Creation of relativistic fermionium in collisions of electrons with atoms

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The cross section for creation of bound states of a fermion and an antifermion, fermionium, in high-energy collisions of electrons with atoms is calculated. Applications to positronium and to bound states of a muon and an antimuon, μ fermionium, are discussed.

I. INTRODUCTION

This is the second paper on the creation of relativistic beams of bound fermion-antifermion states. In the previous paper,[1] the creation of positronium by a high-energy photon in the field of an atom was calculated. In this paper we calculate the cross section for the creation of bound fermion-antifermion states in high-energy electron collisions with atoms

$$e+Z \rightarrow e+Z+(f\bar{f}) \; .$$

Previously estimates for positronium production have been made by Meledin, Serbo, and Slivkov[2] and data for positronium production by high-energy electrons on a tungsten target have been reported by Akhundov, Bardin, and Nemenov.[3]

The present paper gives formulas for angular and energy distributions of produced bound fermion-antifermion states $(f\bar{f})$, taking into account the screening of the target atom. It is reasonable to believe that the availability of relativistic positronium beams may be of interest for current experimental studies of positronium as an alternative to the usual experimental studies. Similarly, the possibility of producing bound states of μ fermionium $(\mu\bar{\mu})$ may be of interest.

We calculate separately the incoherent triplet and singlet fermion production cross sections (Fig. 1). Of particular interest for experimental application may be the production of the long-lived triplet positronium.

II. PARA (SINGLET) FERMIONIUM

The singlet bound-state fermion-antifermion is produced by the electron virtual photon in the field of the atom, and the process is thus closely related to the production by a real photon calculated previously.[1]

The cross section is obtained most easily by multiplying the real-photon cross section[1] $d\sigma(\gamma Z \rightarrow (f\bar{f})Z)$ by the number of virtual photons[4] $N(\omega)d\omega/\omega$, where $\omega = E_{f\bar{f}}$, the fermionium energy, and where we neglect the contribution from longitudinal photons which is a good approximation for high energies. The cross section for creation of fermionium in the nth energy s level is thus given by

$$\begin{aligned} d^2\sigma_n(E_{f\bar{f}},\theta) &= d\sigma(\gamma Z \rightarrow (f\bar{f})Z)\frac{dE_{f\bar{f}}}{E_{f\bar{f}}}N(E_{f\bar{f}}) \\ &= 4\pi\frac{Z^2\alpha^6}{n^3}\frac{m_{f\bar{f}}{}^2}{E_{f\bar{f}}{}^4}\frac{[1-F(\mathbf{q})]^2\theta^3 d\theta}{(m_{f\bar{f}}{}^4/4E_{f\bar{f}}{}^4+\theta^2)^2(m_{f\bar{f}}{}^2/E_{f\bar{f}}{}^2+\theta^2)^2} \\ &\quad\times\frac{dE_{f\bar{f}}}{E_{f\bar{f}}}\frac{\alpha}{2\pi}\left\{\left[1+\left[\frac{E_1-E_{f\bar{f}}}{E_1}\right]^2\right]\ln\frac{E_1(E_1-E_{f\bar{f}})m_{f\bar{f}}{}^2}{E_{f\bar{f}}{}^2 m_e{}^2}-2\frac{E_1-E_{f\bar{f}}}{E_1}\right\}, \end{aligned} \tag{1}$$

where E_1 is the initial electron energy, $\mathbf{q}$ the momentum transfer to the atom, $F(\mathbf{q})$ the atomic form factor, $m_{f\bar{f}}$ the fermionium mass, θ the fermionium emission angle, and n the fermionium energy quantum number. From Eq. (1) follows that the angular distribution is the same as for creation by real-photons, which is a result of the smallness of the virtual-photon emission angles, of the order m_e/E_1, compared to the fermionium emission angles, of order $m_{f\bar{f}}/E_{f\bar{f}}$, which are much larger since the virtual-photon energy is considerably smaller than E_1 and also $m_{f\bar{f}} > m_f$ always. We refer to Ref. 1 for a discussion of the angular distribution.

The cross section summed over the fermionium energy states and integrated over angles with a simplified Thomas-Fermi-Molière form factor,

$$[1-F(\mathbf{q})]/\mathbf{q}^2 = 1/(\mathbf{q}^2+\Lambda^2), \quad \Lambda = m_e Z^{1/3}/111 \; , \tag{2}$$

is obtained in a similar way as in Ref. 1:

$$d\sigma_{\text{sing}}(E_{f\bar{f}}) = -4\frac{Z^2\alpha^7}{m_{f\bar{f}}{}^2}\zeta(3)\left\{\tfrac{1}{2}\ln\left[\left[\frac{m_{f\bar{f}}}{E_{f\bar{f}}+p_{f\bar{f}}}\right]^2+\bar{\Lambda}^2\right]+\frac{p_{f\bar{f}}}{E_{f\bar{f}}}\right\}\left[2\ln\frac{E_1 m_{f\bar{f}}}{E_{f\bar{f}}m_e}-1\right]\frac{dE_{f\bar{f}}}{E_{f\bar{f}}} \; , \tag{3}$$

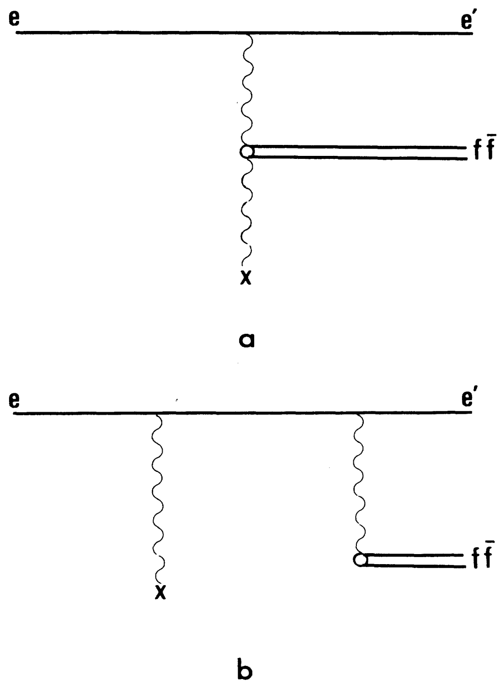

FIG. 1. Feynman diagrams for (a) singlet and (b) triplet production of a bound $f\bar{f}$ state in a collision of an electron with an atom.

where m_e is the mass of the incoming electron, $\zeta(3)=\sum_1^\infty n^{-3}=1.202$ and

$$\bar{\Lambda}=Z^{1/3}m_e/(111m_{f\bar{f}}) .$$

The total cross section for production of singlet state fermionium is obtained by integrating Eq. (3). We find the following approximate simple analytic formulas:

$$\sigma_{\text{sing}}=\int_{m_{f\bar{f}}}^{E_1} dE_{f\bar{f}}\, d\sigma_{\text{sing}}(E_{f\bar{f}})/dE_{f\bar{f}}$$

$$=1.20Z^2\alpha^5 r_f{}^2 I_{\text{sing}} \tag{4}$$

with $r_f=\alpha/m_f$,

$$I_{\text{sing}}=\tfrac{1}{3}L^3-0.6L^2+0.1L \quad \text{for } 1 \ll E_1/m_{f\bar{f}} \lesssim \bar{\Lambda}^{-1} \tag{4a}$$

and

$$I_{\text{sing}}=L(L-1)(L_s+\ln 2-1)-(L-\tfrac{1}{2})L_s{}^2+\tfrac{1}{3}L_s{}^3 \quad \text{for } E_1/m_{f\bar{f}} \gtrsim \bar{\Lambda}^{-1} , \tag{4b}$$

where L and L_s are energy- and screening-dependent factors, respectively:

$$L=\ln(E_1/m_{f\bar{f}}), \quad L_s=-\ln\bar{\Lambda} .$$

The errors in the analytic formulas (4a) and (4b) are of the order of a few percent.

The energy of the produced singlet state fermionium is in most cases considerably smaller than the initial electron energy E_1 especially at very high energies. For example, one finds from Eq. (3) for $E_1/m_{f\bar{f}}=10$ that about 70% of the fermionium particles have energies smaller than $\frac{1}{4}E_1$, and for $E_1/m_{f\bar{f}}=10^3$ about 85% of the particles have energies less than $\frac{1}{10}E_1$.

III. ORTHO (TRIPLET) FERMIONIUM

The cross section for production of triplet fermionium is closely related to the four-body final-state pair-production process

$$e+Z \rightarrow e+\gamma^*+Z \rightarrow e+f+\bar{f}+Z$$

with the cross section $d^8\sigma(\mathbf{p}_2,\mathbf{p}_f,\mathbf{p}_{\bar{f}})$ where $\mathbf{p}_2,\mathbf{p}_f$ and $\mathbf{p}_{\bar{f}}$ are the final-state momenta of the scattered electron and the created fermion and antifermion, respectively. The pair is created by the single virtual photon so if the pair is produced in a bound state it is a spin-1, triplet state. The cross section for creation of bound-state triplet fermionium $d^5\sigma(\mathbf{p}_2,\mathbf{p}_{f\bar{f}})$ is obtained by multiplying the four-particle final-state cross section $d^8\sigma(\mathbf{p}_2,\mathbf{p}_f,\mathbf{p}_{\bar{f}})$ for equal fermion and antifermion momenta $\mathbf{p}_f=\mathbf{p}_{\bar{f}}$ by the momentum-space factor

$$(2\pi)^3\frac{m_{f\bar{f}}}{E_{f\bar{f}}}\left[\frac{E_f}{m_f}\right]^2\frac{d^3p_{f\bar{f}}}{d^3p_f d^3p_{\bar{f}}}N_{f\bar{f}}{}^{-2} , \tag{5}$$

where $N_{f\bar{f}}{}^{-1}$ is the fermionium normalization constant for the nth s state

$$N_{f\bar{f}}{}^{-2}=\alpha^3 m_f{}^3/(8\pi n^3) .$$

We find after a simple calculation for high energies and small angles the triplet-fermionium creation cross section

$$d^5\sigma_{\text{trip}}(\mathbf{p}_2,\mathbf{p}_{f\bar{f}})=-2\frac{Z^2\alpha^7}{(2\pi)^2}\frac{\zeta(3)}{E_1E_2E_{f\bar{f}}}\frac{[1-F(\mathbf{q})]^2}{\mathbf{q}^4}$$
$$\times\left[\frac{\mathbf{q}^2}{D_1D_2}(E_1{}^2+E_2{}^2)-\frac{1}{4}\left[\frac{D_1}{D_2}+\frac{D_2}{D_1}\right]+(2m_e{}^2+m_{f\bar{f}}{}^2)\left[\frac{E_1}{D_2}+\frac{E_2}{D_1}\right]^2\right]d^3p_{f\bar{f}}d^3p_2/dE_2 , \tag{6}$$

where $E_2=E_1-E_{f\bar{f}}$ is the energy of the scattered electron and

$$D_1=m_{f\bar{f}}{}^2-2p_1\cdot p_{f\bar{f}} ,$$
$$D_2=m_{f\bar{f}}{}^2+2p_2\cdot p_{f\bar{f}} . \tag{7}$$

The term $(D_1/D_2+D_2/D_1)$ in Eq. (6) should be noted. It is small compared to the other terms in the large square brackets for $|\mathbf{q}|\sim m_f$. However, for $|\mathbf{q}|\sim q_{\min}$ the three terms are of the same order of magnitude and since the contribution to the cross section from $|\mathbf{q}|\sim q_{\min}$ is of the same order of magnitude as for $|\mathbf{q}| \sim m_f$, the term $(D_1/D_2+D_2/D_1)$ is important and must be kept.

It is convenient as usual in high-energy processes at small angles to introduce the components $\mathbf{u}$ and $\mathbf{v}$ of $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively, perpendicular to the momentum of the other high-energy final-state particle, the fermionium, so that $u=p_1\theta_1$ and $v=p_2\theta_2$. We further introduce the quantities ξ and η by

$$D_1=-2q_{\min}E_2/\xi \ , \qquad D_2=2q_{\min}E_1/\eta \ , \tag{8}$$

with

$$\xi^{-1}=1+\left[\frac{E_{f\bar{f}}}{2E_1E_2}\right]\frac{u^2}{q_{\min}} \ , \qquad \eta^{-1}=1+\left[\frac{E_{f\bar{f}}}{2E_1E_2}\right]\frac{v^2}{q_{\min}} \ , \tag{9}$$

where $q_{\min}$ is the minimum momentum transfer to the atom

$$q_{\min}=(E_1E_2m_{f\bar{f}}^2+E_{f\bar{f}}^2m_e^2)/(2E_1E_2E_{f\bar{f}}) \ . \tag{10}$$

The differential cross section becomes

$$d^4\sigma_{\text{trip}}(\mathbf{p}_2,\mathbf{p}_{f\bar{f}})=\frac{Z^2\alpha^7}{4\pi}\zeta(3)\frac{E_{f\bar{f}}}{E_1^3E_2q_{\min}^2}[1-F(\mathbf{q})]^2 \times\left[\frac{E_1^2+E_2^2}{E_1E_2}\frac{\xi\eta}{\mathbf{q}^2}-\frac{q_{\min}^2}{\mathbf{q}^4}\left[\frac{\xi E_1}{\eta E_2}+\frac{\eta E_2}{\xi E_1}\right]-(2m_e^2+m_{f\bar{f}}^2)\frac{(\xi-\eta)^2}{\mathbf{q}^4}\right]dE_{f\bar{f}}u\,du\,v\,dv\,d\varphi \ . \tag{11}$$

The angular distributions are dominated by the factors ξ, η, and $\mathbf{q}^{-4}$ and the emission angles are consequently of the order

$$\theta_1\sim\theta_2\sim m_{f\bar{f}}/E_{f\bar{f}}$$

as long as the fermionium energy $E_{f\bar{f}}$ is of the same order of magnitude as the incoming electron energy.

Of particular interest is the spectrum of the emitted fermionium. Integration over angles gives as shown in Appendix A for no screening, i.e., for lower energies

$$d\sigma_{\text{trip}}(E_{f\bar{f}})=\frac{Z^2\alpha^7}{8}\zeta(3)\frac{E_{f\bar{f}}}{E_1^3E_2q_{\min}^2} \times\left[\frac{2(E_1^2+E_2^2)q_{\min}}{E_{f\bar{f}}}-\frac{2m_e^2+m_{f\bar{f}}^2}{3}\right]\left[\ln\left[\frac{2E_1E_2}{E_{f\bar{f}}q_{\min}}\right]-1\right]dE_{f\bar{f}}, \quad 1\ll E_{f\bar{f}}/m_{f\bar{f}}\ll\bar{\Lambda}^{-1} \ , \tag{12}$$

while for complete screening, i.e., for higher energies the corresponding formula is

$$d\sigma_{\text{trip}}(E_{f\bar{f}})=\frac{Z^2\alpha^7}{8}\zeta(3)\frac{E_{f\bar{f}}}{E_1^3E_2q_{\min}^2}\left\{\left[\frac{2(E_1^2+E_2^2)q_{\min}}{E_{f\bar{f}}}-\frac{2m_e^2+m_{f\bar{f}}^2}{3}\right]\left[\ln\left[\frac{2E_1E_2q_{\min}}{E_{f\bar{f}}\Lambda^2}\right]+1\right] +\tfrac{1}{9}(2m_e^2+m_{f\bar{f}}^2)\right\}dE_{f\bar{f}}, \quad E_{f\bar{f}}/m_{f\bar{f}}\gg\bar{\Lambda}^{-1} \ , \tag{13}$$

where the screening effect is as above described by the simplified Moliere screening, Eq. (2). The more complete expression for the cross section for arbitrary screening is given in Appendix A.

The close similarity of Eq. (6) to the small-angle, high-energy Bethe-Heitler bremsstrahlung cross section should be noted. In fact, when the factor $\alpha^4\zeta(3)/4$ which is due to the photon propagator and the produced fermionium, is left out and with $m_{f\bar{f}}=m_\gamma=0$ and $p_{f\bar{f}}=k$, we obtain the Bethe-Heitler cross section[5]

$$d^5\sigma_{\text{brems}}(\mathbf{p}_1,\mathbf{k})=-\frac{1}{2}\frac{Z^2\alpha^3}{(2\pi)^2}\frac{1}{E_1E_2\omega}\frac{[1-F(\mathbf{q})]^2}{\mathbf{q}^4} \times\left[\frac{\mathbf{q}^2}{D_1D_2}(E_1^2+E_2^2)-\frac{1}{4}\left[\frac{D_1}{D_2}+\frac{D_2}{D_1}\right]+2m_e^2\left[\frac{E_1}{D_2}+\frac{E_2}{D_1}\right]^2\right]d^3k\,d^3p_2/dE_2 \ . \tag{14}$$

Similarly, from Eqs. (12) and (13) we regain directly the Bethe-Heitler spectrum[6] by introducing $m_{f\bar{f}}=m_\gamma=0$, $q_{\min}=\omega m_e^2/(2E_1E_2)$ and deleting the factor $\alpha^4\zeta(3)/4$. These observations are useful checks on the calculations:

$$d\sigma_{\text{brems}}(\omega)=2Z^2r_e^2\alpha\frac{d\omega}{\omega}\frac{E_2}{E_1}\left[\frac{E_1{}^2+E_2{}^2}{E_1E_2}-\frac{2}{3}\right]\left[2\ln\left[\frac{2E_1E_2}{m_e\omega}\right]-1\right],\quad 1\ll\frac{E_1E_2}{m_e\omega}\ll 111Z^{-1/3}\,, \tag{15}$$

$$d\sigma_{\text{brems}}(\omega)=2Z^2r_e^2\alpha\frac{d\omega}{\omega}\frac{E_2}{E_1}\left[\left[\frac{E_1{}^2+E_2{}^2}{E_1E_2}-\frac{2}{3}\right]2\ln(183Z^{-1/3})+\frac{2}{9}\right],\quad \frac{E_1E_2}{m_e\omega}\gg 111Z^{-1/3}\,. \tag{16}$$

IV. POSITRONIUM CREATION

The positronium mass is $m_{f\bar{f}}=m_p=2m_e$ when we neglect the small binding energy, 6.7 eV. The minimum momentum transfer is with the positronium energy $E_{f\bar{f}}=E_p$ and $x=E_p/E_1$ of the order $m_e(m_e/E_1)$:

$$q_{\min}=(2-x)^2m_e{}^2/[2x(1-x)E_1]\,. \tag{17}$$

For para (singlet) positronium the lifetime is $\tau_{\text{sing}}=1.25\times10^{-10}$ sec giving a decay length for relativistic energies $l_{\text{sing}}=3.75\ (E_p/m_p)$ cm. The corresponding positronium spectrum is from Eq. (3), with $x=E_p/E_1$, $\beta_p=P_p/E_p$, and $r_e=\alpha/m_e$,

$$d\sigma_{p,\text{sing}}(x)=1.20Z^2\alpha^5r_e{}^2\left\{\tfrac{1}{2}\ln\left[\left[\frac{2m_e}{E_1x(1+\beta_p)}\right]^2+\left[\frac{Z^{1/3}}{222}\right]^2\right]+\beta_p\right\}(2\ln x+1)dx/x\,. \tag{18}$$

This cross section is shown in Fig. 2 for specific elements and energies.

The total cross section is from Eq. (4):

$$\sigma_{p,\text{sing}}=1.20Z^2\alpha^5r_e{}^2I_{p,\text{sing}}\,, \tag{19}$$

where $I_{p,\text{sing}}$ is given by Eqs. (4a) or (4b) for $m_{f\bar{f}}=2m_e$. Typical cross sections are for 10 MeV, $\sigma=1.9\times10^{-33}$ cm^2 for Cu and $\sigma=1.9\times10^{-32}$ cm^2 for U and for 1 GeV, $\sigma=1.2\times10^{-31}$ cm^2 for Cu $\sigma=1.1\times10^{-30}$ cm^2 for U.

As pointed out in Sec. II the energy of the produced singlet fermionium is usually considerably smaller than the initial electron energy. Thus, even for high electron energies the decay length of the singlet positronium would not be very large. Typically, as discussed at the end of Sec. II, for 10-MeV initial electron energy, 70% of the positronium particles would have a decay length smaller than 9.4 cm and for 1 GeV, 85% of the positronium particles would have a decay length less than 3.8 m.

For ortho (triplet) positronium the lifetime is considerably longer, $\tau_{\text{trip}}=1.39\times10^{-7}$ sec, which gives a decay length for relativistic positronium $l_{\text{trip}}=42(E_p/m_p)$ m. This fact combined with the large triplet positronium energy $E_p\sim E_1$ may make this process promising for the production of high-energy positronium beams.

The cross sections given by Eqs. (12) and (13) become, when $q_{\min}$ Eq. (17) is introduced,

$$d\sigma_{p,\text{trip}}=0.30Z^2\alpha^5r_e{}^2\left[1+\left[\frac{x}{2-x}\right]^4\right]\times\left[2\ln\left[\frac{2(1-x)E_1}{(2-x)m_e}\right]-1\right]x\,dx \tag{20a}$$

for no screening ($E_p/m_e\ll444/Z^{1/3}$) and

$$d\sigma_{p,\text{trip}}=0.30Z^2\alpha^5r_e{}^2\left\{\left[1+\left[\frac{x}{2-x}\right]^4\right]\left[2\ln\left[\frac{(2-x)111}{xZ^{1/3}}\right]+1\right]+\frac{4x^2(1-x)}{3(2-x)^4}\right\}x\,dx \tag{20b}$$

for complete screening ($E_p/m_e\gg444/Z^{1/3}$). The cross section for arbitrary screening in given in Appendix A. These cross sections are shown in Fig. 3 for specific elements and energies.

The total cross section is obtained as

$$\sigma_{\text{p,trip}}=1.20z^2\alpha^5r_e{}^2I_{p,\text{trip}}$$

with

$$I_{p,\text{trip}}=0.303\ln(E_1/m_e)-0.542 \tag{21a}$$

for no screening, and

$$I_{p,\text{trip}}=0.303\ln(111Z^{-1/3})+0.362 \tag{21b}$$

for complete screening.

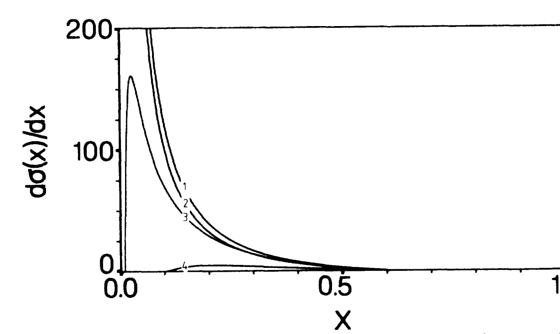

FIG. 2. Singlet-positronium spectra in units of $1.20Z^2\alpha^5r_e{}^2$. Curves 1, 3, and 4 refer to an Al target for electron energies E_1 1 GeV, 100 MeV, and 10 MeV, respectively. Curve 2 is for a U target for 1 GeV.

V. μ-FERMIONIUM CREATION

The μ-fermionium mass $m_{\mu\bar{\mu}}=2m_\mu$. We only discuss the case of no screening here. Screening will only be important for energies as high as

$$E_1 \sim 111Z^{-1/3}m_{\mu\bar{\mu}}^2/m_e \sim 1 \text{ TeV} .$$

The minimum momentum transfer is for most of the spectrum

$$q_{\min}=m_{\mu\bar{\mu}}^2/2E_{\mu\bar{\mu}}=2m_\mu^2(xE_1)^{-1} .$$

However, close to the upper end of the spectrum the more accurate formula is needed:

$$q_{\min}=m_{\mu\bar{\mu}}^2[1-x+x^2(m_e/m_{\mu\bar{\mu}})^2][2x(1-x)E_1]^{-1}$$
$$\approx m_{\mu\bar{\mu}}^2[1-x+(m_e/m_{\mu\bar{\mu}})^2][2x(1-x)E_1]^{-1} ,$$

where we have used the fact that $m_e/m_{\mu\bar{\mu}}$ is only of importance compared to $1-x$ when x is very close to one.

The singlet μ-fermionium spectrum is given by Eq. (3) with $m_{f\bar{f}}=2m_\mu$ and no screening

$$d\sigma_{\mu,\text{sing}}(x)=-1.20Z^2\alpha^5 r_\mu^2[\ln(E_1/2m_\mu)+\ln x(1+\beta_{\mu\bar{\mu}})-\beta_{\mu\bar{\mu}}](2\ln x+1)dx/x,\quad E_{\mu\bar{\mu}} \ll 2.2Z^{-1/3}\text{ TeV} . \tag{22}$$

As discussed in Sec. II the energies xE_1 are in general much lower than the initial electron energy. The total cross section is given by Eqs. (4) and (4a), with $m_{f\bar{f}}=2m_\mu$.

The triplet μ-fermionium spectrum is from Eq. (12):

$$d\sigma_{\mu,\text{trip}}=0.30Z^2\alpha^5 r_\mu^2\frac{x(1-x)dx}{[1-x+(m_e/m_{\mu\bar{\mu}})^2]^2}(1-x+\tfrac{1}{3}x^2)\{2\ln(E_1/m_\mu)+2\ln(1-x)-\ln[1-x+(m_e/m_{\mu\bar{\mu}})^2]-1\} \tag{23}$$

for no screening $E_1 \ll 2.2Z^{-1/3}$ TeV. The cross section is shown in Fig. 4 for various energies. The total cross section is given by

$$\sigma_{\mu,\text{trip}}=1.20Z^2\alpha^5 r_\mu^2 I_{\mu,\text{trip}}$$

with

$$I_{\mu,\text{trip}}=1.79\ln(E_1/m_\mu)-6.12 . \tag{24}$$

For μ-fermionium the singlet and triplet lifetimes are of the same order of magnitude. The singlet 2γ decay lifetime is the positronium lifetime scaled by the factor m_e/m_μ, $\tau_{\mu,\text{sing}}=0.62\times10^{-12}$ sec. The triplet lifetime of μ-fermionium is on the other hand determined by the decay $(\mu\bar{\mu})\rightarrow e^+e^-$ and obtained as $\tau_{\mu,\text{trip}}=6/(m_\mu\alpha^5)=1.8\times10^{-12}$ sec. The corresponding decay lengths are

$$l_{\mu,\text{sing}}-0.018(E_{\mu\bar{\mu}}/m_{\mu\bar{\mu}})\text{ cm},\quad l_{\mu,\text{trip}}=3l_{\mu,\text{sing}} .$$

VI. DISCUSSION

In calculating the number of fermionium particles produced, it is important to take into account the breakup of the created fermionium. In fact the effective thickness for not breaking up in the target is

$$t_b=1/(N\sigma_b) ,$$

where σ_b is the total cross section for breaking up the fermionium

$$(f\bar{f})+Z\rightarrow f+\bar{f}+Z$$

and N is the number of atoms per cm^3. The number of fermionium particles produced in the target per sec, with momentum $p_{f\bar{f}}$ is then

$$dN_{f\bar{f}}/dt=(dN_e/dt)(d^3\sigma_{f\bar{f}}/d^3p_{f\bar{f}})Nt_b$$
$$=(dN_e/dt)(d^3\sigma_{f\bar{f}}/d^3p_{f\bar{f}})/\sigma_b , \tag{25}$$

where dN_e/dt is the number of incident electrons on the target per sec.

When we use Mrowczynski's values[7] for σ_b we find the number of positronium and μ-fermionium particles produced in the triplet state, for electron energies $E_1 \gtrsim 500$ MeV,

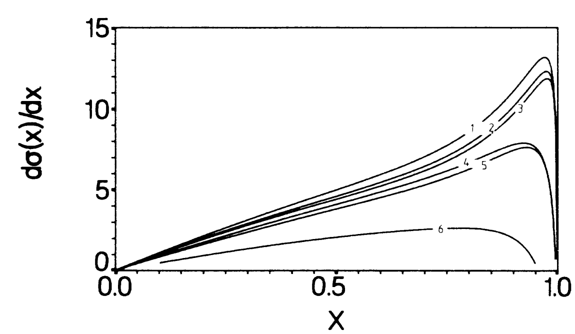

FIG. 3. Triplet positronium spectra in units of $0.30Z^2\alpha^5 r_e^2$. Curves 1, 2 , and 3 refer to an electron energy $E_1=1$ GeV for Al, Sn, and U targets, respectively. Curves 4 and 5 are for $E_1=100$ MeV for Al and U targets and curve 6 is for $E_1=10$ MeV.

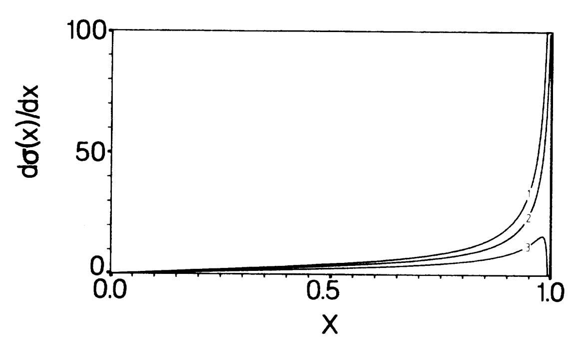

FIG. 4. Triplet μ-fermionium spectra in units of $0.30Z^2\alpha^5 r_\mu^2$. Curves 1, 2, and 3 refer to electron energies $E_1=10$, 5, and 2 GeV, respectively.

$$dN_{p,\text{trip}}/dt=(dN_e/dt)_{\text{mA}}0.35Z^{2/3}(0.30\ln 111Z^{-1/3}+0.36)\ , \tag{26}$$

where we have approximated Mrowczynski's values for σ_b for $(e\bar{e})$ by

$$\sigma_b \approx 3.5Z^{4/3}10^{-20}\ \text{cm}^2\ .$$

In (26), $dN_{p,\text{trip}}/dt$ is in particles per sec, and $(dN_e/dt)_{\text{mA}}$ is in mA. As an example one finds for a 1-mA current with $E_1>100$ MeV approximately nine positronium triplet-state particles produced per sec for lead target. These positronium particles are virtually stable in the laboratory system.

The number of μ-fermionium particles produced per second is

$$dN_{\mu,\text{trip}}/dt=(dN_e/dt)_{\text{mA}}0.021[1.79\ln(E_1/m_\mu)-6.12]\ , \tag{27}$$

where we have approximated Mrowczynski's values for σ_b for $(\mu\bar{\mu})$ by

$$\sigma_b \approx 1.3Z^2 10^{-23}\ \text{cm}^2\ .$$

A 1-mA current with energy 10 GeV produces approximately three triplet-μ-fermionium particles per min. The decay length is however only 2.7 cm.

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APPENDIX A

The spectrum for emission of triplet fermionium is obtained by integration of Eq. (11) over angles:

$$d\sigma_{\text{trip}}(E_{f\bar{f}})=\frac{1}{4\pi}Z^2\alpha^7\zeta(3)\frac{E_{f\bar{f}}}{E_1{}^3E_2q_{\min}{}^2}\left[\frac{E_1{}^2+E_2{}^2}{E_1E_2}I_1-2(m_e{}^2+m_{f\bar{f}}{}^2)I_2\right]dE_{f\bar{f}}\ , \tag{A1}$$

where

$$I_1=\int\frac{u\,du\,v\,dv\,d\varphi}{(\mathbf{q}^2+\Lambda^2)^2}(\xi\eta\mathbf{q}^2-q_{\min}{}^2)$$

and

$$I_2=\int\frac{u\,du\,v\,dv\,d\varphi}{(\mathbf{q}^2+\Lambda^2)^2}(\xi-\eta)^2\ .$$

One finds

$$I_1=\pi\frac{E_1E_2q_{\min}}{E_{f\bar{f}}}\left[\ln\frac{2E_1E_2q_{\min}}{E_{f\bar{f}}(q_{\min}{}^2+\Lambda^2)}+1-2\frac{q_{\min}}{\Lambda}\arctan\frac{\Lambda}{q_{\min}}\right] \tag{A2}$$

and

$$I_2=\frac{\pi}{6}\left[\ln\frac{2E_1E_2q_{\min}}{E_{f\bar{f}}(q_{\min}{}^2+\Lambda^2)}+\frac{2}{3}-3\frac{q_{\min}{}^2}{\Lambda^2}\ln\frac{q_{\min}{}^2+\Lambda^2}{q_{\min}{}^2}+4\frac{q_{\min}{}^2}{\Lambda^2}\left[1-\frac{q_{\min}}{\Lambda}\arctan\frac{\Lambda}{q_{\min}}\right]\right]\ . \tag{A3}$$

When (A2) and (A3) are inserted into Eq. (A1) one obtains the spectrum for arbitrary screening, and for the special cases of $q_{\min}{}^2\gg\Lambda^2$ and $q_{\min}{}^2\ll\Lambda^2$, the spectra for no screening, Eq. (12), and complete screening, Eq. (13), in the text.

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[1]Haakon A. Olsen, Phys. Rev. D **33**, 2033 (1986).

[2]G. V. Meledin, V. G. Serbo, and A. K. Slivkov, Pis'ma Zh. Eksp. Teor. Fiz. **13**, 98 (1971) [JETP Lett. **13**, 68 (1971)].

[3]A. A. Akhundov, D. Yu. Bardin, and L. L. Nemenov, Yad. Fiz. **27**, 1542 (1978) [Sov. J. Nucl. Phys. **27**, 812 (1978)]; A. A. Akhundov and D. Yu. Bardin, Report No. JINR R2-9587, 1976 (unpublished). One photon and two photon couplings to $f\bar{f}$ have been given by L. C. Hostler and W. W. Repko, Ann. Phys. (N.Y.) **130**, 329 (1980) and A. Devoto and W. W. Repko, Phys. Lett. **106B**, 501 (1981).

[4]Haakon A. Olsen, Phys. Rev. D **19**, 100 (1979).

[5]See, e.g., H. A. Olsen and L. C. Maximon, Phys. Rev. **114**, 887 (1959).

[6]W. Heitler, *Quantum Theory of Radiation,* 3rd ed. (Clarendon Press, Oxford, 1953), p. 248, Eq. (21) and p. 249, Eq. (26).

[7]St. Mrowczynski, Phys. Rev. A **33**, 1549 (1986).