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Contribution of the $\Delta(1232)$ to $\mu^- p \to n\nu\gamma$

Douglas S. Beder
Department of Physics, University of British Columbia, Vancouver, British Columbia, Canada V6T 2A6

Harold W. Fearing
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

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We examine contributions from the $\Delta(1232)$ to radiative capture (at rest) of a muon by a nucleon. This is motivated by the need for a thorough understanding of all contributing amplitudes in order to determine the pseudoscalar weak current and its coupling g_P from experiments on radiative capture which are now underway or being planned. We find changes as large as 7–8 % in the capture rate at the high-energy end of the photon spectrum and conclude that a precision analysis of the capture process must indeed include Δ effects.

I. INTRODUCTION

Considerable theoretical and experimental effort over a number of years has been directed towards a precise understanding of the so-called "induced pseudoscalar weak interaction nucleon current" and a determination of its coupling strength g_P which is predicted by the Goldberger-Treiman relation. In particular, radiative capture of a muon (at rest) by either a proton or by complex nuclei offers the opportunity to selectively enhance this interaction, and thus g_P, for appropriate kinematics. Assuming dominance of the pseudoscalar interaction by exchange of a virtual pion, one expects such enhancement when the momentum transfer due to the weak interaction is as close as possible to the pole of the pion propagator, i.e., to m_π^2. In radiative muon capture this corresponds to zero neutrino energy and maximum photon energy, where the momentum transfer can reach m_μ^2 and where one obtains almost a factor of 3 enhancement in g_P.

Normally one plans to extract the desired information about g_P by comparing experimental rates for large photon energy (k) with theoretical amplitudes involving nucleon and muon weak currents. However, such precision comparison is reliable only if one knows that no significant amplitudes have been omitted from the calculations. In fact, previous literature[1–3] seems not to have included the simplest amplitudes involving the $\Delta(1232)$ as an intermediate excited state, though Ohta[4] examined some Δ contributions to more complicated two-nucleon currents in nuclei. Thus in the present work we attempt to remedy this omission by including all diagrams in which the dominant nucleon resonance, the $\Delta(1232)$, contributes to the single-nucleon current.

We concentrate on the process $\mu^- p \to n\nu\gamma$, i.e., on capture on a free proton, so in some sense the work is exploratory. To extend the calculation to the experimentally easier capture on a nucleus would be much more difficult because of a number of effects involving interaction of the Δ with the nuclear medium which would have to be included. However the capture on a free proton is not without interest in itself, as there are in fact experiments[5] now in progress or planned which hope to measure the free capture rate.

We find as a result of our calculations that for large photon energies the Δ can enhance the radiative capture rate by of order 7–8 %. In retrospect this should come as no surprise—the $\pi^- p \to \gamma n$ rate is similarly enhanced at threshold by Δ effects, as is $\pi^- p \to \gamma\gamma n$ as well. This implies that such Δ effects must be included in calculations which address future precision experiments.

II. AMPLITUDES

The full amplitude for the radiative capture process $\mu^- p \to n\nu\gamma$ can be written as

$$M_{fi} = M_{fi}^N + M_{fi}^\Delta \ . \tag{1}$$

Here the amplitude M_{fi}^N represents the usual contributions, not involving a Δ, represented by the diagrams of Fig. 1. The explicit expression for this amplitude is given in Eq. (1) of Ref. 1. All notation will be as given in Ref. 1, and in particular the four-momenta will be labeled by

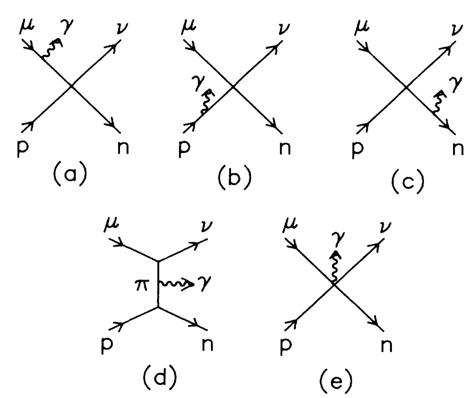

FIG. 1. Standard diagrams contributing to radiative muon capture on a proton. The full weak and electromagnetic interactions as given in the text are used at the vertices.

the particle and satisfy $\mu+p=n+\nu+k$. The standard contributions include radiation from the charge and magnetic moments of the nucleons and from the muon. The weak vertices include vector and weak magnetism couplings obtained via CVC (conservation of vector current), an axial-vector coupling obtained from neutron β decay and the induced pseudoscalar contribution. All form factors except for g_P have been taken as constants, which is a good approximation since the momentum transfer is small. For g_P the form factor is obtained in the usual way from pion pole dominance or PCAC (partial conservation of axial-vector current) and is given explicitly in Ref. 1. It is normalized in such a way that the Goldberger-Treiman prediction for the free nucleon is $g_P(0)=6.6g_A$. Figure 1(d) corresponds to the radiation from this exchanged pion and 1(e) is a contact interaction required to maintain gauge invariance.

The amplitude M_{fi}^{Δ} represents the new contributions which we have included which come from the diagrams involving the Δ of Fig. 2. It is given explicitly by

$$M_{fi}^{\Delta}=-\epsilon_\mu L_\alpha \bar{u}_n \Gamma_{\rm EM}^{\delta\mu}(k)P_{\delta\beta}(n+k)\Gamma_{\rm wk}^{\beta\alpha}(n+k-p,n+k)u_p-\epsilon_\mu L_\alpha \bar{u}_n \Gamma_{\rm wk}^{\beta\alpha}(p-k-n,p-k)P_{\beta\delta}(p-k)\Gamma_{\rm EM}^{\delta\mu}(k)u_p \ . \qquad (2)$$

In this expression $P_{\delta\beta}$ is the Δ propagator in the Rarita-Schwinger formalism given by

$$P_{\delta\beta}(q)=-\left[g_{\delta\beta}-\tfrac{1}{3}\gamma_\delta\gamma_\beta-\frac{2}{3}\frac{q_\delta q_\beta}{m_\Delta{}^2}-\frac{q_\delta\gamma_\beta-q_\beta\gamma_\delta}{3m_\Delta}\right]\frac{q^\alpha\gamma_\alpha+m_\Delta}{q^2-m_\Delta{}^2} \ . \qquad (3)$$

The Δ mass in $q^2-m_\Delta{}^2$ in this equation is actually taken to be complex, i.e., $m_\Delta\rightarrow m_\Delta-i\Gamma/2$ with $m_\Delta=1232$ MeV and $\Gamma=115$ MeV.

The electromagnetic vertex representing the photon-Δ-nucleon coupling is given, up to an overall factor of $\sqrt{4\pi\alpha}$ which has been extracted, by

$$\Gamma_{\rm EM}^{\delta\mu}(k)=g_{\gamma\Delta N}(k^\beta\gamma_\beta g^{\delta\mu}-k^\delta\gamma^\mu)\gamma_5 \ . \qquad (4)$$

Here $g_{\gamma\Delta N}$ is taken as $-2.4/m\simeq 0.36/m_\pi$, where m is the nucleon mass, from the fit to pion photoproduction of Davidson, Mukhopadhyay, and Wittman.[6] Note that we have taken the simplest, but presumedly most important, term in the electromagnetic vertex. There are other more general forms possible, one of which is used in Ref. 6. However by keeping the dominant term only the electromagnetic vertex is consistent via CVC with the weak vertex used below.

The weak-Δ-nucleon vertex is given by

$$\Gamma_{\rm wk}^{\beta\alpha}(q,Q)=g_{V\Delta N}(q^\delta\gamma_\delta g^{\beta\alpha}-q^\beta\gamma^\alpha)\gamma_5 +f_{A\Delta N}(q\cdot Q g^{\beta\alpha}-q^\beta Q^\alpha) +g_{A\Delta N}\left[g^{\beta\alpha}-\frac{q^\beta q^\alpha}{q^2-m_\pi{}^2}\right] \ . \qquad (5)$$

This form has been taken from Llewellyn-Smith.[7] Again it is not the most general form, but it is the one which has been normally used and is based on a survey of fits to neutrino scattering data which have been used to determine the couplings. Such fits indicate that the coefficients of other possible terms in the interactions are consistent with zero.[7] We take $g_{A\Delta N}=-1.2$ and $f_{A\Delta N}=3/m^2$ both from Ref. 7. The second of these is poorly determined, but fortunately our results are not particularly sensitive to it. The vector coupling is given by CVC to be $g_{V\Delta N}=-g_{\gamma\Delta N}$. PCAC has been used explicitly to fix the pion pole part of the $g_{A\Delta N}$ term.

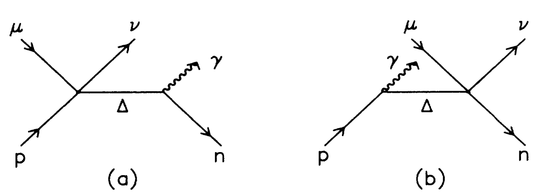

FIG. 2. Contributions from the $\Delta(1232)$ to the single-nucleon interaction. The full weak and electromagnetic interactions as given in the text are used at the vertices.

We have taken considerable care to establish our phase conventions for all of the Feynman-diagram vertices. Hermiticity and charge-conjugation invariance (for the nonweak interactions) were explicitly used. Isospin invariance was exploited to relate vertices for different charge states of the Δ and, as has been noted, CVC and PCAC were used to constrain the weak-interaction coupling. Some care had to be taken in relating nucleon$\rightarrow\Delta$ and $\Delta\rightarrow$nucleon vertices, and some signs originating from this comparison have been absorbed in the above formulas.

Despite this it was necessary to take numerical values of the couplings from the literature, and it is not always clear what conventions were used to obtain the numerical values. Thus an independent check on the phase of the Δ contribution relative to the rest of the amplitude is necessary. The only important Δ term is the $g_{A\Delta N}$ term since the contributions of $g_{V\Delta N}$ and $f_{A\Delta N}$ are small. Thus the only crucial sign is that of the $g_{\gamma\Delta N}g_{A\Delta N}$ term relative to the non-Δ contributions of M_{fi}^{N}. Since the Δ contribution to the rate arises through interference terms it is linear in the product of couplings. Thus this sign determines the sign of the Δ effect. This sign can be fixed as follows. Note that the pion pole part of the full M_{fi} gives just the pion photoproduction amplitude. Thus by dropping the lepton current and extracting the pole piece one gets an amplitude which can be compared with the threshold pion photoproduction amplitude as calculated with Δ and non-Δ contributions. This was done numerically and the sign used above gives an enhancement near threshold arising from the Δ contributions which is in agreement with the result of the chirally invariant Lagrangian of Peccei.[8] Independently,[9] a nucleon plus Δ Born amplitude calculation using the Rarita-Schwinger spin-$\frac{3}{2}$ propagator was compared directly with existing data for $\pi^- p \to \gamma n$ at low energies.[10,11] The sign used above is strongly preferred when results are compared to the backward angle scattering data and also to the observed Panofsky ratio at threshold. The relative sign of the two Δ diagrams Figs. 2(a) and 2(b) is presumedly given correctly by the Feynman rules.

III. AMPLITUDE CALCULATIONAL TECHNIQUE

All calculations have been done relativistically, with no nonrelativistic approximations. The amplitude M_{fi} above is too complicated to square using the usual trace techniques, even using computer programs to do the algebraic manipulations. Thus we have developed a program to numerically calculate the complex 4×4 matrix operators for specified momenta. This includes subroutines to construct the Δ propagator and the various Δ vertices and to perform contractions on Lorentz indices. Using these we construct numerically each contributing 4×4 amplitude, add all such operator amplitudes, and then evaluate between appropriate initial and final spinors (again, numerically constructed). Finally we take squared moduli for all possible particle spins, with appropriate account of the initial atomic spin state.

For liquid H_2 one expects capture to be dominantly from a p-μ-p molecular state with total spin $\frac{1}{2}$. The calculations were thus performed separately for initial singlet and triplet $\mu^- p$ states and then combined appropriately for the p-μ-p molecular case as well as for the statistical case.

IV. PHASE SPACE AND RATE

The photon spectrum is obtained by combining the square of the amplitude obtained as described above with appropriate phase-space and flux factors. It is given by Eq. (2) of Ref. 1 in which four-momentum conservation has been used to eliminate the neutron momentum and the neutrino energy. Realizing that for an unpolarized initial state the spin-summed square of the matrix element must be independent of, say, the photon angles and the photon-neutrino azimuthal angle, one can do all but the integration on the angle between photon and neutrino to obtain

$$\frac{d\Gamma}{dk}=\frac{\alpha G^2|\Phi_\mu|^2 m_n}{(2\pi)^2} \times \int_{-1}^{1} dy \frac{kE_\nu^2}{W_0-k(1-y)}\left[\frac{1}{4}\sum_{\text{spins}}|M_{fi}|^2\right], \quad (6)$$

where $E_\nu=W_0\ (k_{\max}-k)/[W_0-k(1-y)]$, $y=\hat{\mathbf{k}}\cdot\hat{\mathbf{v}}=\cos\theta_{\gamma\nu}$, $k_{\max}=(W_0^2-m_n^2)/2W_0=99.15$ MeV, and $W_0=m_p+m_\mu-$(binding energy of muon)$=1043.9$ MeV. G is the Fermi constant of neutron β decay and Φ_μ is the muon wave function at the origin. As noted above the statistical spin mixture $\frac{1}{4}\sum|M_{fi}|^2$ was actually replaced by appropriate combinations for singlet and triplet states.

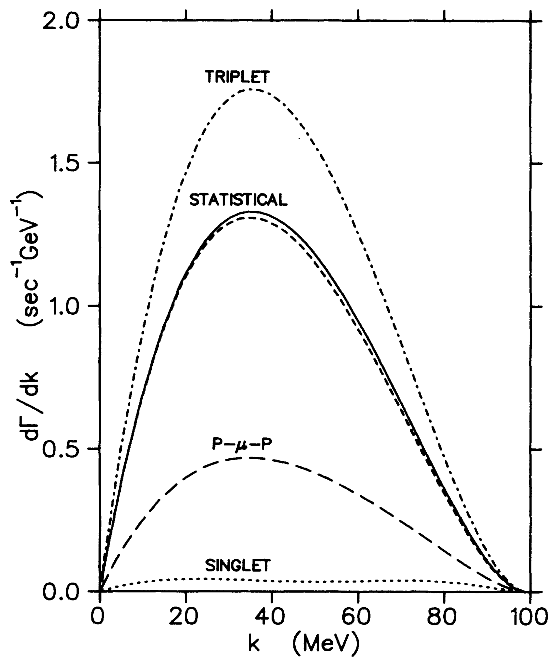

FIG. 3. Photon spectrum for radiative muon capture on a proton including contributions from the Δ(1232) intermediate state. The dotted, long-dashed, solid, and dot-dashed curves correspond, respectively, to the singlet, ortho p-μ-p molecule, statistical, and triplet-spin combinations. The short-dashed curve is the statistical combination without the Δ terms.

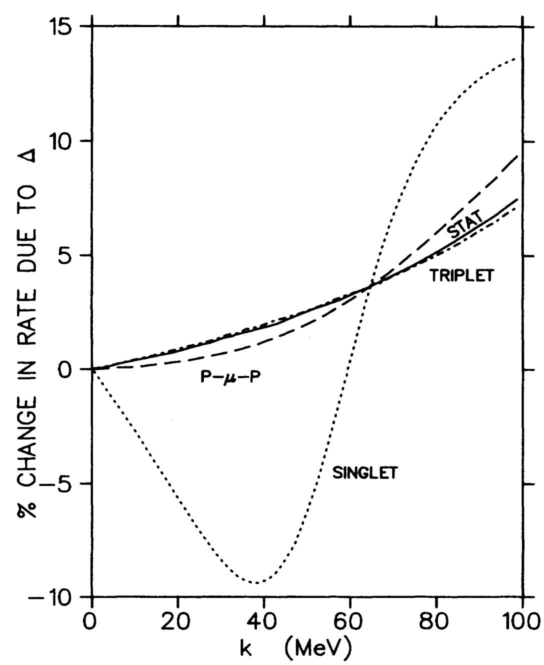

FIG. 4. The percentage change in the photon spectrum produced by the inclusion of the Δ(1232) contributions of Fig. 2. The dotted, dashed, solid, and dot-dashed curves correspond to the singlet, ortho p-μ-p molecule, statistical, and triplet spin combinations.

V. RESULTS

We have calculated the photon spectrum both with and without the Δ contributions considered here. Figure 3 shows the spectrum with the Δ included. The curves correspond to (from the bottom) singlet, ortho p-μ-p molecule, statistical, and triplet-spin states. The short dashed curve is the statistical case with no Δ. Clearly on this scale the Δ effects are small. A more informative way of looking at the results is given in Fig. 4, which shows the percentage change in the spectrum when the Δ contributions are added. The effect is largest in the singlet case which is unfortunately probably unmeasurable. For the practical cases the effect is of the order of 7–8 % at the experimentally accessible upper end of the spectrum.

This result is dominated by the $g_{A\Delta N}$ part of the weak-Δ-nucleon current. The poorly known coupling $f_{A\Delta N}$ is responsible for less than 10% of the full Δ effect. The vector part of the weak-Δ-nucleon current $g_{V\Delta N}$ is negligible as is the contribution from the Δ width.

VI. DISCUSSION

Thus we have seen that effects due to the Δ of the type considered here enhance the rate over the experimentally accessible upper part of the photon spectrum by the order of 7–8 %. This is clearly not a large effect but is probably one which should be considered for precision experiments. If one were to fit "data" containing the Δ and using the canonical values of the couplings with a theory which does not contain the Δ, the value of g_P extracted would be too high by the order of 10%, i.e., by $0.5-0.7g_A$. This is another way of stating the uncertainty introduced by these Δ contributions and the errors in the extracted value of g_P which results from leaving them out.

As noted above this calculation is exploratory in the sense we consider only capture on the free proton. The intent was to get an idea of the size of the effect. Clearly it is of importance for the free proton only for precision experiments. Capture on a nucleus is much easier experimentally since the rate is much higher so it is interesting to ask what the Δ effects would be in that case. However uncertainties in the nuclear physics make precision interpretations there unlikely, and so if the Δ effects in a nucleus were to be of the same order as for the free proton, it is problematical that they would ever be seen. However in a nucleus other physical effects enter. The Δ is a strongly interacting particle and so effects due to propagation in the nuclear medium may be important. In at least one calculation[12] the authors find large effects due to modification of the muon propagator, though this has been challenged by others,[13] and have looked at effects of the nucleon propagating in the medium. Effects involving the Δ originating in a two-nucleon current such as those considered by Ohta[4] might also contribute. Hence the magnitude and character of effects due to the Δ might be quite different in a nucleus than we have found for a free proton. In any case one must do a different type of calculation, one beyond the scope of this paper, but one which could be quite interesting.

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