

## Supersymmetric-particle production at electron-positron colliders

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We present cross-section formulas for the production of scalar fermions, photinos, and  $W$  and  $Z$  gauginos (including heavy  $W$  and  $Z$  gauginos) by electron-positron collisions. We have considered general mixings, so that most of our results are general and not dependent on any particular model (except that we assume a relatively light photino). The mass eigenstates in the chargino sector have been studied in particular detail. Numerical results for beam energies varying from present-day DESY PETRA energies to beyond CERN LEP II energies are presented.

### I. INTRODUCTION

The realization<sup>1</sup> that the remarkable ultraviolet properties<sup>2</sup> of supersymmetric<sup>3</sup> quantum field theories may provide a solution to the "technical" aspect of the hierarchy problem of grand unified theories (GUT's), has aroused a great deal of interest in supersymmetric model building. Much of the interest in supersymmetry phenomenology<sup>4</sup> stems from the fact that for supersymmetry (SUSY) to tame the radiative corrections that lead to the technical hierarchy problem, the effective SUSY-breaking scale in the low-energy  $SU(3) \times SU(2) \times U(1)$  theory cannot be larger than  $\sim 1$  TeV. Thus, even if SUSY is broken, the superpartners of the quarks and leptons (whose masses determine this effective SUSY-breaking scale) may be accessible at various accelerators that either exist or are scheduled to operate in the near future.

On the experimental side, the DESY and SLAC electron-positron colliders<sup>5</sup> PETRA and PEP have provided us with lower limits on the masses of supersymmetric particles. Since the supersymmetric particles are produced in pairs, direct mass limits are typically a little below the beam energy ( $\leq 23$  GeV) (Ref. 6). For the mass limit on the  $Z$  gaugino ( $\tilde{Z}$ ) (Ref. 7), one can do slightly better since the  $\tilde{Z}$  can be produced in association with a light photino ( $\tilde{\gamma}$ ) via  $e^+e^- \rightarrow \tilde{Z}\tilde{\gamma}$  provided the scalar electron ( $\tilde{e}$ ) is not too heavy. In addition, there are indirect mass limits<sup>8</sup> such as that on the scalar-electron mass ( $m_{\tilde{e}} \leq 50$  GeV for  $m_{\tilde{\gamma}}=0$ ) obtained by the ASP experiment by studying the reaction  $e^+e^- \rightarrow \tilde{\gamma}\tilde{\gamma}\gamma$  which is mediated by the  $t$ -channel scalar-electron exchange.

The situation at the CERN  $\bar{p}p$  collider is not as clear. Various calculations<sup>9</sup> of jet(s) + missing transverse momentum ( $\not{p}_T$ ) signals indicate that scalar quarks and gluinos, if light enough, would lead to an observable event rate. It has been pointed out,<sup>10</sup> however, that such signals are also present within the standard model, and that this background must be understood before any conclusions can be drawn. A conclusive absence of a sufficient number of jet(s) +  $\not{p}_T$  events would translate<sup>11</sup> into lower bounds of  $m_{\tilde{q}} \geq 65-75$  GeV ( $m_{\tilde{g}} > m_{\tilde{q}}$ ) or  $m_{\tilde{g}} > 60-70$

GeV ( $m_{\tilde{g}} < m_{\tilde{q}}$ ). It has also been suggested that by a detailed analysis of gaugino and scalar-lepton production via  $W^\pm$  and  $Z_0$  decays, it may be possible to obtain improved mass limits of  $\sim 35-40$  GeV on the scalar-lepton<sup>12</sup> and  $W$ - and  $Z$ -gaugino<sup>13</sup> masses. We emphasize that these are theoretical expectations and that mass limits can only be put by the UA1 and UA2 Collaborations after an analysis of their data.

In spite of only negative experimental results, the phenomenology of supersymmetry has been a subject of continued interest. In a previous Letter,<sup>14</sup> we had considered the total cross section for producing SUSY particles at  $e^+e^-$  colliders ranging from PETRA to CERN LEP II energies. For reasons of brevity, we had confined ourselves to a graphical presentation of the results. In this paper, we present a complete compilation of the cross-section formulas for reactions considered in Ref. 14 along with cross-section formulas for other reactions that may be relevant at LEP. We are aware that some of these formulas are by now available in the literature, but felt that it would be useful to have all the formulas listed in one paper.

We have improved on our previous Letter in the following respects.

(i) We consider heavy- $W$ -gaugino and heavy- $Z$ -gaugino processes that had been previously neglected.

(ii) Unlike as in our previous work, we have not confined our analysis to the simple tree-breaking model of Ref. 15. In particular, we have done a rather careful analysis of the charged-gaugino-Higgs-fermion sector where the effect of more general mixing is most prominent.

(iii) We have included the effect of a nonzero photino mass and also the correction to the mixing angles induced by this.

(iv) Finally, we have attempted to incorporate mass thresholds for scalar quarks and leptons as given by supergravity models. We should mention that in this paper we have not concentrated on signatures for individual SUSY processes. These have been considered elsewhere in the literature.

The organization of this paper is as follows. In Sec. II we set up the general framework for the couplings and masses in minimal  $SU(3) \times SU(2) \times U(1)$  supergravity theories.<sup>16</sup> Section III is devoted to a study of particle production in the gaugino–Higgs-fermion sector for a wide range of mixing angles. Scalar-quark and scalar-lepton production is considered in Sec. IV. We end with some general remarks in Sec. V.

## II. THEORETICAL CONSIDERATIONS IN SUPERGRAVITY MODELS: COUPLINGS AND MASSES

In this section we discuss the framework we use for the couplings and masses for the SUSY particles. Motivated by the fact that globally supersymmetric models with supersymmetry broken at a scale  $\lesssim 1$  TeV (so that SUSY can be used to stabilize the hierarchy of masses in a GUT) lead to phenomenological problems,<sup>17</sup> we work within the framework of  $N=1$  supergravity models<sup>16</sup> currently in vogue. In these models, supersymmetry is broken in a sector of the theory (the hidden sector) at a scale  $\mu \sim 10^{11}$  GeV. This sector (and hence the Goldstone fermion) couples to matter, gauge, and Higgs multiplets only via gravitational interactions, leading to an (effective) supersymmetry-breaking scale  $\mu_s$  in the effective low-energy theory of  $\sim \mu^2/M_P \sim 10^3$  GeV, where  $M_P \sim 10^{19}$  GeV is the Planck mass.

It has been shown that the effective low-energy theory obtained from these models is just a globally supersymmetric  $SU(3) \times SU(2) \times U(1)$  gauge theory with additional soft-SUSY-breaking terms<sup>18,16</sup> that parametrize the effect of spontaneous SUSY breaking in the hidden sector. In this sense, *local* SUSY plays no direct role in the determination of the couplings. A particularly nice feature of these models is that the breaking of SUSY in the hidden sector, because of gravitational interactions, drives the breaking of  $SU(2) \times U(1)$ , with the weak scale being  $\mu^2/M_P \sim 1$  TeV.

In addition to the quarks and leptons and their superpartners, the weakly interacting sector contains the gauge and Higgs supermultiplets. All SUSY models contain at least two Higgs doublets since it is not possible to give masses both to the  $T_{3L} = +\frac{1}{2}$  and  $T_{3L} = -\frac{1}{2}$  fermions with just one Higgs field. Thus the minimal content of the gaugino–Higgs-fermion sector is the  $SU(2)$  and  $U(1)$  gauge fermions  $\lambda$  and  $\lambda_0$  along with the Higgs fermion doublets  $h$  and  $h'$  whose scalar partners give masses to the  $T_3 = +\frac{1}{2}$  and  $T_3 = -\frac{1}{2}$  fermions, respectively. We now turn to the calculation of the mass eigenstates and their couplings.

The couplings of the gauge and Higgs fermions to electroweak gauge bosons and to quarks and leptons and their superpartners are all determined by  $SU(2) \times U(1)$  and SUSY. Unlike the left- and right-handed scalar quarks ( $\tilde{q}_L$  and  $\tilde{q}_R$ ) and scalar leptons ( $\tilde{l}_L$  and  $\tilde{l}_R$ ) (Ref. 19) which are themselves mass eigenstates in the limit of negligible quark and lepton masses, the gauginos and Higgs fermions of the same charge mix once  $SU(2) \times U(1)$  is broken. The mixing angles, and hence the mass eigenstates are, in general, model dependent, but as we will

shortly see, it is possible to parametrize the couplings and masses of SUSY particles in all supergravity models in terms of relatively few parameters.

The mass terms for the gauge and Higgs fermions take the form

$$(\bar{\lambda}_-, \bar{\chi}_-)(M_{(\text{charge})} P_L + M_{(\text{charge})}^T P_R) \begin{pmatrix} \lambda_- \\ \chi_- \end{pmatrix} \quad (1a)$$

for the charged sector and

$$\frac{1}{2}(\bar{h}^0, \bar{h}'^0, \bar{\lambda}_3, \bar{\lambda}_0)(M_{(\text{neutral})} P_L + M_{(\text{neutral})}^T P_R) \begin{pmatrix} h^0 \\ h'^0 \\ \lambda_3 \\ \lambda_0 \end{pmatrix} \quad (1b)$$

for the neutral sector. The spinors  $h^0$ ,  $h'^0$ ,  $\lambda_3$ , and  $\lambda_0$  are self-conjugate, whereas the Dirac spinors in Eq. (1a) are defined by

$$\lambda_- \equiv \frac{1}{\sqrt{2}}(\lambda_1 + i\lambda_2), \quad \chi_- \equiv P_L h' - P_R h,$$

where  $h$  ( $h'$ ) and  $h^0$  ( $h'^0$ ) are the charged and neutral members of the fermion doublet  $h$  ( $h'$ ), respectively. The mass matrices in Eqs. (1a) and (1b) are

$$M_{(\text{charge})} = \begin{pmatrix} \mu_2 & gv' \\ gv & 2m_1 \end{pmatrix} \quad (2a)$$

and

$$M_{(\text{neutral})} = \begin{pmatrix} 0 & -2m_1 & \frac{1}{\sqrt{2}}gv & -\frac{1}{\sqrt{2}}g'v \\ -2m_1 & 0 & -\frac{1}{\sqrt{2}}gv' & \frac{1}{\sqrt{2}}g'v' \\ \frac{1}{\sqrt{2}}gv & -\frac{1}{\sqrt{2}}gv' & \mu_2 & 0 \\ -\frac{1}{\sqrt{2}}g'v & \frac{1}{\sqrt{2}}g'v' & 0 & \mu_1 \end{pmatrix} \quad (2b)$$

and  $P_L$  ( $P_R$ ) is the left (right) chirality projector. In Eq. (2),  $2m_1$  is the supersymmetric Higgs-fermion mixing mass term,  $v$  and  $v'$  are the vacuum expectation values of the Higgs fields  $h^0$  and  $h'^0$ , and  $\mu_1$  and  $\mu_2$  are soft-SUSY-breaking  $U(1)$  and  $SU(2)$  gaugino masses. In a GUT with a common gaugino mass at the unification scale,  $\mu_1$  and  $\mu_2$  satisfy

$$\frac{\mu_1}{\mu_2} = \frac{5}{3} \tan^2 \theta_W. \quad (3)$$

We see that all the masses and mixing angles in the gauge and Higgs-fermion sector are determined in terms of the parameters  $\mu_2$ ,  $2m_1$ , and  $v'/v$ . [Recall  $v$  and  $v'$  are not independent since  $M_W^2 = \frac{1}{2}g^2(v^2 + v'^2)$ .] In the class of supergravity models in which  $SU(2) \times U(1)$  breaking is radiatively driven<sup>16</sup> by the Yukawa coupling of a top quark of mass  $\sim 40$ – $50$  GeV (Ref. 20) or in the simple model of Ref. 15, where  $SU(2) \times U(1)$  is broken at the tree level,<sup>21</sup>  $v'/v \simeq 1$ . In this case, the number of parameters is further

reduced and the diagonalization is even simpler. We will see that the contribution of the neutralinos to the total cross section is rather small. For this reason, we have diagonalized this sector only for the favored  $v'=v$  case. The charginos make a large contribution to the cross section and hence we have examined their production for all values of  $v'/v$ . The analysis of these mass matrices has been carried out by a number of authors.<sup>22</sup> Our analysis is presented here for completeness.

The mass matrix (2a) is diagonalized by a biunitary transformation. The eigenvalues  $m_{\pm}$  are given by

$$m_{\pm}^2 = \frac{1}{2}[(4m_1^2 + 2M_W^2 + \mu_2^2) \pm \zeta], \quad (4)$$

where

$$\zeta^2 = (4m_1^2 - \mu_2^2)^2 + 4M_W^2(M_W^2 \cos^2 2\alpha + 4m_1^2 + \mu_2^2 + 4m_1\mu_2 \sin 2\alpha). \quad (5a)$$

To obtain the mass eigenstates, the left- and right-handed components are rotated by different angles  $\gamma_L$  and  $\gamma_R$  ( $0 \leq \gamma_L, \gamma_R \leq 180^\circ$ ) given by

$$\tan \gamma_L = x_-^{-1}, \quad (5b)$$

$$\tan \gamma_R = y_-^{-1}, \quad (5c)$$

with

$$x_- = \frac{(4m_1^2 - \mu_2^2 - 2M_W^2 \cos 2\alpha) - \zeta}{2\sqrt{2}M_W(\mu_2 \sin \alpha + 2m_1 \cos \alpha)} \quad (5d)$$

and

$$y_- = \frac{(4m_1^2 - \mu_2^2 + 2M_W^2 \cos 2\alpha) - \zeta}{2\sqrt{2}M_W(\mu_2 \cos \alpha + 2m_1 \sin \alpha)}. \quad (5e)$$

The angle  $\alpha$  in Eq. (5) is given by

$$\tan \alpha = v'/v. \quad (6)$$

Knowing the mixings, the couplings can be readily obtained from the Lagrangian involving the current eigenstates  $\lambda$ ,  $\lambda_0$ ,  $h$ , and  $h'$  since the couplings of these are fixed by  $SU(2) \times U(1)$  and SUSY. Before proceeding to a discussion of this, it is worthwhile to briefly discuss some features of Eq. (4) for the chargino masses.

The first thing to note is that for a given value of  $\mu_2$ , and hence the photino mass [see Eq. (12c)], the mass  $m_-$  of the lighter chargino cannot be arbitrarily large. Although this state is not the SUSY partner of the  $\tilde{W}$  boson, we will refer to it as the  $\tilde{W}$  gaugino ( $\tilde{W}$ ). Also, we will refer to the other chargino state as the "heavy  $\tilde{W}$  gaugino" ( $\tilde{W}_h$ ). Shown in Fig. 1(a) is the maximum  $\tilde{W}$  mass (for a fixed value of  $m_{\tilde{\gamma}}$ ) as a function of the ratio  $v'/v$ . The reason for the symmetry of the graph is the  $\alpha$  dependence in Eq. (5a). From the absence of  $\tilde{W}$  signals at PETRA,<sup>6</sup> one may infer the allowed values of  $v'/v$  for various photino masses.<sup>23</sup>

The dependence of the  $\tilde{W}$  mass of the Higgs-fermion mixing term is shown in Fig. 1(b) for several values of  $v'/v$ . [We are aware that  $v'/v \leq 1$  for  $SU(2)_L \times U(1)_Y$  breaking in models with either (i) top-quark masses of  $\sim 40$  GeV or tree-breaking models or (ii) models with a

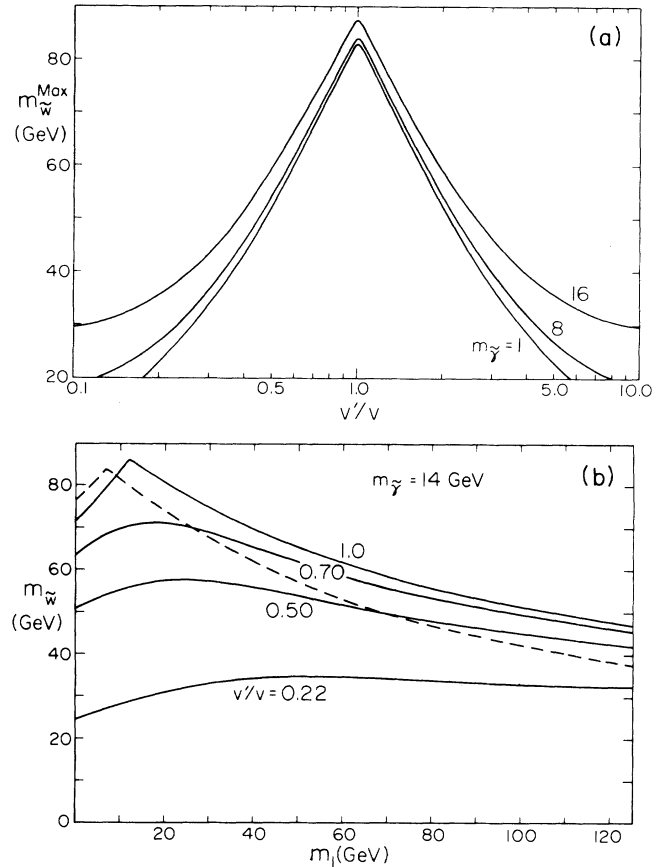


FIG. 1. (a) The maximum  $\tilde{W}$  mass in GeV allowed by (4) as a function of  $v'/v$  and the mass of the photino. ( $m_{\tilde{\gamma}}$  in GeV.) (b) The  $\tilde{W}$  mass, as a function of the parameter  $m_1$  (the supersymmetric Higgs-fermion mixing mass term), for various values of  $v'/v$ . The photino mass was fixed at 14 GeV, rather than 8 GeV as used in the rest of the paper, to make the graph easier to read. The dashed line is for  $m_{\tilde{\gamma}} = 8$  GeV with  $v'/v = 1.0$ .

heavy top quark. In this paper, however, we have analyzed  $v'/v > 1$  also, for the sake of completeness.] We see that a given  $\tilde{W}$  mass, in general, corresponds to two values of  $m_1$ . Reducing  $m_{\tilde{\gamma}}(\mu_2)$  makes the graph peak closer to  $m_1 = 0$  until for  $m_{\tilde{\gamma}} = 0$  [as shown by the dashed line in Fig. 1(b) which is for  $m_{\tilde{\gamma}} = 8$  GeV and  $v'/v = 1$ ] the graph is monotonically decreasing. These double solutions for  $m_{\tilde{\gamma}} \neq 0$  imply that the mixing angles  $\gamma_L$  and  $\gamma_R$  (and hence the couplings) are not completely determined by  $m_{\tilde{W}}$ . This effect is particularly important for values of  $v'/v$  substantially different from unity since the curve is quite flat and so the same  $\tilde{W}$  mass corresponds to very different values of  $m_1$ . For the curves shown in the paper we do not encounter this difficulty since for the chosen values of parameters we have only one solution for  $m_1$ . In the case of a double solution one may tend to slightly favor the larger value of  $m_1$  (arguing the mass parameters in the superpotential all have the scale  $\mu_s$ , the effective SUSY-breaking scale) unless one has a specific model that requires  $m_1 \simeq 0$ .

Before proceeding to the diagonalization of the neutralino sector we note that the diagonalization of (1a) is particularly simple for  $v'=v$ . We then find that the eigenstates  $\tilde{W}_{(\pm)}$  with eigenvalues  $m_{\pm}$  are given by

$$\begin{pmatrix} \tilde{W}_{(+)} \\ \gamma_5 \tilde{W}_{(-)} \end{pmatrix} = \begin{pmatrix} f_- & f_+ \\ f_+ & -f_- \end{pmatrix} \begin{pmatrix} \lambda_- \\ \chi_- \end{pmatrix} \quad (7a)$$

with

$$f_{\pm} = \left[ \frac{m_{\pm} \mp \mu_2}{m_+ + m_-} \right]^{1/2}. \quad (7b)$$

In this case we have  $\gamma_L = \gamma_R = \gamma$ , with  $f_+ = \sin\gamma$  and  $f_- = -\cos\gamma$  and the  $\tilde{W}$  and heavy- $\tilde{W}$  masses reduce to

$$-i\gamma_5 \tilde{Z}_{(-)}^{(0)} = \left[ \frac{\mu_-}{2(\mu_+ + \mu_-)} \right]^{1/2} \left[ -h^0 + h'^0 + \frac{\sqrt{2}M_Z}{\mu_-} (\cos\theta_W \lambda_3 - \sin\theta_W \lambda_0) \right], \quad (8b)$$

$$\tilde{Z}_{(+)}^{(0)} = \left[ \frac{\mu_+}{2(\mu_+ + \mu_-)} \right]^{1/2} \left[ h^0 - h'^0 + \frac{\sqrt{2}M_Z}{\mu_+} (\cos\theta_W \lambda_3 - \sin\theta_W \lambda_0) \right], \quad (8c)$$

and

$$-i\gamma_5 \tilde{h}^{(0)} = \frac{1}{\sqrt{2}} (h^0 + h'^0) \quad (8d)$$

with eigenvalues 0,  $\mu_-$ ,  $\mu_+$ , and  $2m_1$ , respectively. The  $\gamma_5$  transformations are required to get positive masses and the factor  $i$  ensures the states are Majorana. The eigenvalues  $\mu_+, \mu_-$  are not independent of  $m_{\tilde{W}}$  (or  $m_-$ ) and are given by

$$\mu_- = (m_1^2 + M_Z^2)^{1/2} - m_1 \quad (9a)$$

and

$$\mu_+ \mu_- = M_Z^2. \quad (9b)$$

The fact that for  $m_{\tilde{\gamma}}=0$  we have the states  $\tilde{W}$  and  $\tilde{Z}_{(-)}^{(0)}$ , respectively, lighter than the  $W$  and  $Z$  bosons is a general feature<sup>15,24</sup> and not, in any way particular to the model. We may thus expect to see at least these states at CERN LEP if the photino is not too heavy. We note also that the states  $\tilde{Z}_{(\pm)}^{(0)}$  and  $\tilde{\gamma}^{(0)}$  all contain substantial gaugino pieces. In the tree-breaking model<sup>15</sup> there is an additional gauge-singlet field<sup>21</sup>  $U$  which mixes with the fields  $h$  and  $h'$  but not with the gauginos. In addition to the states  $\tilde{\gamma}^{(0)}$  and  $\tilde{Z}_{\pm}^{(0)}$  which are left unaltered there are two other mass eigenstates, neither of which [as  $\tilde{h}^{(0)}$  in Eq. (8d)] have any gaugino component. Because the Higgs sector is the most model-dependent sector of the theory, in this paper we do not consider the production of these ‘‘Higgs-fermion’’ states but concentrate on the relatively model-independent states (8a)–(8c).

We now turn to the effects of the gaugino masses  $\mu_2$  and  $\mu_1$ . Since these enter only the gauge sector, the states  $\tilde{\gamma}^{(0)}$  and  $\tilde{Z}_{\pm}^{(0)}$  mix with one another (but not with the Higgs fermions) to form the mass eigenstates  $\tilde{\gamma}$ ,  $\tilde{Z}_{(-)}$ ,

$$m_{\pm} = \left[ \left[ m_1 - \frac{\mu_2}{2} \right]^2 + M_W^2 \right]^{1/2} \pm \left[ m_1 + \frac{\mu_2}{2} \right]. \quad (7c)$$

[In Eq. (7c), we have assumed  $(2m_1 + \mu_2) > 0$ , i.e., we have chosen the solution with the larger value of  $m_1$ .]

We now turn to the diagonalization of the neutralino sector which, as has already been mentioned, has been diagonalized only for  $v=v'$ . The diagonalization is rather tedious and has been done only for values of the SUSY-breaking gaugino masses that are much smaller than  $M_W$ . For  $\mu_1 = \mu_2 = 0$ , the eigenstates are given by

$$-i\gamma_5 \tilde{\gamma}^{(0)} = \cos\theta_W \lambda_0 + \sin\theta_W \lambda_3 \quad (8a)$$

( $\tilde{\gamma}^{(0)}$  is the superpartner of the photon and is massless),

and  $\tilde{Z}_{(+)}$ . Treating  $\mu_1$  and  $\mu_2$  to lowest order in perturbation theory, these are related to  $\tilde{\gamma}^{(0)}$  and  $\tilde{Z}_{(\pm)}^{(0)}$  by

$$\begin{pmatrix} \tilde{Z}_{(-)}^{(0)} \\ \tilde{Z}_{(+)}^{(0)} \\ \tilde{\gamma}^{(0)} \end{pmatrix} = \begin{pmatrix} 1 & \delta & \epsilon_1 \\ -\delta & 1 & \epsilon_2 \\ -\epsilon_1 & -\epsilon_2 & 1 \end{pmatrix} \begin{pmatrix} \tilde{Z}_{(-)} \\ \tilde{Z}_{(+)} \\ \tilde{\gamma} \end{pmatrix} \quad (10)$$

with

$$\epsilon_1 = \sqrt{2} N_1 \frac{M_Z}{\mu_-} \sin\theta_W \cos\theta_W (\mu_2 - \mu_1), \quad (11a)$$

$$\epsilon_2 = -\sqrt{2} N_2 \frac{M_Z}{\mu_+} \sin\theta_W \cos\theta_W (\mu_2 - \mu_1), \quad (11b)$$

$$\delta = \frac{2N_1 N_2}{\mu_+ + \mu_-} (\mu_2 \cos^2\theta_W + \mu_1 \sin^2\theta_W), \quad (11c)$$

and

$$N_{1,2} = \left[ \frac{(\mu_-, \mu_+)}{2(\mu_+ + \mu_-)} \right]^{1/2}. \quad (11d)$$

The corresponding masses are

$$m_{\tilde{Z}_{(-)}} = \mu_- + \frac{\mu_+}{\mu_+ + \mu_-} \tilde{M}, \quad (12a)$$

$$m_{\tilde{Z}_{(+)}} = \mu_+ - \frac{\mu_-}{\mu_+ + \mu_-} \tilde{M}, \quad (12b)$$

and

$$m_{\tilde{\gamma}} = |\mu_2 \sin^2 \theta_W + \mu_1 \cos^2 \theta_W|, \quad (12c)$$

where

$$\tilde{M} = |\mu_2 \cos^2 \theta_W + \mu_1 \sin^2 \theta_W|. \quad (12d)$$

Since we have defined  $\tilde{\gamma}^{(0)}$  in Eq. (8a) with a  $\gamma_5$ , we

choose  $\mu_1$  and  $\mu_2$  as negative numbers. As in the case of the chargino sector, we refer to the states  $\tilde{Z}_{(-)}$  and  $\tilde{Z}_{(+)}$  as the  $Z$  gaugino ( $\tilde{Z}$ ) and the heavy  $Z$  gaugino ( $\tilde{Z}_h$ ), respectively.

The relevant Lagrangian can now be readily worked out using the standard-model couplings for the current eigenstates. We find

$$\mathcal{L} = \mathcal{L}_{\text{standard}} + \mathcal{L}_{\text{gaugino}} + \mathcal{L}_{\text{scalar fermions}} + \mathcal{L}_{\text{gaugino-scalar-fermion}}, \quad (13)$$

where

$$\mathcal{L}_{\text{standard}} = -e \sum_f q_f \bar{f} \gamma_\mu f A_\mu + e \sum_f \bar{f} \gamma_\mu (\alpha_f + \beta_f \gamma_5) f Z_\mu, \quad (14a)$$

$$\begin{aligned} \mathcal{L}_{\text{gaugino}} = & e (\bar{W} \gamma^\mu \tilde{W} + \bar{W}_h \gamma^\mu \tilde{W}_h) A_\mu - e \cot \theta_W \bar{W} \gamma^\mu (x_C - y_C \gamma_5) \tilde{W} Z_\mu - e \cot \theta_W \bar{W}_h \gamma^\mu (x_s - y_s \gamma_5) \tilde{W}_h Z_\mu \\ & - \frac{1}{2} e (\cot \theta_W + \tan \theta_W) \bar{W} \gamma^\mu (x - y \gamma_5) \tilde{W} Z_\mu - \frac{1}{2} e (\cot \theta_W + \tan \theta_W) \bar{W}_h \gamma^\mu (x - y \gamma_5) \tilde{W}_h Z_\mu, \end{aligned} \quad (14b)$$

where

$$x_C = 1 - \frac{1}{4} \sec^2 \theta_W (\cos^2 \gamma_L + \cos^2 \gamma_R), \quad y_C = \frac{1}{4} \sec^2 \theta_W (\cos^2 \gamma_R - \cos^2 \gamma_L),$$

$$x_s = 1 - \frac{1}{4} \sec^2 \theta_W (\sin^2 \gamma_L + \sin^2 \gamma_R), \quad y_s = \frac{1}{4} \sec^2 \theta_W (\sin^2 \gamma_R - \sin^2 \gamma_L),$$

$$x = \frac{1}{2} (\theta_x \sin \gamma_L \cos \gamma_L - \theta_y \sin \gamma_R \cos \gamma_R), \quad y = \frac{1}{2} (\theta_x \sin \gamma_L \cos \gamma_L + \theta_y \sin \gamma_R \cos \gamma_R).$$

Here,  $\theta_x = +1$  for  $x_- > 0$  and  $-1$  for  $x_- < 0$  where  $x_-$  is given in Eq. (5d) and similarly  $\theta_y$ . Notice that for  $v = v'$  the  $\tilde{W} \tilde{W} Z$  and  $\tilde{W}_h \tilde{W}_h Z$  couplings become purely vector whereas the  $\tilde{W} \tilde{W}_h Z$  coupling becomes purely axial vector. Continuing, we have

$$\mathcal{L}_{\text{scalar fermion}} = -ie \sum_f q_f (\tilde{f}_L^\dagger \partial_\mu \tilde{f}_L + \tilde{f}_R^\dagger \partial_\mu \tilde{f}_R) A^\mu + \text{H.c.} + ie \sum_f [(\alpha_f - \beta_f) \tilde{f}_L^\dagger \partial_\mu \tilde{f}_L + (\alpha_f + \beta_f) \tilde{f}_R^\dagger \partial_\mu \tilde{f}_R] Z^\mu + \text{H.c.} \quad (14c)$$

and finally

$$\begin{aligned} \mathcal{L}_{\text{gaugino-scalar-fermion}} = & +ie B_L \tilde{e}_L^\dagger \tilde{\gamma} \frac{1-\gamma_5}{2} e + ie B_R \tilde{e}_R^\dagger \tilde{\gamma} \frac{1+\gamma_5}{2} e + \text{H.c.} + ie C_L \tilde{e}_L^\dagger \tilde{Z} \frac{1-\gamma_5}{2} e + ie C_R \tilde{e}_R^\dagger \tilde{Z} \frac{1+\gamma_5}{2} e + \text{H.c.} \\ & + e D_L \tilde{e}_L^\dagger \tilde{Z}_h \frac{1-\gamma_5}{2} e + e D_R \tilde{e}_R^\dagger \tilde{Z}_h \frac{1+\gamma_5}{2} e + \text{H.c.} \\ & + g \sin \gamma_R \tilde{\nu}^\dagger \tilde{W} \frac{1-\gamma_5}{2} e - \theta_y g \cos \gamma_R \tilde{\nu}^\dagger \tilde{W}_h \frac{1-\gamma_5}{2} e + \text{H.c.}, \end{aligned} \quad (14d)$$

where

$$B_L = -\sqrt{2} + \frac{N_1 M_Z}{\mu_-} (t - c) \epsilon_1,$$

$$B_R = -\sqrt{2} + \frac{2N_1 M_Z}{\mu_-} t \epsilon_1,$$

$$C_L = \frac{N_1 M_Z}{\mu_-} (t - c) + \sqrt{2} \epsilon_1,$$

$$C_R = \frac{2N_1 M_Z}{\mu_-} t + \sqrt{2} \epsilon_1,$$

$$D_L = -\frac{N_2 M_Z}{\mu_+} (t - c),$$

and

$$D_R = \frac{2N_2 M_Z}{\mu_+} t.$$

Here  $t \equiv \tan \theta_W$  and  $c \equiv \cot \theta_W$ . The term proportional to  $\epsilon_1$  comes in from the correction to the neutralino mixing angles due to  $m_{\tilde{\gamma}} \neq 0$ . We have ignored the numerically tiny corrections (proportional to  $\delta$  and  $\epsilon_2$ ) coming from the mixing of  $\tilde{Z}_{(+)}^{(0)}$  with  $\tilde{\gamma}^{(0)}$  and  $\tilde{Z}_{(-)}^{(0)}$  [see Eq. (10)]. The parameters that appear in Eqs. (14a) and (14c) are listed in Table I. This completes our discussion of the SUSY-particle couplings. We now turn to the final aspect

TABLE I. The definition of the parameters that determine the couplings in the Lagrangian, Eqs. (14a) and (14c).

$f$	$q_f$	$\alpha_f$	$\beta_f$
$l$	$-1$	$+\frac{1}{4}(3t - c)$	$+\frac{1}{4}(t + c)$
$u$	$\frac{2}{3}$	$-\frac{5}{12}t + \frac{1}{4}c$	$-\frac{1}{4}(t + c)$
$d$	$-\frac{1}{3}$	$+\frac{1}{12}t - \frac{1}{4}c$	$+\frac{1}{4}(t + c)$

of the theoretical framework, viz., the SUSY-particle masses.

We have already seen that the masses of the photino, the  $W$  gauginos ( $\tilde{W}$  and  $\tilde{W}_h$ ) and the  $Z$  gauginos ( $\tilde{Z}$  and  $\tilde{Z}_h$ ) are determined in terms of two parameters  $\mu_2$  and  $m_1$  which we may eliminate in favor of  $m_{\tilde{\gamma}}$  and  $m_{\tilde{W}}$ , respectively. The remaining masses and mixings are then completely determined. The only remaining arbitrariness is in the scalar-fermion masses. In the supergravity Lagrangian these have a universal mass  $m_0$  at the unification scale (apart from  $D$ -term contributions). This supergravity Lagrangian is considered to be an effective Lagrangian with the parameters (masses and couplings) renormalized at the unification scale  $M_p$ . In order to use the couplings thus obtained at LEP energies, one needs to sum the large logarithms that result from the difference in the LEP and Planck energy scales. This has been done using renormalization-group methods, and one has, for the scalar-fermion masses (for three generations in the  $\beta$  function and for  $\sin^2\theta_W=0.22$ ) (Ref. 25),

$$\begin{aligned} m^2(\tilde{d}_L) &= m_0^2 + 0.43rM_Z^2 + 30.2m_{\tilde{\gamma}}^2, \\ m^2(\tilde{d}_R) &= m_0^2 + 0.07rM_Z^2 + 28.4m_{\tilde{\gamma}}^2, \\ m^2(\tilde{u}_L) &= m_0^2 - 0.36rM_Z^2 + 30.2m_{\tilde{\gamma}}^2, \\ m^2(\tilde{u}_R) &= m_0^2 - 0.14rM_Z^2 + 28.4m_{\tilde{\gamma}}^2, \\ m^2(\tilde{e}_L) &= m_0^2 + 0.28rM_Z^2 + 2.2m_{\tilde{\gamma}}^2, \\ m^2(\tilde{e}_R) &= m_0^2 + 0.22rM_Z^2 + 0.6m_{\tilde{\gamma}}^2, \\ m^2(\tilde{\nu}) &= m_0^2 - 0.50rM_Z^2 + 2.2m_{\tilde{\gamma}}^2, \end{aligned} \quad (15)$$

with  $r \equiv (v^2 - v'^2)/(v^2 + v'^2)$ . In Eq. (15) the second term arises from the  $D$  terms (notice it vanishes for  $v = v'$ ) while the last term comes from the renormalization-group evolution of the mass.

This brings us to an end of the discussion of the model. We are now in a position to calculate various SUSY processes relevant at SLC or LEP.

### III. GAUGINO PRODUCTION AT ELECTRON-POSITRON COLLIDERS

In this section, we present analytic formulas along with our results for the production cross sections for  $\tilde{W}$ ,  $\tilde{W}_h$ ,

$$\begin{aligned} \sigma^{(\pm)}(x,y) &= \frac{e^4}{16\pi s} \frac{k}{\sqrt{s}} \left[ 1 + \frac{x^2 + y^2 - 2m_{\tilde{e}}^2}{2k\sqrt{s}} \ln \left[ \frac{s - x^2 - y^2 + 2m_{\tilde{e}}^2 + 2k\sqrt{s}}{s - x^2 - y^2 + 2m_{\tilde{e}}^2 - 2k\sqrt{s}} \right] - \frac{m_{\tilde{e}}^2(x^2 + y^2) - m_{\tilde{e}}^4 - x^2y^2}{[\frac{1}{2}(s - x^2 - y^2) + m_{\tilde{e}}^2]^2 - k^2s} \right. \\ &\quad \left. \pm \frac{xy\sqrt{s}}{k} \frac{1}{(s - x^2 - y^2) + 2m_{\tilde{e}}^2} \ln \left[ \frac{s - x^2 - y^2 + 2m_{\tilde{e}}^2 + 2k\sqrt{s}}{s - x^2 - y^2 + 2m_{\tilde{e}}^2 - 2k\sqrt{s}} \right] \right] \end{aligned} \quad (19)$$

with

$$k = [s^2 + (x^2 - y^2)^2 - 2s(x^2 + y^2)]^{1/2} / 2\sqrt{s}.$$

In Eq. (19) we have neglected the small splitting between

$\tilde{Z}$ ,  $\tilde{Z}_h$ , and  $\tilde{\gamma}$  in  $e^+e^-$  collisions. The relevant processes in the neutralino sector are

$$\begin{aligned} e^+e^- &\rightarrow \tilde{Z}\tilde{\gamma}, \quad e^+e^- \rightarrow \tilde{Z}\tilde{Z}, \\ e^+e^- &\rightarrow \tilde{Z}_h\tilde{\gamma}, \quad e^+e^- \rightarrow \tilde{Z}\tilde{Z}_h, \end{aligned} \quad (16)$$

and

$$e^+e^- \rightarrow \tilde{Z}_h\tilde{Z}_h,$$

whereas in the charged sector we have

$$e^+e^- \rightarrow \tilde{W}\tilde{W}, \quad e^+e^- \rightarrow \tilde{W}\tilde{W}_h + \tilde{W}_h\tilde{W},$$

and

$$e^+e^- \rightarrow \tilde{W}_h\tilde{W}_h.$$

The cross sections can be readily calculated using the couplings given in the previous section.

For  $v'/v=1$  [a value favored by both the tree-breaking<sup>15</sup> and radiative breaking models<sup>16,18</sup> with a top-quark mass of  $\sim 40$  GeV (Ref. 20)], neutralino production takes place only via the  $t$ -channel exchange of a scalar electron since neither the photon nor the  $Z^0$  couple to the neutralinos. In this case, the cross sections for the reactions (16) are given by

$$\sigma(e^+e^- \rightarrow \tilde{Z}\tilde{\gamma}) = (B_L^2 C_L^2 + B_R^2 C_R^2) \sigma^{(-)}(m_{\tilde{Z}}, m_{\tilde{\gamma}}), \quad (18a)$$

$$\sigma(e^+e^- \rightarrow \tilde{Z}_h\tilde{\gamma}) = (B_L^2 D_L^2 + B_R^2 D_R^2) \sigma^{(+)}(m_{\tilde{Z}_h}, m_{\tilde{\gamma}}), \quad (18b)$$

$$\sigma(e^+e^- \rightarrow \tilde{Z}\tilde{Z}_h) = (C_L^2 D_L^2 + C_R^2 D_R^2) \sigma^{(+)}(m_{\tilde{Z}}, m_{\tilde{Z}_h}), \quad (18c)$$

$$\sigma(e^+e^- \rightarrow \tilde{Z}\tilde{Z}) = \frac{1}{2}(C_L^4 + C_R^4) \sigma^{(-)}(m_{\tilde{Z}}, m_{\tilde{Z}}), \quad (18d)$$

and

$$\sigma(e^+e^- \rightarrow \tilde{Z}_h\tilde{Z}_h) = \frac{1}{2}(D_L^4 + D_R^4) \sigma^{(-)}(m_{\tilde{Z}_h}, m_{\tilde{Z}_h}). \quad (18e)$$

In Eq. (18)

$\tilde{e}_L$  and  $\tilde{e}_R$  masses. [Since left- and right-handed scalar-electron exchanges do not interfere, as is evident from Eq. (18), the formula with  $m_{\tilde{e}_L} \neq m_{\tilde{e}_R}$  can be readily read off.] Also,  $\sqrt{s}$  is the center-of-mass energy of the machine.

The reason for the difference in sign in Eqs. (18b) and (18c) and Eqs. (18a), (18d), and (18e) is the change in sign of the interference term between the  $t$ - and  $u$ -channel graphs coming from the factor  $i$  on the  $\tilde{Z}$  and  $\tilde{\gamma}$  couplings (but not on the  $\tilde{Z}_h$  coupling) in Eq. (14d). We have checked that our formulas involving  $\sigma^{(-)}$  agree with those of Dawson *et al.*,<sup>4</sup> Eq. (A2) [modulo mixings which factor out in Eq. (18)]. The difference in sign for  $\sigma^{(+)}$  is for the reason just discussed.

Before proceeding to a discussion of our results, we should say a few words about the way we have chosen to present these. Throughout this paper, as in Ref. 14, we have shown  $R$  defined by

$$R(e^+e^- \rightarrow A+B) = \frac{\sigma(e^+e^- \rightarrow A+B)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}, \quad (20)$$

where  $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  is the radiatively uncorrected muon-pair production cross section. We have chosen to present this ratio rather than the absolute cross section since this gives us a direct comparison between the rates for the SUSY process and a typical standard-model process. Throughout our calculation we have used a  $Z^0$  width of 2.64 GeV (the value in the standard model) plus the additional contributions expected in a supersymmetric theory with the SUSY-particle masses as in the particular case being considered. In Fig. 2 we have plotted  $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  for the standard model so that the reader can conveniently convert the  $R$  value to an event rate. The SUSY contributions alter this curve only very near the  $Z^0$  peak. For the largest SUSY contribution, the change is a factor 2 at the peak, but only 20% 1 GeV off the peak and 3% 3 GeV away from the peak.

The  $R$  value for the sum total of the neutralino processes in Eq. (16) is shown in Fig. 3 for three values of  $\tilde{W}$  masses and for  $v'/v=1$  and for a photino mass nominally fixed at 8 GeV. We have fixed<sup>11</sup> the scalar-electron mass by requiring that the average scalar-quark mass squared be  $(100 \text{ GeV})^2$  [see Eq. (15)]. The following features are worth noting.

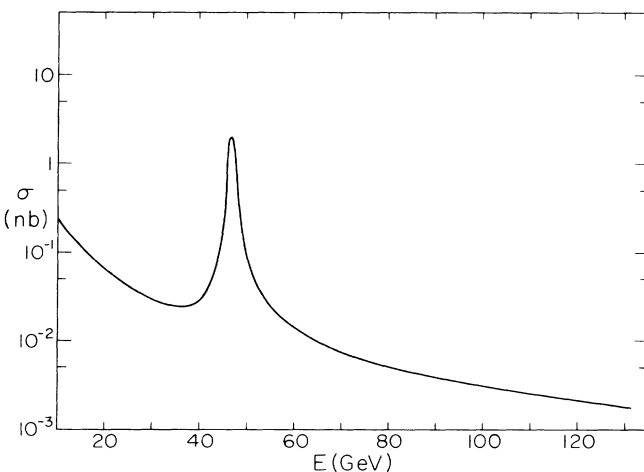


FIG. 2. The cross section in nanobarns for  $\mu^+\mu^-$  production including both  $\gamma$  and  $Z^0$  exchanges. For this figure the  $Z$  width was taken as 2.64 GeV.

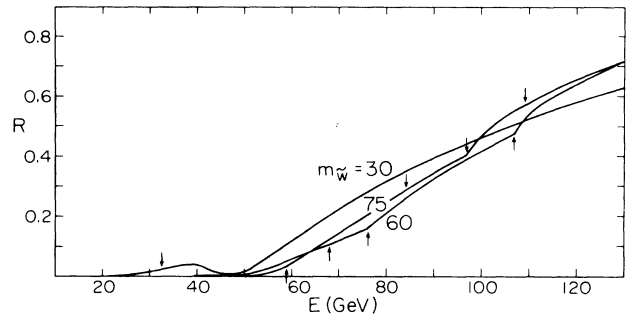


FIG. 3. The  $R$  ratio for production of  $\tilde{Z}_h\tilde{\gamma} + \tilde{Z}\tilde{Z} + \tilde{Z}_h\tilde{\gamma} + \tilde{Z}\tilde{Z}_h + \tilde{Z}_h\tilde{Z}_h$  for  $m_{\tilde{\gamma}}=8$  GeV and  $v'/v=1$ . The other parameters are fixed by the value of the  $\tilde{W}$  mass. For  $m_{\tilde{W}}=30$  GeV,  $m_{\tilde{Z}}$  is 32.9 GeV,  $m_{\tilde{Z}_h} \simeq 412$  GeV; for  $m_{\tilde{W}}=60$  GeV,  $m_{\tilde{Z}}$  is 68.2 GeV, while  $m_{\tilde{Z}_h}$  is 144.7 GeV; for  $m_{\tilde{W}}=75$  GeV,  $m_{\tilde{Z}}$  is 84.4 GeV and  $m_{\tilde{Z}_h}$  is 109.6 GeV. The scalar masses are fixed such that the average scalar-quark mass squared is  $(100 \text{ GeV})^2$ . The small arrows show the thresholds for the various processes.

(i) The dip at  $E=M_Z/2$  is due to the fact  $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  is resonance enhanced whereas the neutralino production is not.

(ii) For the smallest value of  $m_{\tilde{Z}}$  (corresponding to  $m_{\tilde{W}}=30$  GeV,  $m_{\tilde{Z}_h}=412$  GeV, so that the  $\tilde{Z}_h$  is never produced. For heavier  $\tilde{Z}$ 's,  $m_{\tilde{Z}_h}$  is smaller and hence  $\tilde{Z}_h\tilde{\gamma}$ ,  $\tilde{Z}_h\tilde{Z}$ , and (only for  $m_{\tilde{W}}=75$  GeV)  $\tilde{Z}_h\tilde{Z}_h$  thresholds also open up. It is for this reason the  $m_{\tilde{W}}=30$  GeV curve falls below the other curves for large beam energies. The crossover of the  $m_{\tilde{W}}=60$  GeV and  $m_{\tilde{W}}=75$  GeV is for similar reasons.

(iii) The magnitude of the neutralino cross section is rather small and dependent on the mass of the scalar electron in the  $t$  channel. For  $m_{\tilde{e}}=45$  GeV instead of  $\sim 90$  GeV as in the figure the  $R$  value increases by a factor which depends on  $E$  but is around 2. In spite of the smallness of the cross section the process  $e^+e^- \rightarrow \tilde{Z}\tilde{\gamma}$  may be very important since it may well be the only SUSY reaction accessible at LEP I or Stanford Linear Collider (SLC) energies. It leads to a rather characteristic signature, which has been discussed in some detail in Ref. 26.

Because the cross section for neutralino production is rather small compared with that for charginos and scalar fermions (see forthcoming discussion) we have not calculated the cross section for  $v \neq v'$ . We should mention, however, that for  $v \neq v'$ , the  $Z^0$  can directly couple to the neutralinos (via their Higgs-fermion components) and so some of the cross sections could increase particularly at the  $Z^0$  pole. Since for small  $m_{\tilde{\gamma}}$ , the  $\tilde{\gamma}$  is almost a pure gaugino, it would couple only weakly but the  $\tilde{Z}$  and  $\tilde{Z}_h$  couplings could be significant (depending on  $v'/v$ ); the  $R$  value could be significantly affected, particularly in the

vicinity of the  $Z^0$  pole.<sup>27</sup> We will not discuss this point any further in this paper but now turn to a discussion of the charged sector.

The production of  $\tilde{W}$  and  $\tilde{W}_h$  pairs takes place via  $s$ -

channel  $\gamma$  and  $Z^0$  exchanges and via  $t$ -channel scalar-neutrino exchange whereas the photon contribution is absent for  $\tilde{W}\tilde{W}_h + \tilde{W}_h\tilde{W}$  production. The cross sections for these processes are given by

$$\begin{aligned}
\sigma(e^+e^- \rightarrow \tilde{W}\tilde{W}) &= \frac{e^4}{128\pi s} \frac{p}{E} \left\{ \frac{32}{3} \left[ 1 + \frac{2m_{\tilde{W}}^2}{s} \right] \right. \\
&+ \frac{32 \cot^2 \theta_W s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \left[ (x_c^2 + y_c^2)(\alpha_e^2 + \beta_e^2) \frac{s + 2m_{\tilde{W}}^2}{3} - 2y_c^2(\alpha_e^2 + \beta_e^2)m_{\tilde{W}}^2 \right] \\
&+ \frac{\sin^4 \gamma_R}{\sin^4 \theta_W} \left[ 2 - \frac{m_{\tilde{\nu}}^2 - m_{\tilde{W}}^2}{Ep} \ln A + \frac{2(m_{\tilde{\nu}}^2 - m_{\tilde{W}}^2)^2}{(m_{\tilde{\nu}}^2 - m_{\tilde{W}}^2)^2 + m_{\tilde{\nu}}^2 s} \right] \\
&- \frac{64}{3} \cot \theta_W (s - m_Z^2) \alpha_e x_c \frac{s + 2m_{\tilde{W}}^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \\
&+ \frac{4 \cot \theta_W}{\sin^2 \theta_W} \sin^2 \gamma_R \frac{(s - M_Z^2)(\alpha_e - \beta_e)s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \\
&\times \left[ (x_c - y_c) \left[ 1 - \frac{2(m_{\tilde{\nu}}^2 - m_{\tilde{W}}^2)}{s} + \frac{(m_{\tilde{\nu}}^2 - m_{\tilde{W}}^2)^2 + m_{\tilde{W}}^2 s}{2Eps} \ln A \right] + y_c \frac{m_{\tilde{W}}^2}{Ep} \ln A \right] \\
&- \left. \frac{4 \sin^2 \gamma_R}{\sin^2 \theta_W} \left[ 1 - \frac{2(m_{\tilde{\nu}}^2 - m_{\tilde{W}}^2)}{s} + \frac{(m_{\tilde{\nu}}^2 - m_{\tilde{W}}^2)^2 + m_{\tilde{W}}^2 s}{2Eps} \ln A \right] \right\}, \tag{21}
\end{aligned}$$

where  $p$  is the momentum of the  $\tilde{W}$  [ $p = (E^2 - m_{\tilde{W}}^2)^{1/2}$ ], and

$$A = \frac{2E(E+p) + m_{\tilde{\nu}}^2 - m_{\tilde{W}}^2}{2E(E-p) + m_{\tilde{\nu}}^2 - m_{\tilde{W}}^2}.$$

The cross section for  $\tilde{W}_h$  pair production can be obtained from Eq. (21) by replacing  $(x_c, y_c)$  by  $(x_s, y_s)$ , and  $m_{\tilde{W}}$  and  $p$  by the  $\tilde{W}_h$  mass and momentum, respectively. We have checked that Eq. (21) reduces to our previous answer<sup>14</sup> for  $v = v'$ . Finally, for  $\tilde{W}\tilde{W}_h$  production, we have

$$\begin{aligned}
\sigma(e^+e^- \rightarrow \tilde{W}\tilde{W}_h) &= \sigma(e^+e^- \rightarrow \tilde{W}\tilde{W}_h) \\
&= \frac{e^4}{128\pi} \frac{p}{E} \left\{ \frac{8(c+t)^2(\alpha_e^2 + \beta_e^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \left[ (x^2 + y^2) \left[ E^2 - \Delta^2 - m_{\tilde{W}}m_{\tilde{W}_h} + \frac{p^2}{3} \right] + 2x^2m_{\tilde{W}}m_{\tilde{W}_h} \right] \right. \\
&+ \frac{\sin^2 \gamma_R \cos^2 \gamma_R}{\sin^4 \theta_W} \frac{1}{s} \left[ 2 - \frac{2a}{p} \ln \frac{E+p+a}{E-p+a} + \frac{a^2 - \Delta^2}{p} \left[ \frac{1}{E-p+a} - \frac{1}{E+p+a} \right] \right] \\
&- \frac{2\theta_y(c+t)\sin \gamma_R \cos \gamma_R (s - m_z^2)(\alpha_e - \beta_e)}{\sin^2 \theta_W (s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \\
&\times \left[ (x-y) \frac{1}{2Ep} \left[ 2Ep - 2ap + (a^2 - m_{\tilde{W}}m_{\tilde{W}_h} - \Delta^2) \ln \frac{E+p+a}{E-p+a} \right] \right. \\
&+ \left. \frac{xm_{\tilde{W}}m_{\tilde{W}_h}}{Ep} \ln \frac{E+p+a}{E-p+a} \right] \left. \right\}, \tag{22}
\end{aligned}$$



with  $p$  being the  $\tilde{W}$  momentum,  $\Delta = (m_{\tilde{W}_h}^2 - m_{\tilde{W}}^2)/4E$ , and  $a = (2E\Delta + m_{\tilde{\nu}}^2 - m_{\tilde{W}_h}^2)/2E$ . The other constants that appear in Eqs. (21) and (22) are defined in the previous section. Equation (21) agrees with the  $\tilde{W}\tilde{W}$  production cross section given in the recent paper of Bartl *et al.*<sup>28</sup> In Eq. (22) all terms involving  $m_{\tilde{W}}m_{\tilde{W}_h}$  have superficially an opposite sign from those in Ref. 28. We note, however, that in our couplings we have explicitly made the transformation  $\tilde{W} \rightarrow \gamma, \tilde{W}$  (Ref. 29) [see Eq. (7a)] whereas the authors of Ref. 28 have not. This flips the sign of all  $m_{\tilde{W}}m_{\tilde{W}_h}$  terms so that we are in agreement with Eq. (10) of Ref. 28.

The  $R$  value from the  $\tilde{W}$  and  $\tilde{W}_h$  processes is shown in Figs. 4(a)–4(c) for  $m_{\tilde{W}} = 30, 60,$  and  $75$  GeV and for three values of  $v'/v$ . Apart from the “favored” value  $v'/v = 1$ , we have used the extreme value of  $v'/v$  allowed by the constraint  $m_{\tilde{W}} \leq m_{\tilde{W}}^{\max}$  [see Fig. 1(a)] for  $m_{\tilde{W}} = 8$  GeV. As in Fig. 3, we have fixed the scalar-neutrino mass in each case by requiring that  $\langle m_{\tilde{q}}^2 \rangle = (100 \text{ GeV})^2$ . The values of the ratio  $v'/v$ , the heavy- $\tilde{W}$  and scalar-neutrino masses and the mixing angles  $\gamma_L$  and  $\gamma_R$  are shown in Table II. The following features are worth noting.

(i) The total cross section for chargino production is quite large even for relatively large  $\tilde{W}$  masses. In the case of large values of  $m_{\tilde{W}}, m_{\tilde{W}_h}$  is small (compared to  $m_{\tilde{W}_h}$  for smaller  $\tilde{W}$  masses) and the contribution of heavy  $\tilde{W}$  pairs to the chargino cross section is significant for larger beam energies, particularly for  $v'/v < 1$ . The largeness of the cross section is due to the large (weak isovector) coupling of the gauginos to  $Z^0$ . As explained earlier our numerical results are presented only for  $\mu_2 \ll m_1$  [although Eqs. (21) and (22) and the diagonalization of the chargino mass matrix of the previous section is valid for all values of  $\mu_2$ ]. Some numerical results for  $\mu_2 \gg 2m_1$  have been considered in Ref. 27 whereas energy and angular distributions of the decay products of the  $\tilde{W}$  have been studied in Ref. 28.

(ii) We warn the reader of one feature of  $\tilde{W}$  production

TABLE II. The masses and mixings for different values of  $m_{\tilde{W}}$  and  $v'/v$  that determine the chargino cross section in Fig. 4.

$m_{\tilde{W}}$ (GeV)	$v'/v$	$m_{\tilde{W}_h}$ (GeV)	$m_{\tilde{\nu}}$ (GeV)	$\tan\gamma_L$	$\tan\gamma_R$
30	0.234	128	66	−0.51	−117 718
	1.0	407	91	−5.07	−5.07
	4.27	128	111	−11 023	−0.51
60	0.574	105	78	−0.23	−38 360
	1.0	135	91	−1.79	−1.79
	1.74	105	102	−3821	−0.23
75	0.808	93	86	−0.19	−12 247
	1.0	99	91	−1.35	−1.35
	1.24	93	96	−1408	−0.18

that is not at all evident from the figure. For the  $v'/v \leq 1$  curves, the contribution of  $\tilde{W}$  pairs to  $R$  ( $R_{\tilde{W}\tilde{W}}$ ) has a shallow valley, i.e., it rises to a maximum, then rises again (it is this second rise that is not evident). For example, in Fig. 4(b), for  $E = 130$  GeV,  $R_{\tilde{W}\tilde{W}} = 1.23$  (1.77) for  $v'/v = 1.0$  (1.74). In other words, the contribution of  $R_{\tilde{W}\tilde{W}_h}$  is not as large as it looks in the figure, particularly for  $v'/v = 1$ .

(iii)  $\tilde{W}\tilde{W}_h$  pair production is, in general, small. For small values of  $v'/v$ , it is essentially negligible and is largest for  $v'/v = 1$ . This can be understood if we recognize that  $\tilde{W}\tilde{W}_h$  pairs are dominantly produced by scalar-

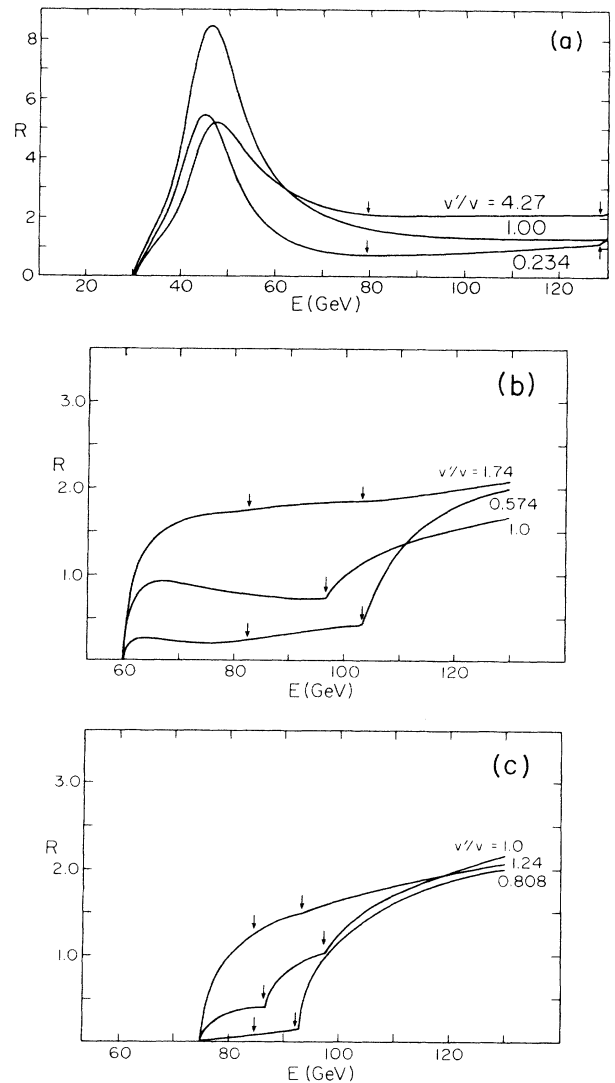


FIG. 4. The  $R$  ratio for production of  $\tilde{W}\tilde{W} + \tilde{W}\tilde{W}_h + \tilde{W}_h\tilde{W}_h$  for various values of  $v'/v$  for (a)  $m_{\tilde{W}} = 30$  GeV; (b)  $m_{\tilde{W}} = 60$  GeV; and (c)  $m_{\tilde{W}} = 75$  GeV. The small arrows show the thresholds for  $\tilde{W}\tilde{W}_h$  and  $\tilde{W}_h\tilde{W}_h$ . The heavy- $\tilde{W}$  mass depends on the value of  $v'/v$  but can be deduced from the position of the  $\tilde{W}\tilde{W}_h$  threshold.

neutrino exchange. Since the coupling of  $\tilde{W} (\tilde{W}_h)$  to  $\tilde{\nu}$  is proportional to  $\sin\gamma_R (\cos\gamma_R)$  it follows from Table II that for  $v'/v$  small, only  $Z^0$  exchange contributes to this process. Moreover,  $\sin\alpha_R \cos\alpha_R$  [in Figs. 4(b) and 4(c)] is largest for  $v'/v=1$  which explains the ordering of the  $\tilde{W} \tilde{W}_h$  contributions in Fig. 4.

(iv) From Table II and Eq. (14d) it is clear that the  $t$ -channel scalar-neutrino exchange gives large contributions to  $\tilde{W} (\tilde{W}_h)$  pair production when  $v'/v$  is small (large), which accounts for the shapes of the extreme curves in Figs. 4(b) and 4(c). The  $v'/v=1$  curves can be understood as an intermediate situation.

(v) Regarding  $\tilde{W} \tilde{W}$  production in Fig. 4(a) ( $E < m_{\tilde{W}_h}$ ), we first note that the interference term between the  $Z^0$  and the  $t$ -channel scalar neutrino is positive below the pole and negative above. Since the scalar-neutrino couplings are much bigger for the  $v'/v$  small case, its effect is to enhance the cross section below  $E=M_Z/2$  and suppress it above, thereby accounting for the crossover of the  $v'/v=0.234$  and  $4.27$  curves. The  $v'/v=1$  curve has the largest cross section since both left- and right-handed

$\tilde{W}$ 's are largely gauginos (see Table II) and so the  $Z\tilde{W}\tilde{W}$  vertex is largest for this case. This concludes our discussion of the gaugino production cross sections. In the next section we turn to a discussion of scalar-fermion production.

#### IV. SCALAR-FERMION PRODUCTION

In this section, we present cross section formulas for the production of scalar quarks, scalar electrons, scalar muons, and scalar  $\tau$ 's and also the rates for the various processes. Scalar-quark, scalar-muon, and scalar- $\tau$  production takes place via the  $s$ -channel  $\gamma$  and  $Z^0$  annihilation graphs whereas scalar-electron pair production also occurs via  $t$ -channel exchanges of the  $\tilde{\gamma}$ , the  $\tilde{Z}$ , and the  $\tilde{Z}_h$  (Refs. 31–33). Unlike the  $s$ -channel graphs which lead to  $\tilde{f}_L \tilde{f}_L$  or  $\tilde{f}_R \tilde{f}_R$  ( $f=e, \mu, \tau, \text{ or } q$ ) pairs, the  $t$ -channel exchanges also lead to  $\tilde{e}_L \tilde{e}_R$  and  $\tilde{e}_R \tilde{e}_L$  pair production. The cross section for the production of scalar muons, scalar  $\tau$ 's, or scalar quarks is given by ( $f=\mu, \tau, u, d$  and  $i=L, R$ )

$$\sigma(e^+e^- \rightarrow \tilde{f}_i \tilde{f}_i) = \frac{N_f e^4 \beta^3}{192\pi} \left[ \frac{4q_f^2}{s} + \frac{A_f^2(\alpha_e^2 + \beta_e^2)s - 4\alpha_e q_f A_f(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \right], \quad (23)$$

where  $N_f=1$  (3) for scalar leptons (scalar quarks),  $\beta \equiv (1 - 4m_{\tilde{f}}^2/s)^{1/2}$ , and  $A_f=2(\alpha_f - \beta_f)$  or  $2(\alpha_f + \beta_f)$  for left- and right-handed scalar fermions, respectively. The other parameters are as in Table I. The cross section for the production of left- (right-) handed scalar electron pairs is given by ( $i=L, R$ )

$$\begin{aligned} \sigma(e^+e^- \rightarrow \tilde{e}_i \tilde{e}_i) = & \frac{e^4}{384\pi} \beta^3 \left\{ \frac{8}{s} + \frac{2A_e^2(\alpha_e^2 + \beta_e^2)s + 8\alpha_e A_e(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} - C_i^4 F(a_{\tilde{Z}}) - B_i^4 F(a_{\tilde{\gamma}}) - D_i^4 F(a_{\tilde{Z}_h}) - 2C_i^2 G(a_{\tilde{Z}}) \right. \\ & - 2B_i^2 G(a_{\tilde{\gamma}}) - 2D_i^2 G(a_{\tilde{Z}_h}) - \frac{A_e(\alpha_e + \beta_e)s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} [C_i^2 G(a_{\tilde{Z}}) + B_i^2 G(a_{\tilde{\gamma}}) + D_i^2 G(a_{\tilde{Z}_h})] \\ & + \frac{2B_i^2 C_i^2 E^2}{m_{\tilde{Z}}^2 - m_{\tilde{\gamma}}^2} [G(a_{\tilde{\gamma}}) - G(a_{\tilde{Z}})] + \frac{2B_i^2 D_i^2 E^2}{m_{\tilde{Z}_h}^2 - m_{\tilde{\gamma}}^2} [G(a_{\tilde{\gamma}}) - G(a_{\tilde{Z}_h})] \\ & \left. + \frac{2C_i^2 D_i^2 E^2}{m_{\tilde{Z}_h}^2 - m_{\tilde{Z}}^2} [G(a_{\tilde{Z}}) - G(a_{\tilde{Z}_h})] \right\} \quad (24) \end{aligned}$$

with

$$F(x) = \frac{3}{p^2} - \frac{3}{2} \frac{E+x}{p^3} \ln \frac{E+p+x}{E-p+x}, \quad (25a)$$

$$G(x) = \frac{3}{4E} \left[ \frac{2}{p^2} (E+x) + \frac{1}{p} \ln \left[ \frac{E+p+x}{E-p+x} \right] - \frac{(E+x)^2}{p^3} \ln \left[ \frac{E+p+x}{E-p+x} \right] \right], \quad (25b)$$

and

$$a_x = (m_x^2 - m_{\tilde{e}_i}^2) / 2E. \quad (25c)$$

In Eqs. (24) and (25),  $E$  is the beam energy and  $p$  is the momentum of the produced scalar fermion. The minus (plus) sign in Eq. (24) refers to production of left- (right-) handed scalar electrons. For the  $\gamma$  and  $\tilde{\gamma}$  exchanges our result agrees with Ref. 31. Finally, the cross section for the production of  $\tilde{e}_L \tilde{e}_R$  or  $\tilde{e}_R \tilde{e}_L$  pairs is given by

$$\begin{aligned}
\sigma(e^+e^- \rightarrow \tilde{e}_L \tilde{e}_R) &= \sigma(e^+e^- \rightarrow \tilde{e}_L \tilde{e}_R) \\
&= \frac{e^4}{128\pi} \frac{p}{sE} \left[ \frac{2B_L^2 B_R^2 m_{\tilde{\gamma}}^2}{m_{\tilde{\gamma}}^2 + a_{\tilde{\gamma}}^2} + \frac{2C_L^2 C_R^2 m_{\tilde{Z}}^2}{m_{\tilde{Z}}^2 + a_{\tilde{Z}}^2} \right. \\
&\quad + \frac{2D_L^2 D_R^2 m_{\tilde{Z}_h}^2}{m_{\tilde{Z}_h}^2 + a_{\tilde{Z}_h}^2} + \frac{2B_L B_R C_L C_R m_{\tilde{Z}} m_{\tilde{\gamma}}}{p(a_{\tilde{\gamma}} - a_{\tilde{Z}})} \ln \frac{(E + a_{\tilde{\gamma}} - p)(E + a_{\tilde{Z}_h} + p)}{(E + a_{\tilde{Z}} - p)(E + a_{\tilde{\gamma}} + p)} \\
&\quad + \frac{2B_L B_R D_L D_R m_{\tilde{\gamma}} m_{\tilde{Z}_h}}{p(a_{\tilde{\gamma}} - a_{\tilde{Z}_h})} \ln \frac{(E + a_{\tilde{\gamma}} - p)(E + a_{\tilde{Z}_h} + p)}{(E + a_{\tilde{Z}_h} - p)(E + a_{\tilde{\gamma}} + p)} \\
&\quad \left. + \frac{2C_L C_R D_L D_R m_{\tilde{Z}} m_{\tilde{Z}_h}}{p(a_{\tilde{Z}} - a_{\tilde{Z}_h})} \ln \frac{(E + a_{\tilde{Z}} - p)(E + a_{\tilde{Z}_h} + p)}{(E + a_{\tilde{Z}_h} - p)(E + a_{\tilde{Z}} + p)} \right]. \tag{26}
\end{aligned}$$

The constants  $B_i$ ,  $C_i$ , and  $D_i$  that appear in Eqs. (24)–(26) are as defined in Sec. II.

We now turn to a discussion of the numerical results for the scalar-fermion cross sections. The total cross section, of course, depends on the values of the masses of the various scalars which, in turn, are determined by the average scalar-fermion mass  $m_0$ ,  $v'/v$ , and  $m_{\tilde{\gamma}}$  [see Eq. (15) and the following discussion]. As with the rest of this paper we have chosen  $m_{\tilde{\gamma}} = 8$  GeV. Shown in Fig. 5 is the total contribution to  $R$  for the favored case  $v'/v = 1$  and for three different values of average scalar-lepton (-quark) mass for three charged scalar leptons and five scalar quarks. We have not included the scalar-neutrino cross section in the figure because light scalar neutrinos are ex-

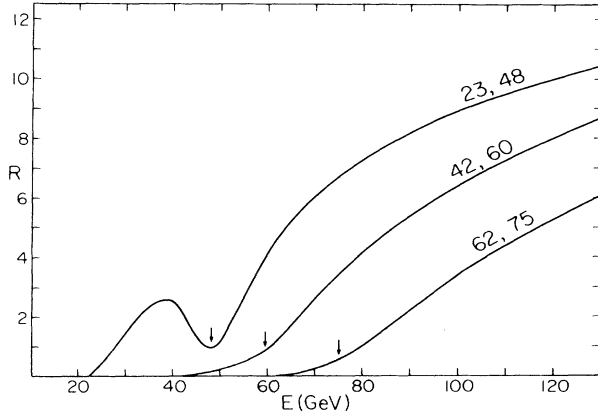


FIG. 5. The  $R$  ratio for the production of supersymmetric partners of quarks and leptons for  $v'/v = 1$ . The curves are labeled by three choices for the average mass of the scalar lepton ( $\bar{m}_l$ ) and average mass of the scalar quark ( $\bar{m}_q$ ) in GeV. The other parameters are those of the  $m_{\tilde{W}} = 75$  GeV case;  $m_{\tilde{\gamma}} = 8$  GeV,  $m_{\tilde{Z}} = 84$  GeV,  $m_{\tilde{Z}_h} = 110$  GeV. For lighter  $m_{\tilde{Z}}$  this scalar contribution increases slightly for  $E$  above about 100 GeV.

pected to decay invisibly via the  $\tilde{\nu} \rightarrow \nu \tilde{\gamma}$  mode<sup>34</sup> nor have we included the top scalar quark although, in principle, one combination of  $\tilde{t}_L$  and  $\tilde{t}_R$  may be light.<sup>37</sup> The following remarks are in order.

(i) More than half the contribution to  $R$  comes from scalar-electron pair production which has a large contribution to  $R_{\tilde{e}_i \tilde{e}_i}$  from the  $t$ -channel exchange of a light photino. Except where threshold effects are important, the total scalar-quark contribution is  $\sim \frac{2}{3}$  that of the scalar electron.

(ii) The bump below  $E = M_Z/2$  for the  $\bar{m}_l = 23$  GeV curve is also due to the large scalar-electron cross section—over 80% of the total for  $E \sim 35$  GeV. At the  $Z^0$  pole all the scalar leptons contribute almost the same and the  $R$  value is suppressed by a factor  $\beta^3$  and a spin factor.

(iii) The contribution of  $\tilde{e}_L \tilde{e}_R + \tilde{e}_R \tilde{e}_L$  pairs is always smaller than 25% of the total  $\tilde{e} \tilde{e}$  contribution. This is quite sensitively dependent on the photino mass.

(iv) The value of  $R$  is independent of the gaugino masses except for scalar-electron production. The curves shown are for  $m_{\tilde{W}} = 75$  GeV (corresponding to  $m_{\tilde{Z}} = 84$  GeV and  $m_{\tilde{Z}_h} = 110$  GeV). For lighter values of  $\tilde{Z}$  masses, the curves change only slightly for  $E < 90$  GeV. At higher energies, the  $R$  value increases slightly—by about 0.5 for the extreme case of  $m_{\tilde{Z}} = 30$  GeV<sup>7</sup> and  $E = 130$  GeV. (See also the  $v'/v = 1$  curve in Fig. 6 for which  $m_{\tilde{W}} = 30$  GeV, corresponding to  $m_{\tilde{Z}} = 33$  GeV and  $m_{\tilde{Z}_h} = 412$  GeV.)

(v) We mention here that because of the effect of additional mixings due to nonzero photino mass, the  $e \tilde{e}_L \tilde{\gamma}$  and  $e \tilde{e}_R \tilde{\gamma}$  couplings are not exactly equal. Thus,  $\tilde{e}_L \tilde{e}_L$  and  $\tilde{e}_R \tilde{e}_R$  production cross sections are not exactly equal. By the same token, the cross section for  $e^+ e^- \rightarrow \tilde{\gamma} \tilde{\gamma}$  gets different contributions from  $\tilde{e}_L$  and  $\tilde{e}_R$  exchanges. This could be significant for single-photon experiments<sup>8</sup> particularly if only one of  $\tilde{e}_L$  or  $\tilde{e}_R$  is light enough to be important.<sup>36</sup>

(vi) Both  $\tilde{W}$  and scalar-lepton pair production lead to acollinear lepton pairs in the event. Scalar leptons can be

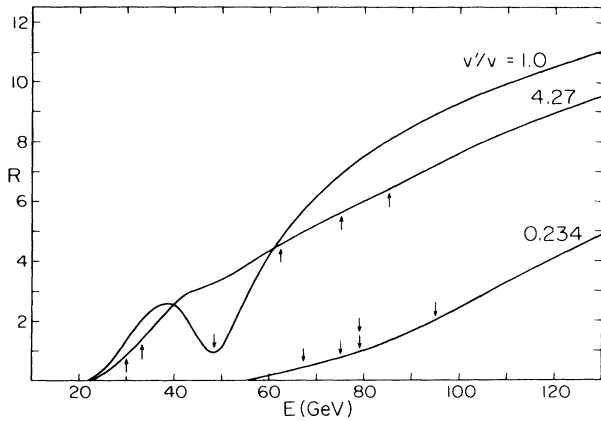


FIG. 6. The  $R$  ratio for the production of supersymmetric scalars for several values of  $v'/v$ . The other parameters are as for  $m_{\tilde{W}}=30$  GeV with these  $v'/v$  values. The small arrows indicate the various thresholds for the three cases. For  $v'/v=0.234$  the masses of  $\tilde{d}_L$ ,  $\tilde{d}_R$ ,  $\tilde{u}_R$ ,  $\tilde{u}_L$ ,  $\tilde{e}_L$ , and  $\tilde{e}_R$  are 95, 78, 67, 55, 79, and 75 GeV, respectively. For  $v'/v=4.27$  the same masses are 33, 62, 75, 85, 22, and 30 GeV. For  $v'/v$  equal to the scalar-quark masses are all 48 GeV, while  $\tilde{e}_L$  and  $\tilde{e}_R$  are 24 and 22 GeV.

distinguished from  $\tilde{W}$ 's by the fact that there are equal numbers of  $e\bar{\mu} + \mu\bar{e}$  pairs as  $e\bar{e} + \mu\bar{\mu}$  pairs in the  $\tilde{W}$  case, and from the fact that the energy distribution of a lepton coming from the decay of a scalar lepton is flat.<sup>32</sup> Details of the energy and angular distributions can be found in Refs. 30 and 31.

We now turn to a brief discussion of the  $v'/v \neq 1$  case. As has been emphasized in Sec. II, our couplings for the neutralinos are valid only for  $v'=v$  and so it would appear that a complete reanalysis would be necessary. We note, however, that  $\tilde{\gamma}^{(0)}$  is a zero-mass eigenstate of the neutralino mass matrix even for  $v'/v \neq 0$  provided  $\mu_1 = \mu_2 = 0$ . Thus, up to the modifications due to nonzero gaugino masses, our photino couplings are still valid. Also, we saw that the  $\tilde{Z}$  and  $\tilde{Z}_h$  contributions to scalar-electron pair production were small. Thus our couplings of Sec. II may be used to get an estimate of the scalar-fermion cross section even for  $v'/v \neq 1$ . The dominant effect of  $v'/v$  shows itself through the scalar-fermion masses [see Eq. (15)]. This would effect the thresholds at which the various particles come in.

To illustrate this, in Fig. 6 we have plotted the  $R$  value from scalar fermions for the extreme range of  $v'/v$  values we have considered as acceptable in this paper. We have chosen the parameters that determine the masses in the following fashion: (i) the lightest charged SUSY particle should be heavier than 23 GeV and (ii)  $m_{\tilde{\nu}}^2 > 0$  so as not to break lepton number. The second condition is important only for  $v'/v < 1$ , and for the extreme case we have considered, causes the other SUSY-particle masses to be rather large (for  $m_{\tilde{\nu}} = 1$  GeV the lightest charged particle is  $\tilde{u}_L$  with a mass of 55 GeV). This accounts for the smallness of  $R$  for this case. We make the following remarks.

(i) The largeness of the  $v'/v=4.27$  curve near the  $Z^0$  pole as compared with the  $v'/v=1$  case is because only the former receives contributions from three flavors of the down-type left-handed scalar quark. There would be a large cross section for  $Z^0 \rightarrow \text{jet(s)} + \text{missing energy}$  at LEP I for this case.

(ii) For the  $v'/v=1$  case all the scalar quarks (and also all the scalar leptons) were degenerate whereas the masses are spread out when  $v'/v \neq 1$ . Therefore, for  $v'/v=1$ , all the scalar quarks come in at a common value causing an abrupt rise in  $R$ . This is much more gradual for the  $v'/v \neq 1$  case and so accounts for the crossover of the  $v'/v=4.27$  and  $v'/v=1$  curves near  $E=60$  GeV. Notice also that the individual thresholds do not show up as abrupt kinks on the curve.

(iii) Excluding the  $\tilde{Z}$  and  $\tilde{Z}_h$  changes the  $v'/v \neq 1$  curves in Fig. 6 by a maximum of 5%. This "justifies" the use of the  $v'/v=1$  couplings for  $\tilde{Z}$  and  $\tilde{Z}_h$  as explained earlier.

## V. CONCLUDING REMARKS

In this paper we have presented a complete set of cross-section formulas for SUSY processes that may be relevant to SLC, LEP I, and LEP II energies. We have also shown component cross sections for the production of neutral and charged states in the gaugino-Higgs-fermion sector and for scalar fermions. Apart from expanding on our earlier Letter,<sup>14</sup> we have extended the analysis presented here in several ways as discussed in Sec. I. In particular, we have presented a fairly detailed analysis of the chargino sector for a wide variety of models. It is important to note that for a given value of  $m_{\tilde{\nu}}$ , the PETRA limit<sup>6</sup> on the  $\tilde{W}$  mass already restricts the allowed range of  $v'/v$  (in the two doublet models considered here) as shown in Fig. 1(b). An improvement in the lower limit on the  $\tilde{W}$  mass would thus translate into restrictions on the couplings in the model.

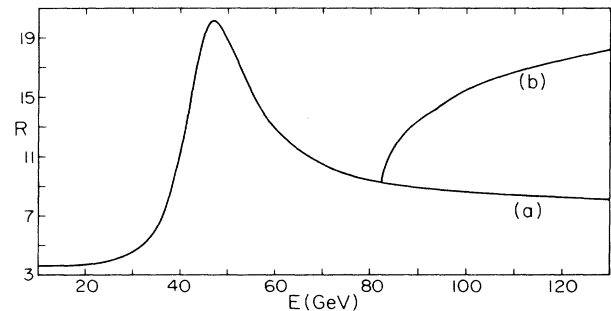


FIG. 7. The  $R$  ratio in the standard model. Curve (a) is for the production of quarks only and includes a 50-GeV top quark. Curve (b) includes the quarks of (a) plus production of  $WW$  and  $ZZ$ . We have not included the production of  $Z\gamma$  which has a very large cross section because it may be possible to identify this process and subtract it out. The  $Z$  width is 2.64 GeV. The  $\mu\mu$  production cross section, used in the denominator of  $R$ , includes  $Z$  exchange as well as photon exchange.

Our result for the  $R$  values from different SUSY sources are shown in Figs. 3–6. We see that for all values of  $v'/v$  and  $m_{\tilde{W}}$ , the gaugino–Higgs-fermion contribution to  $R$  is greater than nearly two units somewhere in the energy range  $E < 130$  GeV. This is a rather model-independent result (except we have restricted our analysis to light photinos). Scalar-fermions may also contribute substantially to  $R$ , depending on their mass values. Within the framework of the supergravity mass relations (15), the behavior of  $R$  is shown in Figs. 5 and 6. For the convenience of the reader we have shown the  $R$  value in the standard model<sup>37</sup> including a 50-GeV top quark in Fig. 7. We note that the cross section for pair production of gauge bosons (which would lead to qualitatively similar signatures as the SUSY processes considered here) is much larger than the SUSY cross sections, and so, one may well be forced to search for SUSY below the  $W$ -pair threshold.

We recognize that identification of SUSY particles at very-high-energy  $e^+e^-$  colliders ( $2E \gg M_Z$ ) will rely on

a detailed analysis<sup>36</sup> of various signatures, since the cross sections are not very large. We have not considered these here, but there already exist various analyses of these in the literature.<sup>4,26,28–33</sup> It will have to be seen whether the machines scheduled to go into operation at KEK, Stanford, or CERN turn up signatures for supersymmetry.

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