In this regard, we note that our bound from perturbative unification is somewhat smaller than the bound found by Drees (in the paper cited above). We chose to define perturbative unification by requiring that $\lambda^2/\pi < 1$ at the Planck scale; he required (we believe) that a Landau pole not be reached by the unification scale. These differing definitions give a 20% difference in the Higgs mass bound, accounting for most of the discrepancy between our bounds.

ACKNOWLEDGMENTS

We thank Tom Banks, Graham Ross, and Probir Roy for useful conversations, and the SLAC Theory Group for their hospitality. This work was supported in part by the U.S. Department of Energy and the National Science Foundation.

*Present address: Washington University, St. Louis, MO 63130.

[1]E. Witten, Nucl. Phys. **B185**, 513 (1981); S. Dimopoulos and H. Georgi, *ibid.* **B193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981).

[2]K. Inoue, A. Kakuto, A. Komatsu, and S. Takeshita, Prog. Theor. Phys. **67**, 1889 (1982); **68**, 927 (1982); **70**, 330(E) (1983); **71**, 413 (1984); R. Flores and M. Sher, Ann. Phys. (N.Y.) **148**, 95 (1983).

[3]D. Chang and A. Kumar, Phys. Rev. D **31**, 2698 (1985).

[4]M. B. Green and J. H. Schwarz, Nucl. Phys. **B218**, 43 (1983); D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985); Nucl. Phys. **B256**, 253 (1986).

[5]P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985).

[6]M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, Nucl. Phys. **B259**, 549 (1985); M. Mangano, Z. Phys. C **28**, 613 (1985); J. D. Breit, B. A. Ovrut, and G. C. Segre, Phys. Lett. **158B**, 33 (1985); J. P. Derendinger, L. E. Ibanez, and H. P. Nilles, *ibid.* **155B**, 65 (1985); Nucl. Phys. **B267**, 365 (1986); E. Cohen, J. Ellis, K. Enqvist, and D. V. Nanopoulos, Phys. Lett. **165B**, 76 (1985); **161B**, 85 (1985); F. del Aguila, G. Blair, M. Daniel, and G. G. Ross, Nucl. Phys. **B272**, 413 (1986); M. Drees, N. Falck, and M. Gluck, Phys. Lett. **167B**, 187 (1986); J. Ellis, K. Enqvist, D. V. Nanopoulos, and F. Zwirner, Nucl. Phys. **B276**, 14 (1986); L. E. Ibanez and H. P. Nilles, Phys. Lett. **169B**, 354 (1986); L. E. Ibanez and J. Mas,

Report No. CERN-TH.4426/86, 1986 (unpublished).
[7]It turns out that a very narrow region of parameter space exists for which our bounds are violated. See the note added in proof.
[8]P. Binetruy, S. Dawson, I. Hinchliffe, and M. Sher, Nucl. Phys. **B273**, 501 (1986).
[9]H. P. Nilles, Phys. Rep. **110**, 1 (1984).
[10]J. F. Gunion and H. E. Haber, Nucl. Phys. **B272**, 1 (1986).
[11]S. K. Soni and H. A. Weldon, Phys. Lett. **126B**, 215 (1983).
[12]We would like to thank M. Drees and M. Gluck for discussions on this point, and for pointing out an error in the original version of this paper.
[13]K. S. Babu and E. Ma, Phys. Rev. D **33**, 3334 (1985).
[14]J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. **B106**, 292 (1976).
[15]J. F. Gunion and H. E. Haber, Nucl. Phys. **B278**, 449 (1986).