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Mass splittings in quasi Nambu-Goldstone models

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Spontaneous internal-global-symmetry breaking in a supersymmetric theory is nonlinearly realized by Goldstone superfields. We discuss a mechanism by which the components of the Goldstone supermultiplets are given nonzero mass splittings by a generalized current-current coupling to a hidden O'Raifeartaigh sector. Particular attention is focused on a U(6)/U(4)×SU(2) model.

I. INTRODUCTION

The introduction of composite structure for quarks and leptons is an attempt to better understand their complicated mass patterns and the replication of generations. All such preon models, however, face an immediate serious constraint. Since no substructure of leptons or quarks has been yet discovered, it follows that the momentum scale Λ of the preonic binding energy must be at least of order 100 GeV$-$1 TeV which is much larger than the physical masses of the quarks and leptons. For gauge theories to give satisfactory explanation of this behavior, some symmetry must be imposed to force the composite states to be light.

One such symmetry mechanism[1] utilizes the spontaneous breakdown of a global internal symmetry in the presence of a rigid supersymmetry (SUSY). More specifically, when a supersymmetric preon theory with internal-symmetry group G is spontaneously broken down to a subgroup H via preon condensate formations, Goldstone supermultiplets are generated as preon bound states. In particular, the fermionic partners of Goldstone bosons, which are massless as a result of the supersymmetry, are identified with the quarks and leptons. The Goldstone multiplet interactions are determined by constructing the nonlinear realization of the $G\rightarrow H$ breakdown. SUSY requires that the Goldstone superfields take values on a Kählerian coset space.[2-5]

Such SUSY preon models will not be complete until a mechanism is introduced to break supersymmetry and generate the nonzero-mass spectrum for quarks and leptons while at the same time lifting all the scalar partner masses above the present experimental limits.[6] In this paper we study a model involving a hidden O'Raifeartaigh sector[7] to accomplish these goals. Imagine a G-invariant supersymmetric sector where supersymmetry is spontaneously broken via an O'Raifeartaigh mechanism. The idea is to construct an interaction connecting the hidden O'Raifeartaigh sector with the observable Goldstone sector so that the Goldstone supermultiplets acquire nonzero masses once the supersymmetry is broken. This technique is reminiscent of the method used to break SUSY in supersymmetric extensions of the standard model using a hidden $N=1$ supergravity sector.

We shall illustrate the mechanism in the context of a specific model[8] based on the coset $G/H=\mathrm{U}(6)/\mathrm{U}(4)\times\mathrm{SU}(2)$. The super-Kähler potential describing the interactions of the Goldstone supermultiplets in the unbroken SUSY case are reviewed in Sec. II. In Sec. III we introduce the hidden O'Raifeartaigh model spontaneously breaking the SUSY as well as explicitly but softly breaking the internal G symmetry. When this sector is coupled to the Goldstone multiplets, we find, at the tree level, all bosonic degrees of freedom and exotic fermions acquire large nonzero masses while one generation of fermions remains massless. Thus, there is an encouraging indication that in this class of supersymmetric preon models, it is possible for the composite fermions to remain light, while the boson masses are made large.

For clarity, we shall indicate the fields in the Goldstone sector by the lower case letters ϕ,a,ψ,f, while the upper case letters Φ,A,Ψ,F are reserved for the fields in the O'Raifeartaigh sector.

II. U(6)/U(4)×SU(2) SUPERSYMMETRIC NONLINEAR REALIZATION

Zumino[2] has shown that the construction of supersymmetric extensions of nonlinear models requires the introduction of a super-Kähler potential. The coset manifold G/H is demanded to be Kählerian such that the two-form

$$\omega=\frac{i}{2}g_{i\bar{j}}(\mathbf{A}^{\dagger},\mathbf{A})dA^{i}\wedge dA^{\dagger j} \tag{2.1}$$

is closed, $d\omega=0$. Here A^i is a complex scalar field and the metric $g_{i\bar{j}}$ is Hermitian. A G-invariant and supersymmetric action can then be constructed as

$$I=\int dV K(\boldsymbol{\phi}^{\dagger},\boldsymbol{\phi}) , \tag{2.2}$$

where

$$dV=d^4x\, d^2\theta\, d^2\bar{\theta} , \tag{2.3}$$

and the super-Kähler potential is such that

$$g_{i\bar{j}}(\boldsymbol{\phi}^{\dagger},\boldsymbol{\phi})=\frac{\partial^2K(\boldsymbol{\phi}^{\dagger},\boldsymbol{\phi})}{\partial\phi^{\dagger i}\partial\phi^{j}}\equiv K_{\bar{i}j} . \tag{2.4}$$

The (anti)chiral Goldstone superfields $(\boldsymbol{\phi}^{\dagger})\boldsymbol{\phi}$ have as components the Goldstone bosons, quasi-Goldstone bosons (from the complexification of G/H coset), and quasi-Goldstone fermions—the supersymmetric fermionic partners. It is these quasi-Goldstone fermions which are to be identified with the composite quarks and leptons, the massless preon bound states. To accommodate the standard quarks and leptons with appropriate quantum numbers, one must carefully choose the group structure G/H. Buchmüller, Peccei, and Yanagida[8] (BPY) suggested the coset manifold $U(6)/U(4)\times SU(2)$ to account for eight left-handed fermions of one generation of quarks and leptons.

We begin with the U(6) algebra which is given in standard form ($A,B,\ldots=1,\ldots,6$) by

$$[T_B^A,T_D^C]=\delta_D^A T_B^C-\delta_B^C T_D^A , \tag{2.5}$$

where the matrix generators in the fundamental representation are taken to be ($I,J=1,\ldots,6$)

$$(T_B^A)_{IJ}=\delta_{IB}\delta_{JA} . \tag{2.6}$$

There are $16+3=19$ unbroken generators ($a,b=1,\ldots,4$; $i,j=5,6$) given by

$$L_a^{\dagger b}=L_b^a\equiv T_b^a , \tag{2.7a}$$

$$L_i^{\dagger j}=L_j^i\equiv T_j^i-\tfrac{1}{2}\delta_j^i T_k^k , \tag{2.7b}$$

and $36-19=17$ broken generators denoted by

$$X_a^{\dagger i}=X_i^a\equiv T_i^a , \tag{2.8a}$$

$$X_i^{\dagger a}=X_a^i\equiv T_a^i , \tag{2.8b}$$

$$X_0^\dagger=X_0\equiv\frac{1}{\sqrt{2}}T_k^k . \tag{2.8c}$$

The U(6) algebra can then be expressed explicitly in the form

$$[L_B^A,L_D^C]=\delta_D^A T_B^C-\delta_B^C T_D^A , \tag{2.9}$$

$$[L_b^a,X_i^c]=-\delta_b^c X_i^a , \tag{2.9a}$$

$$[L_b^a,X_c^i]=\delta_c^a X_b^i , \tag{2.9b}$$

$$[L_j^i,X_k^a]=\delta_k^i X_j^a-\tfrac{1}{2}\delta_j^i X_k^a , \tag{2.9c}$$

$$[L_j^i,X_a^k]=-\delta_j^k X_a^i+\tfrac{1}{2}\delta_j^i X_a^k , \tag{2.9d}$$

$$[L_b^a,X_0]=[L_j^i,X_0]=0 , \tag{2.9e}$$

$$[X_i^a,X_j^b]=[X_a^i,X_b^j]=0 , \tag{2.9f}$$

$$[X_i^a,X_b^j]=\delta_b^a L_j^i-\delta_i^j L_b^a+\frac{1}{\sqrt{2}}\delta_b^a\delta_i^j X_0 , \tag{2.9g}$$

$$[X_0,X_i^a]=\frac{1}{\sqrt{2}}X_i^a , \tag{2.9h}$$

$$[X_0,X_a^i]=-\frac{1}{\sqrt{2}}X_a^i . \tag{2.9i}$$

Corresponding to the broken generators, there are 17 Goldstone bosons. We can include 16 of these as the complex scalar components of the ϕ_i^a Goldstone superfields. On the other hand, the superfield ϕ_0 must be complexified to contain one Goldstone boson and one quasi-Goldstone boson. We can thus realize the SUSY with 17 Goldstone bosons, 1 quasi-Goldstone boson, and $(17+1)/2=9$ quasi-Goldstone fermions. Eight of the fermions are to be identified with the left-handed quarks and leptons at one family. The ninth fermion, coming from the superfield ϕ_0, was called "novino" by BPY. The Killing vectors defined by

$$\frac{1}{i}[T_B^A,\phi_i^a]=\mathbf{A}_{Bi}^{Aa},\quad \frac{1}{i}[T_B^A,\phi_0]=\mathbf{A}_B^A , \tag{2.10}$$

are determined as

$$\frac{1}{i}[X_i^a,\phi_j^b]=\left[\frac{1}{i}[X_a^i,\phi_b^{\dagger j}]\right]^\dagger=\phi_j^a\phi_i^b , \tag{2.10a}$$

$$\frac{1}{i}[X_a^i,\phi_j^b]=\frac{1}{i}[X_i^a,\phi_b^{\dagger j}]=\delta_j^i\delta_a^b , \tag{2.10b}$$

$$\frac{1}{i}[X_i^a,\phi_0]=\left[\frac{1}{i}[X_a^i,\phi_0^\dagger]\right]^\dagger=\frac{i}{\sqrt{2}}\phi_i^a , \tag{2.10c}$$

$$\frac{1}{i}[X_a^i,\phi_0]=\frac{1}{i}[X_i^a,\phi_0^\dagger]=0 , \tag{2.10d}$$

$$\frac{1}{i}[X_0,\phi_i^a]=\left[\frac{1}{i}[X_0,\phi_a^{\dagger i}]\right]^\dagger=\frac{-i}{\sqrt{2}}\phi_i^a , \tag{2.10e}$$

$$\frac{1}{i}[X_0,\phi_0]=\frac{1}{i}[X_0,\phi_0^\dagger]=1 , \tag{2.10f}$$

$$\frac{1}{i}[L_b^a,\phi_j^c]=i\delta_b^c\phi_j^a , \tag{2.10g}$$

$$\frac{1}{i}[L_b^a,\phi_c^{\dagger j}]=-i\delta_c^a\phi_b^{\dagger j} , \tag{2.10h}$$

$$\frac{1}{i}[L_j^i,\phi_k^c]=-i\delta_k^i\phi_j^c+\frac{i}{2}\delta_j^i\phi_k^c , \tag{2.10i}$$

$$\frac{1}{i}[L_j^i,\phi_c^{\dagger k}]=i\delta_j^k\phi_c^{\dagger i}-\frac{i}{2}\delta_j^i\phi_c^{\dagger k} , \tag{2.10j}$$

$$\begin{aligned}\frac{1}{i}[L_b^a,\phi_0]&=\frac{1}{i}[L_j^i,\phi_0]\\&=\frac{1}{i}[L_b^a,\phi_0^\dagger]=\frac{1}{i}[L_j^i,\phi_0^\dagger]=0 .\end{aligned} \tag{2.10k}$$

The totally G-invariant super-Kähler potential satisfying $\delta K_0=0$ can then be secured via a power-series expansion:

$$\begin{aligned}K_0(\boldsymbol{\phi}^\dagger,\boldsymbol{\phi})=&\sqrt{2}i{v_1}^2(\phi_0-\phi_0^\dagger)\\&-\frac{{v_2}^2}{2}(\phi_0-\phi_0^\dagger)^2+{v_1}^2\phi_a^{\dagger i}\phi_i^a\\&+\frac{i}{\sqrt{2}}v_2^2\phi_a^{\dagger i}\phi_i^a(\phi_0-\phi_0^\dagger)+\cdots ,\end{aligned} \tag{2.11}$$

where ${v_1}^2$ and ${v_2}^2$ are free parameters. [In general, only the G invariance of the D measure of the solution K is demanded to construct the supersymmetric action $I=\int dV K(\boldsymbol{\phi}^\dagger,\boldsymbol{\phi})$. The totally G-invariant solution K_0 differs from general solution K by a chiral superfield and an antichiral superfield:[9] $K_0(\boldsymbol{\phi}^\dagger,\boldsymbol{\phi})=K(\boldsymbol{\phi}^\dagger,\boldsymbol{\phi})-F(\boldsymbol{\phi})-F^\dagger(\boldsymbol{\phi}^\dagger)$.]

To achieve sensible results we need to expand the potential around its minimum obtained when $\langle\phi_0-\phi_0^\dagger\rangle=\sqrt{2}i{v_1}^2/{v_2}^2$, and then, by shifting the fields, $(\phi_0-\phi_0^\dagger)\to(\phi_0-\phi_0^\dagger)-\sqrt{2}i{v_1}^2/{v_2}^2$, and dropping the constant, we obtain the effective super-Kähler potential

$$K_0^{\rm eff}(\phi^\dagger,\phi)=-\frac{\nu_2^2}{2}(\phi_0-\phi_0^\dagger)^2 + \frac{i}{\sqrt{2}}\nu_2^2(\phi_0-\phi_0^\dagger)\phi_a^{\dagger i}\phi_i^a+\cdots \quad (2.12)$$

Note that, up to the cubic term, $K_0^{\rm eff}$ is the degenerate limit, $\nu_1^2\to 0$, of K_0. A shift of field,

$$\mathrm{Im}\phi_0\,|_{\theta=0}\to\mathrm{Im}\phi_0\,|_{\theta=0}+\frac{i}{\sqrt{2}}\frac{\nu_1^2}{\nu_2^2},$$

in $K_0^{\rm eff}$ can recover the original K_0. This reflects the fact that the imaginary part of $\phi_0\,|_{\theta=0}$ is the (only) quasi-Goldstone boson generated from the complexification of the superfield ϕ_0.

Using the Killing vectors, we can also calculate the global-internal-symmetry Goldstone current superfields[9]

$$j_B^A=-\frac{i}{2}\left[\frac{\partial K_0^{\rm eff}}{\partial\phi_a^{\dagger i}}\bar{A}\,^{Ai}_{Ba}-\frac{\partial K_0^{\rm eff}}{\partial\phi_0^\dagger}\bar{A}\,^A_B\right] \quad (2.13)$$

as

$$j_b^a=\text{cubic and higher terms}\,, \quad (2.13a)$$

$$j_j^i=\text{cubic and higher terms}\,, \quad (2.13b)$$

$$j_a^i=\frac{\nu_2^2}{\sqrt{2}}(\phi_0-\phi_0^\dagger)\phi_a^{\dagger i}+\cdots\,, \quad (2.13c)$$

$$j_i^a=\frac{\nu_2^2}{\sqrt{2}}(\phi_0-\phi_0^\dagger)\phi_i^a+\cdots\,, \quad (2.13d)$$

$$j_0\equiv\frac{1}{\sqrt{2}}j_k^k=i\nu_2^2(\phi_0-\phi_0^\dagger)-\frac{\nu_1^2-\nu_2^2}{\sqrt{2}}\phi_c^{\dagger k}\phi_k^c+\cdots\,. \quad (2.13e)$$

III. U(6) O'RAIFEARTAIGH HIDDEN SECTOR

In this section we will construct the hidden sector responsible for the supersymmetry breakdown of the theory. The only constraint we impose on this hidden sector is that the supersymmetry is spontaneously (or explicitly but softly) broken and no massless particles appear in the mass spectrum. Without any guidance from the underlying dynamics, we take the simplest O'Raifeartaigh model with only three superfields, to limit the number of independent coupling constants.

We introduce an U(6) singlet and two U(6) fundamental chiral superfields carrying neutral U(1) charges as

$$\Phi_0,\quad \Phi_1=\begin{bmatrix}\Phi_1^1\\ \Phi_1^2\\ \Phi_1^3\\ \Phi_1^{3c}\\ \Phi_1^{2c}\\ \Phi_1^{1c}\end{bmatrix},\quad \Phi_2=\begin{bmatrix}\Phi_2^1\\ \Phi_2^2\\ \Phi_2^3\\ \Phi_2^{3c}\\ \Phi_2^{2c}\\ \Phi_2^{1c}\end{bmatrix}, \quad (3.1)$$

where c indicates charge conjugation. The chiral superpotential gives rise to the Lagrangian

$$\begin{aligned}L_P&\equiv P(\Phi)\,|_F+\text{H.c.}\\ &=\left[\lambda\Phi_0+m\Phi_1\Phi_2+\frac{g}{2}\Phi_0\Phi_1\Phi_1\right]_F+\text{H.c.}\end{aligned} \quad (3.2)$$

with the charge-conjugation matrices in multiplications suppressed.

By solving the auxiliary field equations we obtain the effective bosonic potential (we employ the notation by Wess and Bagger[10]):

$$\begin{aligned}V&=\sum F^*F\\ &=\left|\lambda+\frac{g}{A}A_1^2\right|^2+|mA_2+gA_0A_1|^2+|mA_1|^2\,.\end{aligned} \quad (3.3)$$

An examination of the minimum of this scalar potential yields

$$\langle A_0\rangle=\langle A_1\rangle=\langle A_2\rangle=0 \text{ and } \langle V\rangle=|\lambda|^2>0\,.$$

Thus supersymmetry is spontaneously broken while the U(6) symmetry survives.

To break the U(6) symmetry we complicate the model a little by introducing the explicit but soft-gauge-breaking terms $b_1\Phi_1+b_2\Phi_2$ in the superpotential. Here the couplings $b_{1,2}$ represent the column vectors

$$b_1=\begin{bmatrix}b_1^1\\ b_1^2\\ b_1^3\\ b_1^4\\ b_1^5\\ b_1^6\end{bmatrix},\quad b_2=\begin{bmatrix}b_2^1\\ b_2^2\\ b_2^3\\ b_2^4\\ b_2^5\\ b_2^6\end{bmatrix}. \quad (3.4)$$

Thus, the interaction Lagrangian is given by

$$\begin{aligned}L_{P-b}=\Big[&\lambda\Phi_0+m\Phi_1\Phi_2+\frac{g}{2}\Phi_0\Phi_1\Phi_1\\ &+b_1\Phi_1+b_2\Phi_2\Big]_F+\text{H.c.}\,,\end{aligned} \quad (3.5)$$

where m can be chosen as diagonal. The resultant minima of the effective potential,

$$\begin{aligned}V_b=&\left|\lambda+\frac{g}{2}A_1^2\right|^2+|b_1+mA_2+gA_0A_1|^2\\ &+|b_2+ma_1|^2\,,\end{aligned} \quad (3.6)$$

satisfies the equations

$$b_1+mA_2+gA_0A_1=0\,, \quad (3.7a)$$

$$\tfrac{1}{2}|g|^2|A_1|^2A_1^*+(|m|^2+g\lambda^*)A_1^*+b_2^*m=0\,. \quad (3.7b)$$

If we choose

$$\tfrac{1}{2}|g|^2\langle A_1\rangle^2\ll\big|\,|m|^2+g\lambda^*\big|$$

and $|m|^2 \gg |\lambda g|$, these equations yield the simple solution

$$\langle A_0 \rangle = 0 , \tag{3.8a}$$

$$\langle A_1 \rangle \simeq \frac{-b_2}{m'} \equiv \frac{-b_2}{m + \dfrac{\lambda g^*}{m^*}} \simeq \frac{-b_2}{m} , \tag{3.8b}$$

$$\langle A_2 \rangle = -\frac{b_1}{m} . \tag{3.8c}$$

which has the property that $\langle A_1 \rangle \to 0$ as $b_2 \to 0$ and $\langle A_2 \rangle \to 0$ as $b_1 \to 0$.

To expand the potential around the minimum, we define the new fields

$$\Phi_0 \to \Phi_0 , \tag{3.9a}$$

$$\Phi_1 \to \Phi_1 + \frac{b_2}{m} , \tag{3.9b}$$

$$\Phi_2 \to \Phi_2 + \frac{b_1}{m} , \tag{3.9c}$$

and rewrite the interaction Lagrangian in terms of the new fields as

$$L_{P-b} = \left\{ \left[\lambda + \frac{g}{2} \left[\frac{b_2}{m} \right]^2 \right] \Phi_0 + \frac{\lambda g^*}{m^* m} b_2 \Phi_2 - \frac{g b_2}{m} \Phi_0 \Phi_1 + m \Phi_1 \Phi_2 + \frac{g}{2} \Phi_0 \Phi_1 \Phi_1 \right\}_F + \text{H.c.} \tag{3.10}$$

and the minimum of the scalar potential is

$$\langle V_b \rangle \simeq |\lambda'|^2 \equiv \left| \lambda + g \left[\frac{b_2}{m} \right]^2 \right|^2 . \tag{3.11}$$

One difficulty of this model is the appearance of a massless Goldstone fermion as the result of spontaneous supersymmetry breakdown. To check this, we calculate the fermionic mass matrix

$$M_F = \begin{bmatrix} 0 & g\dfrac{b_2}{m} & 0 \\ g\dfrac{b_2}{m} & 0 & m \\ 0 & m & 0 \end{bmatrix} . \tag{3.12}$$

Clearly, $\det M_F = 0$ implies that a massless fermion, the Goldstone fermion, exists in the spectrum. Since this sector is supported to be hidden we must give a mass to the Goldstone fermion. This can be accomplished by adding a soft-SUSY-breaking term

$$L_{\text{soft}} = \tfrac{1}{2} \mu \Psi_0{}^2 \tag{3.13}$$

and choosing $\mu < m$. This breaking term does not affect the auxiliary field equations or tree-level vacuum expectation values (VEV's) of the model and serves only to give the Goldstone fermion a nonzero mass.

To couple this O'Raifeartaigh sector to the Goldstone superfields we need the relevant symmetry currents. For the O'Raifeartaigh sector, they take the form[11]

$$J_B^A = \sum \overline{\Phi} T_B^A \Phi \tag{3.14}$$

which in terms of the shifted fields are

$$J_B^A = \left[\frac{b_2^{*B} b_2^A}{|m|^2} - \frac{b_2^A}{m} \Phi_1^{\dagger B} - \frac{b_2^{*B}}{m^*} \Phi_1^A + \Phi_1^{\dagger B} \Phi_1^A \right] + 1 \leftrightarrow 2 . \tag{3.15}$$

IV. O'RAIFEARTAIGH-GOLDSTONE COUPLING

We are now in the position to construct the interaction between the O'Raifeartaigh and Goldstone sectors which can be used to generate tree-level mass terms for the Goldstone fields. We shall see that obtaining nonzero masses for the Goldstone fields requires both SUSY breaking and explicit G symmetry breaking. This is signaled, respectively, by having nontrivial VEV's of the scalar potential $\langle F \rangle$ and the currents $\langle J \rangle$ arising in the O'Raifeartaigh sector.

For completeness, we consider the leading terms of the most general SUSY and G-invariant vector coupling Lagrangian given by [with $\overline{\Phi} \equiv (\Phi^\dagger)^c$]

$$\begin{aligned} L_G &\equiv G(\Phi^\dagger, \Phi; \phi^\dagger, \phi)|_D \\ &= [c_0 + c_1(\Phi_0 + \Phi_0^\dagger) + c_2(\Phi_0^\dagger \Phi_0 + \overline{\Phi}_1 \Phi_1 + \overline{\Phi}_2 \Phi_2) \\ &\quad + c_3(\overline{\Phi}_1 \Phi_2 + \overline{\Phi}_2 \Phi_1)](c_4 + K_0^{\text{eff}})(c_5 + J_B^A j_B^A)|_D \end{aligned} \tag{4.1}$$

which is a generalization of the current-current coupling of Ref. 11. We normalize the O'Raifeartaigh superfields and the super-Kähler potential so that $c_2 c_4 c_5 = 1$ and $c_0 c_5 = 1$. For simplicity, we shall assume $c_3 / c_2 \ll 1$ so that the equations of motion in the hidden sector require no modification.

The general mass matrix from this Lagrangian contains pure Goldstone terms as well as mixing between the Goldstone and O'Raifeartaigh superfields. The pure Goldstone-field mass terms arise from the combination of the following two conditions: (1) non-zero VEV of the superfields or currents from the O'Raifeartaigh sector and (2) quadratic terms from the Goldstone sector. Condition (1) is met by the terms $J_B^A|_{0,F,\bar{F},D}$, $\Phi_0|_F + \text{H.c.}$, $(\Phi_0^\dagger \Phi_0 + \overline{\Phi}_2 \Phi_2)_D$, while condition (2) is satisfied by all bosonic components (except D terms) $j_B^A|_{0,F,\bar{F}}$; $K^{\text{eff}}|_{0,F,\bar{F}}$.

The preceding arguments lead us to the following combinations for pure Goldstone-field mass couplings:

$$\text{(i)} \quad \langle J_B^A \rangle |_D j_B^A |_0 , \tag{4.2a}$$

$$\text{(ii)} \quad \langle \Phi_0^\dagger \Phi_0 + \overline{\Phi}_2 \Phi_2 \rangle |_D K_0^{\text{eff}} |_0 , \tag{4.2b}$$

$$\text{(iii)} \quad \langle \Phi_0^\dagger \Phi_0 + \overline{\Phi}_2 \Phi_2 \rangle |_D \langle J_B^A \rangle |_0 j_B^A |_0 , \tag{4.2c}$$

$$\text{(iv)} \quad \langle \Phi_0^\dagger \rangle |_{\bar{F}} K_0 |_F + \text{H.c.} \tag{4.2d}$$

Searching for mass contributions term by term, we find the following.

(i) To extract the mass terms from $\langle J_B^A \rangle |_D j_B^A |_0$, we first secure the VEV from $\langle J_B^a \rangle |_D$ to be

$$\langle J_B^A\rangle|_D=\langle\Phi_2^{\dagger B}\Phi_2^A\rangle|_D$$
$$=\langle F_2^{*B}F_2^A\rangle=\left|\frac{\lambda g}{m^2}\right|^2 b_2^B b_2^{*A}\,. \tag{4.3}$$

Then we can write down all mass couplings which are given by

$$\langle J_i^a\rangle|_D j_i^a|_0=\left[\sqrt{2}iv_2^2\left|\frac{\lambda g}{m^2}\right|^2 b_2^i b_2^{*a}\right]a_i^a\mathrm{Im}a_0+\cdots\,, \tag{4.4a}$$

$$\langle J_a^i\rangle|_D j_a^i|_0=\left[\sqrt{2}iv_2^2\left|\frac{\lambda g}{m^2}\right|^2 b_2^{*i}b_2^a\right]a_a^{*i}\mathrm{Im}a_0+\cdots\,, \tag{4.4b}$$

$$\langle J_0\rangle|_D j_0|_0=\left[\frac{v_2^2}{2}\left|\frac{\lambda g}{m^2}\right|^2 b_2^k b_2^{*k}\right]a_a^{*i}a_i^a+\cdots\,, \tag{4.4c}$$

where

$$J=\frac{1}{\sqrt{2}}J_k^k\,.$$

(ii) Since

$$\langle\Phi_0^\dagger\Phi_0+\overline{\Phi}_2\Phi_2\rangle|_D\to\langle V_b\rangle$$

[refer to Eqs. (3.6) and (3.12)] it follows that

$$\langle\Phi_0^\dagger\Phi_0+\overline{\Phi}_2\Phi_2\rangle|_D K_0^{\mathrm{eff}}|_0=\langle V_b\rangle[-v_2^2(\mathrm{Im}a_0)^2]+\cdots$$
$$\simeq-|\lambda'|^2v_2^2\mathrm{Im}a_0\mathrm{Im}a_0+\cdots\,. \tag{4.5}$$

(iii) Similarly, using

$$\langle\Phi_0^\dagger\Phi_0+\overline{\Phi}_2\Phi_2\rangle\simeq|\lambda'|^2 \tag{4.6}$$

and

$$\langle J_B^A\rangle|_0=\frac{1}{|m|^2}(b_1^Ab_1^{*B}+b_2^Ab_2^{*B}) \tag{4.7}$$

the mass couplings can be easily found as

$$\langle\Phi_0^\dagger\Phi_0+\overline{\Phi}_2\Phi_2\rangle|_D\langle J_i^a\rangle|_0 j_i^a|_0=\left[\sqrt{2}iv_2^2\left|\frac{\lambda'}{m}\right|^2(b_1^{*i}b_1^a+1\leftrightarrow2)\right]a_i^a\mathrm{Im}a_0+\cdots\,, \tag{4.8a}$$

$$\langle\Phi_0^\dagger\Phi_0+\overline{\Phi}_2\Phi_2\rangle|_D\langle J_a^i\rangle|_0 j_a^i|_0=\left[-\sqrt{2}iv_2^2\left|\frac{\lambda'}{m}\right|^2(b_1^ib_1^{*a}+1\leftrightarrow2)\right]a_a^{*i}\mathrm{Im}a_0+\cdots\,, \tag{4.8b}$$

$$\langle\Phi_0^\dagger\Phi_0+\overline{\Phi}_2\Phi_2\rangle|_D\langle J_0\rangle|_0 j_0|_0=\left[\frac{v_2^2}{2}\left|\frac{\lambda'}{m}\right|^2(b_1^kb_1^{*k}+1\leftrightarrow2)\right]a_a^{*i}a_i^a+\cdots\,. \tag{4.8c}$$

(iv) In this combination, $K_0^{\mathrm{eff}}|_F$ will give the only fermionic mass terms for the pure Goldstone sector:

$$\Phi_0^\dagger|_F K_0^{\mathrm{eff}}|_F+\mathrm{H.c.}=-\lambda'\frac{v_2^2}{2}\psi_0\psi_0+\mathrm{H.c.} \tag{4.9}$$

Collecting these results we obtain the mass terms in the pure Goldstone sector to be (using the identity $b^Ab^{*B}=\overline{b}T_B^Ab$ and $b_\beta=\sum_{\beta=1}^{\beta=2}b_\beta$)

$$L_{\text{diag pure mass}}=\tfrac{1}{2}a_a^{*i}a_i^a\left[c_0v_2^2\left|\frac{\lambda'}{m}\right|^2\overline{b}_\beta X_0b_\beta+\frac{c_0^2}{c_2}v_2^2\left|\frac{\lambda g}{m^2}\right|^2\overline{b}_2X_0b_2\right]$$
$$+\tfrac{1}{2}\mathrm{Im}a_0\mathrm{Im}a_0\left[\frac{-2c_2}{c_0}|\lambda'|^2v_2^2\right]+\tfrac{1}{2}\psi_0\psi_0\left[\frac{-c_1}{c_0}\lambda'v_2^2\right]+\mathrm{H.c.}\,, \tag{4.10}$$

$$L_{\text{off-diag pure mass}}=a_i^a\mathrm{Im}a_0\left[\sqrt{2}i\frac{c_0^2}{c_2}v_2^2\left|\frac{\lambda g}{m^2}\right|^2\overline{b}_2X_i^ab_2+\sqrt{2}ic_0v_2^2\left|\frac{\lambda'}{m}\right|^2\overline{b}_\beta X_i^ab^\beta\right]+\mathrm{H.c.} \tag{4.11}$$

As we can see, the only pure mass terms for quasi-Goldstone fermions (QGF's) are

$$-\left[\left[\frac{c_1}{c_0}\right]\lambda'\left[\frac{v_2^2}{2}\right]\psi_0\psi_0+\mathrm{H.c.}\right]\,.$$

The procedure for finding the mixed mass terms of O'Raifeartaigh and Goldstone sectors is similar to that used for the pure Goldstone particles, except the second condition for pure Goldstone masses is replaced by the following two conditions: (1) linear terms from the O'Raifeartaigh sector, e.g., $\Phi_0+\Phi_0^\dagger$ and

$$J_B^A = -\frac{b_2^A}{m}\Phi_1^{\dagger B} - \frac{b_2^{*B}}{m^*}\Phi_1^A - 1\leftrightarrow 2 ,$$

and (2) linear terms from the Goldstone sector, e.g., $j_0 = i\nu_2^2(\phi_0 - \phi_0^\dagger) - \cdots$.

Clearly the current-current coupling in the O'Raifeartaigh-Goldstone Lagrangian is the central ingredient for obtaining the mixed mass couplings. In fact, there are only two possible combinations which can contribute.

Case (i): $c_0(\Phi_0^\dagger\Phi_0 + \overline{\Phi}_2\Phi_2)J_B^A j_B^A$. We shall examine various ways to pick the overall D component:

$$\langle \Phi_0^\dagger\Phi_0 + \overline{\Phi}_2\Phi_2\rangle\,|_D J_B^A\,|_0 j_B^A\,|_0 \simeq \frac{|\lambda'|^2}{\sqrt{2}}\left[-\frac{b_2^k}{m}A_1^{*k} - \frac{b_2^{*k}}{m^*}A_1^k - 1\leftrightarrow 2\right]2i\nu_2^2\,\mathrm{Im}a_0 + \cdots , \tag{4.12}$$

$$\langle \Phi_0^\dagger\Phi_0 + \overline{\Phi}_2\Phi_2\rangle\,|_{\theta\bar\theta\theta}\langle J_B^A\rangle\,|_0 j_B^A\,|_\theta + \mathrm{H.c.} = -\sqrt{2}\lambda'\Psi_0\left[\left|\frac{b_1^k}{m}\right|^2 + 1\leftrightarrow 2\right]i\nu_2^2\psi_0 + \mathrm{H.c.} + \cdots . \tag{4.13}$$

Case (ii): $(c_0c_1/c_2)(\Phi_0^\dagger/\Phi_0)J_B^A j_B^A$. To pick the D measure, repeat the routine in case (i) with $\Phi_0^\dagger\Phi_0$ replaced by Φ_0 or $\Phi_0^\dagger$ so that

$$\langle \Phi_0^\dagger\rangle\,|_F J_B^A\,|_\theta j_B^A\,|_\theta + \mathrm{H.c.} = \lambda'\left[\frac{b_2^{*k}}{m^*}\Psi_1^k + 1\leftrightarrow 2\right]i\nu_2^2\psi_0 + \mathrm{H.c.} + \cdots , \tag{4.14a}$$

$$\begin{aligned}\langle \Phi_0^\dagger\rangle\,|_F J_B^A\,|_F j_B^A\,|_0 + \mathrm{H.c.} = &-\lambda'\sqrt{2}\left[\frac{b_2^{*k}}{m^*}\left[m^*A_2^{*k} - g^*\frac{b_2^{*k}}{m^*}A_0^*\right] + b_1^{*k}A_1^{*k}\right]i\nu_2^2\,\mathrm{Im}a_0 \\ &+ 2\sqrt{2}g^*\frac{b_2^*}{m^*}A_1^*\frac{b_1^{*k}}{m^*}\left[\frac{\lambda g^*}{|m|^2}\right]^* b_2^{*k}i\nu_2^2\,\mathrm{Im}a_0 + \mathrm{H.c.} + \cdots ,\end{aligned} \tag{4.14b}$$

$$\Phi_0^\dagger\,|_{\bar\theta}\langle J_B^A\rangle\,|_F j_B^A\,|_{\bar\theta} + \mathrm{H.c.} = -\Psi^\dagger\frac{b_1^{*k}}{m^*}\left[\frac{\lambda g^*}{|m|^2}\right]^* b_2^{*k}i\nu_2^2\psi_0^\dagger + \mathrm{H.c.} + \cdots . \tag{4.14c}$$

So we have mixed mass terms for not only bosons but also fermions. Since eight of the QGF's are expected to be the lightest in our spectrum, we need to study the fermionic spectrum more carefully to check whether this is achieved. First, we collect all fermionic mixed mass terms as

$$\begin{aligned}L_{\text{fermion mix mass}} = &\Psi_0\Psi_0\left[\frac{i}{2}\nu_2^2\left[-c_0\sqrt{2}\lambda'\frac{|b_\beta^k|^2}{|m|^2} + \frac{c_0c_1}{c_2}\frac{\lambda g^*}{|m|^1}\frac{b_1^kb_2^k}{|m|^2}\right]\right] \\ &+ \Psi_1^k\psi_0\left[\frac{i}{2}\nu_2^2\frac{c_0c_1}{c_2}\lambda'\frac{b_2^{*k}}{m^*}\right] + \Psi_2^k\psi_0\left[\frac{i}{2}\nu_2^2\frac{c_0c_1}{c_2}\lambda'\frac{b_1^{*k}}{m^*}\right] + \mathrm{H.c.}\end{aligned} \tag{4.15}$$

These fermionic mixing mass couplings are then combined with the fermionic mass coupling in the O'Raifeartaigh sector and the fermionic mass terms in the pure Goldstone sector to form the complete fermionic mass matrix. We can facilitate the diagonalization of this mass matrix, while maintaining its general feature, if we assume the G-symmetry-breaking vectors b to be degenerate, i.e., $b_1^k \equiv b_2^k \equiv b$. The resultant fermionic mass matrix can then be written as

$$M_F = \begin{array}{c} \\ \Psi_0 \\ \Psi_1 \\ \Psi_2 \\ \psi_0 \\ \psi_i^a \end{array}\begin{array}{c}\begin{array}{ccccc}\Psi_0 & \Psi_1 & \Psi_2 & \psi_0 & \psi_i^a\end{array} \\ \begin{bmatrix} \mu & \frac{gb}{m} & 0 & P & 0 \\ \frac{gb}{m} & 0 & m & Q & 0 \\ 0 & m & 0 & Q & 0 \\ P & Q & Q & R & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\end{array} \tag{4.16}$$

with

$$P = i\nu_2^2\left[-2\sqrt{2}c_0\lambda'\left|\frac{b}{m}\right|^2 + \frac{c_0c_1}{c_2}\frac{\lambda g^*}{|m|^2}\frac{b^2}{m}\right], \tag{4.17a}$$

$$Q = \frac{i}{2}\nu_2^2\frac{c_0c_1}{c_2}\lambda'\frac{b^*}{m^*} , \tag{4.17b}$$

$$R = -\frac{c_1}{c_0}\lambda'\nu_2^2 . \tag{4.17c}$$

Under the assumption $m^2 > \mu^2 \gg |g\lambda| \gg |gb| \gg mR \gg mP, mQ$, we are able to diagonalize this mass matrix as

$$M_F^{\text{diag}} = \begin{bmatrix} \mu & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & -m & 0 & 0 \\ 0 & 0 & 0 & R & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} . \tag{4.18}$$

The lightest modes, ψ_i^a, which have all zero entries in the naive mass matrix, are recognized to be the eight left-handed quarks and leptons.

V. CONCLUSIONS AND DISCUSSIONS

We have demonstrated how by breaking SUSY in an O'Raifeartaigh model and then coupling to the Goldstone sector, nonzero masses could be generated for the components of Goldstone supermultiplets. The mechanism was demonstrated by constructing the resulting mass spectrum in a U(6)/U(4)$\times$SU(2) model. Summarizing the results, we found the following.

(1) By adjusting the model parameters, the mass scales of the bosonic partners and the exotic fermions, including the novino, could be lifted well above the GeV order. Moreover the bosonic mass scale can be split from that of fermions without any difficulty. One way to achieve this is to assign a smaller value to v^2 relative to the combinations, with the same dimension of v^2, of other coupling parameters, since all fermionic mass squares are proportional to ${v_2}^4$ while all bosonic mass squares are proportional to ${v_2}^2$, and

$$\frac{(M_{\text{fermions}})^2}{(M_{\text{bosons}})^2}=\frac{{v_2}^2}{\text{combination of other parameters}} . \tag{5.1}$$

(2) As the explicit gauge breaking terms vanish, $b\to 0$, only the mass terms for the novino and its quasibosonic partner survive. That is,

$$M_{\psi_0}=\frac{c_0}{c_1}\lambda' {v_2}^2 , \tag{5.2a}$$

$$M_{\text{Im}a_0}{}^2=\frac{-2c_2}{c_0}\,|\lambda'|^2{v_2}^2 , \tag{5.2b}$$

and the rest particles in the Goldstone sector, particularly all Goldstone bosons, remain massless.

(3) The eight light fermions turn out massless in the tree level. These fermions can still acquire masses by radiative corrections from O'Raifeartaigh-Goldstone interactions. Since the masses of the supermultiplets of the O'Raifeartaigh sector are split by supersymmetry breakdown, one can no longer expect the cancellation between bosonic loops and fermionic loops in the radiative corrections of mass renormalizations. These Goldstone-fermion masses, as a result of radiative corrections, will definitely be much lower than the tree-level Goldstone-boson masses.

It is worthwhile to examine the mass sum rule[12] in a nonlinear theory such as we proposed. The total Lagrangian can be written as

$$L_T=G(\Phi_I^\dagger,\Phi_I;\phi_i^\dagger,\phi_i)\,|_D+[P(\Phi_I)\,|_F+\text{H.c.}] , \tag{5.3}$$

where the upper case Φ_I indicates the O'Raifeartaigh superfields while the lower case ϕ_i indicates the Goldstone superfields. G and P can be some arbitrary vector potential and scalar potential, respectively.

The superfield equations of motion are [use the notation of (2.4)]

$$\bar D^2G_i=0 , \tag{5.4a}$$

$$\bar D^2G_I+P_I=0 , \tag{5.4b}$$

and their Hermitian conjugation. We then shift the fields $\Phi\to\Phi+\langle\Phi\rangle$, $\phi\to\phi+\langle\phi\rangle$ and expand the superfield equations about the physical vacuum:

$$\bar D^2(\langle G_{i\bar j}\rangle\phi_j^\dagger+\langle G_{i\bar J}\rangle\Phi_J^\dagger+\langle G_{ij}\rangle\phi_j+\langle G_{iJ}\rangle\Phi_J+\langle G_{ij\bar k}\rangle\phi_j\phi_k^\dagger+\langle G_{ij\bar K}\rangle\phi_j\Phi_K^\dagger+\langle G_{iJ\bar k}\rangle\Phi_j\phi_k^\dagger+\langle G_{iJ\bar K}\rangle\Phi_J\Phi_K^\dagger$$
$$+\langle G_{iJ\bar k\bar L}\rangle\Phi_J\phi_k^\dagger\Phi_L^\dagger+\tfrac12\langle G_{iJ\bar K\bar L}\rangle\Phi_J\Phi_K^\dagger\Phi_L^\dagger+\cdots)=0 , \tag{5.5a}$$

$$\bar D^2(\langle G_{I\bar j}\rangle\phi_j^\dagger+\langle G_{I\bar J}\rangle\Phi_J^\dagger+\langle G_{Ij}\rangle\phi_j+\langle G_{IJ}\rangle\Phi_J+\langle G_{Ij\bar k}\rangle\phi_j\phi_k^\dagger+\langle G_{Ij\bar K}\rangle\phi_j\Phi_K^\dagger+\langle G_{IJ\bar k}\rangle\Phi_J\phi_k^\dagger+\langle G_{IJ\bar K}\rangle\Phi_J\Phi_K^\dagger$$
$$+\langle G_{IJ\bar k\bar L}\rangle\Phi_J\phi_k^\dagger\Phi_L^\dagger+\tfrac12\langle G_{iJ\bar K\bar L}\rangle\Phi_J\Phi_K^\dagger\Phi_L^\dagger+\cdots)+\langle P_I\rangle+\langle P_{IJ}\rangle\Phi_J+\tfrac12\langle P_{IJK}\rangle\Phi_J\Phi_K+\cdots=0 . \tag{5.5b}$$

From now on, we will drop the angular brackets on G and P. For convenience, we shall assume $G_{\alpha\bar\beta}\simeq\delta_{\alpha\beta}$, with the greek letters α,β representing either the upper case $I,J,\ldots$ or the lower case $i,j,\ldots$.

Successively applying D_α on the superfield equations (5.5) and evaluating at $\theta=0$, we can obtain the component equations of motion by eliminating the auxiliary fields:

$$i\partial\bar\psi_i-G_{ij\bar K}P_K\psi_j-G_{iJ\bar K}P_K\Psi_J+\cdots=0 , \tag{5.6a}$$

$$i\partial\bar\Psi_I-G_{Ij\bar K}P_K\psi_j-G_{IJ\bar K}P_K\Psi_J+P_{IJ}\Psi_J+\cdots=0 , \tag{5.6b}$$

and

$$\Box a_i^*+G_{iL\bar K}P_KP_{LJ}^*A_J^*+G_{iL\bar K}P_L^*P_{KJ}A_J+G_{iK\bar L\bar j}P_K^*P_La_j^*+G_{iK\bar L\bar J}P_K^*P_LA_J^*+\cdots=0 , \tag{5.7a}$$

$$\Box A_i^*+G_{IL\bar K}P_KP_{LJ}^*A_J^*+G_{IL\bar K}P_L^*P_{KJ}A_J+G_{IK\bar L\bar j}P_K^*P_La_j^*+G_{IK\bar L\bar J}P_K^*P_LA_J^*-P_{IL}P_{LJ}^*A_J^*-P_{IJK}P_K^*A_J+\cdots=0 . \tag{5.7b}$$

From Eqs. (5.6) and (5.7) we can read off the fermion and boson mass matrices:

$$M_F = \begin{array}{c} \\ i \\ I \end{array} \begin{array}{c} \begin{array}{cc} \quad j \quad & \quad J \quad \end{array} \\ \begin{bmatrix} -G_{ij\bar{K}}P_K & -G_{iJ\bar{K}}P_K \\ -G_{Ij\bar{K}}P_K & -G_{IJ\bar{K}}P_K P_{IJ} \end{bmatrix} \end{array} , \tag{5.8a}$$

$$M_B^2 = \begin{array}{c} \\ i \\ I \end{array} \begin{array}{c} \begin{array}{cc} \quad j \quad & \quad J \quad \end{array} \\ \begin{bmatrix} -G_{iK\bar{L}\bar{j}}P_K^* P_L & -G_{iL\bar{K}}P_K P_{LJ}^* - G_{iK\bar{L}\bar{J}}P_K^* P_L \\ -G_{IK\bar{L}\bar{j}}P_K^* P_L & -G_{IL\bar{K}}P_K P_{LJ}^* - G_{IK\bar{L}\bar{J}}P_K^* P_L + P_{IL}P_{LJ}^* \end{bmatrix} \end{array} . \tag{5.8b}$$

The mass relation is given by the graded trace of the squared mass matrices:

$$\begin{aligned} \mathrm{Tr}M^2 &\equiv \sum(-1)^{2J}(2J+1){M_J}^2 = \mathrm{Tr}{M_B}^2 - 2\,\mathrm{Tr}M_F M_F^* \\ &= -2G_{\alpha\bar{\alpha}K\bar{L}}P_L^* P_K - 2G_{\beta\alpha\bar{K}}P_K G_{\bar{\alpha}\bar{\beta}L}P_L^* + (G_{IL\bar{K}}P_K P_{LI}^* + \mathrm{H.c.})\ , \end{aligned} \tag{5.9}$$

where P_K are exactly the VEV of the auxiliary fields $\langle F_K \rangle$, the signature of spontaneous SUSY breaking.

Thus, the mass sum rule, $\mathrm{Tr}M^2=0$, which holds for renormalizable linear theories would not, in general, be satisfied for nonlinear effective Lagrangians retaining only some of the degrees of freedom. This allows more freedom to split the mass degeneracy of the fermions and bosons in the effective theory. The simplified spectrum without the gauge-breaking terms [cf. conclusion (2)] can serve as a quick check. In fact, all the mass couplings in Sec. IV can be derived alternatively by using Eq. (5.9) with higher-order terms.

Finally, it has come to our attention that the use of spontaneous broken SUSY to generate quasi-Goldstone-fermion masses has also been employed by Goity.[13] His approach differs from ours in the nature of the coupling of the Goldstone sector to the SUSY-breaking sector. We agree, however, on the emergence of light quasi-Goldstone-fermion modes.

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