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Observational limits on the time evolution of extra spatial dimensions

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Limits are imposed upon the possible rate of change of extra spatial dimensions in Kaluza-Klein and superstring theories by considering the consequences of such changes for primordial nucleosynthesis and the positioning of nuclear resonance levels in samarium, carbon, and oxygen. Previous limits from nucleosynthesis had not included strong-interaction effects. The strongest limits constrain the present rate of change of the mean radius of any additional spatial dimensions to be less than about $10^{-19}\ \mathrm{yr}^{-1}$.

One of the most interesting features of particle-physics theories of the Kaluza-Klein and superstring variety is that the extra spatial dimensions required may have observable consequences within the three spatial dimensions in which our observations are made. These higher-dimensional theories typically possess a space-time of the form $M^4 \times C^D$ where M^4 is four-dimensional space-time, C^D is a D-dimensional compact space whose isometry group generates a low-energy quantum field theory of the Yang-Mills type. In the prototype of Kaluza-Klein theory, C^D was S^1 and the associated isometry group is the U(1) of electromagnetism.[1] If there exist $4+D$ space-time dimensions then the gauge coupling constants of the associated Yang-Mills theories in four dimensions will be inversely proportional to the size of the compact D-dimensional space. The true bare constants of nature would be defined in $4+D$ dimensions. Any cosmological evolution of the extra D spatial dimensions would result in the usual "constants" of nature defined in the observed three-dimensional space possessing a variation in time (and/or space) determined by the geometric mean scale factor $R(t,\mathbf{x})$ of the cosmological evolution of the additional D spatial dimensions.[2]

In Kaluza-Klein theories, the fine[3] ($\alpha = e^2$), weak ($\alpha_w = G_F{}^2 m^2$), and strong ($\alpha_s = g_s^2$) gauge coupling "constants" observed in our four-dimensional space-time would evolve as

$$\alpha \propto \alpha_w \propto \alpha_s \propto R^{-2} . \tag{1}$$

The Newtonian gravitational "constant" would be observed to vary as the volume of D-dimensional space:

$$G \propto R^{-D} . \tag{2}$$

In the currently popular ten-dimensional superstring theories[4] it is predicted that[5]

$$\alpha \propto \alpha_w \propto \alpha_s \propto G \propto R^{-6} . \tag{3}$$

Kolb, Perry, and Walker[5] have attempted to place limits on the extent to which extra spatial dimensions could have undergone cosmological time evolution in these types of theory by investigating the effects on the primordial nucleosynthesis of ^{4}He of varying the electroweak and gravitational coupling constants. This enables limits to be placed upon the amount by which the geometric-mean scale factor of the extra dimensions could have changed between the epoch of nucleosynthesis, t_{ns}, and the present time $t_0 \sim (15 \pm 2) \times 10^9$yr.

Cosmological nucleosynthesis is the earliest event in the evolution of the Universe about which we have decisive observational evidence and detailed theoretical predictions.[6] There is some chance that residual effects from compactification at very high energies ($\sim 10^{19}$ GeV) might still persist at the epoch when nucleosynthesis occurs. This evolution might take the form of damped oscillations about the compactified state.

Current observations indicate that there is precise agreement between the predictions of the standard hot big-bang theory and the observational data with respect to the abundances of ^{4}He, ^{3}He, deuterium, and ^{7}Li. ^{4}He is a good diagnostic of possible aberrations to the standard big-bang model of the early Universe because its abundance is predicted to be exponentially sensitive to deviations from expansion isotropy,[7] uncertainties in the neutron half-life and the presence of additional fermion species or gravitational waves.[6] Kolb, Perry, and Walker,[5] examined the limits that could be imposed on the evolution of extra dimensions by determining the constraints imposed upon time variation in the electromagnetic, weak, and gravitational interaction strengths between the period of neutron-proton "freeze-out" ($t_* \sim 1$ sec) and the present. In what follows we would like to point out that the effects of a small change in the strength of the strong interaction can be more important than those considered in Ref. 5. Furthermore, the strong-interaction effects work in the opposite sense to those produced by variations in the strength of the other interactions. Also, we point out some additional effects of changing the electroweak couplings at the epoch of nucleosynthesis.

Kolb, Perry, and Walker,[5] identify three important effects on ^{4}He nucleosynthesis which arise from any difference in the value of the three-dimensional "constants" α, α_w, or G between the time of nucleosynthesis and the present.

(a) A small increase in the Newtonian gravitational coupling G alters the temperature-time evolution of the universal expansion and increases the temperature T_* at which the neutron-proton fraction is frozen out of weak interaction equilibrium. This increases the resulting neutron-proton ratio and the final ^{4}He abundance.

(b) A small increase in the weak-interaction strength decreases the freeze-out temperature T_* and thereby the final ^{4}He abundance.

(c) An increase in the electromagnetic interaction strength alters the electromagnetic contribution to the neutron-proton mass difference.[8] This decreases the neutron-proton fraction frozen out of equilibrium and thereby lowers the final ^{4}He abundance.

The dependence of the frozen-out neutron-proton number ratio $(n/p)_*$ on the Fermi coupling constant G_F, G, and the neutron-proton mass difference $\Delta m = m_n - m_p$ is $(n/p)_* = \exp(-\Delta m / T_*)$, where

$$T_* \propto G_F^{-2/3} G^{1/6}, \quad \Delta m \propto \alpha \ . \tag{4}$$

There is an exponential sensitivity to each of the variations (a), (b), and (c). They give constraints on the change of additional dimensions between $t(T_*) \sim 1$ sec and the present.

We wish to consider the additional effects of the variation of the strong coupling constant α_s according to either (1) or (3). Specifically, the following effects will result from its variation.

(d) A slight decrease in the strong coupling will tend to unbind the deuteron which is the most weakly bound nucleus with a binding energy per nucleon of only ~ 1 MeV.

(e) A slight increase in the strong coupling can allow the dineutron and diproton to exist.

(f) The rate of the proton capture $p + n \rightarrow {}^2\mathrm{H} + \gamma$, a key step in the synthesis of neutrons and protons into ^{4}He, will be altered by any change in the strong-coupling strength.

There is also a dramatic consequence in addition to (c) if the electromagnetic coupling is changed by enough to alter the electromagnetic contribution to the neutron-proton mass difference so that the following occurs.

(g) The neutron-proton mass difference falls to a value less than the electron mass ~ 0.511 MeV. If this occurs the β decay of the neutron is no longer energetically possible and is replaced by the inverse radioactive decay of protons into neutrons via $p + e^- \rightarrow p + \nu_e$.

The easiest new effect to evaluate is (d). If the strong coupling is weak enough to prevent the existence of the deuteron at the time of nucleosynthesis ($t_N \sim 100$ sec) then all the neutrons frozen-out of weak equilibrium will β decay before many-body nuclear reactions can fuse them into ^{4}He. Deuterium is the first step in the chain of reactions that make ^{4}He fusion possible. If deuterium cannot exist when the temperature is high enough to overcome Coulomb barriers then the final primordial ^{4}He fraction will be essentially zero no matter how high the prior neutron-proton ratio is fixed by changes in the electroweak and gravitational coupling constants. The primordial deuterium abundance will be zero by definition in such circumstances and the abundances of ^{3}He and ^{7}Li will also be observationally indistinguishable from zero today. All these eventualities would be in conflict with observation.[6]

The binding energy of the deuteron is $E = -2.226$ MeV. It exists only in the ground state; no excited states are known. The strength and range of the nuclear binding force are well defined when we specify a model potential. In order to get a reasonable estimate of how large a change in the strong-interaction strength would be necessary to unbind the deuteron we use the simple square-well potential[9] of constant depth $V < 0$ and radius r. This is justified by the fact that the deuteron has a very small quadrupole moment $\sim 2.74 \times 10^{-27}$ cm^2. We are neglecting the nonspherically symmetric contributions to the two-nucleon interaction arising from spin-spin interactions and the tensor potential contributions. These nonspherical factors contribute only about 4% to the ground-state wave function. A textbook calculation[10] gives the energy E of the bound state as the solution of

$$\tan[\sqrt{q(1-\beta)}] = -\frac{\sqrt{1-\beta}}{\beta}, \quad \beta \equiv \frac{E}{V}, \ q \equiv -r^2 V M \ , \tag{5}$$

where M is the reduced mass of the two-nucleon system. For the triplet S state, using the known binding energy $E = -2.226$ MeV with $r = 2.02$ fm, the well depth is given by $V = -36.2$ MeV. (The singlet S state just fails to be bound.) Now, keeping r fixed, the binding energy will be reduced to zero in (4) with $V = -25.1$ MeV. Thus a 31% reduction in the strength of the strong coupling V will suffice to unbind the deuteron according to this model potential.

If R_0 is the present mean radius of the extra dimensions and $R_{\rm ns}$ their mean radius at the epoch of nucleosynthesis then in order to avoid unbinding the deuteron at the epoch of nucleosynthesis we require that $R_{\rm ns}/R_0 < 1.20$ in Kaluza-Klein theories and $R_{\rm ns}/R_0 < 1.06$ in superstring theories.

Next we consider the effects (e) and (f) resulting from an increase in the strong-coupling strength at the time of nucleosynthesis compared with the present value. The most peculiar effect would appear to be (e). The dineutron and the diproton (He^2) only fail to exist as bound states by ~ 72 keV. In the pp system the Coulomb repulsion prevents nuclear binding (although the exclusion principle would enter to prevent the existence of the diproton even if this were not the case). The pp and nn interactions differ from those in the deuteron because the exclusion principle stops identical nucleons residing in the same triplet S states. Therefore the forces acting in the S state of the pp and nn systems should be compared with those in the singlet S state of the deuteron which does not produce a bound state in the pn system. Using the square-well model introduced above with $r = 2.59$ fm to fit the low-energy scattering data, Eq. (5) requires $V = -14.0$ MeV. An increase of just 9% to $V = -15.3$ MeV would be required to bind the dineutron. An increase of about 13% would be required to overcome the additional Coulomb repulsion of about 0.56 MeV and bind the diproton.

To avoid binding the diproton during nucleosynthesis we require $R_{\rm ns}/R_0 > 0.94$ in Kaluza-Klein and $R_{\rm ns}/R_0 > 0.98$ with superstring theories. To avoid binding the dineutron the respective limits are $R_{\rm ns}/R_0 > 0.96$ and $R_{\rm ns}/R_0 > 0.99$.

The effects on ^{4}He nucleosynthesis of binding the pp

and nn systems are accompanied by an increased binding of the deuteron and a speed-up of the crucial $p+n \to {}^2\mathrm{H}+\gamma$ capture. The latter precedes the rapid synthesis of ^{4}He by reactions whose rates are too fast relative to the cosmological expansion rate to be much influenced by small changes in the strong-coupling strength. The creation of new nn and pp bound states increases the efficiency with which protons and neutrons are fused into ^{4}He by providing more reaction pathways for two-body captures[11] and decays such as $p+p \to {}^2\mathrm{He}+\gamma$ and ${}^2\mathrm{He} \to {}^2\mathrm{H}+e^+ + \nu_e$. The increased rate of $p+n$ capture has the same qualitative effect. The effects of binding energy changes on ^{4}He itself are of lower order than those on the pn system because ^{4}He is so tightly bound with a binding energy per nucleon ~ 7 MeV. The principal effect of these rate changes on the He fraction result from the increased rate of $p+n$ capture. The inverse photodissociation of deuterium stops earlier than usual so there has been less time available for the β decay of the frozen neutron-proton ratio. In the standard scenario the n/p fraction falls from about $\frac{1}{6}$ to $\frac{1}{7}$ during the period from freeze-out at T_* to the temperature $T_N \sim 0.1$ MeV when $p+n$ capture reactions proceed faster than their inverse dissociations. This temperature is determined by the condition that the relative number of photons in the Wien tail of the radiation sea with energies exceeding the deuteron binding energy fall below unity; that is roughly by the condition

$$\eta^{-1} \simeq \exp(E_D/T_N) , \tag{6}$$

where $\eta \sim 10^{-10}$ is the number of baryons per photon in the Universe and E_D is the deuteron binding energy. An increase in E_D is equivalent to an increase in $\ln(\eta^{-1})$ and produces a logarithmic increase in the final abundance of ^{4}He. The n/p ratio at the temperature of nucleosynthesis, T_N, is related to that at freeze-out T_* roughly by

$$\left[\frac{n}{P}\right]_N = \left[\frac{n}{p}\right]_* \exp\left[-\frac{t_N}{\tau_n}\right] , \tag{7}$$

where $\tau_n \propto G_F^{-2}$ is the neutron half-life and $t_N \propto G^{-1/2} T_N^{-2}$ is[12] the time at which the Friedmann model has cooled to the temperature T_N. Thus, the lower the value of T_N the smaller the value of $(n/p)_N$ that survives from the β decay of the frozen-out neutrons. The dependence on coupling constants of the β-decay term in (7) is, as $E_D \propto \alpha_s$,

$$\frac{t_N}{\tau_n} \propto G^{-1/2} \alpha_s^2 G_F^2 . \tag{8}$$

The final ^{4}He mass fraction Y can be estimated accurately from the value of $(n/p)_N$ as

$$Y = \frac{2(n/p)_N}{1+(n/p)_N} . \tag{9}$$

If we combine (1)—(7) we can determine the total effect on $(n/p)_N$, and hence on Y via (8), of simultaneous variations in the strong, weak, electromagnetic, and gravitational couplings. In superstring theories,

$$\left[\frac{n}{P}\right]_N^{\mathrm{ss}} = \exp\left[\frac{-\Delta m}{T_*}\frac{R_{\mathrm{ns}}^9}{R_0^9} - \frac{t_N}{\tau_n}\frac{R_0^3}{R_{\mathrm{ns}}^3}\right] , \tag{10}$$

while in D-dimensional Kaluza-Klein theories we have

$$\left[\frac{n}{P}\right]_N^{\mathrm{KK}} = \exp\left[\frac{-\Delta m}{T_*}\frac{R_{\mathrm{ns}}^{(20-D)/6}}{R_0^{(20-D)/6}} - \frac{t_N}{\tau_n}\frac{R_0^{D/2}}{R_{\mathrm{ns}}^{D/2}}\right] . \tag{11}$$

In (10) and (11) the quantities Δm, T_*, T_N, and τ_n ($=10.6$ min) are the standard values which obtain in the three-dimensional universe model when the extra dimensions do not evolve ($R_{\mathrm{ns}} = R_0$). Substituting the standard values in (10) and (11) we can calculate the maximum deviation of R_{ns}/R_0 from unity that is compatible with the observed value[6,13] of $Y = 0.24 \pm 0.01$. If we write

$$\frac{R_{\mathrm{ns}}}{R_0} = 1+\epsilon , \tag{12}$$

then we find that observation requires

$$|\epsilon|_{\mathrm{ss}} < 0.002 \tag{13}$$

in superstring theories,

$$|\epsilon|_{K2} < 0.007 \tag{14}$$

in $D=2$ Kaluza-Klein theories, and

$$|\epsilon|_{K7} < 0.011 \tag{15}$$

in $D=7$ Kaluza-Klein theories. In each case an increase of R_{ns}/R_0 above unity produces a lower ^{4}He abundance than is predicted by the standard big-bang model with no varying constants. Conversely a decrease of R_{ns}/R_0 below unity raises the ^{4}He abundance. We have assumed here that $T_* > T_N$. If this is not true then the neutron-proton ratio which undergoes nucleosynthesis is $\exp(-\Delta m/T_N)$ and not $\exp(-\Delta m/T_*)$. Thus, using (6), we expect

$$\left[\frac{n}{p}\right]_N = \exp[-\Delta m E_D^{-1} \ln(\eta^{-1})] = \eta \Delta m/E_D . \tag{16}$$

Now $\Delta m/E_D \sim \alpha/\alpha_s$ will be constant under changes in the extra dimensions of the form (1)—(3) as long as Δm does not fall below $m_e \sim 0.511$ MeV. This ensures that the n/p ratio is independent of the changing constants in the case $T_* > T_N$. The invariant $\Delta m/E_D = 0.581$ and $\eta \sim 10^{-10}$ in the observed Universe so $(n/p)_N \sim 10^{-5.8} \simeq 0$. Therefore, when $T_* > T_N$ the nucleosynthesis will result in essentially 100% hydrogen. Since

$$\left[\frac{T_*}{T_N}\right]_{KK} \propto R^{1/6(20-D)} \tag{17}$$

and

$$\left[\frac{T_*}{T_N}\right]_{ss} \propto R^9 \tag{18}$$

and $T_* \sim 10 T_N \sim 1$ MeV in the standard model ($R_{\mathrm{ns}} = R_0$), R would have to decrease by more than $10^{1/9} \sim 1.29$ in superstring theories to produce this situation and changes by factors of 2.15 and 2.89 would be necessary in two- and seven-dimensional Kaluza-Klein theories, respectively. Note that a decrease in R_{ns}/R_0 produces a decrease in the ^{4}He abundance, as found by Kolb, Perry, and Walker,[5] but the effect of this decrease on the strong interaction is to increase the He abundance. The consequences of these changes for the synthesis of deuterium, ^{3}He, and ^{7}Li are difficult to estimate accurately without the use of a numerical code.[14] However, we note that the existence of stable pp and nn states would lead to a decrease in neutrons and protons being incorporated into nuclei heavier than hydrogen at the epoch of nucleosynthesis. Subsequently the strong coupling must weaken in order to be consistent with its currently observed value and so the diproton and dineutron states would unbind following nucleosynthesis and, following β decay of the neutrons, show up as an augmentation of the hydrogen abundance today: that is, as a decrease in the mass fraction of light elements heavier than hydrogen.

The effect (g) listed above would result in only neutrons emerging from the nucleosynthesis process. Subsequently, the increase in α that would have to occur in order for the observed value of Δm to be obtained today would result in the β decay of all these primordial neutrons when $\Delta m \propto \alpha$ rises to exceed m_e. The result would be no primordial nuclei heavier than hydrogen. This situation is avoided if $(R_{ns}/R_0)_{KK} < 1.59$ and $(R_{ns}/R_0)_{ss} < 1.17$.

Prior to the consideration of higher-dimensional theories there existed no way of considering the observational effects of varying all "constants" in a self-consistent way. There existed theories[15] for the self-consistent variation of the gravitational coupling constant but these did not admit the variation of any other coupling constants. Because of this problem the limits on the allowed variations of constants, which are derived on the assumption that only one "constant" of nature varies, have no real basis. A good example of this problem is provided by the strongest limits cited claimed to limit the possible time variation of the strong, weak, and electromagnetic coupling constants. These limits arise from a detailed study of the events which took place 1.8×10^9 yr ago on the current site of an open-pit uranium mine at Oklo in the West African Republic of Gabon.[16] This site gave rise to a natural nuclear reactor when it went critical for a period about 1.8×10^9 yr ago. In order to preserve the fine-tuning of resonances in the neutron absorption cross sections necessary for this sequence of events Shylakhter[17] has argued that the strong, weak, and electromagnetic contributions to the nuclear binding energy of samarium and europium must not have altered by more than about 0.05 eV during the past 1.8×10^9 yr. It is not possible to calculate the position of the neutron absorption resonances exactly in terms of the fundamental coupling constants but if one models the potential of these heavy ($A\sim150$) nuclei to a first approximation by a square well of a depth of about 50 MeV and uses estimates of the weak and electromagnetic contributions to the binding then the following limits are obtained[18] by considering the effects of varying *one* of the coupling constants α, α_w, α_s, *while the other two are left constant:*

$$\frac{\dot{\alpha}_s}{\alpha_s}\le 5\times10^{-19}\ \text{yr}^{-1},\quad \frac{\dot{\alpha}}{\alpha}\le 10^{-17}\ \text{yr}^{-1},\qquad \frac{\dot{\alpha}_w}{\alpha_w}\le 2\times10^{-12}\ \text{yr}^{-1}. \tag{19}$$

However, if the three coupling constants vary simultaneously [as required by the higher-dimensional cosmological models (1) and (3)] then a linear weighting of the strong, weak, and electromagnetic contributions to the nuclear binding energy (and hence the resonance levels) of the form implicitly assumed by Shylakhter allows no individual limit to be placed upon the possible variation of any of the individual couplings. Using (1) and (3) we find that in Kaluza-Klein and superstring theories the mean scale factor of the additional dimensions is only limited significantly by the strong and electromagnetic contributions. The Oklo samples constrain its rate of change to satisfy the limits

TABLE I. Summary of the limits derived in the text upon allowed changes in the geometric mean radius R of additional spatial dimensions between the epoch of primordial nucleosynthesis (ns) and the present (0). Also tabulated are the limits on the maximum rate of change imposed by the presence of particular nuclear resonance levels in samarium within the Oklo natural reactor and in carbon and oxygen nuclei.

Constraint $R_{ns}/R_0=1+\epsilon$	Superstring $D=10$	Kaluza-Klein $D=2$ $D=7$
^{2}H bound	$\epsilon<0.06$	$\epsilon<0.20$
^{2}He unbound	$\epsilon>-0.02$	$\epsilon>-0.06$
^2n unbound	$\epsilon>-0.01$	$\epsilon>-0.04$
$Y=0.24\pm0.01$	$\lvert\epsilon\rvert<0.002$	$\lvert\epsilon\rvert_2<0.007$, $\lvert\epsilon\rvert_7<0.011$
No weak p decay	$\epsilon<0.17$	$\epsilon<0.59$
Oklo reactor	$\left\lvert\frac{\dot{R}}{R}\right\rvert<8.8\times10^{-20}\ \text{yr}^{-1}$	$\left\lvert\frac{\dot{R}}{R}\right\rvert<1.9\times10^{-19}\ \text{yr}^{-1}$
^{12}C and ^{16}O	$\left\lvert\frac{\dot{R}}{R}\right\rvert<8.3\times10^{-14}\ \text{yr}^{-1}$	$\left\lvert\frac{\dot{R}}{R}\right\rvert<1.9\times10^{-13}\ \text{yr}^{-1}$

$$\left|\frac{\dot{R}}{R}\right|_{\mathrm{KK}} \le 1.9\times 10^{-19}\ \mathrm{yr}^{-1}\ ,$$
$$\left|\frac{\dot{R}}{R}\right|_{\mathrm{ss}} \le 8.8\times 10^{-20}\ \mathrm{yr}^{-1}\ . \tag{20}$$

These are considerably stronger limits than those imposed by primordial nucleosynthesis.

There is one further site where the positioning of nuclear resonance levels is known to have played a crucial role in the past. Hoyle[19] first pointed out that the existence of carbon in the products of stellar nucleosynthesis was the consequence of a twofold natural "coincidence." The fact that the 7.6549-MeV level in the C^{12} nucleus lies just below the energy of Be^8 plus He^4 at the temperature in stellar interiors allows resonance to occur in the beryllium-catalyzed carbon-producing reaction $3He^4 \rightarrow Be^8 + He^4 \rightarrow C^{12} + 2\gamma$. Furthermore, the O^{16} nucleus has an energy level at 7.1187 MeV which lies just above the total energy of $C^{12}+He^4$ (=7.1616 MeV). Thus, the burning of carbon to oxygen via $C^{12}+He^4 \rightarrow O^{16}$ just fails to be resonant. The presence and absence of the carbon and oxygen levels in positions which admit a resonance, respectively, is probably a necessary factor for our own existence. It is certainly necessary to explain the existence of carbon in the Universe at the observed levels. If we assume that the position of these levels is dominated by the strong coupling and is linearly dependent upon it with a square-well potential of the sort used to model the samarium and europium resonance shifts, any time variation in the mean scale factor of additional spatial dimensions is constrained over a period roughly equal to that of the oldest stars ($\sim 15\times 10^9$ yr) by

$$\left|\frac{\dot{R}}{R}\right|_{\mathrm{KK}} \le (1.1\pm 0.8)\times 10^{-13}\ \mathrm{yr}^{-1}\ ,$$
$$\left|\frac{\dot{R}}{R}\right|_{\mathrm{ss}} \le (4.6\pm 3.7)\times 10^{-14}\ \mathrm{yr}^{-1}\ . \tag{21}$$

In conclusion, we have determined the constraints that can be placed upon the cosmological evolution of additional spatial dimensions by considering the effects upon the process of primordial nucleosynthesis. This analysis extends previous work by including the strong-interaction effects and additional electromagnetic effects as well as providing accurate analytic determinations of the ^{4}He abundance. Although the limits obtained from nucleosynthesis are impressively strong and limit the maximum rate of change of extra dimensional scale factors to less than about 5% of the current Hubble rate, they are dramatically superseded by those obtained by considering resonance levels in the Oklo natural reactor and in stellar nucleosynthesis of carbon and oxygen. These constraints are summarized in Table I.

I would like to thank P. Evans, B. Pagel, R. J. Tayler, and especially J. P. Elliott for helpful discussions

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[8]It is argued in Ref. 5 that the dominant contribution to the neutron-proton mass difference is electromagnetic in origin; we shall assume this to be the case also.

[9]The adequacy of this model was pointed out to the author by Professor J. P. Elliott to whom he is most grateful for supplying the experimentally fitted parameters for the two-nucleon systems. An improved model using the Yukawa potential does not give a signficantly different result.

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[11]F. Dyson, Sci. Am. **225**, 51 (1971), was the first to suggest that the binding of the diproton would have effects upon stellar nucleosynthesis. However, it is not clear that it would reduce stellar lifetimes as dramatically as was suggested because of the compensating opacity effects within stellar interiors which would tend to create a new hydrogen-burning equilibrium.

[12]We assume that there are no particle mass thresholds closely adjacent to the temperatures T_N and T_* which would result in a change the effective number of spin states contributing to the total radiation density of the Universe at the moment of neutron-proton freeze-out when considering the models with evolving extra dimensions.

[13]B. E. J. Pagel, R. Terlevich, and J. Melnick, talk delivered at the UCL Symposium, Windsor, 1986, by B. Pagel (unpublished). They report a primordial ^{4}He fraction of $Y=0.236\pm0.007$ when the spectral analysis of 200 blue compact galaxies are extrapolated to zero ^{14}N or ^{16}O abundance. Pagel also argued that when the object IIZw40 (which had not previously been accurately corrected for various *in situ* ionization effects and strongly weights the data set) is excluded from their results the earlier determination of primordial He by D. Kunth and W. L. W. Sargent, Astrophys. J **273**, 81 (1983), yielding $Y=0.243\pm0.010$, becomes $Y=0.234\pm0.008$. There is then excellent agreement between the two sets of observations as well as with the primordial value of 0.230 ± 0.004 claimed by J. Lequenx, M. Peimbert, J. F. Rayo, and S. Torres-Peimbert, Astron. Astrophys. **80**, 155 (1979).

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