Constraint on additional neutral gauge bosons from electroweak radiative corrections

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Quantum loop corrections to the quark mixing matrix resulting from additional neutral gauge bosons are computed. Agreement between the finding $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9984\pm0.0021$ and the unitarity value of 1 is used to provide a generic bound on masses and couplings of such bosons. For grand unified models of the type $SO(10)\to SU(3)_C\times SU(2)_L\times U(1)\times U(1)_\chi$, we find $m_{Z_\chi}\gtrsim 266$ GeV at 90% confidence limit. That bound also applies to the extra boson in some specific E_6 superstring-motivated models. In the general case of $E_6\to SU(3)_C\times SU(2)_L\times U(1)\times U(1)_\eta\times U(1)_{\eta'}$ with $m_{Z_{\eta'}}\geq m_{Z_\eta}$, we obtain the bound $m_{Z_{\eta'}}\gtrsim 254$ GeV at 90% C.L.

The existence of additional neutral gauge bosons beyond the usual γ and Z of the standard $SU(2)_L\times U(1)$ model would have major ramifications for present-day theory as well as ongoing attempts to construct grand unified theories (GUT's). Such bosons may eventually be produced and detected at high-energy colliders if their masses are not too large and their couplings to quarks and leptons are not too feeble.[1] For typical gauge-theory couplings, the upgraded CERN $p\bar{p}$ collider with $\sqrt{s}=630$, GeV will be capable of searching up to masses of about 200 GeV. That regime may be extended to $\simeq$400 GeV by the Fermilab $p\bar{p}$ collider with $\sqrt{s}=2$ TeV if sufficiently high luminosity, $L\simeq10^{30}/\text{cm}^2\,\text{sec}$, is attained. Finally, the proposed Superconducting Super Collider (SSC) pp collider with $\sqrt{s}=40$ TeV and $L=10^{33}/\text{cm}^2\,\text{sec}$ would be able to push the search as far as masses in the 5–10-TeV range, an exciting possibility. Experiments at those facilities are, however, expected in the not so near future. Therefore, one would like to search for indications of additional neutral gauge bosons in existing low-energy data. In that way, constraints can be placed on their allowed masses and couplings, or perhaps even a hint of their existence may be revealed. Furthermore, if direct evidence for an additional gauge boson is ever uncovered at future collider experiments, one will surely need constraints from low-energy phenomenology to sort out its properties and determine its proper place in theory.

So far, only global fits to existing neutral-current data, direct collider searches, and $W^\pm$ and Z masses have been used to bound the masses and couplings of additional neutral gauge bosons.[2-6] Those fits generally give bounds on the masses of such bosons in the range 100–300 GeV, depending on their couplings. Here, we illustrate a novel constraint on such bosons provided by weak-charged-current phenomenology at the quantum loop level. In that case, additional neutral gauge bosons only give rise to $O(\alpha)$ corrections to the standard-model tree-level amplitudes; however, that suppression is compensated by the high precision of existing charged-current data. In particular, quark-lepton charged-current universality has now been tested to about 0.1% at the amplitude level in the comparison of hadronic β decays and muon decay. We will show how the very good agreement found between theory and experiment can lead to rather strong bounds on the masses of additional neutral gauge bosons for some interesting models.

To establish notation and normalization, we begin by giving the fermion–gauge-boson interaction Lagrangian of the standard $SU(2)_L\times U(1)$ model:

$$L_{\text{int}}=\frac{-g}{\sqrt{2}}W^\mu(x)J_\mu^{\text{CC}}(x)+\text{h.c.}-eA^\mu(x)J_\mu^{\text{em}}(x)-\frac{g}{\cos\theta_W}Z^\mu(x)J_\mu^{\text{NC}}(x)\ , \tag{1}$$

where W^μ, A^μ, and Z^μ are gauge-boson fields, $\cos\theta_W\equiv m_W/m_Z$ and $g=e/\sin\theta_W$ is the $SU(2)_L$ gauge coupling, and the fermion currents are

$$J_\mu^{\text{CC}}=\sum_{l=e,\mu,\tau}\bar{\nu}_{l_L}\gamma_\mu l_L+\sum_{\substack{q=u,c,t\\ q'=d,s,b}}\bar{q}_L V_{qq'}\gamma_\mu q'_L\ , \tag{2a}$$

$$J_\mu^{\text{em}}=\sum_f Q_f\bar{f}\gamma_\mu f\ , \tag{2b}$$

$$J_\mu^{\text{NC}}=\sum_f (T_{3f}\bar{f}_L\gamma_\mu f_L-\sin^2\theta_W Q_f\bar{f}\gamma_\mu f)\ , \tag{2c}$$

where $f_L\equiv[(1-\gamma_5)/2]f$. In those expressions, the sum is over all three generations of fermions $f=\nu_e,e,u,d,\nu_\mu,\mu,c,s,\nu_\tau,\tau,t,b$ with Q_f and T_{3f} the electric charge and weak $SU(2)_L$ isospin of fermion f, i.e., $Q_e=-1$, $T_{3e}=-\frac{1}{2}$. The $V_{qq'}$ in Eq. (2a) are matrix elements of the 3×3 quark mixing matrix

$$V=\begin{bmatrix} V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{bmatrix} \tag{3a}$$

which must satisfy the unitarity conditions

$$\sum_i V_{ij}^* V_{ik} = \sum_i V_{ji}^* V_{ki} = \delta_{jk} \tag{3b}$$

if there are no additional quark-mixing effects (such as a fourth generation).

The first row of V in Eq. (3a) has been very well determined experimentally by comparing hadronic β decays with the precisely measured muon decay rate. After accounting for the $O(\alpha)$ radiative corrections of the standard model, one finds[7-9]

$$|V_{ud}| = 0.9747 \pm 0.0010 \,, \tag{4a}$$

$$|V_{us}| = 0.220 \pm 0.002 \,, \tag{4b}$$

$$|V_{ub}| < 0.012 \,. \tag{4c}$$

Taken together and combining the errors in quadrature, those constraints give

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0021 \tag{5}$$

which is to be compared with the three-generation unitarity value of 1 implied by Eq. (3b). The good agreement between theory and experiment is very impressive, particularly since without the radiative corrections,[10,11] i.e., at the tree level, Eq. (5) would have given 1.039, an apparent violation of unitarity. So, the experimental measurements in Eq. (4) constitute a quantum loop amplitude test of the 3-generation standard model at the level of $\simeq 0.1\%$.

New physics can be constrained by the disturbance it causes in Eq. (5). For example, if a new heavy b' quark with charge $-\frac{1}{3}$ coming from a fourth generation[11] or E_6 multiplet[12] mixes with the other quarks, one finds, from Eq. (5) and the unitarity requirement $\sum_j |V_{uj}|^2 = 1$ (sum over all charge $-\frac{1}{3}$ quarks),

$$|V_{ub'}| \le 0.065 \;\; (90\% \text{ C.L.}) \,. \tag{6}$$

Similarly, bounds can be placed on heavy-neutrino mixing, composite mass scales, supersymmetry loop corrections,[13] etc., using unitarity and Eq. (5). In the case of additional $O(\alpha)$ radiative corrections to the V_{uj}, beyond those of the standard model that have already been accounted for, they cause a further shift Δ in Eq. (5) to

$$|V_{ud}|^2 + |V_{us}|^2 + V_{ub}|^2 = 0.9984 \pm 0.0021 + \Delta \,. \tag{7}$$

Three-generation unitarity then implies the bounds

$$\Delta \ge -0.0011 \;\; (90\% \text{ C.L.}) \,, \qquad \Delta \le 0.0043 \;\; (90\% \text{ C.L.}) \,. \tag{8}$$

If extra neutral gauge bosons exist, they can contribute to Δ through $O(\alpha)$ loop corrections. One can, therefore, constrain the masses and couplings of such bosons via the bounds on Δ. The rest of this paper is devoted to discussing the resulting constraints.

General analysis. We begin by assuming a simple extension of the standard model to an effective $SU(2)_L \times U(1) \times \prod_{i=1}^N U(1)_i$ gauge theory with N additional gauge bosons[14] Z_i with masses m_{Z_i}. Their couplings to fermions are conveniently parametrized by

$$L_{\text{int}} = \frac{-3g}{2\sqrt{10}} \sum_{i=1}^{N} C_i Z_i^{\mu}(x) J_{\mu}^i(x) \,, \tag{9a}$$

where g is the $SU(2)_L$ coupling and

$$J_{\mu}^i = \sum_f (Q_{f_L}^i \bar{f}_L \gamma_\mu f_L + Q_{f_R}^i \bar{f}_R \gamma_\mu f_R) \,, \tag{9b}$$

such that $f_{R,L} = [(1 \pm \gamma_5)/2] f$. The normalization of the coupling in Eq. (9a) may appear peculiar, but it is motivated by our subsequent discussion of SO(10) and E_6 grand unification[5,6] and a desire to make it similar in magnitude to the neutral-current couplings in Eq. (1). In any case, since the C_i and $Q_{f_{L,R}}^i$ are for now arbitrary, our analysis is so far completely general. Of course, the $SU(2)_L$ symmetry requires $Q_{e_L}^i = Q_{\nu_{eL}}^i$, $Q_{u_L}^i = Q_{d_L}^i$, etc. In addition, we shall assume generation universality, i.e., $Q_{\mu_L}^i = Q_{e_L}^i$, $Q_{d_L}^i = Q_{s_L}^i = Q_{b_L}^i$ etc.

To determine the $O(\alpha)$ corrections to Δ in Eq. (7) implied by the additional Z_i bosons, one must compute the one-loop radiative corrections to quark β-decay amplitudes from which the $|V_{uj}|, j = d,s,b$ are extracted as well as muon decay which normalizes those amplitudes. Ignoring corrections of $O(\alpha m_f^2/m_W^2)$, one finds that all relevant $O(\alpha)$ corrections come from the box diagrams in Fig. 1. A straightforward calculation of those diagrams (using $Q_{\mu_L}^i = Q_{e_L}^i = Q_{\nu_{\mu L}}^i = Q_{\nu_{eL}}^i$ and $Q_{u_L}^i = Q_{d_L}^i = Q_{s_L}^i = Q_{b_L}^i$) gives

$$\Delta = \frac{-27\alpha}{40\pi \sin^2\theta_W} \sum_{i=1}^{N} |C_i|^2 Q_{e_L}^i (Q_{e_L}^i - Q_{d_L}^i) \frac{\ln x_i}{x_i - 1} \,, \qquad x_i \equiv m_{Z_i}^2 / m_W^2 \,, \tag{10}$$

where $g^2/4\pi \simeq \alpha / \sin^2\theta_W$ has been employed. Several features of our result should be commented on. (1) Since we ignore $O(\alpha m_f^2/m_W^2)$ corrections, the fractional correction induced by the extra Z_i is the same for all $|V_{uj}|^2$. (2) Only the left-handed fermion couplings Q_L^i enter into Eq. (10) because the W-boson couplings in Fig. 1 are purely left-handed and we are ignoring fermion mass effects. (3) The summation in Eq. (10) is over the Z_i mass eigenstates. Effects of mixing[15] are included through the C_i and Q^i in that expression. (4) Because the diagrams in Fig. 1 are dominated by high-frequency loop momenta, QCD strong-interaction corrections to the hadronic β-decay amplitudes are calculable. Following the analysis in Refs. 10 and 16, we find those corrections

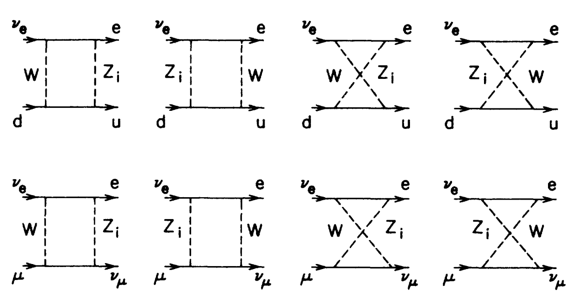

FIG. 1. Box-diagram corrections to quark and muon β decays involving Z_i bosons. For the case of V_{us} and V_{ub}, $d \to s$, or b in the above diagrams.

reduce the magnitude of Eq. (10) by $\simeq -4.4\%$. That reduction is more than compensated by a $\simeq 7\%$ enhancement which results when $\alpha(m_W)\simeq\frac{1}{128}$ (appropriate for short-distance amplitudes[17]) is used in place of $\alpha\simeq\frac{1}{137}$. Therefore, in subsequent numerical analysis, we ignore both of those effects and employ $\alpha=\frac{1}{137}$ in Eq. (10).

For a given model, knowing the couplings of the extra Z_i determines the C_i, $Q^i_{e_L}$, and $Q^i_{d_L}$ in Eq. (10). The bounds on Δ in Eq. (8) can then be used to constrain the x_i or m_{Z_i}. To get a feeling for the types of bounds that can be expected, consider the case of one extra Z' boson appended to the standard model such that $|C'|^2 Q'_{e_L}(Q'_{e_L}-Q'_{d_L})\simeq 0.5$, i.e., its couplings are similar to the U(1) weak hypercharges of the standard model. In that case, we find from Eqs. (8) and (10), $m_{Z'}\gtrsim 215$ GeV, a good constraint. For more specific GUT models, we shall see that the bounds are often somewhat better. Of course, in cases where $Q^i_{e_L}=Q^i_{d_L}$ or $Q^i_{e_L}=0$, we obtain no constraint at all.

To further illustrate the applicability of our result in Eq. (10), we examine several GUT examples with potentially low-mass additional gauge bosons. First consider the model[2,5]

$$E_6\to SO(10)\times U(1)_\psi \to SU(3)_C\times SU(2)_L\times U(1)\times U(1)_\psi$$

with one extra low-mass Z_ψ boson. Since Z_ψ stems from $SO(10)\times U(1)_\psi$, it must have a universal coupling to all members of complete SO(10) representations. Indeed, one finds (for a normalization where $C_\psi=1$ at E_6 grand unification)

$$Q^\psi_{e_L}=Q^\psi_{d_L}=\sqrt{5/27} \tag{11}$$

so, Δ gets no contribution from Z_ψ and we have no bound on m_{Z_ψ}. We fare much better for the model

$$SO(10)\to SU(5)\times U(1)_\chi \to SU(3)_C\times SU(2)_L\times U(1)\times U(1)_\chi\ .$$

In that case[2,5,6] [for $C_\chi=1$ at the SO(10) unification scale]

$$Q^\chi_{e_L}=-3Q^\chi_{d_L}=1 \tag{12}$$

and one finds from Eqs. (8) and (10) (using $\sin^2\theta_W=0.23$)

$$|C_\chi|^2\frac{\ln x_\chi}{x_\chi-1}\le 0.12\ . \tag{13}$$

Although the value of C_χ has been normalized such that $C_\chi=1$ at the SO(10) unification mass scale, at lower energy μ, it is the ratio of the effective $U(1)_\chi$ and $SU(2)_L$ couplings

$$C_\chi(\mu)=g_\chi(\mu)/g(\mu)\ . \tag{14a}$$

That ratio is expected to be less than one at low energies, since $g(\mu)$ generally increases while $g_\chi(\mu)$ must decrease as one goes to lower energies. A renormalization-group analysis leads to[6]

$$C_\chi(\mu)=(\tfrac{5}{3})^{1/2}\tan\theta_W(\mu)\sqrt{\lambda_\chi}\ , \tag{14b}$$

where $\sqrt{\lambda_\chi}$ is the ratio of the $U(1)_\chi$ and ordinary U(1) couplings at low energies. [It is understood that the ordinary U(1) coupling has been normalized to equal $g(\mu)$ at unification.] Taking $\sqrt{\lambda_\chi}\simeq 1$ (a rather good assumption), one finds, by combining Eqs. (13) and (14),

$$\frac{\ln x_\chi}{x_\chi-1}\le 0.24 \tag{15a}$$

or employing $m_W=81$ GeV,

$$m_{Z_\chi}>266\ \text{GeV}\ \ (90\%\ \text{C.L.})\ . \tag{15b}$$

That result is somewhat higher than previous bounds on m_{Z_χ} obtained from neutral-current tree-level constraints.[2,5,6] Of course, if $\lambda_\chi>1$, our bound increases. In some low-energy supersymmetry scenarios with direct

$$SO(10)\to SU(3)_C\times SU(2)_L\times U(1)\times U(1)_\chi$$

breaking and right-handed neutrinos with GUT masses, one finds λ_χ may be as big as 1.22. In that case, our bound becomes $m_{Z_\chi}\ge 307$ GeV.

From the two examples given above, one can start to see under what conditions our Δ constraint is useful and when it is not. Clearly, we can say nothing about additional neutral gauge bosons with right-handed or universal quark-lepton couplings. However, the resulting constraint is quite good when $Q^i_{e_L}(Q^i_{e_L}-Q^i_{d_L})$ is ~ 1 as in the $SO(10)\to SU(5)\times U(1)_\chi$ example. A closer inspection of the relevant couplings in that case [see Eq. (12)] indicates that they are proportional to $B-L$ (baryon number $-$lepton number) and consequently $Q^\chi_{e_L}(Q^\chi_{e_L}-Q^\chi_{d_L})=\frac{4}{3}$, a relatively large value. In fact, for most GUT's, it is the $B-L$ current content of the J^i_μ in Eq. (9) that determines the contribution of the Z_i to Δ. However, the effect of the $B-L$ current will generally be distributed among the Z_i due to mixing. To illustrate this last point, we consider the model[2]

$$E_6\to SU(3)_C\times SU(2)_L\times U(1)\times U(1)_\eta\times U(1)_{\eta'}$$

with two extra relatively light flavor-diagonal neutral gauge bosons Z_η and $Z_{\eta'}$ that are orthogonal linear combinations of the Z_ψ and Z_χ previously discussed:[2]

$$Z_\eta=Z_\psi\sin\phi+Z_\chi\cos\phi\ , \tag{16a}$$

$$Z_{\eta'}=Z_\psi\cos\phi-Z_\chi\sin\phi\ , \tag{16b}$$

where by definition we take $m_{Z_\eta}\le m_{Z_{\eta'}}$, $-\pi/2<\phi\le\pi/2$.

Using $Z^\mu_\eta J^\eta_\mu+Z^\mu_{\eta'}J^{\eta'}_\mu=Z^\mu_\psi J^\psi_\mu+Z^\mu_\chi J^\chi_\mu$ at unification and recalling Eqs. (11) and (12), one finds

$$Q^\eta_{e_L}=\cos\phi+(\tfrac{5}{27})^{1/2}\sin\phi\ ,\qquad Q^\eta_{d_L}=-\tfrac{1}{3}\cos\phi+(\tfrac{5}{27})^{1/2}\sin\phi\ , \tag{17a}$$

$$Q^{\eta'}_{e_L}=-\sin\phi+(\tfrac{5}{27})^{1/2}\cos\phi\ ,\qquad Q^{\eta'}_{d_L}=\tfrac{1}{3}\sin\phi+(\tfrac{5}{27})^{1/2}\cos\phi\ , \tag{17b}$$

which leads to

$$Q_{e_L}^{\eta}(Q_{e_L}^{\eta}-Q_{d_L}^{\eta})=\tfrac{4}{3}\cos^2\phi[1+(\tfrac{5}{27})^{1/2}\tan\phi]\ , \qquad (18a)$$

$$Q_{e_L}^{\eta'}(Q_{e_L}^{\eta'}-Q_{d_L}^{\eta'})=\tfrac{4}{3}\sin^2\phi[1-(\tfrac{5}{27})^{1/2}\cot\phi]\ . \qquad (18b)$$

[Note that $Q_{e_L}^{\eta}(Q_{e_L}^{\eta}-Q_{d_L}^{\eta})+Q_{e_L}^{\eta'}(Q_{e_L}^{\eta'}-Q_{d_L}^{\eta'})=Q_{e_L}^{\chi}(Q_{e_L}^{\chi}-Q_{d_L}^{\chi})$ independent of ϕ.] Making the reasonable assumption $|C_\eta|^2\simeq|C_{\eta'}|^2\simeq\frac{5}{3}\tan^2\theta_W\simeq 0.5$ in Eq. (10), the constraint in Eq. (8) becomes for this model

$$\cos^2\phi[1+(\tfrac{5}{27})^{1/2}\tan\phi]\frac{\ln x_\eta}{x_\eta-1} + \sin^2\phi[1-(\tfrac{5}{27})^{1/2}\cot\phi]\frac{\ln x_{\eta'}}{x_{\eta'}-1}\le 0.24\ . \qquad (19)$$

If Z_η and $Z_{\eta'}$ are nearly degenerate, this reduces to Eq. (15a) and we find m_{Z_η}, $m_{Z_{\eta'}}\ge 266$ GeV. If we use constraints from the UA1 and UA2[18] Collaborations as well as neutral-current analyses[2,3] which give $m_{Z_\eta}\gtrsim 125$ GeV (for essentially any ϕ), then since $\cos^2\phi+(\frac{5}{27})^{1/2}\sin\phi\cos\phi\ge -0.0443$ (corresponds to $\phi\simeq-78.4°$), the heavier $m_{Z_{\eta'}}$ must satisfy (independent of ϕ)

$$\frac{\ln x_{\eta'}}{x_{\eta'}-1}<0.26 \qquad (20a)$$

or

$$m_{Z_{\eta'}}\ge 254\ \text{GeV} \qquad (20b)$$

the bound quoted in the abstract.

In some of the specific superstring scenarios considered in the literature, $m_{Z_{\eta'}}$ is very heavy $\simeq 10^{10}-10^{18}$ GeV, and ϕ is fixed for one reason or another. Three examples are $\tan\phi=0$, $\sqrt{5/27}$, and $-\sqrt{5/3}$ (corresponding to $\phi=0°$, 23.3°, and $-52.2°$). The first two[19] have the virtue of allowing a large Majorana mass term for one of the neutral leptons in the E_6 27-plet and thus accommodating light neutrinos in a natural manner. In both cases we find, from Eq. (19),

$$m_{Z_\eta}\ge 266\ \text{GeV}\ \ (\phi=0°,\ 23.3°)\ . \qquad (21)$$

Of course, $\tan\phi=0$ corresponds to the $SO(10)\to SU(3)_C\times SU(2)_L\times U(1)\times U(1)_\chi$ model previously considered; however, the $\tan\phi=\sqrt{5/27}$ model is novel.

The third possibility $\tan\phi=-\sqrt{5/3}$ is perhaps the most popular of the E_6 superstring scenarios.[4,19,20] It can be obtained directly from E_6 via non-Abelian Wilson-loop symmetry breaking. In that case, we find from Eq. (18)

$$\frac{1}{6}\frac{\ln x_\eta}{x_\eta-1}+\frac{5}{6}\frac{\ln x_{\eta'}}{x_{\eta'}-1}\le 0.24\ , \qquad (22a)$$

or since $x_{\eta'}$ is very large

$$\frac{1}{6}\frac{\ln x_\eta}{x_\eta-1}\le 0.24\ , \qquad (22b)$$

$$m_{Z_\eta}\ge 55\ \text{GeV}\ \ (\phi=-52.2°)\ , \qquad (22c)$$

not much of a constraint. As previously mentioned, neutral-current phenomenology as well as direct searches by the UA1 and UA2 Collaborations already give $m_{Z_\eta}\ge 125$ GeV for this model.

As a generalization of the above examples, we give in Fig. 2 bounds on m_{Z_η} as a function of ϕ. Note, that for a rather broad range $-23°\lesssim\phi\lesssim 47°$ we obtain the bound $m_{Z_\eta}\ge 200$ GeV.

For GUT's larger than E_6, the result in Eq. (19) generalizes to

$$\sum_{i=1}^{N} a_i\frac{\ln x_i}{x_i-1}\le 0.24\ , \qquad (23a)$$

where assuming all $|C_i|^2\simeq 0.5$ at low energies,

$$\sum_{i=1}^{N} a_i=1\ . \qquad (23b)$$

[Deviations from 1 in Eq. (23b) are in principle calculable via the renormalization group.] The magnitude of each a_i is a measure of the $B-L$ current content of the J_μ^i in Eq. (9a). In fact, even the large -3.6% photonic correction in the standard model can be viewed as coming from the $B-L$ current component of J_μ^{em}. For left-handed quark and lepton fields, the electric charge in Eq. (2b) can be decomposed as

$$Q_f=T_{3f}+(B-L)/2\ ; \qquad (24)$$

so we see that J_μ^{em} has a relatively large $B-L$ component. In the case of the ordinary Z boson, Eq. (2c) indicates that its $B-L$ component is somewhat suppressed by $\sin^2\theta_W$. Therefore, in comparing the contribution of the standard and additional Z's to quark-lepton universality violation, there are two competing factors: the former is suppressed by an additional $\sin^2\theta_W$ factor while the latter are inhibited by their larger masses.

The above examples illustrate the important role Δ already plays as a constraint or possible indicator for new physics phenomena. If the bounds on Δ could be further improved to $\simeq\pm 0.0005$, it would provide an even better probe of additional neutral gauge bosons up to masses of about 450 GeV. What are the prospects for improving the

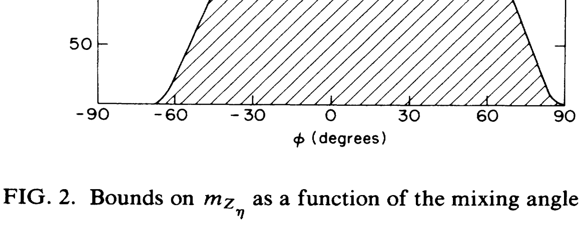

FIG. 2. Bounds on m_{Z_η} as a function of the mixing angle ϕ.

bounds on Δ in Eq. (8)? To do better requires a reduction in the error on V_{ud}, since it dominates the uncertainty in Eq. (5). That error is primarily due to hadronic structure uncertainties coming from charged-current axial-vector loop effects.[10,11] To reduce the quoted error would require better theoretical calculations for the heavy nuclear β-decays from which $|V_{ud}|$ is presently derived. That possibility is being studied. Alternatively, new experiments involving simpler decays could be undertaken in which case theoretical uncertainty can probably be reduced. With regard to the latter possibility, there are two good candidates. The first is a high-precision measurement of the pion β decay rate $\Gamma(\pi^+ \to \pi^0 e^+ \nu_e)$. Unfortunately, improving existing determinations[21] of that rate to better than 0.2% accuracy is very difficult because the branching ratio $\simeq 10^{-8}$ is so small. Nevertheless, the very small theoretical uncertainty in the decay-rate calculations suggests that such a measurement should be attempted at one of the high-flux pion facilities. A second possibility involves neutron β decay $n \to p e \bar{\nu}_e$. In that case, precision measurements of both the lifetime and g_A/g_V must be made.[22] The advent of dedicated cold neutron facilities may make those experiments technically feasible in a few years. We, of course, strongly advocate such measurements.

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