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Analysis and interpretation of a large body of ^{76}Ge zero-neutrino double-β-decay data

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Data from five recent ^{76}Ge double-β-decay experiments were combined in a single spectrum equivalent to counting for 0.4 yr with 1770-cm^3 fiducial volume of Ge. The composite spectrum was analyzed using an approximate analytical form of the likelihood function. The maximum likelihood 68%-C.L. values for the half-life and Majorana ν mass are $T_{1/2}^{0\nu}(^{76}\text{Ge}) \gtrsim 4.1\times10^{23}$ yr and $\langle m_\nu\rangle \lesssim 2.4$ eV, respectively, when accounting for the slight depression in the spectrum, and 2.4×10^{23} yr and $\langle m_\nu\rangle \lesssim 3.2$ eV when neglecting the depression. It is clearly demonstrated that experiments with current levels of background will soon reach their point of diminishing returns.

The importance of double-beta ($\beta\beta$) decay as a probe of Majorana ν mass and right-handed ν currents has been extensively reviewed in the literature.[1] Recently, a number of improved, lower background ^{76}Ge $\beta\beta$-decay experiments have been reported,[2] all of which have comparable background levels in the energy region of the anticipated 0ν $\beta\beta$-decay peak at $\sim$2041 keV.

The purpose of this paper is severalfold: to report the results of the analysis of a composite background spectrum formed from the data of five individual experiments, to demonstrate several interpretations of maximum-likelihood analyses, and finally to predict the sensitivity of the current "world experiment" for several longer counting times. The parameters characterizing each individual experiment, as well as those of the collective experiment, are given in Table I. The composite spectrum is shown in Fig. 1.

The experimental limit on the half-life is given by

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \gtrsim (\ln 2)Nt/c \ , \qquad (1)$$

where N is the number of ^{76}Ge atoms in the fiducial volume of the detectors, t is the counting time, and c is the upper limit on the number of counts in the background spectrum that can be attributed to 0ν $\beta\beta$ decay. For the present "world experiment," $Nt = 2.45\times10^{24}$ yr, and $T_{1/2}^{0\nu} \gtrsim 1.7\times10^{24}/c$. There is no precise or unique method for obtaining the parameter c, but there are a number of popular approaches which are statistical estimators, usually numerical, and the results obtained can differ significantly from one to another. If maximum-likelihood analyses are used, it would be very helpful if likelihood functions and their interpretations were presented; however, this is not common practice.

The simplest estimator is the statistical fluctuation in the total count rate in a known region of the expected peak. For example, there are 36.4 counts in the central three channels of the spectrum shown in Fig. 1. The Poisson fluctuation is $\sim$6. Those channels comprise 64% of the peak area; hence $c \simeq 9.4$ and $T_{1/2}^{0\nu} \gtrsim 1.8\times10^{23}$ yr. This is, however, a very conservative limit and does not reflect the knowledge of the response function of the detector; for that reason most authors prefer maximum-likelihood analyses. There are, however, several ways to interpret the resulting likelihood functions, and these are

TABLE I. Summary of five recent ^{76}Ge $\beta\beta$-decay experiments (Ref. 2).

Experiment	$(BG/Nt)\times10^{23}$	Nt (10^{23} yr)
Caltech	0.50[a]	2.08
Guelph	0.50	6.56
Milano	0.68	2.64
PNL/USC	0.40	4.07
UCSB/LBL	0.35	9.19
Total	0.45	24.54

[a]Given in counts keV^{-1} per 10^{23} ^{76}Ge atoms per yr.

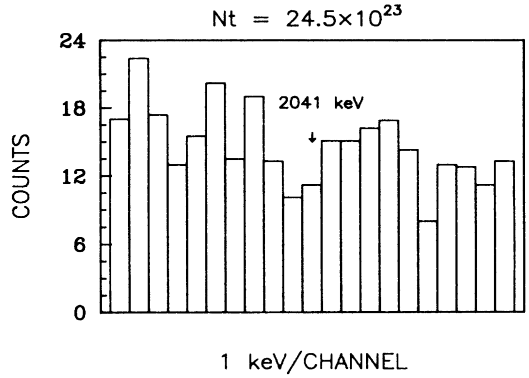

FIG. 1. The spectrum formed from the sum of the individual spectra from the five experiments cited in Ref. 2.

TABLE II. Results of various interpretations of the likelihood function. A: Direct interpretation, c is computed directly from $L(\lambda)$ integrated from λ_0. These overly optimistic limits clearly demonstrate interpretation A should not be used when $\lambda_0<0$. B: Offset interpretation, $L(\lambda)$ is integrated from $\lambda=0$. C: Offset neglected, c is computed as $\sigma_L, 2\sigma_L$, etc.

$T_{1/2}^{0\nu}$ (limit; 10^{23} yr)	Confidence level	c	Interpretation
6.0	90%	2.8	A
3.2	95%	5.4	A
1.4	99.7%	12.6	A
4.1	68%	4.2	B
1.5	95%	11.2	B
2.4	68%	7.0	C
1.2	95%	14.0	C

demonstrated using an approximate analytical technique introduced earlier.[3]

Given an experimental spectrum with n energy bins (or channels), $\{X_i\}$, having N_i counts in the ith channel, and a mean background m per channel, we define $Z_i \equiv N_i - m$. The *a posteriori* probability of observing the data points $\{N_i\}$ when there exists a Gaussian peak containing λ counts, is expressed by the likelihood function

$$L(\lambda) \equiv \prod_{i=1}^{n} f_\lambda(X_i Y_i Z_i) \ . \tag{2}$$

In Eq. (2), $Y_i = Y_i(\lambda)$ is the Gaussian spectral line (at 2041 keV in the present case). A significant simplification occurs if normal statistics are used with the following nonstandard definition of the individual *a posteriori* probabilities:

$$f_\lambda(X_i Y_i Z_i) \equiv (\sigma_y \sqrt{2\pi})^{-1} \times \int_{Z_i}^{Z_i+\Delta Z_i} \exp[-(Z_i - Y_i)^2/2\sigma_y^2] dZ_i \ , \tag{3}$$

where σ_y, the statistical fluctuation in N_i, is treated as a constant. Equation (3) is tantamount to defining $f_\lambda(i)$ as the *a posteriori* probability that the ith data point will fall on the interval Z_i to $Z_i + \Delta Z_i$, when the Gaussian peak contains λ real events. These probabilities are approximately proportional to their integrands when $\Delta Z_i \ll \sigma_y$. With these approximations, one arrives at a simple closed expression for the likelihood function which gives results in good agreement with elaborate numerical codes based on more conventional definitions of $f_\lambda(i)$.

It was shown[3] that using the approximations above,

$$L(\lambda) \propto e^{a^2/2b} e^{-b(\lambda-\lambda_0)^2/2} \ , \tag{4}$$

where

$$a \equiv (\sigma_y^2 \sqrt{2\pi})^{-1} \sum_{i=1}^{n} Z_i \eta_i \ , \tag{5}$$

$$\eta_i \equiv e^{-(X_i - X_0)^2/2\sigma_x^2} \ , \tag{6}$$

and

$$b \equiv (2\pi\sigma_y^2 \sigma_x)^{-1} \sum_{i=1}^{n} \eta_i^2 \ . \tag{7}$$

It was also shown in Ref. 3 that the most probable number of counts in the peak, implied by the data, is given by $\lambda_0 = a/b$ and $\sigma_L = b^{-1/2}$. Application of Eqs. (5)–(7) to the spectrum shown in Fig. 1 yields $m = 14.7 \pm 0.8$, $\sigma_x = 1.64$, $a = -0.174$, $b = 0.019$, $\lambda_0 = -9.1$, and $\sigma_L = 7.0$. The results of several interpretations of the likelihood function $L(\lambda)$ are given in Table II.

It is occasionally the case that a depression or dip occurs in the spectrum at the energy of the anticipated peak. This results in $\lambda_0 < 0$. If this offset is interpreted to be statistically insignificant, the limit c is obtained from the expression

$$\int_{\lambda_0}^{c} L(\lambda) d\lambda \Big/ \int_{\lambda_0}^{\infty} L(\lambda) d\lambda = \text{C.L.} \ , \tag{8}$$

where C.L. is the desired confidence limit and $\lambda_0 < 0$ corresponds to the maximum of $L(\lambda)$. This corresponds to interpretation A in Table II. In the present case, $\lambda_0 = -9.1$ or $-1.3\sigma_L$, and this interpretation is very questionable. In fact the first three cases shown in Table II clearly indicate that this interpretation can give unjustifiably optimistic results when $\lambda_0 < 0$, and we strongly recommend against its use unless $\lambda_0 > 0$.

A more conservative interpretation when $\lambda_0 < 0$ results from evaluating c using the expression

$$\int_{0}^{c} L(\lambda) d\lambda \Big/ \int_{0}^{\infty} L(\lambda) d\lambda = \text{C.L.} \tag{9}$$

This is referred to as interpretation B in Table II. We favor this interpretation because it properly reflects the connection between the probability of observing $\lambda_0 < 0$ and that for having a given number of real events hiding in the background. In addition, it always results in positive-definite values for the denominator of Eq. (1).

Finally, the most conservative interpretation of $L(\lambda)$, when $\lambda_0 < 0$, is to neglect the offset and set c equal to the multiple of σ_L corresponding to the appropriate C.L. This is essentially a square-root analysis, but one which accounts for the line shape of the detector response function. Whatever interpretation is used, it is clear that it should be specified completely when $\beta\beta$-decay results are presented.

In any case, regardless of which interpretation is favored, the parameters of the likelihood function should also be presented so that the results can be used independently of the choice of interpretation by the original authors.

In the present case, it seems appropriate to adopt interpretation B, and that yields the limit $T_{1/2}^{0\nu} \gtrsim 4.1 \times 10^{23}$ yr. This corresponds to an upper limit on the Majorana ν mass $\langle m_\nu \rangle \lesssim 2.4$ eV, using the nuclear-structure calculations of Haxton, Stephenson, and Strottman.[4] This half-life limit also corresponds to $\langle m_\nu \rangle \lesssim 2.1$ eV, $\langle m_\nu \rangle \lesssim 1.7$ eV, and $\langle m_\nu \rangle \lesssim 1.0$ eV, using the Osaka,[5] Tübingen-Jülich,[6] and Heidelberg[7] calculations, respectively. We shall use the theoretical results given in Ref. 4 in what follows.

It is interesting to ask how long the present five experiments would have to count in order to achieve substantially new levels of sensitivity. To make such projections, it is not appropriate to assume that a much longer counting time will still yield a negative value of λ_0, because if it is indeed statistical it may well disappear. For the particular purpose of future projections, we arbitrarily adopt the conservative interpretation of the likelihood function, which yields a "world limit" of $T_{1/2}^{0\nu}(^{76}\mathrm{Ge}) \gtrsim 2.4 \times 10^{23}$ yr, derived from an experiment equivalent to counting for 0.4 yr with 1770 cm^3 of Ge with a specific background in the 2041-keV region of 0.5 counts per keV per 10^{23} ^{76}Ge atoms. The corresponding limit on the Majorana ν mass is $\langle m_\nu \rangle \lesssim 3.2$ eV. With the background level shown in Fig. 1, the limit on $T_{1/2}^{0\nu}$ will increase approximately with $t^{1/2}$. Using this simple prescription, it will require almost 7 yr of counting to reach the sensitivity of $T_{1/2}^{0\nu} \gtrsim 10^{24}$ yr and $\langle m_\nu \rangle \lesssim 1.6$ eV. To achieve a limit $\langle m_\nu \rangle \lesssim 1.0$ eV, which corresponds to $T_{1/2}^{0\nu} \gtrsim 2.4 \times 10^{24}$ yr, would require 40 yr of counting with the present total volume and background. This could be reduced to 4 yr by lowering background rates a factor of 100. Present studies with the Pacific Northwest Laboratory/University of South Carolina prototype detector indicate this should be possible, and pursuit of this goal is in progress. In any case, it is abundantly clear that efforts to place limits on the Majorana ν mass, with present background levels, are rapidly approaching the point of diminishing returns. Significant background reduction, coupled with increased detector volume, is the key to achieving interesting new levels of sensitivity in these experiments.

In summary, the combination of a large body of data from five ^{76}Ge, 0ν $\beta\beta$-decay experiments ($Nt = 2.45 \times 10^{24}$ yr) implies a lower limit $T_{1/2}^{0\nu}(^{76}\mathrm{Ge}) \gtrsim 4.1 \times 10^{24}$ yr and the corresponding limit $\langle m_\nu \rangle \lesssim 2.4$ eV at the 68% maximum likelihood C.L. when using the nuclear-structure calculations of Haxton and Stephenson. Several possible interpretations of likelihood functions are demonstrated when the most probable number of counts in the 0ν $\beta\beta$-decay peak is negative, and the serious pitfalls of the most direct one are evident in this case. Finally, it is clearly demonstrated that even large volume ^{76}Ge experiments will soon reach their point of diminishing returns if restricted to present levels of background.

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