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Proposed experiment addressing *CP* and *CPT* violation in the K^0-$\bar{K}^0$ system

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An experiment utilizing the decay $\phi \to K_S K_L$ is proposed for measuring the ratio ϵ'/ϵ of *CP*-violating parameters in the kaon system. It appears one can probe values of ϵ'/ϵ down to 10^{-3} with 10^9 ϕ's. An asymmetry measurement of the relative times of $\pi^+\pi^-$ and $\pi^0\pi^0$ decays is capable of testing the phase difference $\phi_{+-}-\phi_{00}$ to $\pm 1.6^0$ (3σ), with 10^{10} ϕ's. Far fewer ϕ's (perhaps 10^8) can be useful in constraining some parameters of the kaon system associated with CPT violations.

I. INTRODUCTION

Since the important discovery of *CP* violation,[1] no significant new discoveries on discrete symmetry breakings have been wrested from elusive nature. As yet, the K^0-$\bar{K}^0$ system has proved itself to be the only one delicate enough to display *CP* violation.[2]

The kaon system so far has not provided one very important piece of information. All *CP* violation at present can be described in terms of a single parameter ϵ which serves to express the mass eigenstates K_S and K_L in terms of *CP* eigenstates K_1 and K_2:

$$K_S = K_1 + \epsilon K_2 + O(\epsilon^2) ,$$
$$K_L = K_2 + \epsilon K_1 + O(\epsilon^2) . \quad (1)$$

If ϵ were the only source of *CP* violation, the manifestations of this violation in two-pion decays of the K_L would be the same for charged and neutral pion pairs. A parameter ϵ' describes possible differences in $\pi^+\pi^-$ and $\pi^0\pi^0$ amplitudes. Specifically,

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^- \mid K_L \rangle}{\langle \pi^+\pi^- \mid K_S \rangle} \equiv \mid \eta_{+-} \mid e^{i\phi_{+-}} = \epsilon + \epsilon' , \quad (2a)$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0 \mid K_L \rangle}{\langle \pi^0\pi^0 \mid K_S \rangle} \equiv \mid \eta_{00} \mid e^{i\phi_{00}} = \epsilon - 2\epsilon' . \quad (2b)$$

[Here we have expressed the amplitude ratios η_{+-} and η_{00}, characterizable by four real parameters, in terms of two complex numbers ϵ and ϵ'; we do not introduce $\langle I=2 \mid K_S \rangle / \langle I=0 \mid K_S \rangle$ amplitude ratios, in contrast with (e.g.) the first three references of Ref. 2.] If ϵ' were zero, η_{00} and η_{+-} would be equal. The search for a ratio $\mid \eta_{00}/\eta_{+-} \mid$ differing from unity has been of great experimental interest recently.[3–5] These searches have mainly concentrated on comparisons between K_L and K_S produced in high-energy accelerator beams at Fermilab,[3] Brookhaven,[4] and CERN.[5] There are also studies being planned in low-energy antiproton annihilations.[6]

In this work we study a different process for producing kaons: the decay $\phi \to K_S K_L$. The ϕ can be produced very cleanly in e^+e^- annihilations. Its quantum numbers $J^{PC} = 1^{--}$ ensure, as we shall see, that *only* $K_S K_L$ is formed in the final state, even in the presence of *CP* or *CPT* violation in the kaon system. There are interesting time-dependent correlations in the final states when both K_S and K_L decay to two pions.[7–10] The possibility of a ϕ factory for studying *CP* violation in the kaon system is, in fact, receiving intense scrutiny by experimenters at present.[11–14]

Our specific results, ignoring detector inefficiencies, are as follows (similar results have been found in Refs. 13 and 14).

(1) We find that with about $10^9 \phi$'s, one can perform a useful measurement of ϵ'/ϵ down to a level of 10^{-3}.

(2) We expect that the phase difference $\phi_{+-} - \phi_{00}$ can be tested to $\pm 1.6°$ (3σ) with such an experiment, with $10^{10}\phi$'s.

(3) We find that such experiments are particularly suited to tests for large *imaginary* values of ϵ'/ϵ. These are not expected if *CPT* is valid, but have not yet been excluded by experiments. Indeed, questions have been raised repeatedly over the years about *CPT* invariance.[15] Such tests can be helpful with far fewer ϕ's than discussed above; one can begin to make a useful contribution with about 10^8 ϕ's. We review in Sec. II, the definition and present experimental status of ϵ'/ϵ.

In Sec. III we introduce the $\phi \to K_S K_L$ process as an interesting ϵ'/ϵ probe. From e^+e^- machines it may be feasible to expect as many as 4×10^{10} ϕ's (Ref. 16) in the process $e^+e^- \to (\gamma_V) \to \phi$. The *C*-even $K\bar{K}$ background is found to be negligible in e^+e^- annihilations.

Section IV presents the general formalism for interference in ϕ decays. We will be mainly interested in the decay $\phi \to K_S K_L \to (\pi^+\pi^-, \pi^0\pi^0)$.

We relate, in Sec. V, complex values of ϵ'/ϵ to asymmetries of the rates $\phi \to K_S K_L \to (\pi^+\pi^-, \pi^0\pi^0)$ at various time slices, and discuss the required number of ϕ's to observe those asymmetries.

In Sec. VI we quote the times which minimize the number of ϕ's required for any given ϵ'/ϵ. We shall discuss two classes of experiments.

(a) For $\mathrm{Re}(\epsilon'/\epsilon) = O(\mid \epsilon'/\epsilon \mid)$, the quantities of interest are the number of $(\pi^+\pi^-, \pi^0\pi^0)$ decays when the $\pi^+\pi^-$ decay occurs *before* $\pi^0\pi^0$, compared to the number obtained when $\pi^+\pi^-$ decay occurs *after* $\pi^0\pi^0$. We require 2×10^9 ϕ for $\epsilon'/\epsilon = 10^{-3}$ (to 3σ accuracy).

(b) For essentially imaginary ϵ'/ϵ and $\mid \epsilon'/\epsilon \mid \lesssim 0.05$, we find that a finite-time comparison minimizes the required number of ϕ's. Specifically, we compare the number of $(\pi^+\pi^-, \pi^0\pi^0)$ decays occurring at

$$2.5\tau_S \geq \Delta t = t_{00} - t_{+-} \geq 0 \tag{3}$$

to the number of decays occurring at

$$0 \geq \Delta t = t_{00} - t_{+-} \geq -2.5\tau_S \ . \tag{4}$$

With 10^{10} ϕ's we probe $|\mathrm{Im}(\epsilon'/\epsilon)| \leq 10^{-2}$ (assuming Re $\epsilon'/\epsilon \approx 0$), corresponding to $|\phi_{+-} - \phi_{00}| \lesssim 1.6°$ (3σ accuracy).

Section VII shows the utility of time-dependent intensity curves, when the $|\eta_{+-}/\eta_{00}|$ experiments and our asymmetry measurement fail to measure an ϵ'/ϵ.

A comparison of experimental methods is given in Sec. VIII. There it is found that the $|\eta_{+-}/\eta_{00}|$ experiments practically map out the same allowed ϵ'/ϵ regions as time-asymmetry experiments of class (a), mentioned above. However, for testing the $\phi_{+-} - \phi_{00}$ phase difference our method (b) probes a very different region of the parameter space.

In Sec. IX we exhibit the possibility of measuring rare K_S decays, due to the wildly disparate lifetime of the accompanying K_L. This can be utilized to address the question of *CP* violation in the K_S system, as of yet unobserved. In the superweak model 10^9 ϕ's suffice to observe *CP* violation in $K_S \to 3\pi^0$ and K_S semileptonic asymmetry. Section X is a summary of our main findings.

II. REVIEW OF PRESENT STATUS

The two experimental groups[3,4] which have measured $|\eta_{+-}/\eta_{00}|$ actually quote their results as if $\mathrm{Im}(\epsilon'/\epsilon) \approx 0$, in which case they find the following.

Chicago-Saclay group (Ref. 3):

$$\frac{\epsilon'}{\epsilon} = -0.0046 \pm 0.0053 \pm 0.0024 \ ; \tag{5a}$$

BNL-Yale group (Ref. 4):

$$\frac{\epsilon'}{\epsilon} = 0.0017 \pm 0.0072 \pm 0.0043 \ . \tag{5b}$$

The average of these two values is

$$\frac{\epsilon'}{\epsilon} = -0.0025 \pm 0.0047 \ . \tag{6}$$

This translates to a value of

$$\left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 = 0.985 \pm 0.028 \ , \tag{7}$$

which we shall sometimes use in what follows.

The neglect of $\mathrm{Im}(\epsilon'/\epsilon)$ is motivated by constraints of *CPT* invariance, which imply that the phases of ϵ and ϵ' are approximately equal.[17] However, independent tests of *CPT* invariance are always welcome. Would the subtle K^0-$\bar{K}^0$ system be a good place to harbor tiny *CPT* violations as well? Indeed Christenson *et al.*[18] reported

$$\phi_{+-} - \phi_{00} = (-12.6 \pm 6.2)^0 \ , \tag{8}$$

a 2σ violation of *CPT* invariance. If the nonzero differ-

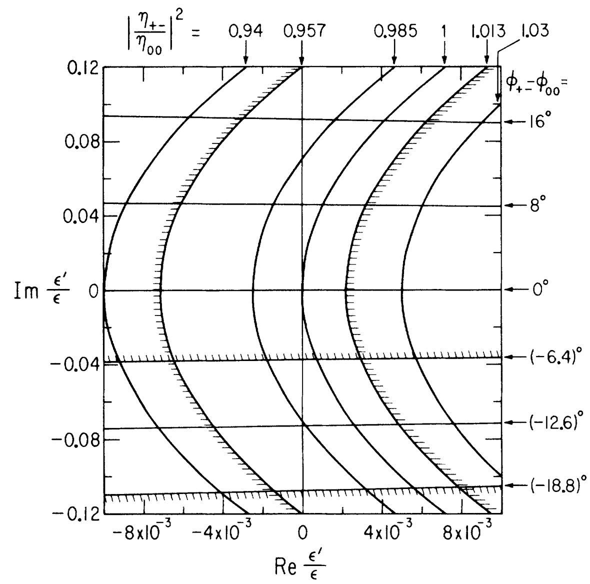

FIG. 1. Contours of $|\eta_{+-}/\eta_{00}|^2$ and $\phi_{+-} - \phi_{00}$ on the complex ϵ'/ϵ plane. The experimental limit (1σ) is the unshaded area. Note the large $|\epsilon'/\epsilon|$ values still unprobed.

ence could be taken seriously, large values of $|\epsilon'/\epsilon|$ are compatible with $|\eta_{+-}/\eta_{00}| \approx 1$, as shown in Fig. 1. One cannot overemphasize the importance of measuring the phases of η_{+-} and η_{00} accurately.

Experiments are indeed contemplated to measure $\phi_{+-} - \phi_{00}$ more accurately. Refinements and extensions[19] of a Fermilab experiment[3] will measure this difference to better than $\pm 5°$ (1σ) within two years, and to $\pm 1°$ (1σ) ultimately (by the early 1990s). The LEAR experiment mentioned earlier[6] hopes to achieve an accuracy of $\pm 2°$ (1σ) in this figure.

Just as an example, take $|\eta_{+-}| \approx |\eta_{00}|$ (Refs. 3 and 4) and $\phi_{+-} - \phi_{00} = -10°$ (Ref. 18). We obtain

$$\frac{\epsilon'}{\epsilon} \approx 5.8 \times 10^{-2} e^{i270°} \ , \tag{9}$$

a value which is imaginary and an order of magnitude larger in magnitude than the experimental results quoted in Refs. 3 and 4. This would indicate *CPT* violation. Limits on *CPT* from existing data can be found in Ref. 20. A phase difference $|\phi_{+-} - \phi_{00}| \lesssim 1\%$ can be accommodated with *CP* violation alone.[21]

Clearly we need an experiment which measures $\eta_{+-} - \eta_{00} = 3\epsilon'$ directly. Here we present an outline of such an experiment.

III. $\phi \to K_S K_L$ LABORATORY

In the process

$$e^+e^- \to (\text{virtual photon}) \to \phi \ , \tag{10}$$

when ϕ decays to $K^0\bar{K}^0$, we shall show that the final state is always $K_S K_L$, even if *CP* or *CPT* is violated in kaon decays. The basic idea of the experiment is then to observe both $K_S \to 2\pi$ and the *CP*-violating process $K_L \to 2\pi$.

When both kaons decay to $\pi^+\pi^-$ or to $\pi^0\pi^0$, Bose statistics requires the amplitude to vanish when both dipion systems are emitted at equal times after kaon production. However, when one kaon decays to $\pi^+\pi^-$ and the other to $\pi^0\pi^0$, the amplitude at equal decay times is then proportional to[8–10]

$$\eta_{+-} - \eta_{00} = 3\epsilon' . \tag{11}$$

Thus a ϕ factory will "sense" directly the complex number ϵ'. Creating ϕ's (from e^+e^- annihilation) has the advantage that the branching ratio to $K_S K_L$ is large, $B(\phi \to K_S K_L) = (34.3 \pm 0.9)\%$ (Ref. 22). In this paper we shall discuss the practical implementation of this idea, via studies of asymmetries in the distributions of time differences Δt between emission of the $\pi^+\pi^-$ and $\pi^0\pi^0$ systems.

We first concentrate on ϕ production in the process (10). The optimal luminosity on top of the ϕ resonance could be as high as 10^{33} cm^{-2}s^{-1} (Ref. 16), "pushing" present capabilities. Since $\sigma_\phi \sim 4\mu$b, an experiment could obtain 4×10^{10} ϕ's and $1.4\times10^{10}\phi \to K_S K_L$ decays in a run of 10^7 s. A 1987 upgrade of the Novosibirsk e^+e^- machine is expected to result in a luminosity greater than 10^{31} cm^{-2}s^{-1} at the ϕ meson.[12] Thus "interesting" numbers of produced ϕ's range from about 5×10^8 to more than 10^{11}. We shall find that to observe a three-standard-deviation effect of nonzero ϵ', we need to produce 4×10^8 ϕ's when $\epsilon'/\epsilon = (\frac{1}{20})e^{i270^\circ}$, and 2×10^9 ϕ's when $\epsilon'/\epsilon = 10^{-3}$. The distributions with respect to Δt will be quite different in the two cases, however.

We now discuss the quantum numbers of the $K^0\bar{K}^0$ system formed in e^+e^- annihilations. When *CPT* is broken, the eigenvectors of the mass matrix are arbitrary:

$$|K_S\rangle \equiv p'|K^0\rangle + q'|\bar{K}^0\rangle , \tag{12a}$$

$$|K_L\rangle \equiv p|K^0\rangle - q|\bar{K}^0\rangle , \tag{12b}$$

where we take $|p'|^2 + |q'|^2 = |p|^2 + |q|^2 = 1$. (Requiring *CPT* invariance we get $p'=p$ and $q'=q$.)

Assuming C is conserved in strong and electromagnetic interactions we observe that the initial $K\bar{K}$ state immediately after ϕ decay is

$$\begin{aligned} |i\rangle &= |K^0\bar{K}^0(C=\text{odd})\rangle \\ &= \frac{1}{\sqrt{2}}[\,|K^0(\hat{\mathbf{z}})\bar{K}^0(-\hat{\mathbf{z}})\rangle - |\bar{K}^0(\hat{\mathbf{z}})K^0(-\hat{\mathbf{z}})\rangle] , \end{aligned} \tag{13}$$

where we choose the $\hat{\mathbf{z}}$ axis as the direction of the momenta of the kaons in the c.m. system; $(\hat{\mathbf{z}})$ means that the particle moves in the positive z direction and $(-\hat{\mathbf{z}})$ means that the particle moves in the negative z direction. The time evolution can be readily read off when we substitute Eqs. (12) into $|i\rangle$ [Eq. (13)]:

$$\begin{aligned} |i\rangle = \frac{1}{\sqrt{2}(qp'+q'p)}[\,&|K_L(\hat{\mathbf{z}})K_S(-\hat{\mathbf{z}})\rangle \\ &- |K_S(\hat{\mathbf{z}})K_L(-\hat{\mathbf{z}})\rangle] . \end{aligned} \tag{14}$$

Note that *CPT* invariance in kaon decay has *not* been assumed in deriving this equation. The critical minus sign[8–10] in Eq. (14) is just the reflection of the fact that ϕ is a C-odd particle and conserves C while decaying. In similar fashion we obtain the $K^0\bar{K}^0$ C-even background. It is easily proven to be a linear combination of $K_S K_S$ and $K_L K_L$, and relinquishing *CPT* invariance[10] also of $K_S K_L + K_L K_S$:

$$\begin{aligned} |b\rangle &= |K^0\bar{K}^0(C=\text{even})\rangle \\ &= \frac{1}{\sqrt{2}}[\,|K^0(\hat{\mathbf{z}})\bar{K}^0(-\hat{\mathbf{z}}) + |\bar{K}^0(\hat{\mathbf{z}})K^0(-\hat{\mathbf{z}})\rangle] . \end{aligned} \tag{15}$$

We demonstrate that the C-even background produced via

$$e^+e^- \to 2\gamma \to K^0\bar{K}^0 \tag{16}$$

can be neglected. Using unitarity bounds,[23] one finds

$$\sigma(e^+e^- \to K^0\bar{K}^0, J^P=0^+) \gtrsim \alpha^2\pi\frac{m_e^{\,2}}{s}\left[2+\ln\frac{s}{m_e^{\,2}}\right]^2 \sigma(\gamma\gamma \to K^0\bar{K}^0, J^P=0^+) . \tag{17}$$

With the experimental data[24] near threshold

$$\sigma(\gamma\gamma \to K^0\bar{K}^0, J^P=0^+) \approx 16\pm6 \text{ nb} , \tag{18}$$

we obtain the ratio

$$\frac{\sigma(e^+e^- \to K^0\bar{K}^0, J^P=0^+)}{\sigma(e^+e^- \to \phi \to K_S K_L)} \gtrsim 3.6\times10^{-10} . \tag{19}$$

We expect the inequality to be saturated within an order of magnitude in analogy[23] with the $K_L \to l^+l^-$ decay analyses. Another background, of the same order in perturbation theory but even less important, comes from $e^+e^- \to e^+e^-K^0\bar{K}^0$ (with the final e^+e^- undetected). Since we are discussing experiments with at most about 10^{11} detected ϕ decays, we neglect the $K^0\bar{K}^0$ even charge-parity background in what follows.

IV. INTERFERENCE IN ϕ DECAYS

Choose any two final states f_1 (i.e., $\pi^+\pi^-$, $\pi^0\pi^0$, $3\pi^0$, etc.) and f_2 (i.e., $\pi^+\pi^-$, $\pi l\nu, \ldots$) and observe the decay amplitude

$$\begin{aligned} &\langle f_1(t_1,\hat{\mathbf{z}}), f_2(t_2,-\hat{\mathbf{z}})\,|\,i\rangle \\ &= \frac{1}{\sqrt{2}(qp'+q'p)}[\langle f_1(t_1)|K_L\rangle\langle f_2(t_2)|K_S\rangle \\ &\qquad - \langle f_1(t_1)|K_S\rangle\langle f_2(t_2)|K_L\rangle] , \end{aligned} \tag{20}$$

where $f_2(t_2,-\hat{\mathbf{z}})$ indicates that the kaon moving in the $-\hat{\mathbf{z}}$ direction decays at time t_2 into the f_2 final state. One observes the possibility of interference, which is the crux of this paper.

The "complex masses" of the $K_{S,L}$ are

$$\lambda_{S,L} \equiv m_{S,L} - \frac{i}{2}\gamma_{S,L} . \tag{21}$$

Define also

$$\Delta\lambda \equiv \lambda_L - \lambda_S = \Delta m - \frac{i}{2}\Delta\gamma ,$$
$$\Delta m \equiv m_L - m_S , \quad (22)$$
$$\Delta\gamma \equiv \gamma_L - \gamma_S ,$$
$$\gamma \equiv \frac{\gamma_L + \gamma_S}{2} . \quad (23)$$

For the kaons we have $\Delta m > 0$, $\Delta\gamma \equiv \gamma_L - \gamma_S \approx -\gamma_S < 0$, and $\Delta\gamma / 2\Delta m \approx -1$. (More precise experimental values are given in Appendix A.) A $K_{S,L}$ evolves in time, being an eigenvector of the mass matrix, as

$$| K_{S,L}(t) \rangle = e^{-i\lambda_{S,L} t} | K_{S,L}(t=0) \rangle . \quad (24)$$

For convenience we suppress the time argument of $| K_{S,L}(t=0) \rangle$ in subsequent discussions, and write only $| K_{S,L}(t=0) \rangle \equiv | K_{S,L} \rangle$.

We generalize Eqs. (2) to read for any final state f_i:

$$\eta_i \equiv \frac{\langle f_i | K_L \rangle}{\langle f_i | K_S \rangle} . \quad (25)$$

Note that this will be the inverse of the conventional notation for the 3π modes. We obtain

$$\langle f_1(t_1,\hat{\mathbf{z}}), f_2(t_2,-\hat{\mathbf{z}}) i \,|\, \rangle = \frac{1}{\sqrt{2}(qp'+q'p)} \langle f_1 | K_S \rangle \langle f_2 | K_S \rangle e^{-i(\lambda_L+\lambda_S)t/2} \times \left[(\eta_1 - \eta_2)\cos\left[\frac{\Delta\lambda \Delta t}{2}\right] + i(\eta_1+\eta_2)\sin\left[\frac{\Delta\lambda\Delta t}{2}\right] \right] , \quad (26)$$

where

$$\Delta t \equiv t_2 - t_1 , \quad (27a)$$
$$t \equiv (t_1 + t_2) . \quad (27b)$$

An illustrative example is in order. For equal times $\Delta t = 0$ we have a vanishing amplitude for identical final states of the two kaons $f_1 = f_2 = f$ (i.e., $f = \pi^+\pi^-$, $\pi^0\pi^0, \ldots$). However, at equal times for $f_1 \neq f_2$, e.g., $f_1 = \pi^+\pi^-$, $f_2 = \pi^0\pi^0$, we have an amplitude[8–10] proportional to $\eta_{+-} - \eta_{00} = 3\epsilon'$. One could judiciously choose time slices to maximize signal to background and statistics, comparing $\{\pi^+\pi^-, \pi^+\pi^-\}$ to $\{\pi^+\pi^-, \pi^0\pi^0\}$.

A more direct approach is to compare the decays $\{\pi^+\pi^-, \pi^0\pi^0\}$ to themselves at suitably chosen times. In this paper we mainly concentrate on the latter, more direct approach. This same approach is being studied independently in Ref. 14. We have by no means exhausted all the possibilities; we will merely be able to quote some "good choices." Our conclusion, mentioned earlier, is that to observe a 3σ effect for $\epsilon'/\epsilon = \frac{1}{20} e^{i270^\circ}$ one needs $4\times 10^8 \phi$, and for $\epsilon'/\epsilon = 10^{-3}$ real one requires $2\times 10^9 \phi$. In the two cases, one chooses different time slices, however.

V. REQUIRED NUMBER OF ϕ's AS A FUNCTION OF ϵ'/ϵ

Here we outline one way to calculate the required number of ϕ's (N_ϕ) to observe a given complex value of ϵ'/ϵ. The rate corresponding to Eq. (26) is

$$| \langle f_1(t_1,\hat{\mathbf{z}}), f_2(t_2,-\hat{\mathbf{z}}) | i \rangle |^2 = \frac{1}{2\,| qp'+q'p |^2} | \langle f_1 | K_S \rangle |^2 | \langle f_2 | K_S \rangle |^2 e^{-\gamma t} \times \left[\left| \cos\frac{\Delta\lambda\Delta t}{2} \right|^2 | \eta_2 - \eta_1 |^2 + \left| \sin\frac{\Delta\lambda\Delta t}{2} \right|^2 | \eta_1 + \eta_2 |^2 - 2\,\mathrm{Im}\left[(\eta_1+\eta_2)(\eta_1^* - \eta_2^*) \sin\frac{\Delta\lambda\Delta t}{2} \cos^* \frac{\Delta\lambda\Delta t}{2} \right] \right] . \quad (28)$$

Note that in general $\Delta\lambda$ is a complex number and so we deal with hyperbolic functions as well as trigonometric ones. We will develop this formalism in general for any f_1 and f_2 and $\lambda_{L,S}$, and so our results will be applicable to analogous resonances [e.g., $\psi'' \to D^0\bar{D}^0$, $\Upsilon(4S) \to B_d \bar{B}_d$]. However, having foremost ϵ'/ϵ in mind, we choose

$$f_1 = \pi^+\pi^- , \quad f_2 = \pi^0\pi^0 \quad (29)$$

and this will be our choice until noted otherwise. We integrate over all "accessible" times $t = t_1 + t_2$ keeping the temporal distance $\Delta t = t_2 - t_1$ a constant. This we then denote as our "intensity" $I(\Delta t)$

$$I(\Delta t) \equiv \tfrac{1}{2} \int_{|\Delta t|}^{\infty} dt \, | \langle f_1(t_1,\hat{\mathbf{z}}), f_2(t_2,-\hat{\mathbf{z}}) | i \rangle |^2 . \quad (30)$$

Here the factor of $\frac{1}{2}$ is the Jacobian for the transformation from (t_1, t_2) to $(t, \Delta t)$. Note that $t_1, t_2 \geq 0$ and so the integration begins at $|\Delta t| \geq 0$. In Fig. 2 we display the time-dependent intensity plots. We note that for $\epsilon'/\epsilon = 0$ [Fig. 2(a)] the plot is symmetric with respect to Δt reflection ($\Delta t \to -\Delta t$). Asymmetries arise for $\epsilon'/\epsilon \neq 0$; to dramatize them we have shown in Figs. 2(b)—2(d) and 2(f) cases that are ruled out experimentally. The intensities are roughly constant over $10 \lesssim \Delta t/\tau_S \lesssim 100$, a fact easily understood in terms of the disparate lifetimes of a K_S and K_L. (The lifetimes of a K_S and K_L from ϕ decay at rest translate to mean decay paths[22] of 0.6 cm and 3.4 m, respectively.) We define an intensity asymmetry for any given Δt as

$$A_I(\Delta t) \equiv \frac{I(\Delta t)-I(-\Delta t)}{I(\Delta t)+I(-\Delta t)} = \frac{2\left[\eta_{\mathrm{re}}\sinh\frac{\Delta\gamma\Delta t}{2} - \eta_{\mathrm{im}}\sin\Delta m\,\Delta t\right]}{(\eta_- + \eta_+)\cosh\frac{\Delta\gamma\Delta t}{2} + (\eta_- - \eta_+)\cos\Delta m\,\Delta t}\,, \quad (31)$$

where

$$\begin{aligned} \eta_{\mathrm{re}} &\equiv \mathrm{Re}[(\eta_1+\eta_2)(\eta_1^* - \eta_2^*)]\,, \\ \eta_{\mathrm{im}} &\equiv \mathrm{Im}[(\eta_1+\eta_2)(\eta_1^* - \eta_2^*)]\,, \\ \eta_+ &\equiv |\eta_1+\eta_2|^2\,, \\ \eta_- &\equiv |\eta_1-\eta_2|^2\,. \end{aligned} \quad (32)$$

In the case we are interested in [Eq. (29)], $\eta_1 \equiv \eta_{+-}$, $\eta_2 = \eta_{00}$, we obtain

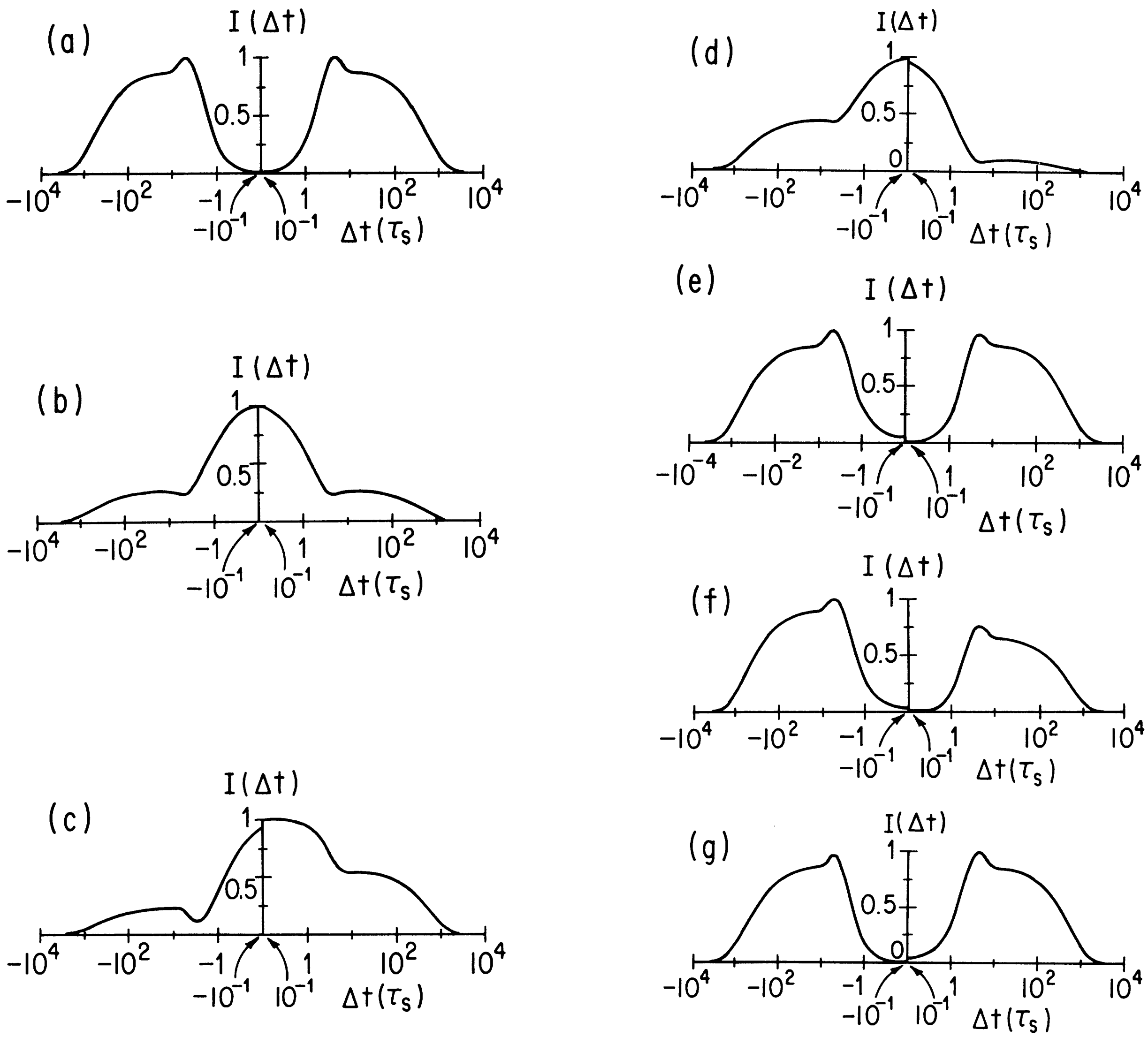

FIG. 2. The time-dependent intensity plot [Eq. (30)] in arbitrary units as a function of Δt, for various ϵ'/ϵ. Note that cases (a) and (b) exhibit no interference; cases (b)—(d) and (f) are ruled out experimentally. Also, since the Δt axis (horizontal) is logarithmic, the apparent discontinuity arises because of the comparison between $\Delta t = 10^{-1}$ and $\Delta t = -10^{-1}$. (a) For $\epsilon'/\epsilon = 0$ (or equivalently $\eta_1 = \eta_2$, so also for $\pi^+\pi^-, \pi^+\pi^-$). (b) For $\epsilon'/\epsilon = 2$ (or equivalently $\eta_{+-} = -\eta_{00}$). (c) For $\epsilon'/\epsilon = 1e^{i90^\circ}$. (d) $\epsilon'/\epsilon = 1$. (e) $\epsilon'/\epsilon = 0.05e^{i270^\circ}$. (f) $\epsilon'/\epsilon = 0.05e^{i0^\circ}$. (g) $\epsilon'/\epsilon = 0.05e^{i90^\circ}$.

$$\eta_{+}=|\epsilon|^2\left[4-4\,\mathrm{Re}\frac{\epsilon'}{\epsilon}+\left|\frac{\epsilon'}{\epsilon}\right|^2\right],$$

$$\eta_{-}=|\epsilon|^2\left[9\left|\frac{\epsilon'}{\epsilon}\right|^2\right],$$

$$\eta_{\mathrm{re}}=|\epsilon|^2\left[6\,\mathrm{Re}\frac{\epsilon'}{\epsilon}-3\left|\frac{\epsilon'}{\epsilon}\right|^2\right], \quad (33)$$

$$\eta_{\mathrm{im}}=|\epsilon|^2\left[-6\,\mathrm{Im}\left[\frac{\epsilon'}{\epsilon}\right]\right].$$

Then

$$\lim_{\Delta t\to\infty}A_I(\Delta t)=\frac{3\left[-2\,\mathrm{Re}\frac{\epsilon'}{\epsilon}+\left|\frac{\epsilon'}{\epsilon}\right|^2\right]}{2-2\,\mathrm{Re}\frac{\epsilon'}{\epsilon}+5\left|\frac{\epsilon'}{\epsilon}\right|^2}. \quad (34)$$

Nature tells us that $|\epsilon'/\epsilon|\lesssim 0.05$ is tiny (see discussion in Sec. II). Therefore, for $\mathrm{Re}(\epsilon'/\epsilon)=0$, it is more powerful to limit Δt to only a finite range where the asymmetry is appreciable [Fig. 3(a)]. For $\mathrm{Re}(\epsilon'/\epsilon)=O(|\epsilon'/\epsilon|)$, however, it is statistically more powerful to include all events $\Delta t\in[0,\infty)$ [see Figs. 3(b) and 3(c)].

Define the rate for the decay into f_1 and f_2 to occur anywhere between $\tau_2>\Delta t>-\tau_1$ as

$$\Gamma[\tau_2,-\tau_1]\equiv\int_{-\tau_1}^{\tau_2}d(\Delta t)I(\Delta t)\,. \quad (35)$$

Appendix A contains the explicit formula for this function. The total decay rate is

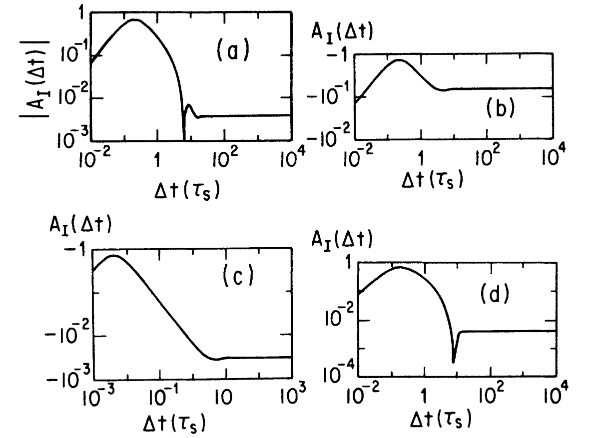

FIG. 3. The intensity asymmetry [Eq. (31)] plotted as a function of Δt (in K_S-lifetime units), for various ϵ'/ϵ. (a) $\epsilon'/\epsilon=0.05e^{i270°}$. For $\Delta t<6.0$ the asymmetry is negative, while for $\Delta t>6.0$ it is positive. We plot the absolute value of the asymmetry. The maximal asymmetry -0.67 occurs at $\Delta t_{\max}=0.22$; $A_I(\Delta t=\infty)=3.7\times10^{-3}$. (b) $\epsilon'/\epsilon=0.05$. The maximal asymmetry -0.72 occurs at $\Delta t_{\max}=0.22$; $A_I(\infty)=-0.15$. (c) $\epsilon'/\epsilon=10^{-3}$. The maximal asymmetry -0.72 occurs at $\Delta t_{\max}=4.5\times10^{-3}$; $A_I(\infty)=-3\times10^{-3}$. (d) $\epsilon'/\epsilon=0.05e^{i90°}$. The maximal asymmetry $+0.71$ occurs at 0.22; $A_I(\infty)=3.7\times10^{-3}$.

$$\Gamma[\infty,-\infty]=\int_0^\infty dt_1\times\int_0^\infty dt_2\,|\langle f_1(t_1,\hat{\mathbf{z}}),f_2(t_2,-\hat{\mathbf{z}})|i\rangle|^2\,. \quad (36)$$

We define a rate asymmetry

$$A_\Gamma[\tau,0]\equiv\frac{\Gamma[\tau,0]-\Gamma[0,-\tau]}{\Gamma[\tau,0]+\Gamma[0,-\tau]}\,. \quad (37)$$

Defining $z=\Delta m/\gamma$, $y=\Delta\gamma/2\gamma$, we obtain the limiting case of $\tau\to\infty$ (see Appendix A)

$$A_\Gamma[\tau=\infty,0]=\frac{6\left[\left[2\,\mathrm{Re}\frac{\epsilon'}{\epsilon}-\left|\frac{\epsilon'}{\epsilon}\right|^2\right]y(1+z^2)+\left[2\,\mathrm{Im}\frac{\epsilon'}{\epsilon}\right]z(1-y^2)\right]}{9\left|\frac{\epsilon'}{\epsilon}\right|^2(2+z^2-y^2)+\left[4-4\,\mathrm{Re}\frac{\epsilon'}{\epsilon}+\left|\frac{\epsilon'}{\epsilon}\right|^2\right](z^2+y^2)}, \quad (38a)$$

which simplifies to

$$A_\Gamma[\tau=\infty,0]\approx\frac{3y(1+z^2)}{2(z^2+y^2)}\left[\left[2\,\mathrm{Re}\frac{\epsilon'}{\epsilon}-\left|\frac{\epsilon'}{\epsilon}\right|^2\right]-\frac{4\gamma_L}{\gamma_S}\mathrm{Im}\frac{\epsilon'}{\epsilon}\right] \quad (38b)$$

under the assumptions that $|\epsilon'/\epsilon|\ll1$ and that $z\approx-y\approx1$. Only on a time scale of the first few K_S lifetimes does the $\eta_{\mathrm{im}}\sim2\,\mathrm{Im}\epsilon'/\epsilon$ term contribute appreciably to the intensity asymmetry [Eq. (31) and Fig. 3]. At later times we observe that $\eta_{\mathrm{re}}\sim2\,\mathrm{Re}(\epsilon'/\epsilon)-|\epsilon'/\epsilon|^2$ dominates, as seen in Eq. (34). Therefore we anticipate that in the rate asymmetry [Eqs. (38)] we find the η_{im} terms suppressed by $O(\gamma_L/\gamma_S)$ relative to the η_{re} term.

Given any ϵ'/ϵ (complex number) we are now in a position to quote the number of required ϕ's (N_ϕ) to observe this ϵ'/ϵ to $N\sigma$ accuracy, when we select events with $\tau\geq\Delta t\geq0$. This number is

$$N_\phi(\tau)\approx2220\times\frac{\Gamma[\infty,-\infty]}{\Gamma[\tau,0]}\times\max\left\{1,\frac{N^2}{2}(1+A_\Gamma)\frac{(1-A_\Gamma{}^2)}{A_\Gamma{}^2}\right\}. \quad (39)$$

We now explain the factors in this relation.

The factor 2220 is the inverse of the branching ratio of $\phi \to \pi^+\pi^- + \pi^0\pi^0$:

$$B(\phi \to \pi^+\pi^- + \pi^0\pi^0) = B(\phi \to K_S K_L) \times B[K_S K_L(C=\text{odd}) \to \pi^+\pi^- + \pi^0\pi^0] \approx \frac{1}{2220} \ . \quad (40)$$

One might naively expect that

$$B[K_S K_L(C=\text{odd}) \to \pi^+\pi^- + \pi^0\pi^0] = B(K_S \to \pi^+\pi^-)B(K_L \to \pi^0\pi^0) + B(K_L \to \pi^+\pi^-)B(K_S \to \pi^0\pi^0) \ , \quad (41)$$

which in fact happens to be the correct answer. For the correct quantum-mechanical treatment we refer the reader to Appendix B. The ratio

$$\frac{\Gamma[\infty, -\infty]}{\Gamma[\tau, 0]}$$

is the ratio of the total number of $\pi^+\pi^- + \pi^0\pi^0$ events to the number of $\pi^+\pi^- + \pi^0\pi^0$ events under the constraint

$$\tau \geq \Delta t\,(=t_{00} - t_{+-}) \geq 0 \ . \quad (42)$$

Finally, we require to know the number[25] of $\pi^+\pi^- + \pi^0\pi^0$ events under the constraint of Eq. (42) to observe an $N\sigma$ effect of the asymmetry A_Γ in Eq. (37). Since we demand at least one such event we take the maximum in Eq. (39).

VI. MINIMIZING THE NUMBER OF REQUIRED ϕ

In experiments utilizing the rate asymmetry (37), one wants to find the τ for any given ϵ'/ϵ which minimizes the required number of ϕ's, N_ϕ. As expected [see the discussion after Eq. (34)] for ϵ'/ϵ small in magnitude and imaginary, we obtain a global $\tau_{\text{min}} \sim 2.5\tau_S$ [Figs. 4(b) and 4(e)]. (A local minimum will be present at $\tau \sim 2.5\tau_S$ for phases near 90°, 270°.) For $\text{Re}(\epsilon'/\epsilon) = O(|\epsilon'/\epsilon|)$ it is statistically more powerful to take $\tau = \infty$ [Figs. 4(a), 4(c), 4(d), and 4(f)—4(i)].

Denote α as the phase of ϵ'/ϵ:

$$\frac{\epsilon'}{\epsilon} = \left|\frac{\epsilon'}{\epsilon}\right| e^{i\alpha} \ . \quad (43)$$

In Table I we display for various ϵ'/ϵ the rate asymmetries [Eq. (37)] and the number of ϕ's required for a 3σ asymmetry [Eq. (39)] for time slices $\tau = \infty > \Delta t \geq 0$. We read off Table I that for a fixed magnitude of ϵ'/ϵ all phases (α) with a sizable real part of $e^{i\alpha}$ require essentially the same N_ϕ (up to factors of 3). A remarkable increase of N_ϕ is predicted for imaginary ϵ'/ϵ in agreement with the discussion after Eq. (34). We note that for $86° \lesssim \alpha \lesssim 92°$ and $268° \lesssim \alpha \lesssim 274°$ the local minimum of $N_\phi(\tau)$ at $\tau \sim 2.5\tau_S$ is a global one (see Table II); for those phases it is most powerful to restrict oneself to finite time slices $\tau \approx 2.5\tau_s \geq \Delta t \geq 0$. It is perhaps worthwhile pointing out that there are some special cases [e.g., $|\epsilon'/\epsilon| = 0.05e^{i\alpha}, \alpha = (85,86,268,270)°$], such that the rate

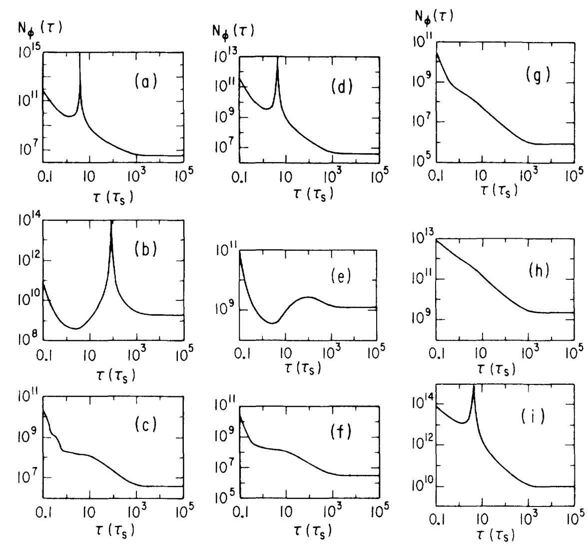

FIG. 4. Number of ϕ's, N_ϕ [Eq. (31)], plotted as a function of τ [in K_S-lifetime units ($\tau \geq \Delta t \geq 0$)]. (a) $\epsilon'/\epsilon = 0.05 \times e^{i240°}$. N_ϕ ($\tau_{\text{local min}} = 1.4$) $= 6.12 \times 10^9$, $N_\phi(\tau = 4.0) = \infty$, $N_\phi(\infty) = 3.5 \times 10^6$. (b) $\epsilon'/\epsilon = 0.05 \times e^{i270°}$. $N_\phi(\tau_{\text{min}} = 2.5) = 0.38 \times 10^9$, $N_\phi(\tau \approx 87) = \infty$, $N_\phi(\infty) = 2.0 \times 10^9$. (c) $\epsilon'/\epsilon = 0.05 \times e^{i300°}$. $N_\phi(\infty) = 3.8 \times 10^6$. (d) $\epsilon'/\epsilon = 0.05 \times e^{i60°}$. $N_\phi(\tau_{\text{local min}} = 1.6) = 3.8 \times 10^9$, $N_\phi(\tau = 4.5) = \infty$, $N_\phi(\tau = \infty) = 3.9 \times 10^6$. (e) $\epsilon'/\epsilon = 0.05e^{i90°}$. $N_\phi(\tau_{\text{min}} = 2.6) = 0.33 \times 10^9$, $N_\phi(\tau_{\text{loc max}} = 87.1) = 2.6 \times 10^9$, $N_\phi(\tau = \infty) = 1.1 \times 10^9$. (f) $\epsilon'/\epsilon = 0.05 \times e^{i120°}$. $N_\phi(\tau = \infty) = 3.4 \times 10^6$. (g) $\epsilon'/\epsilon = 0.05$. $N_\phi(\tau = \infty) = 8.4 \times 10^5$. (h) $\epsilon'/\epsilon = 10^{-3}$. $N_\phi(\tau = \infty) = 2.2 \times 10^9$. (i) $\epsilon'/\epsilon = 10^{-3}e^{i60°}$. $N_\phi(\tau_{\text{local min}} = 1.4) = 1.1 \times 10^{13}$, $N_\phi(\tau = 4.3) = \infty$, $N_\phi(\tau = \infty) = 9.1 \times 10^9$.

asymmetry, Eq. (37), flips sign (see Table II). In fact, this explains why $N_\phi(\tau = \infty)$ for the $\alpha = 270°$ case is larger, at times by an order of magnitude, than for $\alpha = 90°$ (see Table I). A remnant of this sign flip is the functional dependence on ϵ'/ϵ as found in Eq. (38).

VII. A PEDAGOGICAL CASE

We saw that comparing $\pi^+\pi^- + \pi^0\pi^0$ decays to themselves at different time slices leads to detectable effects. However, there are cases when neither the method outlined in Secs. V and VI or the approach relying entirely on the magnitudes of $|\eta_{+-}|$ and $|\eta_{00}|$ will work. For a pedagogical example choose $|\eta_{+-}| = |\eta_{00}|$ and $\phi_{+-} - \phi_{00} = 180°$ (which appears to be ruled out experimentally). The resulting intensity graph [see Fig. 2(b)] shows no asymmetry in the sense of Eq. (31), since indeed no interference occurs [in Eq. (26) $\eta_{+-} - \eta_{00} \neq 0$, $\eta_{+-} + \eta_{00} = 0$]. So our method of Sec. V is not applicable here. The time-dependent intensity plot is a function of the complex cosine. Contrast this situation with $\epsilon'/\epsilon = 0$ (i.e., $\eta_{+-} - \eta_{00} = 0, \eta_{+-} + \eta_{00} \neq 0$). Here also no asymmetry results, and the time-dependent intensity plot [Fig. 2(a)] is drastically different from Fig. 2(b), dictated now by the complex sine. Therefore, looking at the time-dependent intensity curve of $(\pi^+\pi^-,\pi^0\pi^0)$ [Eq. (30)] we gain additional insight about the relative phases of η_{+-} and η_{00}.

VIII. COMPARISON OF EXPERIMENTAL METHODS

We display the contours of constant number of ϕ's, $N_\phi(\tau=\infty)$, and asymmetries $A_\Gamma[\tau=\infty,0]$ for the decays $\pi^+\pi^-$ later than $\pi^0\pi^0$ vs $\pi^+\pi^-$ earlier than $\pi^0\pi^0$, in Fig. 5. We call to the attention of the reader that comparison with Fig. 1 shows that indeed the use of ϕ's in the limit $\tau=\infty$ measures essentially the same as the $|\eta_{+-}/\eta_{00}|$ experiments. That is readily understood, as follows. Neglecting

$$\left[\mathrm{Re}\frac{\epsilon'}{\epsilon}\right]^2 \ll \mathrm{Re}\frac{\epsilon'}{\epsilon}\,, \tag{44}$$

we obtain

$$\mathrm{Re}\frac{\epsilon'}{\epsilon} \approx \frac{1}{2}\left[\mathrm{Im}\frac{\epsilon'}{\epsilon}\right]^2 - \frac{1}{6}\left[1-\left|\frac{\eta_{+-}}{\eta_{00}}\right|^2\right], \tag{45a}$$

$$\mathrm{Im}\frac{\epsilon'}{\epsilon} \approx \tfrac{1}{3}\sin(\phi_{+-}-\phi_{00})\left[1-2\,\mathrm{Re}\frac{\epsilon'}{\epsilon}+O\left[\left[\frac{\epsilon'}{\epsilon}\right]^2\right]\right]. \tag{45b}$$

These were just the relations plotted in Fig. 1.

To obtain Fig. 5 we neglect

$$\left|\frac{\epsilon'}{\epsilon}\right|^2 \ll 1, \quad \left|\mathrm{Re}\frac{\epsilon'}{\epsilon}\right| \ll 1 \tag{46}$$

and we use the known lifetimes and mass difference of the $K^0\bar{K}^0$ system, Eq. (A2), to obtain

$$\mathrm{Re}\frac{\epsilon'}{\epsilon} \approx \frac{2\gamma_L}{\gamma_S}\mathrm{Im}\frac{\epsilon'}{\epsilon} + \frac{1}{2}\left[\mathrm{Im}\frac{\epsilon'}{\epsilon}\right]^2 - \frac{1}{3}A_\Gamma[\tau=\infty,0]\,. \tag{47}$$

TABLE I. The rate asymmetries [Eq. (38) and the number of ϕ's required (N_ϕ), Eq. (39)] with $\tau=\infty(\tau\geq\Delta t\geq 0)$, for various $\epsilon'/\epsilon=|\epsilon'/\epsilon|e^{i\alpha}$. The values with asterisks (*) are excluded experimentally.

(a) $A_\Gamma[\tau=\infty,0]$

α (degrees)	$\left\|\frac{\epsilon'}{\epsilon}\right\|=0.05$	$\left\|\frac{\epsilon'}{\epsilon}\right\|=0.01$	$\left\|\frac{\epsilon'}{\epsilon}\right\|=0.005$	$\left\|\frac{\epsilon'}{\epsilon}\right\|=10^{-3}$
0	-0.15*	-3.0×10^{-2}	-1.5×10^{-2}	-3.0×10^{-3}
30	-0.13*	-2.6×10^{-2}	-1.3×10^{-2}	-2.6×10^{-3}
60	-0.072*	-1.5×10^{-2}	-7.4×10^{-3}	-1.5×10^{-3}
90	$+4.2\times10^{-3}$	2.5×10^{-4}	8.9×10^{-5}	1.2×10^{-5}
120	7.7×10^{-2}*	1.5×10^{-2}	7.6×10^{-3}	1.5×10^{-3}
150	0.13*	2.6×10^{-2}	1.3×10^{-2}	2.6×10^{-3}
180	0.15*	3.0×10^{-2}	1.5×10^{-2}	3.0×10^{-3}
210	0.13*	2.6×10^{-2}	1.3×10^{-2}	2.6×10^{-3}
240	7.6×10^{-2}*	1.5×10^{-2}	7.5×10^{-3}	1.5×10^{-3}
270	3.2×10^{-3}	4.7×10^{-5}	-1.4×10^{-5}	-8.8×10^{-6}
300	-7.3×10^{-2}*	-1.5×10^{-2}	-7.5×10^{-3}*	-1.5×10^{-3}
330	-0.13*	-2.6×10^{-2}	-1.3×10^{-2}	-2.6×10^{-3}

(b) $N_\phi(\tau=\infty)$

α (degrees)	$\left\|\frac{\epsilon'}{\epsilon}\right\|=0.05$	$\left\|\frac{\epsilon'}{\epsilon}\right\|=10^{-2}$	$\left\|\frac{\epsilon'}{\epsilon}\right\|=5\times10^{-3}$	$\left\|\frac{\epsilon'}{\epsilon}\right\|=10^{-3}$
0	8.4×10^{5}*	2.2×10^{7}	9.0×10^{7}	2.2×10^{9}
30	1.2×10^{6}*	3.0×10^{7}	1.2×10^{8}	3.0×10^{9}
60	3.9×10^{6}*	9.2×10^{7}	3.7×10^{8}	9.1×10^{9}
90	1.1×10^{9}	3.2×10^{11}	2.6×10^{12}	1.4×10^{14}
120	3.4×10^{6}*	8.8×10^{7}	3.5×10^{8}	8.9×10^{9}
150	1.2×10^{6}*	3.0×10^{7}	1.2×10^{8}	3.0×10^{9}
180	9.4×10^{5}*	2.3×10^{7}	9.0×10^{7}	2.3×10^{9}
210	1.2×10^{6}*	3.0×10^{7}	1.2×10^{8}	3.0×10^{9}
240	3.5×10^{6}*	9.0×10^{7}	3.6×10^{8}	9.1×10^{9}
270	2.0×10^{9}	9.2×10^{12}	1.0×10^{14}	2.6×10^{14}
300	3.8×10^{6}*	9.0×10^{7}	3.6×10^{8}	8.9×10^{9}
330	1.2×10^{6}*	3.0×10^{7}	1.2×10^{8}	3.0×10^{9}

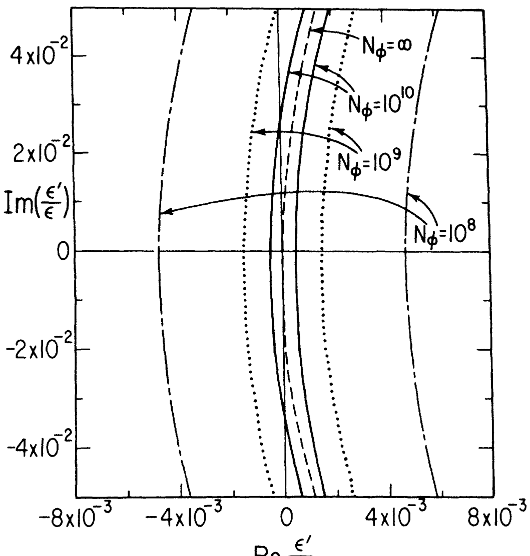

FIG. 5. Contours of constant $N_\phi(\tau=\infty)$, and the corresponding asymmetry $A_\Gamma[\tau=\infty,0]$. Dashed curve, $N_\phi=\infty$, $A_\Gamma=0$; solid curve, $N_\phi=10^{10}$, $A_\Gamma=\pm 1.4\times 10^{-3}$; dotted curve, $N_\phi=10^9$, $A_\Gamma=\pm 4.5\times 10^{-3}$; dashed-dotted curve, $N_\phi=10^8$, $A_\Gamma=\pm 1.4\times 10^{-2}$. Here the three right-hand-most contours have negative asymmetries; all others are positive.

From Eqs. (46) and (A2) we have

$$\frac{\Gamma[\infty,-\infty]}{\Gamma[\infty,0]}\approx 2,\quad A_\Gamma[\tau=\infty,0]\lesssim 10^{-2}\,. \tag{48}$$

If in addition we assume $|\epsilon|=2.28\times 10^{-3}$, which is necessary to obtain $B(\phi\to\pi^+\pi^-+\pi^0\pi^0)\approx 2220$, we find

$$\mathrm{Re}\frac{\epsilon'}{\epsilon}\approx\frac{2\gamma_L}{\gamma_S}\mathrm{Im}\frac{\epsilon'}{\epsilon}+\frac{1}{2}\left[\mathrm{Im}\frac{\epsilon'}{\epsilon}\right]^2\pm\left[\frac{2220}{N_\phi(\tau=\infty)}\right]^{1/2}. \tag{49}$$

As a result of the small ratio γ_L/γ_S, the contour of constant $|\eta_{+-}/\eta_{00}|$ [Eq. (45a)] is to a good approximation also a $N_\phi(\tau=\infty)$ contour [Eq. (49)]. Thus $N_\phi(\tau=\infty)$ experiments probe essentially the same contours as conventional $|\eta_{+-}/\eta_{00}|$ experiments.

Another experiment, the one we advocate as being particularly suited to the measurement of $\mathrm{Im}(\epsilon'/\epsilon)$, is to observe final states with a finite cut on τ. The corresponding asymmetries $A_\Gamma[\tau,0]$ for $\tau=2.5\tau_s$ are shown in Fig. 6(a). The contours of the numbers of ϕ's needed to observe these asymmetries are shown in Fig. 6(b). In fact for many cases one can optimize the minimum number of ϕ's required by choosing a slightly different $\tau_{\min}$, with a benefit of as much as a factor of 2. An example is shown in Table II for $\epsilon'/\epsilon=10^{-3}e^{i60^\circ}$. We see that these experiments are primarily sensitive to the phase difference $\phi_{+-}-\phi_{00}$. For example, $10^{10}\phi$'s would permit the measurement of $|\mathrm{Im}(\epsilon'/\epsilon)|$ to $\lesssim 10^{-2}$ [for $\mathrm{Re}(\epsilon'/\epsilon)\approx 0$] corresponding to $|\phi_{+-}-\phi_{00}|\lesssim 1.6^\circ$. The experiment would require a vacuum pipe at the interaction region with an inner radius of perhaps 10—20 K_S^0 lifetimes (6—12 cm) in order to prevent regeneration. Tracking, calorimeters, and photon conversion layers would be necessary to identify both final states $\pi^+\pi^-$ and $\pi^0\pi^0$ and find their decay vertices.

TABLE II. The local minimum of $N_\phi(\tau)$ [Eq. (39)] and the minimizing $\tau_{\min}$ compared with $N_\phi(\tau=\infty)$ for various phases (α) of $\epsilon'/\epsilon=|\epsilon'/\epsilon|e^{i\alpha}$. The values in part (b) with a dagger (†) do not correspond to a local minimum of N_ϕ.

(a) $|\epsilon'/\epsilon|=0.05$

α (degrees)	τ(min)	$N_\phi(\tau_{\min})$	$A_\Gamma[\tau_{\min},0]$	$N_\phi(\tau=\infty)$	$A_\Gamma[\tau=\infty,0]$
85	2.4	4.2×10^8	0.17	2.6×10^8	-8.8×10^{-3}
86	2.4	4.0×10^8	0.17	5.3×10^8	-6.2×10^{-3}
90	2.6	3.3×10^8	0.18	1.1×10^9	4.2×10^{-3}
92	2.6	3.0×10^8	0.18	2.3×10^8	9.4×10^{-3}
268	2.4	4.2×10^8	−0.17	2.9×10^8	8.4×10^{-3}
270	2.5	3.8×10^8	−0.17	2.0×10^9	3.2×10^{-3}
274	2.6	3.1×10^8	−0.18	3.9×10^8	-7.2×10^{-3}
275	2.6	3.0×10^8	−0.18	2.1×10^8	-9.8×10^{-3}

(b) $|\epsilon'/\epsilon|=10^{-3}$

α (degrees)	$\tau_{\min}$	N_ϕ	$A_\Gamma[\tau_{\min},0]$	$N_\phi(\tau=\infty)$	$A_\Gamma[\tau=\infty,0]$
60	1.4	1.1×10^{13}	2×10^{-3}	9.1×10^9	-1.5×10^{-3}
60	2.5†	1.7×10^{13}†	8.2×10^{-4}†	9.1×10^9	-1.5×10^{-3}
86	2.4	1.1×10^{12}	3.5×10^{-3}	5.2×10^{11}	-2.0×10^{-4}
87	2.4	1.0×10^{12}	3.5×10^{-3}	9.6×10^{11}	-1.5×10^{-4}
90	2.5	8.7×10^{11}	3.6×10^{-3}	1.4×10^{14}	1.2×10^{-5}
93	2.7	7.7×10^{11}	3.6×10^{-3}	7.1×10^{11}	1.7×10^{-4}
267	2.4	1.0×10^{12}	-3.5×10^{-3}	9.2×10^{11}	1.5×10^{-4}
270	2.5	8.7×10^{11}	-3.6×10^{-3}	2.6×10^{14}	-8.8×10^{-6}
273	2.7	7.7×10^{11}	-3.6×10^{-3}	7.4×10^{11}	-1.7×10^{-4}

It appears experimentally feasible to obtain a limit on the phase difference better than $\pm 1.6°$, corresponding to a rate asymmetry $A_\Gamma[\tau=2.5,0]=\pm 3.4\%$. For example, a systematic error in Δt of $0.01\tau_S$ (corresponding to a 60 μm shift in decay length) would induce an error in $A_\Gamma[\tau=2.5,0]$ of only 0.9% (for $\epsilon'/\epsilon=0$).

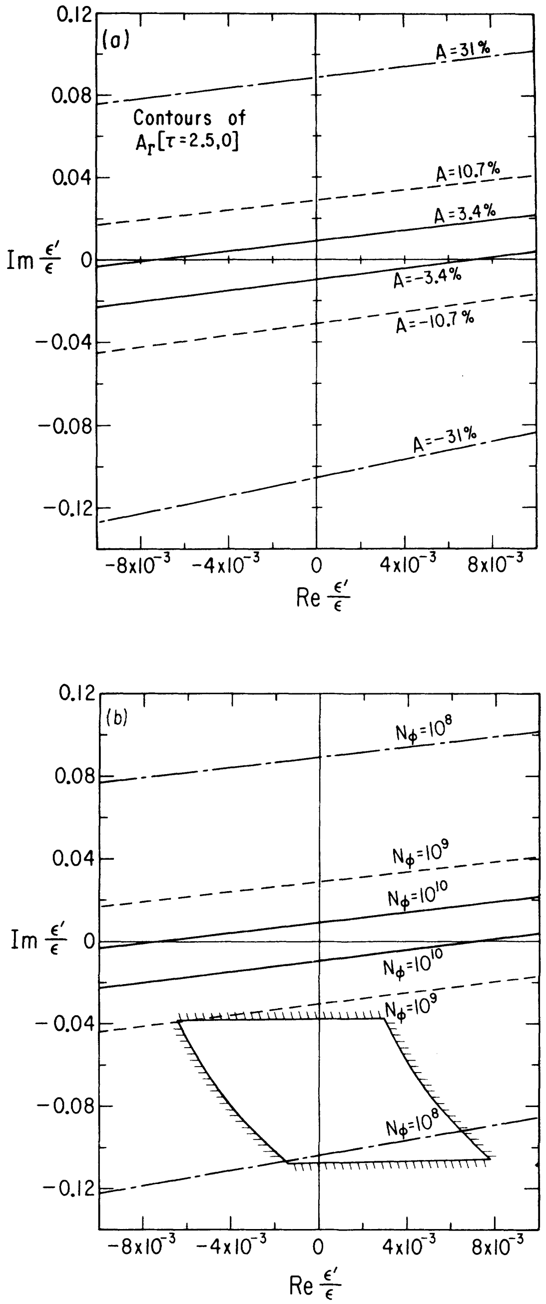

FIG. 6. Contours of constant (a) $A_\Gamma[\tau=2.5\tau_S,0]$; (b) $N_\phi(\tau=2.5\tau_S)$ (the $N_\phi=\infty$ curve is omitted). The experimental limit (1σ) is the unshaded area.

IX. THE K_S: RARE DECAYS AND CP VIOLATION

A K_S decays close to the interaction region, whereas the K_L decays typically hundreds of K_S lifetimes away. This idea can be utilized to measure rare decays of K_S. If the K_L decay has in fact been observed far enough downstream in $\phi\to K_S K_L$, we can in fact be sure the other particle is a K_S.

Using this idea, we can begin to address CP violation in K_S decays with about 10^{10} ϕ's. At present only upper limits exist.[26] The decay $K_S\to 3\pi^0$ is a distinctive signal of CP violation, whereas $K_S\to\pi^+\pi^-\pi^0$ could happen via a CP-conserving amplitude with higher partial angular momenta.

In the superweak ansatz[27] one predicts

$$B(K_S\to 3\pi^0)\approx 1.96\times 10^{-9}\ . \tag{50}$$

Hence, utilizing the branching ratio[22] $B(\phi\to K_S K_L)\approx 34.3\%$ we observe that 1.5×10^9 ϕ's are required to see one $K_S\to 3\pi^0$ decay. The typical geometry is highly favorable. For $K_S\to\pi^+\pi^-\pi^0$ one could utilize carefully chosen times and decay modes for interference effects[6,26] to be seen. This is a viable alternative to the $K_S\to 3\pi^0$ decay also.

Another CP test based on $K\to 3\pi$ decays studies partial-rate asymmetries between such processes as $K^+\to\pi^+\pi^+\pi^-$ and $K^-\to\pi^-\pi^-\pi^+$. (See, e.g., Lee and Wu, Ref. 2.) Such tests are especially suited to a ϕ factory, where there are equal numbers of K^+ and K^-.

X. SUMMARY

Here we have outlined some methods for measuring $\eta_{+-}-\eta_{00}$. In the event that CPT does not hold, these methods promise to be especially advantageous (since we already know that $|\eta_{+-}|\approx|\eta_{00}|$).

For most phases of ϵ'/ϵ considered here, it is statistically more feasible to integrate the intensity in Eq. (30) out from $\Delta t=0$ to $\Delta t=\infty$ and compare it to that for $-\infty$ to 0. One requires 2×10^9 ϕ's to observe $\epsilon'/\epsilon=10^{-3}$ or $\epsilon'/\epsilon=0.05e^{i270°}$ to 3σ accuracy, for full detection efficiencies.

However, for a small island of $\pm 3°$ around $\alpha=90°,270°$ [see Eq. (43)], integrating the intensity out to only a finite time $0\le\Delta t\le\tau\approx 2.5\tau_S$, where $\Delta t=t_{\pi^0\pi^0}-t_{\pi^+\pi^-}$, and comparing with $-2.5\tau_S\le\Delta t\le 0$ appears to be preferable over the first approach. There are also cases (Sec. VII) where no asymmetry will be found. One has the option of comparing the time-dependent curves of $(\pi^+\pi^-+\pi^0\pi^0)$ to $(\pi^+\pi^-+\pi^+\pi^-)$ [Figs. 2(a) and 2(b)].

We also addressed briefly the possibilities of measuring as yet unobserved CP violation in K_S decays. For the superweak ansatz one requires 1.5×10^9 ϕ's to observe one $K_S\to 3\pi^0$, a clear signal of CP violation. Rare decays of the K_S can also be probed to excellent accuracy. All this is due to the fact that a K_S and K_L are so easily distinguished because of their wildly disparate lifetimes.

Note added in proof. Results from a test run of an experiment at Fermilab,[19] when combined with previous data,[3,4] now yield a world average[28] $|\eta_{+-}/\eta_{00}|^2=1.009\pm 0.017$. The corresponding allowed regions in

Figs. 1 and 6 are smaller, but values of $|\text{Im}(\epsilon/\epsilon)| \gg \text{Re}(\epsilon'/\epsilon)|$ still are not excluded.

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APPENDIX A

After some lengthy, albeit straightforward algebra, we obtain

$$\Gamma[\tau_2,-\tau_1]=\frac{|\langle f_1|K_S\rangle|^2|\langle f_2|K_S\rangle|^2}{16|(qp'+q'p)|^2}\{|\eta_1-\eta_2|^2[h_-(\tau_1)+s(\tau_1)+h_-(\tau_2)+s(\tau_2)+2a]$$
$$+|\eta_1+\eta_2|^2[h_-(\tau_1)-s(\tau_1)+h_-(\tau_2)-s(\tau_2)+2b]$$
$$+\text{Re}[(\eta_1+\eta_2)(\eta_1^*-\eta_2^*)]2[h_+(\tau_2)-h_+(\tau_1)]$$
$$+\text{Im}[(\eta_1+\eta_2)(\eta_1^*-\eta_2^*)][\sigma(\tau_2)-\sigma(\tau_1)]\}\ , \tag{A1}$$

where

$$h_\pm(\tau)\equiv\frac{-1}{\gamma\gamma_S}e^{-\gamma_S\tau}\pm\frac{1}{\gamma\gamma_L}e^{-\gamma_L\tau},\quad s(\tau)\equiv\frac{2e^{-\gamma\tau}}{\gamma^2(1+z^2)}(z\sin\Delta m\tau-\cos\Delta m\tau)\ ,$$
$$\sigma(\tau)\equiv\frac{4e^{-\gamma\tau}}{\gamma^2(1+z^2)}(z\cos\Delta m\tau+\sin\Delta m\tau),\quad a\equiv\frac{2(2+z^2-y^2)}{\gamma_L\gamma_S(1+z^2)},\quad b\equiv\frac{2(z^2+y^2)}{\gamma_L\gamma_S(1+z^2)},\quad z\equiv\frac{\Delta m}{\gamma},\quad y\equiv\frac{\Delta\gamma}{2\gamma}\ .$$

For the kaon system we use[22]

$$\frac{\gamma_S}{\gamma_L}=580,\quad a=1.17\times10^3,\quad b=1.16\times10^3,\quad z=0.953,\quad y=-0.997\ . \tag{A2}$$

For the total rate into f_1,f_2 we obtain

$$\Gamma[\infty,-\infty]=\int_0^\infty dt_1\int_0^\infty dt_2\,|\langle f_1(t_1,\hat{\mathbf{z}}),f_2(t_2,-\hat{\mathbf{z}})|i\rangle|^2$$
$$=\frac{|\langle f_1|K_S\rangle|^2|\langle f_2|K_S\rangle|^2}{8|qp'+q'p|^2}(|\eta_1-\eta_2|^2a+|\eta_1+\eta_2|^2b)\ . \tag{A3}$$

We display the rate asymmetry [Eq. (37)] for the limiting case $\tau\to\infty$:

$$A_\Gamma[\tau=\infty,0]=\frac{2[\eta_{\text{re}}y(1+z^2)-\eta_{\text{im}}z(1-y^2)]}{\eta_-(2+z^2-y^2)+\eta_+(z^2+y^2)}\ . \tag{A4}$$

APPENDIX B

The branching ratio of a $C=$odd K_SK_L configuration [Eq. (14)] into $\pi^+\pi^-+\pi^0\pi^0$ is

$$B[K_SK_L(C=\text{odd})\to\pi^+\pi^-+\pi^0\pi^0]=\frac{R(+-,00)}{\sum\limits_{\substack{f_1,f_2\\ \text{only once}}}R(f_1,f_2)}\ . \tag{B1}$$

Here we use a shorthand notation for $\pi^+\pi^-(+-)$ and for $\pi^0\pi^0(00)$. $R(f_1,f_2)$ denotes the total rate into the normalized (f_1,f_2) final state, which is proportional to $\Gamma[\infty,-\infty]$ [Eq. (A3)]. The summation therefore extends over every pair chosen only once. The summation in Eq. (B1) is simplified, when it is realized that $B(K_S\to+-)+B(K_S\to00)\approx100\%$ and $B(K_L\to3\pi)+B(K_L\to\pi^\pm l^\mp\nu)\approx100\%$. To obtain a good estimate we need only to sum the set

$$\{+-,00,3\pi,\pi l\nu\}\ . \tag{B2}$$

Indeed we obtain, utilizing Eq. (A3),

$$R(+-,00)\propto|\langle+-|K_S\rangle|^2|\langle00|K_S\rangle|^2\times(|\eta_{+-}-\eta_{00}|^2a+|\eta_{+-}+\eta_{00}|^2b) \tag{B3a}$$

and

$$\sum_{\substack{f_1,f_2\\ \text{only once}}}R(f_1,f_2)\propto|\langle\text{all}|K_S\rangle|^2|\langle\text{all}|K_L\rangle|^2(a+b)\ . \tag{B3b}$$

Equation (B3b) requires some explanation. We will show what approximation is involved. Take a typical term when we sum over the set [Eq. (B2)] $f_1=+-$, $f_2=3\pi$; then

$$R(+-,3\pi)\propto |\langle +-|K_S\rangle|^2|\langle 3\pi|K_S\rangle|^2 \times(|\eta_{+-}-\eta_{3\pi}|^2a+|\eta_{+-}+\eta_{3\pi}|^2b)\ . \tag{B4}$$

Now it is an experimental fact[22,26] that

$$|\eta_{3\pi}|,\ |\eta_{\pi l\nu}| \gg |\eta_{2\pi}|\ .$$

We caution the reader that we adhere to our definition [Eq. (25)] $\eta_{\pi l\nu}\equiv\langle \pi l\nu|K_L\rangle/\langle \pi l\nu|K_S\rangle$ and $\eta_{3\pi}\equiv\langle 3\pi|K_L\rangle/\langle 3\pi|K_S\rangle$ the last being the inverse of the usual convention.[22] We get $R(+-,3\pi)\propto|\langle +-|K_S\rangle|^2|\langle 3\pi|K_L\rangle|^2(a+b)$. Furthermore for any $f\in\{+-,00,3\pi,\pi l\nu\}$ it is easy to convince oneself that $R(f,f)\ll R(+-,3\pi)$ and so we obtain [Eq. (B3b)]. Thus

$$B[K_SK_L(C=\text{odd})\to+-,00]=\frac{(|\eta_{+-}-\eta_{00}|^2a+|\eta_{+-}+\eta_{00}|^2b)}{(a+b)}\frac{\gamma_S}{\gamma_L}B(K_S\to+-)B(K_S\to 00)$$

$$\approx|\eta_{+-}+\eta_{00}|^2\frac{z^2+y^2}{2(1+z^2)}\frac{\gamma_S}{\gamma_L}B(K_S\to+-)B(K_S\to 00)\approx 1.3\times10^{-3}\ . \tag{B5}$$

The first approximation in Eq. (B5) utilizes the excellent approximate equality of the generalized mixing parameters $a\approx b$ [Eq. (A2)], and the assumption that $|\epsilon'|\ll|\epsilon|$; the last approximation utilizes once more $|\epsilon'|\ll|\epsilon|$ and uses $|\epsilon|\approx 2.28\times10^{-3}$. It is obvious from the first approximation in Eq. (B5) how we obtain Eq. (41). We just utilize $b/(a+b)\approx\frac{1}{2}$ and $|\eta_{+-}+\eta_{00}|^2\approx 2(|\eta_{+-}|^2+|\eta_{00}|^2)$ and we see that Eq. (41) is indeed a lucky accident. It follows whenever one has highly disparate decay rates $\gamma_L\ll\gamma_S$, in which case both a and b are approximately $2/(\gamma_L\gamma_S)$. In such an instance, interference effects can be neglected, allowing one to derive Eq. (41) in two lines. For general γ_L,γ_S the more complete treatment given here is needed.

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