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Momentum-transfer dependence of the elastic-amplitude phase in high-energy hadron scattering

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The phase of the elastic-scattering amplitude is responsible for the probability distribution of events in the impact-parameter space. A rather steep increase of it around $t=0$ can lead to a peripheral behavior of elastic collisions. The experimental data from the interference regions of hadronic and Coulomb scatterings have been shown to be fully consistent with such a peripherality. Other reasons supporting peripherality have been mentioned, also.

Glauber's method, currently used in the description of high-energy elastic scattering of light nuclei, is based on the knowledge of the nucleon-nucleon elastic amplitude at a corresponding energy. The phase of this amplitude is usually taken as weakly varying (with increasing values of the absolute four-momentum transfer squared $|t|$) from a value being determined with the help of an interference between the Coulomb and hadronic components of the total nucleon-nucleon elastic amplitude in the forward direction.

Recently, Franco and Yin[1] have shown that a substantially better agreement between the mentioned Glauber-method predictions and the experimental data can be obtained if a strong t dependence of the nucleon-nucleon amplitude phase is assumed. This t dependence of the phase is not only a question of adjustment of the nucleon-nucleon amplitude in order to obtain an agreement with the data but has a much deeper physical meaning.

It was shown[2] a few years ago that the behavior of the elastic-amplitude phase can be extremely important for the interpretation of elastic scattering in the impact-parameter space. The module of the elastic amplitude and its t dependence can be determined directly from experimental data on the differential cross section while the t dependence of the phase remains, in principle, quite arbitrary. There are no theoretical arguments for its choice, either (in the case of unpolarized scattering). And it is just the t dependence of the phase which determines the probability distribution of individual events in the impact-parameter space.

A constant or weakly t-dependent phase always leads to a central behavior of elastic scattering with its maximum at a zero value of the impact parameter.[2] The use of a nearly constant phase is practically equivalent to the use of an amplitude with dominant imaginary part which is due to the theoretical arguments[3] only valid at asymptotic energies. An equality of the particle-particle and antiparticle-particle differential cross sections can also be derived from these arguments. But the experimental data concerning the pp and $\bar{p}p$ elastic scattering[4] shows us that the dominance of imaginary parts of corresponding amplitudes can be applied only to very small momentum transfers. Thus, there are no theoretical limitations to the choice of the phase t dependence being of practical importance.

Contrary to this fact a majority of published papers still assume that the elastic amplitude has a dominant imaginary part in a rather large t region. Performing the Fourier-Bessel transform of such an amplitude one obtains unavoidably a central behavior of elastic scattering, which can be regarded as a puzzle.[5] A relatively large transparency of the colliding hadrons in head-on collisions is then a logical consequence of such an assumption.[6]

As was stressed before the dominance of the imaginary part of the elastic amplitude can be applied only to very small momentum transfers. Therefore, the central distribution[7] of elastic nucleon-nucleon scattering can hardly be justified. It is also in a contradiction with the diffractive production processes (which are commonly supposed to be produced peripherally[8]) since the dynamics of all the diffractive processes must be regarded as very similar.

The peripheral distribution of high-energy elastic hadron scattering can be obtained only if the phase of the amplitude strongly increases with increasing $|t|$ in a rather close neighborhood of $|t|\sim 0$. This t dependence of the phase cannot be simply linear as it is assumed in Ref. 1 but it must have a more complicated structure. The analysis in Ref. 2 has shown that the t dependence of the phase must lead to the zero value of the imaginary part of the amplitude already at some value of $|t|<0.1$ GeV^2 and not at the diffraction dip (as it is usually assumed). The phase value should reach its maximum at $|t|\sim 0.3$ GeV^2.

The elastic-amplitude phase might have played an important role in the analysis of the data concerning the interference between the Coulomb and hadronic components of the total amplitude in the case of charged particles. In the previous analyses[9,10] (see also Ref. 11), the momentum-transfer distribution in this region has been fitted with the help of the following form of the total amplitude:

$$F(s,t)=\frac{\alpha s}{t}f^2(t)e^{i\alpha\phi}+\frac{\sigma_{\text{tot}}}{4\pi}p\sqrt{s}\,(\rho+i)e^{Bt/2}\,, \qquad (1)$$

where the first term corresponds to the Coulomb component with $\sqrt{s}$ being the total center-of-mass-system (c.m.s.) energy, α is the fine-structure constant, $f^2=[0.71/(0.71-t)]^2$ (t in GeV^2) (the conventional dipole form factor), and $\alpha\phi=-\alpha[\ln(-Bt/2)+\gamma]$, the total West-Yenni[12] phase of the Coulomb amplitude with Euler's constant $\gamma=0.577$. The second term in Eq. (1) describes the hadronic part, where σ_{tot} is the total nucleon-nucleon cross section, p is the total c.m.s. momentum, B is the diffractive slope, and ρ is the ratio of the real and imaginary parts of the hadronic component in the forward direction. It follows from Eq. (1) that the dominance of the imaginary part is included in it.

Three assumptions are involved in the derivation of the preceding formula: (i) spin effects are neglected; (ii) the characteristic exponential dependence of the module of hadronic amplitude in the interference region is assumed; (iii) the same t dependence of the real and imaginary parts of hadronic amplitude is supposed.

The first assumption can be regarded as fully justified. It follows, e.g., from the fact that the various mutually independent methods used for measuring the total cross sections give nearly the same values lying within the experimental errors.[13] The measurements of the momentum-transfer distribution performed in the case of high-energy elastic np scattering[14] allowed us then to establish directly the hadronic part for $|t|<10^{-2}$ GeV^2. Over the whole measured t range covering the used interference region no deviation of the data from the exponential behavior was observed. Taking into account the principle of charge independence of strong interactions and isospin relations one can conclude that the extrapolation schemes used, e.g., in the high-energy elastic scattering from the hadronic amplitude in this interference region, are also fully justified. Therefore, the second assumption is quite acceptable. The third assumption is equivalent to neglecting the t dependence of the phase and there are no reasonable arguments for it.

Thus, the approach proposed recently by Cahn[15] and based on the eikonal approximation seems to provide a more suitable starting point. The following form has been derived for the total amplitude in this case:[15]

$$F(s,t)\sim\frac{\alpha s}{t}f^2(t)+F^N(s,t)\left[1-i\alpha\int_{-\infty}^{0}dt'\ln\frac{t'}{t}\frac{d}{dt'}\left[f^2(t')\frac{F^N(s,t')}{F^N(s,0)}\right]\right]\,. \qquad (2)$$

The hadronic amplitude can be written as

$$F^N(s,t)\sim e^{Bt/2-i\xi(t)}\,, \qquad (3)$$

where the t dependence of the phase can be parametrized in the following manner:

$$\xi(t)=\xi_0+\xi_1\left|\frac{t}{t_0}\right|^{\kappa}e^{\nu|t|}+\xi_2\left|\frac{t}{t_0}\right|^{\lambda},\quad t_0=1\text{ GeV}^2\,. \qquad (4)$$

This parametrization is based on the results of Ref. 2 and allows the peripheral as well as central behaviors of the elastic scattering in the impact-parameter space. For $\xi_1=\xi_2=0$ amplitude (2) reduces in principle to the amplitude (1) used in all previous analysis.

The new formula can now be fitted to the experimental data on differential cross sections being defined by

$$\frac{d\sigma}{dt}=\frac{\pi}{sp^2}|F(s,t)|^2\,. \qquad (5)$$

We have applied it to elastic pp scattering at different energies. For $p_{\text{lab}}=100$—300 GeV/c the data published in Ref. 16 have been used; they concern the momentum-transfer interval $-t\in(0.02,0.04)$ GeV^2 covering the interference regions. The other data for CERN ISR energies $\sqrt{s}=44$—63 GeV and a momentum-transfer interval $-t\in(0.001,0.04)$ GeV^2 have been taken from Ref. 17. Some preliminary results have been given in Ref. 18.

Two types of more detailed fits have been performed at all energies. In the first case (labeled as I) we have chosen $\xi_1=\xi_2=0$ (which is the same as saying that the ρ does not

TABLE I. The results for both groups of fits.

	Fit I				Fit II			
P_{lab} (GeV/c)	σ_{tot} (mb)	B (GeV^{-2})	ρ	χ^2/DF	σ_{tot} (mb)	B (GeV^{-2})	ρ	χ^2
100	38.43	11.78	−0.096	81.15/69	38.49	11.74	−0.090	81.44
150	38.73	12.03	−0.038	74.61/64	38.73	11.86	−0.040	75.14
250	39.26	12.03	−0.043	43.70/60	39.29	11.94	−0.039	43.72
300	39.47	12.16	−0.035	63.18/56	39.53	12.08	−0.035	62.89
1063	41.88	13.10	0.056	59.70/53	41.93	13.10	0.061	51.84
1487	42.38	13.11	0.075	45.51/37	42.38	13.10	0.082	43.06
2081	43.49	13.14	0.086	30.58/30	43.82	13.20	0.089	28.70

TABLE II. The values of free parameters defining the phases of fit II at individual energies.

p_{lab} (GeV/c)	ξ_1	κ	ν	ξ_2	λ
100	2157.1	3.41	−7.82	8.66	0.89
150	3039.4	3.37	−8.20	0.11	0.89
250	2165.2	3.37	−8.00	3.50	1.21
300	2258.4	3.56	−7.92	3.16	0.88
1063	2226.0	3.15	−7.93	0.83	1.18
1487	2852.7	3.11	−8.70	0.76	0.95
2081	2222.0	3.40	−8.22	0.42	0.90

vary with t), while in the other case (labeled as II) all parameters defining the t dependence of the phase in Eq. (4) have been allowed to change. In all cases we have started from the values of σ_{tot} and B determined with the help of other independent approaches; only small deviations (in the limits of given experimental errors) have been allowed for these parameters.

The numerical values of the parameters σ_{tot}, B, and $\rho = \tan\xi_0$ for both types of fits are given in Table I; the χ^2 values obtained in individual cases under the same conditions are shown, also. For fit I the normal central behavior in impact-parameter space is obtained. As can be seen from Table I, the results are equivalent to the previously published ones (cf. Refs. 9, 10, 16, or 17), but in our case the total amplitude defined by Eq. (2) and containing the relative phase between the Coulomb and hadronic components in an implicit integral form has been used.

As to the other free parameters being made use of in the fits of kind II, their values are given in Table II. All these parameters exhibit an energy dependence modifying slightly the corresponding t dependence of the phase. In determining these values a series of constraints imposed on parameters specifying the phase has been applied to in order to obtain the peripheral distribution of elastic scattering in the impact-parameter space. We required the inelastic overlap function defined with the help of unitarity condition (see, e.g., Ref. 19) to be central in the impact-parameter space. Without this constraint one obtains more expressive peripherality of the elastic scattering with slightly different values of the ρ quantity (see Ref. 18).

Some examples of distributions in the impact-parameter space are given in Fig. 1 and the corresponding t dependence of the phase in Fig. 2. We have used a special simple parametrization of the phase [see Eq. (4)] allowing us to include diverse kinds of dependences in the impact-parameter space. Some limitations are imposed, of course, by this parametrization. One can expect that a more expressed peripherality could be obtained with the help of a more suitable one.

One must conclude from the values given in Table I that for fit I the full agreement with the results obtained in Refs. 16 and 17 has been obtained in all cases. As to fit II there exist only very small deviations from fit I. Also, the χ^2 values in both cases are practically the same. This means that the peripherality of elastic scattering is fully consistent with the data. We regard the results of fits I and II as indistinguishable experimentally. Thus one must conclude that the analysis of experimental interference data cannot decide between the two different possibilities. The preference should be given to the peripheral

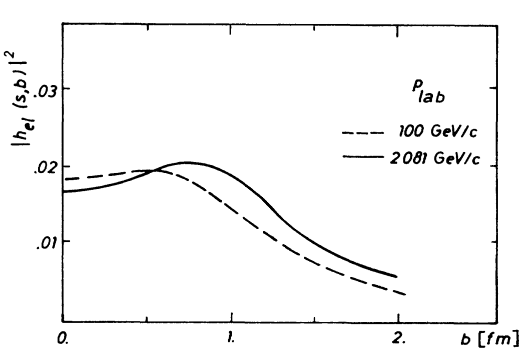

FIG. 1. Some examples of peripheral distributions obtained for fit II.

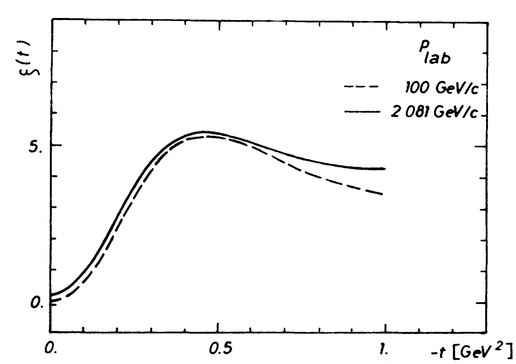

FIG. 2. Some examples of t dependence of the phase obtained for fit II.

interpretation due to logical reasons. Perhaps a more convincing argument will be obtained if our results are combined with possible future results derived with the help of more detailed analyses of light-nuclei scattering.

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