Magnetic moment and form factors of a composite system in a solvable potential model

V. Dananić and D. Tadić
Zavod za Teorijsku Fiziku, Prirodoslovno-Matematički Fakultet, Marulićev trg 19, Zagreb, Croatia, Yugoslavia

M. Rogina
Matematički Odjel, Prirodoslovno-Matematički Fakultet, Marulićev trg 19, Zagreb, Croatia, Yugoslavia

(Received 30 July 1986)

The momentum-transfer (Q) dependence of the electromagnetic form factor is calculated for the charged-fermion—neutral-scalar system in a potential model. The magnetic moment has a major contribution similar to the static bag-model value. Two additional corrections have opposite signs. Their near cancellation is connected with the proper description of the photon vertex. The same description, together with the integration measure deduced from the quasipotential formalism, leads to a form factor which has as low rate of decrease with high-Q values as suggested by the experimental data. This investigation, based on an explicitly solvable model, might give some useful hints for the treatment of the more complex systems.

I. INTRODUCTION

Quark-model calculations of nucleon form factors are usually based on static potential models. In a broader sense the MIT bag model is also included in that category. Starting with a model which describes a situation with zero momentum transfer ($\mathbf{Q}=0$), one has to calculate quantities which are either proportional to $\mathbf{Q}$ (i.e., magnetic moments) or which are continuous functions of $|\mathbf{Q}|$ (i.e., $\mathbf{Q}^2$ dependence of form factors). Instead of static-model states one has to use states which explicitly depend on the total four-momentum P. This means that essentially static-model states have to be suitably boosted. Problems can easily emerge in this transition from $P=0$ to $P\neq 0$ model states.

The calculations of the nucleon magnetic moment in the MIT bag model, for example, did not produce uniform results.[1-9] When form factors were calculated,[1,7,10,11] their falloff with high $\mathbf{Q}^2$ was usually faster then indicated by experiments.

There is some hope that the elucidation of all these difficulties can come from a relativistically covariant potential model.[12] In such a model the total momentum of the system P and its total mass w can be unambiguously defined. Via the quasipotential approach[13-17] *such a model is also connected to the Bethe-Salpeter formalism*.[18] This last connection, as discussed in the Appendix, helps to find a proper manifestly covariant expression for a form factor.

The bound state of a charged spinor and a neutral scalar particle[12] already represents a system which is sufficiently nontrivial for our purposes. The model suggests that the differences in the theoretical values for magnetic moment[1-9] follow from the respective starting assumptions about center-of-mass (c.m.) coordinates. There are two types of corrections to the main contribution. They come, respectively, from the change of the coordinates and from the boosting of the model states. In our simple model these two effects almost completely cancel out.[4,6] As a further illustration one can build a model containing several charged spinor particles which do not interact among themselves. The model contains a neutral scalar particle which interacts with all spinors and in some sense plays a spurion role.[19] Later in the text it will be denoted as the $\{N+1\}$ system. Such a model, which has a kinematic similarity with the soliton-bag model,[4] can be used to illustrate the difference between Refs. 4 and 6 and Ref. 9, respectively.

If the charge form factor is calculated (see Sec. IV) in accordance with prescriptions found in the spinor-scalar model, its Q^2 dependence is in qualitative agreement with the experimentally found behavior. When calculation is carried out by employing the usual conventions,[7,8,10,11] the charge form factor (as shown in Fig. 1) falls off more rapidly with increasing Q^2.

II. TWO-BODY POTENTIAL MODEL

Our model is based on Sec. V of Ref. 12. For simplicity we assume that both particles, or at least the spinor particle 1, are massless. Useful kinematical relations are

$$
\begin{aligned}
&P=p_1+p_2\ ,\\
&p=\frac{\epsilon_2}{w}p_1-\frac{\epsilon_1}{w}p_2\ ,\\
&p_1=p+\frac{\epsilon_1}{w}P,\ \ p_2=-p+\frac{\epsilon_2}{w}P\ ,\\
&\epsilon_1+\epsilon_2=w\ ,
\end{aligned}
\tag{2.1}
$$

Here, p_i and ϵ_i are the momentum and energy of a particle i. According to Ref. 12 the wave function ψ which describes the spinor-scalar system is a four-component spinor which satisfies

$$(\not p_1 - U)\psi = 0 \,, \tag{2.2a}$$

$$P\cdot p\psi = 0 \,. \tag{2.3a}$$

In coordinate space the relevant variables are

$$x = x_1 - x_2 \,,$$
$$y = \frac{\epsilon_1 x_1 + \epsilon_2 x_2}{w} \,, \tag{2.4}$$
$$x_1 = y + \frac{\epsilon_2}{w} x \,.$$

If

$$U = U(x_\perp) \,,$$
$$x_\perp^\mu = \left[-g^{\mu\nu} + \frac{P^\mu P^\nu}{P^2} \right] x_\nu \,, \tag{2.5}$$

the system (2.2a) and (2.3a) is solvable by a factorizable wave function

$$\psi = \phi(y)\eta(x_\perp) \,, \tag{2.6}$$

with

$$\phi(y) = e^{-iP\cdot y} \,, \tag{2.7}$$

one can write (2.2a) as

$$\left[\not p_\perp + \frac{\epsilon_1}{w}\not P - U(x_\perp) \right] \eta(x_\perp) = 0 \,. \tag{2.2b}$$

This equation takes a simple form in the system where center of mass is at rest (c.m.r. system)

$$P = (w, 0) \,,$$
$$U = U(\,|\mathbf{x}|\,) \,. \tag{2.8}$$

The potential U need not be completely specified for our purposes. Where detailed illustrations are useful we will assume that it is an infinite square well or that it is determined by boundary conditions which correspond to the MIT bag model. The condition (2.3a) becomes

$$iw\frac{\partial}{\partial x_0}\eta(\mathbf{x}) = 0 \,, \tag{2.3b}$$

so that the equation (2.2b) assumes the form standard for a stationary problem:

$$\epsilon_1 \eta(\mathbf{x}) = -[\boldsymbol{\gamma}\cdot\mathbf{p} - U(\,|\mathbf{x}|\,)]\eta(\mathbf{x}) \,. \tag{2.2c}$$

Variables $\mathbf{x}$ and $\mathbf{p}$ in (2.8) and (2.2c) are c.m.r. variables. In some expressions below the corresponding quantities will carry the label c.m.r. explicitly. We do not go into the explicit numerical determinations of ϵ_2 and w which are not important for our purposes. The complete solution of such $\{1+1\}$ system is lucidly discussed in Ref. 12. For the case where $m_1 = m_2 = 0$ one finds $\epsilon_1 = \epsilon_2 = w/2$. With $m_1 = 0$ and $m_2 \neq 0$ the system mass w has to be found from the quadratic equation $w^2 - 2\epsilon_1 w - m_2{}^2 = 0$. The quantity ϵ_1 itself is always found by solving (2.2c).

The solution for a c.m.r. system

$$\psi_{\text{c.m.r}} = e^{-iwy_0}\eta(\mathbf{x}) \tag{2.9a}$$

can be boosted to a Lorentz frame whose velocity is $\mathbf{v} = \mathbf{P}/E$ as follows:

$$\psi_{\text{c.m.r.}} \to \psi_P = e^{-iP\cdot y} S(\mathbf{P})\eta \left[\mathbf{x} + \frac{\mathbf{P}(\mathbf{P}\cdot\mathbf{x})}{w(E+W)} - \frac{\mathbf{P}}{w}x_0 \right] . \tag{2.9b}$$

In the simple case discussed here the explicit form of the spinor transformation $S(\mathbf{P})$ has the well-known free-particle form

$$S(\mathbf{P}) = \frac{1}{\sqrt{2w(E+w)}}(\not P\gamma_0 + w) \,. \tag{2.10}$$

In all applications[1–9] the expression (2.10) was used as a reasonable approximation in the calculation of the nucleon (or octet baryon) magnetic moment. It is easy to see that (2.10) connects solutions of Eqs. (2.2b) and (2.2c). The key formulas are

$$U(\,|\mathbf{x}|_{\text{c.m.r.}}) = U(\,|x_\perp|\,) \,,$$
$$(\mathbf{x}_{\text{c.m.r.}})^2 = \left[\mathbf{x} + \frac{\mathbf{P}(\mathbf{P}\cdot\mathbf{x})}{w(E+w)} - \frac{\mathbf{P}}{w}x_0 \right]^2 = -x_\perp^\mu x_{\perp\mu} \,,$$

and

$$S^{-1}(\mathbf{P})\not P S(\mathbf{P}) = w \,,$$
$$S^{-1}(\mathbf{P})\not p_\perp S(\mathbf{P}) = (-)\gamma \left[\mathbf{p}_\perp + \frac{\mathbf{P}(\mathbf{P}\cdot\mathbf{p}_\perp)}{w(E+w)} - \frac{\mathbf{P}}{w}p_{\perp 0} \right]$$
$$= -\boldsymbol{\gamma}(\mathbf{p}_\perp)_{\text{c.m.r}} = -\boldsymbol{\gamma}\cdot\mathbf{p}_{\text{c.m.r.}} \,.$$

Here

$$S^{-1}(\mathbf{P}) = \frac{1}{\sqrt{2w(E+w)}}(\gamma_0 \not P + w) \,.$$

The boosted form (2.9b) satisfies also the subsidiary condition (2.3). That follows trivially from the equality

$$\left[P_0\frac{\partial}{\partial x_0} + \mathbf{P}\frac{\partial}{\partial \mathbf{x}} \right] \left[\mathbf{x} + \frac{\mathbf{P}(\mathbf{P}\cdot\mathbf{x})}{w(E+w)} - \frac{\mathbf{P}}{w}x_0 \right] = 0 \,.$$

The electromagnetic interaction is treated as the first-order perturbation. In the field-theory language that would be a bound-state interaction representation which is commonly used in the quark-model calculations of form factors. This means that strong interactions are included "exactly," which has to be understood in the sense of the potential model building where the bound spinor-scalar state is described by a solution of particular relativistic constrained dynamics.[20] (The nonperturbative treatment of the interactions with an external vector field is a much more complex problem.[20])

In the calculation of the magnetic moment the following expression for the electromagnetic vertex is used:

$$\hat{N}=\int d^4x_1 d^4x_2 \bar{\psi}_{P_f}(x_1,x_2)\mathbf{e}\cdot\gamma e^{-iQx_1}\psi_{P_i}(x_1,x_2)$$
$$=\int d^4x\, d^4y \bar{\psi}_{P_f}(x,y)\mathbf{e}\cdot\gamma \exp\left[-iQ\left[y+\frac{\epsilon_2}{w}x\right]\right]\psi_{P_i}(x,y)\delta(L\cdot x)$$
$$=(2\pi)^4\delta^{(4)}(P_f-P_i-Q)N\ . \tag{2.11a}$$

Here $\mathbf{e}$ is the photon-polarization vector while the integral N is determined by

$$N=\int d^4x\,\eta^\dagger(x,\mathbf{P}_f)S^\dagger(\mathbf{P}_f)\gamma^0\mathbf{e}\gamma \exp\left[-i\frac{\epsilon_2}{w}Q\cdot x\right]S(\mathbf{P}_i)\eta(x,\mathbf{p}_i)\delta(L\cdot x)\ . \tag{2.11b}$$

The expression (2.11) satisfies all general requirements. It openly displays overall energy-momentum conservation in the form of the four-dimensional δ function as demanded in Refs. 7–9. This came naturally from the exponential factor in (2.9). The combination $d^4x\,\delta(L\cdot x)$ constitutes a covariant measure.[10–13,20–23] When

$$P_f=P_i=P\ , \tag{2.12a}$$

$$\mathbf{e}\cdot\gamma\rightarrow\gamma_0\ , \tag{2.12b}$$

the choice $L=P/w=\hat{P}$ allows for a suitable normalization of the wave functions.[20–23] The introduction and the choice of the covariant measure can be, via quasipotential formalism,[14–17,21–23] connected with the Bethe-Salpeter equation.[18] Some details are discussed and described also in the Appendix. General analysis of the physical situation led, already some time ago,[10] to the introduction of such covariant measure, which is always present in the form-factor calculation, even if not always written out explicitly.[7–11] The usual, sometimes hidden, choice is

$$L=\hat{P}_f+\hat{P}_i\ . \tag{2.13}$$

With the overall momentum conservation as in (2.11) a more consistent alternative (see the Appendix) is

$$L=\hat{P}_f(P_f=P_i+Q)\ . \tag{2.14}$$

The low-momentum-transfer behavior, i.e., the value of the magnetic moment, is not influenced by the form of the covariant measure. Its effect is proportional at least to $\mathbf{Q}^2$ so that differences appear at high Q values discussed in Sec. IV of this paper.

The choice of boosting made in (2.11) agrees with physical intuition and with the form of a vertex for free spin-$\frac{1}{2}$ particles. In the free-particle case the vertex would be

$$\int d^4x\,\bar{U}(p_f)e^{ip_fx}\not{e}\,e^{-iq\cdot x}e^{-ip_ix}U(p_i)$$

with Dirac spinors

$$U(p)=S(p)\chi$$

found by boosting Pauli spinors χ and the exponential factors found by Lorentz transforming a wave corresponding to the particle at rest:

$$e^{-iMt}\rightarrow e^{-ipx}\ .$$

If the wave functions η's are bag-model solutions, integration in (2.11) goes over hyperellipsoids whose boundaries depend on the bag radius R and on the four-momenta P_i and P_f.

If one is interested only in the leading terms in $\mathbf{P}_{i,f}$ and $\mathbf{Q}$, the x_0 dependence of (2.11b) can be omitted. An expansion of η's in powers of $\mathbf{P}/wx_0$ shows that the leading contribution after the integration must be proportional to $(\mathbf{P}/w)(\mathbf{R}\cdot\mathbf{P}/E)^2$. In the Breit frame where

$$\mathbf{P}_i=-\mathbf{P}_f=\mathbf{Q}/2 \tag{2.15a}$$

and for the spacelike photon with

$$q=(0,\mathbf{Q}) \tag{2.15b}$$

the exponential factor does not depend on x_0. The choices (2.15) are convenient in the magnetic-moment calculations.[2–7]

In order to find the magnetic moment one can simply replace, in (2.11b),

$$d^4x\,\delta(L\cdot x)\rightarrow d^3x\ ,$$

and integrate over the sphere with the radius $\mathbf{R}$. Detailed justification of such an approximation, valid for the leading low-Q contribution only, can be found by studying formulas in Sec. IV.

The magnetic moment corresponds to the form factor G_M (Refs. 1–9) which always appears multiplied by the factor

$$\sigma\times\mathbf{Q}\ .$$

This $\mathbf{Q}$ can come either from the exponential factor in (2.11) or from the spin rotation [i.e., $S(\mathbf{P})$]. As in Ref. 2 we shall denote the respective contributions μ^S and μ^B so that the result can be written in the general form

$$\mu=\frac{\epsilon_2}{w}\mu^S+\mu^B=\left[1-\frac{\epsilon_1}{w}\right]\mu^S+\mu^B\ . \tag{2.16}$$

Explicit forms for μ^S and μ^B can be found in Ref. 2. Here, μ^S corresponds to the old static-bag-model result while μ^B is a correction due to spinor rotation. It can be relatively large, but as the "standard" contribution μ^S is quenched, the overall result is

$$\mu\sim\mu^S$$

in quantitative agreement with Refs. 4–6.

III. SOME GENERALIZATIONS

In order to learn somewhat more from the result (2.16) one can study the $\{N+1\}$ model, which has already been described in the introduction. Such a model is a simple generalization of a spinor-scalar model. It will be applied to the low-Q limit, or equivalently nonrelativistic limit, so we are not concerned with its relativistic covariance. The structure of the $\{N+1\}$ model is close to the usual[1–9] MIT-model-based calculational schemes. It will be used to illustrate possible choices of so-called "internal coordinates."

One can introduce internal coordinates ρ_i as

$$x_i=\rho_i+y,\quad i=1,\ldots,N\,, \qquad y=\frac{\sum_i \epsilon_i x_i}{w},\quad w=\sum_i \epsilon_i\,. \tag{3.1}$$

Here x_i are "absolute coordinates" while y is the c.m. coordinate. Overall consistency requires

$$\sum_i \epsilon_i\rho_i=0\,. \tag{3.2}$$

In the already discussed spinor-scalar case one finds

$$\rho_1=\frac{\epsilon_2}{w}x,\quad \rho_2=-\frac{\epsilon_1}{w}x\,, \tag{3.3}$$

while the integral (2.11b) can be written as

$$N\sim\int d^3\rho_1\eta^\dagger S^\dagger\gamma^0\mathbf{e}\cdot\gamma S e^{-i\rho_1\cdot\mathbf{Q}}\eta\,. \tag{2.11c}$$

In the boosted bag problem, Refs. 2, 3, 7, and 8 have used for the nucleon bag state a wave function of the general form

$$\psi_B\sim e^{-iPy}\prod_{i=1}^{3}[S(P)\eta(\rho_i,P)]\,, \tag{3.4}$$

which is a straightforward generalization of the wave function (2.9). In the system in which $\mathbf{P}=0$ all exponential factors of the type $\exp(-i\epsilon_i t^0)$ are united in a general factor $\exp(-i\sum_i\epsilon_i t)\sim\exp(-iwy^0)$, $(t^0\to y^0)$. However, Refs. 2, 3, 7, and 8 assume that an absolute coordinate for the ith particle is

$$x_i=\rho_i+y\,. \tag{3.5}$$

This immediately leads to the expressions of the type (2.11e) and to the result

$$\mu=\mu^S+\mu^B\,. \tag{3.6}$$

The important thing is that here y and x_i are treated as independent quantities; y is just a coordinate of the center of the bag. Once such an ansatz is made, Lorentz transformation (i.e., boosting) inevitably leads to (3.6).[9] In a way, such an ansatz seems a natural generalization of a static picture in which the bag is centered in the origin of the coordinate system ($\mathbf{y}=0$). Superficially it seems to be analogous to our model, if one identities ρ_1 as an "internal" bag-model variable. However, if x (2.4) is selected as an analogy for the "internal" bag-model variable, the result of the type (2.16) is obtained, with the quenching factor $(1-\epsilon_i/w)$ analogous to the one suggested by Refs. 4 and 6. As the choice (3.5) ignores physical meaning of the variable y (3.1) and also the consistency condition, (3.6) is highly suspect. Generally speaking, the internal or relative coordinates for the $\{N+1\}$ system are

$$z_i=x_i-x_s,\quad i=1,\ldots,N\,. \tag{3.7}$$

Here x_i,x_s are "absolute" coordinates analogous to x_1 and x_2 from the $\{1+1\}$ case, while z_i's are analogous to the relative coordinate x from (2.4). The binding potentials are functions of z_i and x_S is the coordinate of the scalar particle which here plays the role of a spurion. One can introduce the c.m. coordinate

$$y=\frac{\sum_i\epsilon_i x_i+\epsilon_s x_s}{w},\quad w=\sum_i\epsilon_i+\epsilon_s\,. \tag{3.8}$$

This model closely corresponds to the model described by Ref. 19, where one can find many additional relevant discussions. Coordinate z_i here corresponds to the coordinate ξ_a of Ref. 19. The expression (3.8) is the same as (2.20) from Ref. 19. If the central scalar particle is treated as an unphysical spurion it is possible to arrange that ϵ_i's are eigenvalues of N bound states. Equations (3.7) and (3.8) allow us to express x_i's as functions of z_i's which do not depend on ϵ_S:

$$x_1=y+z_1\left[1-\frac{\epsilon_1}{w}\right]-\frac{1}{w}\sum_{i\neq1}\epsilon_i z_i\,, \tag{3.9}$$

etc. The electromagnetic verticies are described by a straightforward generalization of the formula (2.11a) in which one should make the replacements

$$x\to z_i\,, \qquad \int d^4x\to\sum_{i=1}^{N}\int d^4z_i\,. \tag{3.10a}$$

The electromagnetic field is coupled to a particle, ith spinor and the corresponding contributions are summed over. The overlaps of the other $k\neq i$ spinors give factors one in the nonrelativistic limit. The wave function of the system is a product (properly symmetrized) of the forms (2.9b) and (3.4):

$$\psi_P^{\{N+1\}}=e^{-iPy}\prod_{i=1}^{N}(\text{symm})S_i(\mathbf{P})\eta_i(z_i,\mathbf{P})\,. \tag{3.10b}$$

In the space of three quarks, for example, the operator Γ which describes electromagnetic vertex is

$$\Gamma^\mu=\sum_{\text{perm}\,i,j,k}\gamma_i^\mu\otimes\gamma_j^0\otimes\gamma_k^0\,. \tag{3.10c}$$

Proper relativistic generalization of (3.10c) might lead to nontrivial overlaps of a wave function of spectator spinors and to the redefinition of the electromagnetic vertex.[24] In order to avoid these problems, this section, as mentioned above, does not aspire to the full relativistic covariance.

In that system we have in the photon vertex for the spinor particle $i=1$ an exponential

$$\exp(-i\mathbf{Q}\cdot\mathbf{x}_1)=\exp\left[-i\mathbf{Q}\cdot\mathbf{y}-i\mathbf{Q}\cdot\mathbf{z}_1\left[1-\frac{\epsilon_1}{w}\right]-i\frac{1}{w}\mathbf{Q}\cdot\sum_{i\neq 1}\mathbf{z}_i\epsilon_i\right] . \tag{3.10d}$$

In the magnetic moment calculation the term with y is absorbed in the overall δ function which assures momentum conservation. Only the second term in (3.10d) contributes to μ in the leading order of Q. If all spinor masses are equal one has $\epsilon_i=\epsilon$, so that one has a common factor $(1-\epsilon/w)$ in front of μ^S contributions. With three charged particles, i.e., three quarks, and with properly symmetrized and normalized wave functions for the $\{3+1\}$ system, one finds, with $\epsilon_i=\epsilon$,

$$\mu=\left[1-\frac{\epsilon}{w}\right]\mu^S+\mu^B . \tag{3.11}$$

Here μ^S and μ^B (see Ref. 2, for example) are obtained by summing over three quark contributions which follow from (3.10c). It is not surprising that the result (3.11) is in agreement with the soliton-bag-model-based result[4] as the respective models are obviously analogous.

In order to choose between result (2.16) or (3.11) and (3.6) one has to decide which theoretical model better approximates the real physical composite baryon. A picture in which c.m. coordinate is a function of the absolute variables of the constituents seems intuitively more appealing. It is also favored by the present potential-model-based considerations.

IV. HIGH-MOMENTUM-TRANSFER DEPENDENCE OF A FORM FACTOR

The main aim of this section is to demonstrate how the high-Q^2 behavior of form factors is influenced by the coordinate transformations and by the selection of a covariant measure. The model itself is too simple for the direct comparison with experimental data.

It is sufficient for our purpose to calculate a quantity corresponding to a charge form factor, which is obtainable from formula (2.11) by the replacement (2.12b). A useful form is

$$J_0(\mathbf{Q})=\int d^3x \exp\left[i\alpha\frac{\epsilon_1}{w}\mathbf{Q}\cdot\mathbf{x}\right]\eta\left[\mathbf{x}+\frac{\mathbf{P}_f(\mathbf{P}_f\cdot\mathbf{x})}{w(E+w)}-\frac{\mathbf{P}_f}{wE}\beta\mathbf{P}_f\cdot\mathbf{x}\right]S^{-1}(\mathbf{P}_f)\gamma_0 S(\mathbf{P}_i)\eta\left[\mathbf{x}+\frac{\mathbf{P}_i(\mathbf{P}_i\cdot\mathbf{x})}{w(E+w)}-\frac{\mathbf{P}_i}{wE}\beta\mathbf{P}_i\cdot\mathbf{x}\right] . \tag{4.1}$$

Here parameters α and β are introduced to distinguish various cases:

$$\begin{aligned}&(1)\ \alpha=1,\ \beta=1 ,\\&(2)\ \alpha=1,\ \beta=0 ,\\&(3)\ \alpha=2,\ \beta=1 ,\\&(4)\ \alpha=2,\ \beta=0 .\end{aligned} \tag{4.2}$$

The first case corresponds to (2.14). The fourth case is analogous to the theory adopted by Refs. 7, 8, and 10 and to the coordinate selection leading to the result (3.6) for the magnetic moment. The value $\beta=0$ is associated with the covariant measure (2.13) in the Breit frame (2.15), where

$$\mathbf{L}=\mathbf{P}_i+\mathbf{P}_f=0 .$$

In that frame, with the coordinate substitution

$$\mathbf{z}=\mathbf{x}+\frac{\mathbf{Q}(\mathbf{Q}\cdot\mathbf{x})}{4w(E+w)} \tag{4.3a}$$

and with the notation

$$\mathbf{r}_{\pm}(\beta)=\mathbf{z}\pm\beta\frac{\mathbf{Q}(\mathbf{Q}\cdot\mathbf{z})}{4E^2} , \tag{4.3b}$$

one can write

$$J_0(\mathbf{Q},\alpha,\beta)=\frac{w}{E}\int \exp\left[i\alpha\frac{\epsilon_1}{w}\frac{w}{E}\mathbf{Q}\cdot\mathbf{z}\right]\eta^\dagger(\mathbf{r}_-(\beta))\times\eta(\mathbf{r}_+(\beta))d^3z . \tag{4.3c}$$

According to our potential model $\epsilon_1/w=\frac{1}{2}(\epsilon_1=\epsilon_2=\epsilon;$ $w=2\epsilon)$ so that $\alpha=2$ leads to an exponential factor corresponding to the choice (3.5). The region of integration, according to our bag-model-like boundary condition, is determined by

$$|\mathbf{r}_{\pm}(\beta)|\leq R .$$

The overlap of the functions $\eta^\dagger$ and η exits only over the smaller region

$$|r_+(\beta)|\leq R , \tag{4.3d}$$

which has to be selected for the calculation of the integral J_0. Obviously $\beta=0$ in case (4), for example, leads to an integration over a spherical region, which was indeed used by Refs. 2–7, 10, and 11. The numerical integration is further simplified by introducing

$$|\mathbf{z}|=r,\quad \mathbf{Q}\cdot\mathbf{z}=rQ\xi ,$$

$$f_+(\beta)=\left[1+\beta\frac{Q^2}{4E^2}\left[2+\frac{Q^2}{4E^2}\right]\xi^2\right]^{1/2} , \tag{4.4a}$$

$$r=Ry(f'_+(\beta))^{-1},\quad d^3x\to 2\pi\, dy\, dx\, R(f_+)^{-1} .$$

Here the integration goes over

$$0\leq y\leq 1,\quad -1\leq\xi\leq 1 . \tag{4.4b}$$

The results are normalized by the requirement

$$J_0(0,\alpha,\beta)=1 \tag{4.3e}$$

which is, for case (1), also the correct normalization for the potential-model wave functions.

The numerical values of J_0 were found for

$$\omega = kR = [(\epsilon_1 R)^2 - (m_1 R)^2]^{1/2} = 2.396 \, ,$$
$$\epsilon_1 R = 2.5963 \, , \tag{4.5}$$
$$m_1 R = 1.000 \, .$$

In Table I and Fig. 1 they are shown as functions of

$$t = \frac{|\mathbf{Q}|}{w} \, .$$

The form factor J_0 exhibits well-known zeros[1,25] which are always present when one has model wave functions inside a rigid sphere. The onset of those zeros is model parameter dependent; for our cases (1) and (2) they would appear at higher Q, i.e., t values.

The curve 1, corresponding to case (1), which is the correct relativistically covariant treatment in the potential model, falls much slower with the momentum transfer than other curves. The high-Q behavior is influenced by both the invariant measure and the exponential factor. Of the two, the latter seems to be much more important. Only the selection (3.1) leads to a form factor which does not fall with Q too rapidly. When the selection corresponding to (3.5) ($\alpha = 2$) is made J_0 falls so rapidly with Q that the correspondence curves 3 and 4 are shown in Fig. 1 only for the t values $t < 2$.

One should keep in mind that the calculation of Refs. 7 and 11, for example, lead in general to form factors whose large-Q behavior was unsatisfactory. They were decreasing at high-Q values faster than indicated by the experimental data. It seems very likely that this situation would be improved by using the expression (3.4) instead of one which leads to the result (3.6).

TABLE I. The charge form factor J_0 as a function of $t = |\mathbf{Q}|/w$.

t	Case (1)	Case (2)	Case (3)	Case (4)
0.0	1.000	1.000	1.000	1.000
0.2	0.971	0.974	0.912	0.912
0.4	0.892	0.903	0.699	0.700
0.6	0.785	0.802	0.466	0.499
0.8	0.674	0.687	0.281	0.238
1.0	0.574	0.574	0.159	0.098
1.2	0.491	0.472	0.086	0.022
1.4	0.424	0.385	0.044	−0.011
1.6	0.371	0.314	0.021	−0.021
1.8	0.328	0.256	0.007	−0.019
2.0	0.294	0.211	−0.000	−0.014
2.2	0.266	0.175	−0.004	−0.009
2.4	0.243	0.146	−0.006	−0.004
2.6	0.224	0.123	−0.008	−0.001
2.8	0.207	0.105	−0.008	0.001
3.0	0.193	0.091	−0.008	0.003
3.2	0.181	0.079	−0.008	0.004
3.4	0.170	0.069	−0.008	0.005
3.6	0.161	0.061	−0.008	0.005
3.8	0.152	0.055	−0.008	0.005
4.0	0.144	0.049	−0.007	0.005
4.2	0.138	0.044	−0.007	0.005
4.4	0.131	0.040	−0.007	0.005
4.6	0.125	0.037	−0.007	0.005
4.8	0.120	0.034	−0.006	0.005
5.0	0.115	0.031	−0.006	0.005

V. CONCLUSION

It seems that the main problem in the quark-model calculation of magnetic moment[1-9] and Q dependence of form factors[1,7,10,11] is in the exponential factor (2.11) or (3.10). This factor is actually a photon wave function. Its precise form depends on the electron of the coordinate system used to describe many-body problems. As discussed in Secs. II and III of this paper, this is a question of the model building. A simple covariant potential model provides a well-defined answer described by (2.11). This can be also, as discussed in Sec. III, generalized to a version of the quark model. The factor ϵ/w which appears in the exponent, tends to make the magnetic moment too small. However, this might be the consequence of the MIT bag model wave functions which were used to describe the inner motion of quarks.

On the other hand, the factor ϵ/w in the exponential in (2.11) and (4.1) leads to form factors which fall relatively moderately with Q, as seems to be required by the experimental data.

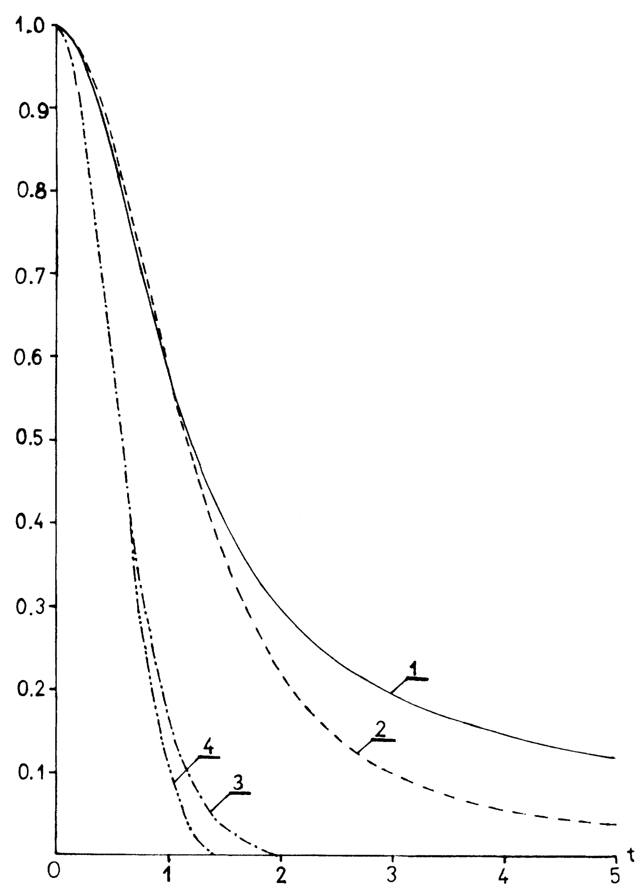

FIG. 1. The values of the charge form factor are calculated for four combinations of parameters α and β defined by (4.2). Curve 1 ($\alpha = 1, \beta = 1$) is given by the exact treatment of the model.

It might be that a suitable potential quark model, with a proper covariant treatment, based on the pseudopotential approximation to the Bethe-Salpeter equation, could provide a satisfactory description for both small and large momentum transfer.

ACKNOWLEDGMENTS

This work has been supported by the joint Yugoslav-USA Programs Nos. JFP-683 and YOR-84/078.

APPENDIX

As stated in Refs. 10, 12, 14—17, and 21—23, the "equal-time surfaces" determined by the condition

$$\langle f \mid \mathscr{P}\cdot x \mid i \rangle = 0 \tag{A1}$$

are associated with the quasipotential equations.[14—17,21—23] Here $\mathscr{P}$ is an operator meaning the total four-momentum of the two-particle system, and x is the relative coordinate. The condition (A1) determines the set of hyperplanes in the space of the relative coordinates. It stays valid even when an external electromagnetic potential is present.[26] The meaning of condition (A1) can be illustrated in the quasipotential approach. One has to study two-time Green's function for two-particle scattering, go to its Fourier transform, and then use the c.m.r. system for the momenta entering the Green's function.[14]

The four-time Green's function (GF) G is connected to the two-time GF $\widetilde{G}$ by

$$\widetilde{G}(x_0\mathbf{x},x_0\mathbf{y},x_0'\mathbf{x}',x_0'\mathbf{y}') = \int \delta(x_0-y_0)\delta(x_0'-y_0')dy_0dy_0'G(x,y,x',y') . \tag{A2}$$

Time integration in (A2) is to be understood as being performed in a c.m.r. system. The covariant form of (A2) is

$$\widetilde{G} = \int \delta\left[\frac{P\cdot(x-y)}{|P|}\right]\delta\left[\frac{P\cdot(x'-y')}{|P|}\right]dy_0dy_0'G . \tag{A3}$$

Here P is the total four-momentum of the system. The coordinates x and x' correspond to one particle, while the coordinates y and y' correspond to another particle.

By using Fourier decomposition for G one finds from (A2):

$$\widetilde{G} = \int d^4p\, d^4q\, d^4p'\, d^4q' \exp[i(-\mathbf{p}\cdot\mathbf{x}-\mathbf{q}\cdot\mathbf{y}+\mathbf{p}'\cdot\mathbf{x}'+\mathbf{q}\cdot\mathbf{y}')]\exp[i(p_0+q_0)x_0-i(p_0'+q_0')x_0']G^f(\mathbf{p},p_0;\mathbf{q},q_0;\mathbf{p}',p_0';\mathbf{q}',q_0') . \tag{A4}$$

By introducing new variables

$$\kappa = p_0+q_0,\quad \kappa' = p_0'+q_0' ,$$
$$\epsilon = q_0,\quad \epsilon' = q_0' , \tag{A5}$$

one can write

$$\widetilde{G} = \int d^3p\, d^3q\, d^3p'\, d^3q'\, d\kappa\, d\kappa'\, d\epsilon\, d\epsilon' \exp[i(-\mathbf{p}\cdot\mathbf{x}-\mathbf{q}\cdot\mathbf{y}+\mathbf{p}'\cdot\mathbf{x}'+\mathbf{q}'\cdot\mathbf{y}')]\exp(i\kappa x_0-i\kappa' x_0')G^f(\mathbf{p},\kappa-\epsilon;\mathbf{q},\epsilon;\mathbf{p}',\kappa'-\epsilon';\mathbf{q}',\epsilon') . \tag{A6}$$

According to Ref. 14 the Fourier transform of the function $\widetilde{G}$ is given by

$$\widetilde{G}^f(\mathbf{p},\kappa;\mathbf{q},\kappa;\mathbf{p}',\kappa';\mathbf{q}',\kappa') = \int d\epsilon\, d\epsilon'\, G^f(\mathbf{p},\kappa-\epsilon;\mathbf{q},\epsilon;\mathbf{p}',\kappa'-\epsilon';\mathbf{q}',\epsilon') . \tag{A7}$$

This expression corresponds to the formula (2.9) of Ref. 14. Our κ is in their notation p_0, etc. In the c.m.r. system, in which (A2) holds, one has

$$\mathbf{p}+\mathbf{q}=\mathbf{p}'+\mathbf{q}'=0,\quad \kappa=\kappa' . \tag{A8}$$

It is sufficient for our aims to study the case of the two scalar particles. It is easy to find an explicit form of (A7) for the case of two free scalar particles whose Green's function is

$$G_0(x,y,x',y) = D(x-x')D(y-y') . \tag{A9}$$

One obtains

$$G_0^f(\mathbf{p},\kappa;\mathbf{q},\kappa;\mathbf{p}',\kappa';\mathbf{q}',\kappa') = g_0^f(\mathbf{p},\mathbf{q},\kappa)\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{q}-\mathbf{q}') . \tag{A10a}$$

Here,

$$g_0^f(\mathbf{p},\mathbf{q},\kappa) \sim \frac{1}{2\omega_p[(\kappa-\omega_p)^2-\omega_q{}^2]} + \frac{1}{2\omega_q[(\kappa+\omega_q)^2-\omega_p{}^2]} ,$$
$$\omega_i{}^2 = \mathbf{p}_i{}^2+m_i{}^2 . \tag{A10b}$$

In the c.m.r. system, in which the whole procedure was carried out, the expression (A10) is actually the kernel in the quasipotential equation, as it was deduced in Ref. 14. As $\omega_p=\omega_q=\omega$ one has

$$g_0^f(\mathbf{p},\kappa) \sim \frac{1}{2\omega}\frac{2}{\kappa^2-4\omega^2} . \tag{A10c}$$

Here,

$$\kappa = E , \tag{A10d}$$

where E is the c.m.r energy of the system (Ref. 17 used the notation $E=w$).

The main aim of this paper is to determine a particular hyperplane in the space of relative coordinates which is suitable for the description of an emission (or absorption) of the photon by a composite two-particle system. Obviously the spinorial properties of particles and fields are

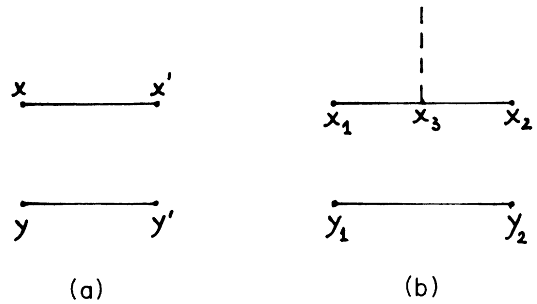

FIG. 2. Solid lines depict scalar particles with mass m. The dashed line corresponds to a massless scalar particle which carries four-momentum s.

not of the paramount importance and one can avoid inessential kinematical complications by studying the emission (or absorption) of a massless scalar field from the particle 1 (which has coordinates x,x').

As shown in Ref. 18 one has to look first at an emission from the free-particle Green's function (A10). The corresponding diagrams are shown in Fig. 2. The vertex in Fig. 2(b) is determined by

$$F(x_1,y_1;x_2,y_2)=\int d^4x_3 D(x_1-x_3) \times e^{iSx_3} D(x_3-x_2) D(y_1-y_2) . \quad \text{(A11)}$$

Omitting some inessential factors one can write the Fourier transform of (A11) as

$$F^f(p_1,q_1,S,p_2,q_2) \sim D(p_1)D(p_2)D(q_1)\delta(p_2+S-p_1)\delta(q_1-q_2) , \quad \text{(A12)}$$

$$D(p_i)=\frac{1}{p_i{}^2-m^2+i0} .$$

The conserved momentum here is

$$P=p_1+q_1=p_2+q_2+S . \quad \text{(A13)}$$

(In our main text notation $P=P_f$.) In order to go to the two-time formalism, i.e., to the quasipotential approach, one can impose the conditions

$$\delta(P\cdot(x_1-y_1)),\ \delta(P\cdot(x_2-y_2)) . \quad \text{(A14a)}$$

Here P is given by (A13) and in the c.m.r. system one has, as before,

$$P=(\kappa,0) . \quad \text{(A14b)}$$

Manipulations are very much simplified if the observed particle is not on the mass shell, i.e.,

$$S=(0,\mathbf{s}),\ S_0=0 . \quad \text{(A15)}$$

(An analogous choice was made in the main text for the photon.) The Fourier transform of the two-time vertex is

$$\tilde{F}^f \sim \int d\epsilon \frac{1}{[(\kappa-\epsilon)^2-\omega_{p_1}{}^2+i0][(\kappa-\epsilon)^2-\omega_{p_2}{}^2-i0](\epsilon^2-\omega_{q_1}{}^2+i0)} \delta(\mathbf{p}_1-\mathbf{p}_2-\mathbf{s})\delta(\mathbf{q}_1-\mathbf{q}_2) . \quad \text{(A16a)}$$

In the c.m.r. system, where

$$\begin{aligned} &\mathbf{p}_1=-\mathbf{q}_1=\mathbf{p},\ \omega_{p_1}=\omega_{q_1}=\omega ,\\ &\omega_{p_2}{}^2=\tilde{\omega}^2=(\mathbf{p}-\mathbf{s})^2+m^2 , \qquad \text{(A16b)}\\ &\omega^2-\tilde{\omega}^2=2\mathbf{p}\cdot\mathbf{s}-\mathbf{s}^2 , \end{aligned}$$

the integration over ϵ in (A16a) can be carried out explicitly and one finds

$$\tilde{F}^f \sim \frac{1}{\omega^2-\tilde{\omega}^2}[g_0^f(\mathbf{p},\mathbf{p},\kappa)-g_0^f(\mathbf{p}-\mathbf{s},\mathbf{p},\kappa)] . \quad \text{(A16c)}$$

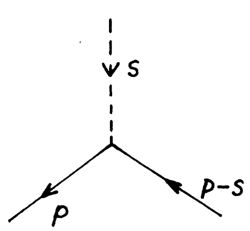

FIG. 3. Solid lines describe a scalar particle with an effective mass M. The dashed line symbolizes a massless scalar particle.

This is analogous to a single-particle vertex, shown in Fig. 3. In momentum space that vertex is of the form

$$V=\frac{1}{(p^2+M^2)[(p-S)^2+M^2]} =\frac{1}{S^2-2p\cdot S}\left[\frac{1}{p^2+M^2}-\frac{1}{(p-S)^2+M^2}\right] . \quad \text{(A17a)}$$

The equality (A17) is analogous to the Ward-Takahashi identity. It is closely connected with (A16). Taking into account (A15) one finds

$$S^2-2p\cdot S=\tilde{\omega}^2-\omega^2 . \quad \text{(A17b)}$$

Thus, obviously, the expression (A16c) describes a scalar vertex for a composite particle which is appropriate for a two-time or quasipotential formalism. The equation (A16c) contains a difference of a two-particle Green's function in the two-time formalism which is divided by the correct factor (A17b). This completes the discussion of the condition (A1), which presently leads to (A14), and which was used in Secs. II and IV.

[1]M. V. Barnhill III, Phys. Rev. D **20**, 729 (1979).
[2]I. Picek and D. Tadić, Phys. Rev. D **27**, 665 (1983).
[3]W-Y. P. Hwang, Z. Phys. C **16**, 327 (1983).
[4]M. Betz and R. Goldflam, Phys. Rev. D **28**, 2848 (1983).
[5]P. A. M. Guichon, Phys. Lett. **29B**, 108 (1983).
[6]Ch. Hajduk and B. Schwesinger, Nucl. Phys. **A423**, 419 (1984).
[7]X. M. Wang and P. Ch. Yin, Phys. Lett. **140B**, 249 (1984).
[8]X. M. Wang, Phys. Lett. **140B**, 413 (1984).
[9]A. O. Gattone and W-Y. P. Hwang, Phys. Rev. D **31**, 2874 (1985); Indiana University report (unpublished).
[10]A. L. Licht and A. Pagnamenta, Phys. Rev. D **2**, 1150 (1970).
[11]N. Barik and M. Das, Phys. Rev. D **33**, 172 (1986).
[12]H. W. Crater and P. Van Alstine, Ann. Phys. (N.Y.) **148**, 57 (1983); Phys. Rev. D **30**, 2585 (1984), and references therein.
[13]H. Sazdjian, Phys. Lett. **156B**, 381 (1985).
[14]A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963); A. A. Logunov, A. N. Tavkhelidze, I. T. Todorov, and O. A. Khrustalev, *ibid.* **30**, 134 (1963).
[15]R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1965).
[16]C. Itzykson, V. G. Kadishevsky, and I. T. Todorov, Phys. Rev. D **1**, 2823 (1970).
[17]I. T. Todorov, Phys. Rev. D **3**, 2351 (1971).
[18]S. Mandelstam, Proc. R. Soc. London **A233**, 248 (1955); G. R. Allcock and D. J. Hoston, Nuovo Cimento **8**, 590 (1958).
[19]T. Biswas and F. Rohrlich, Nuovo Cimento **88A**, 125 (1985).
[20]H. W. Crater and P. Van Alstine, Phys. Rev. Lett. **53**, 1527 (1984); Phys. Rev. D **30**, 2585 (1984).
[21]I. T. Todorov, Ann. Inst. Henri Poincare **28**, 208 (1978).
[22]I. T. Todorov, Communications of the Joint Institute for Nuclear Research, Dubna, Russia, Report No. E2-10125, 1976 (unpublished).
[23]V. V. Molotkov and I. T. Todorov, Communications of the Joint Institute for Nuclear Research, Dubna, Russia, Report No. E2-12770, 1979 (unpublished).
[24]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **26**, 2286 (1982); A. Ilakovac, D. Tadić, F. A. B. Coutinho, and F. Krmpotić, Ann. Phys. (N.Y.) **168**, 181 (1986).
[25]Y. S. Kim and M. E. Noz, Phys. Rev. D **8**, 3521 (1973).
[26]T. Takabayshi and S. Kojima, Prog. Theor. Phys. **57**, 2127 (1977).