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Spin interactions in the flux-tube model and hybrid meson masses

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We present the results of a first exploration of meson spin interactions in the flux-tube model of Isgur and Paton. The model supports the view that long-range spin effects other than kinematical (Thomas) are small. Nonadiabatic effects somewhat enhance the long-range Thomas interaction in quark model mesons, and yield estimates for the spin splittings of the lowest hybrid mesons. We comment on the implications of these spin corrections for previous calculations of exotic hybrid decays.

Recent work has gone a considerable way towards solidifying the basis of the quark potential model in QCD (Ref. 1). It has been argued that the quark model is a low-energy adiabatic approximation for mesons and baryons in which the quark Fock space is truncated at $(q\bar{q})$ or (qqq) and the gluonic field follows the quark motion while remaining frozen into its quantum-mechanical ground state.[2] For the low-lying heavy-quark states this approximation may be justified by general arguments; for light-quark states one must resort to a model of the gluonic degree of freedom to support the validity of the approximation and estimate corrections to it.[3] For light quarks there is the additional problem of relativistic corrections. Again a model must be resorted to, but from the detailed work on relativized models of mesons[4] and baryons[5] it appears that many of the inconsistencies of the nonrelativistic model are removed when plausible relativistic corrections are made.

In addition to the ordinary quark model states, one also predicts nonquark model states called "hybrids" living on the higher adiabatic surfaces, in which the gluonic field is excited.[6] Theoretical evidence for such states in the adiabatic limit of pure gauge theory has been obtained in lattice calculations.[7] We believe it is one of the urgent tasks of experimental hadron spectroscopy to search for unambiguous evidence for such states. Again, models must be resorted to in order to guide this search.[8]

A model for the coarse-grained properties of QCD, which indeed has the quark model as its low-energy adiabatic limit, is the flux-tube model of Isgur and Paton.[2] We have already explored spin-independent nonadiabatic effects in mesons using this model,[3] using for simplicity a nonrelativistic kinetic energy operator for the quarks. (We do not expect that relativistic effects will substantially alter the conclusions.) Nonadiabatic effects are indeed very small for heavy quarks (≈ 1 MeV for the lowest states of $c\bar{c}$), and, although appreciable in light-quark model states, can largely by subsumed in a modification to the centrifugal barrier term in the radial Schrödinger equation to take account of the moment of inertia of the flux tube.

The flux-tube model also yields predictions for the spin-averaged masses of the lowest hybrid states.[2] These are states with total orbital angular momentum $l=1$ and one phonon of transverse flux-tube vibration excited in the lowest mode. The phonon carries one unit of angular momentum about the $q\bar{q}$ axis, $|\lambda|=1$, and may have right- (RH) or left-hand (LH) polarization; parity eigenstates are linear combinations of these polarization states $[(1^+)\pm(1^-)]$. Combined with spin there are therefore eight states of different J^{PC} which are degenerate in the absence of spin interactions. These include several with exotic quantum numbers which provide an unambiguous experimental signal. A model calculation of the likely decays and widths of these states suggests reasons why they have not yet been detected, and indicates where they may be found.[8]

In this paper we report on a first investigation, using the flux-tube model, of spin interactions in mesons. In ordinary quark model mesons we find that the long-range spin-spin interaction is small, so that outside the perturbative region the spin interaction should be dominated by the kinematical Thomas effect. However, the flux-tube model suggests specific nonadiabatic corrections to the canonical Thomas spin splittings. There is no canonical spin interaction in hybrid mesons, since the Fermi-Breit reduction is applicable only to the lowest adiabatic potential surface (ordinary quark model mesons). For the lowest hybrid states we suggest that the dominant spin splitting is kinematical in origin (Thomas precession) and estimate its magnitude.

We have nothing to add to the usual analysis of the short-distance spin interactions in ordinary quark model states. The relativistic $(v/c)^2$ corrections to the static Coulomb potential as given by the Fermi-Breit Hamiltonian are well known.

At larger distances, however, the standard perturbative analysis is invalid. In the confining region, it is generally held that the adiabatic interaction is equivalent to Lorentz scalar in its spin dependence,[9] i.e, that all static spin-spin interactions vanish at large distances, leaving the kinematical Thomas interaction as the only long-range spin effect.

It is interesting to see if this conclusion is consonant with a picture of spin interactions based on the flux-tube model.[10] This requires a prescription for how to couple quark spins to the flux tube. We have investigated[11] the static spin-spin interactions in the flux-tube model using a lattice approximation to $H_{\rm spin}=g(\lambda^a/2)\mathbf{B}^a\cdot\mathbf{s}/M_q$, where $\mathbf{B}^a$ represent the chromomagnetic fields, to give a local interaction of the flux tube with the spin s of the quark at each end. It is natural in lattice theory to take the operator for a component of $(\lambda^a/2)\mathbf{B}^a$ at a lattice site as $[(U_p-U_p^\dagger)/2iga^2]_{\rm av}$, where a is the lattice spacing and the average is over the four plaquettes p adjacent to that site in the perpendicular plane. In the approximation of the flux-tube model, $H_{\rm spin}$ is replaced by an operator with matrix elements only between states with the same flux topology, as defined in Ref. 2. Using the small oscillation harmonic approximation for flux-tube motion,[12] this yields[11]

$$H_{\rm spin}=\left[\frac{b}{32(N+1)a}\right]^{1/2}\sum_{\substack{\rm modes\\ m=1,N}}\left[\frac{2}{a}\sin\frac{\pi m}{2(N+1)}\right]^{1/2}\sin\frac{\pi m}{N+1}\left[\left[\frac{s_1^+}{M_1}+(-1)^m\frac{s_2^+}{M_2}\right](a_{m-}^\dagger+a_{m+})+\text{H.c.}\right], \qquad (1)$$

where b = string tension, a = lattice spacing, and $r=Na$ is the $q\bar q$ separation. $s_i^{\pm},s_2^{\pm}$ are the raising and lowering operators for the $\hat{\mathbf{r}}$ component of the spin of quarks of mass M_1 and M_2, and $a_{m-}^\dagger$ (a_{m+}) creates (annihilates) a LH (RH) vibrational phonon in mode m. The lattice spacing represents a length scale of $O(b^{-1/2})$ below which the flux-tube description of the color field becomes inappropriate, which we take to be ≈ 0.3 fm. In the second order of perturbation theory, $H_{\rm spin}$ gives the following color-magnetic potential:

$$V_{\rm CM}(r)=-\,\text{const}+\frac{b}{8a}\frac{\delta_{N_1}}{M_1M_2}[\mathbf{s}_1\cdot\mathbf{s}_2-(\mathbf{s}_1\cdot\hat{\mathbf{r}})(\mathbf{s}_2\cdot\hat{\mathbf{r}})]-\frac{b(\pi a)^2}{16r^3}\left[\frac{\mathbf{s}_1\cdot\hat{\mathbf{r}}}{M_1{}^2}+\frac{\mathbf{s}_2\cdot\hat{\mathbf{r}}}{M_2{}^2}\right]\sum_{\substack{\rm modes\\ m}}m^2(n_{m+}-n_{m-})\,, \qquad (2)$$

where n_{m+} (n_{m-}) are the number of phonons of flux-tube vibration excited in mode m with RH (LH) polarization.

The second term in (2) gives a spin-spin interaction which vanishes beyond one lattice spacing, and since the flux-tube model breaks down at short range this term may be ignored (or replaced by the usual Fermi-Breit hyperfine and tensor interactions in ordinary quark model mesons). The last term vanishes in ordinary quark model states but gives a long-range interaction in hybrids. This interaction is only meant to be relevant for $r\gg a$, but even ignoring this restriction it is quite small. With $a=0.3$ fm and using the parameters and hybrid wave functions of Ref. 3 ($b=0.18$ GeV^2, $M_u=M_d=330$ MeV), the expectation value $\langle b(\pi a)^2/16M_q{}^2r^3\rangle$ is only 15 MeV in the lowest-lying $u\bar u$ hybrid state and less for the heavier hybrids.

This analysis within the flux-tube model therefore lends support to the view that the long-distance interaction is kinematical (Thomas). The Thomas interaction term is

$$H_{\rm Th}=\boldsymbol{\omega}_{1\rm Th}\cdot\mathbf{s}_1+\boldsymbol{\omega}_{2\rm Th}\cdot\mathbf{s}_2\,, \qquad (3)$$

where

$$\boldsymbol{\omega}_{i\rm Th}=\frac{1}{2}\frac{d^2\mathbf{r}_i}{dt^2}\wedge\frac{d\mathbf{r}_i}{dt} \qquad (4)$$

to order $(v/c)^2$. For ordinary quark model states this gives the standard expression

$$H_{\rm Th}=-\frac{1}{2}\left[\frac{\mathbf{s}_1}{M_1{}^2}+\frac{\mathbf{s}_2}{M_2{}^2}\right]\cdot\mathbf{L}\frac{1}{r}\frac{dV}{dr} \qquad (5)$$

in the adiabatic approximation. To estimate nonadiabatic corrections we add $H_{\rm Th}$ to the flux-tube model Hamiltonian H. Because of terms in H involving flux-tube variables, Eq. (5) no longer follows from Eq. (4). In terms of a spherical polar basis $(\hat{\mathbf{r}},\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\phi}})$, defined with respect to the quark-antiquark relative coordinate $\mathbf{r}=r\hat{\mathbf{r}}$, we have, for quarks of equal mass M_q (Ref. 3),

$$\mathbf{r}_{\substack{1\\(2)}}=\mp\frac{r}{2}\hat{\mathbf{r}}+\frac{2r\sqrt{b}}{br+2M_q}\times\sum_{\substack{\rm odd\\ \rm modes}}\frac{(a_{m-}^\dagger-a_{m+})\hat{\boldsymbol{\theta}}_+ +(a_{m+}^\dagger-a_{m-})\hat{\boldsymbol{\theta}}_-}{(\pi m)^{3/2}} \qquad (6)$$

where $\hat{\boldsymbol{\theta}}_\pm=\mp(\hat{\boldsymbol{\theta}}\pm i\hat{\boldsymbol{\phi}})/\sqrt{2}$. The operators $a_{m\pm}^\dagger,a_{m\pm}$ allow transitions between the adiabatic surfaces, i.e., corrections to the adiabatic limit. The first term alone of Eq. (6) survives in the adiabatic limit $M_q\to\infty$; it leads to Eq. (5) for ordinary quark model mesons. The second term in Eq. (6) arises because the quark-antiquark axis does not pass through the overall center of mass of the system.

To calculate the perturbative effect of $H_{\rm Th}$ we use the identity

$$\langle f\,|\,\ddot{\mathbf{r}}_i\wedge\dot{\mathbf{r}}_i\,|\,i\rangle=-i\sum_I(E_I-E_f)^2(E_I-E_i)\times\langle f\,|\,\mathbf{r}_i\,|\,I\rangle\wedge\langle I\,|\,\mathbf{r}_i\,|\,i\rangle \qquad (7)$$

and evaluate the matrix elements of $\mathbf{r}_i$ using Eq. (6) and the meson and hybrid wave functions of Ref. 3. The details of this calculation are contained in Ref. 11.

In ordinary quark model states, the diagonal matrix elements of $H_{\rm Th}$ have the form

$$\langle jls\rho\,|\,H_{\rm Th}\,|\,jls\rho\rangle=\langle jls\,|\,\mathbf{L}\cdot\mathbf{S}\,|\,jls\rangle\langle\rho l||H_{\rm Th}||\rho l\rangle\,, \qquad (8)$$

where ρ is the radial quantum number. If the intermediate states $|\,I\rangle$ of Eq. (7) are restricted to contain no pho-

TABLE I. Thomas precession reduced matrix elements and spin splittings. (a) Ordinary quark model $l=1$ mesons. (b) Lowest-lying vibrational hybrid mesons.

(a)

State	$\langle l=1\|\|H_{\rm Th}\|\|l=1\rangle$/MeV $\Delta\lambda=0$[a]	$\Delta\lambda=\pm1$[b]	Thomas splitting[b]/MeV
$u\bar{u}$	-280	$-90(-140)$	$-370(-420)\langle \mathbf{L}\cdot\mathbf{S}\rangle$
$s\bar{s}$	-140	$-40(-60)$	$-180(-200)\langle \mathbf{L}\cdot\mathbf{S}\rangle$
$c\bar{c}$	-22	$-4(-6)$	$-26(-28)\langle \mathbf{L}\cdot\mathbf{S}\rangle$
$b\bar{b}$	-8	$-0.4(-0.4)$	$-8(-8)\langle \mathbf{L}\cdot\mathbf{S}\rangle$

(b)

State (M_0/GeV before spin splitting)	δm_1[c]/MeV $\Delta\lambda=0$	δm_1[c]/MeV $\Delta\lambda=\pm1$[b]	δm_2[d]/MeV	Thomas splitting/MeV (J^{PC})			
$u\bar{u}(2.0)$	14	$-90(-100)$	-60	$0(1^{++})$,	$280(0^{+-})$,	$140(1^{+-})$,	$-140(2^{+-})$
				$0(1^{--})$,	$40(0^{-+})$,	$20(1^{-+})$,	$-20(2^{-+})$
$s\bar{s}(2.1)$	7	$-45(-55)$	-23	$0(1^{++})$,	$125(0^{+-})$,	$65(1^{+-})$,	$-65(2^{+-})$
				$0(1^{--})$,	$35(0^{-+})$,	$15(1^{-+})$,	$-15(2^{-+})$
$c\bar{c}(4.2)$	0.8	$-6(-6)$	-1	$0(1^{++})$,	$12(0^{+-})$,	$6(1^{+-})$,	$-6(2^{+-})$
				$0(1^{--})$,	$8(0^{-+})$,	$4(1^{-+})$,	$-4(2^{-+})$
$b\bar{b}(10.8)$	0.05	$-0.8(-1)$	-0.05	$0(1^{\pm\pm})$,	$2(0^{\pm\mp})$,	$1(1^{\pm\mp})$,	$-1(2^{\pm\mp})$

[a] $(-1/2M_q^2r)(dV/dr)$ for quark model states (see text).
[b] Where two figures are given, the main figure only involves phonons in the fundamental mode ($m=1$) in the intermediate state $|I\rangle$, while the figure in parentheses is a sum over all modes. Thus there is a small uncertainty associated with the cutoff.
[c] $\delta m_1=\langle l=1,(1^+)\|H_{\rm Th}\|l=1,(1^+)\rangle$.
[d] $\delta m_2=\langle l=1,(1^-)\|H_{\rm Th}\|l=1,(1^+)\rangle$.

nons, then the reduced matrix element $\langle \rho l\|H_{\rm Th}\|\rho l\rangle$ reproduces the adiabatic result, Eq. (5). The nonadiabatic correction to this involves intermediate states $|I\rangle$ with one phonon. Table I(a) shows these two contributions for the $1P$ mesons, and it is seen that the nonadiabatic correction may increase the Thomas splitting by up to 50% for light-quark states. The presence of this nonadiabatic effect is of considerable interest in principle. However, it would not be easy to disentangle, since there are other uncertainties associated with light-quark spin effects. In the relativized quark model of Ref. 4, in which spin interactions are smeared and contain kinematical factors of $m/E<1$, there is an empirical suppression of the standard $\mathbf{L}\cdot\mathbf{S}$ interaction by about a factor of 2 (Ref. 13).

We turn now to spin splittings in the lowest vibrational hybrids. Since the hybrid wave functions are small at the origin, the short-distance contribution to the spin splitting is likely to be small, and we do not consider it further. We assume that the long-distance interaction in vibrational hybrids is dominated by $H_{\rm Th}$, since it was argued earlier that the long-range spin-spin interaction is likely to be small. We recall that in the absence of spin splitting there is an exact degeneracy of eight states of $J^{PC}=1^{\pm\pm},0^{\pm\mp},1^{\pm\mp},2^{\pm\mp}$ for equal mass $q\bar{q}$ pairs. The first pair of states, being spin singlets, are not affected by the Thomas interaction. The splitting between the remaining states is determined by two reduced matrix elements of $H_{\rm Th}$: the diagonal element $\delta m_1=\langle 1P(1^\pm)\|H_{\rm Th}\|1P(1^\pm)\rangle$, and the element $\delta m_2=\langle 1P(1^-)\|H_{\rm Th}\|1P(1^+)\rangle$ which splits eigenstates of opposite parity. Thus the eigenstates $1/\sqrt{2}\{|j,l=s=1,(1^+)\rangle\pm|j,l=s=1,(1^-)\rangle\}$, which have parity ±1, are shifted in energy by $(\delta m_1\pm\delta m_2)\langle \mathbf{L}\cdot\mathbf{S}\rangle$. Table I(b) shows these reduced matrix elements and the resultant spin splittings. We note that the $\mathbf{L}\cdot\mathbf{S}$ splitting is largest in states of quark model parity $(-1)^{l+1}$, and inverted, as is the Thomas splitting in ordinary quark model states.

Finally we comment briefly on the implications of these spin corrections for previous predictions of hybrid decay.[8] It was pointed out that there are three nonets of low-lying vibrational hybrids with exotic quantum numbers, $J^{PC}=2^{+-}$, 1^{-+}, and 0^{+-}, containing in total nine neutral numbers, but except for four these all are likely to be

TABLE II. Masses (including estimated spin splittings) and dominant decays of the most readily observable exotic hybrid mesons. x, y, and z denote the flavor states $(u\bar{u}-d\bar{d})/\sqrt{2}$, $(u\bar{u}+d\bar{d})/\sqrt{2}$, and $s\bar{s}$. The subscript on a state is J, the superscripts P and C_n.

Hybrid state (mass[a]/MeV)	Main decay channels	Partial width[b]/MeV
$y_2^{+-}(1900)$	πB	500
$z_2^{+-}(2100)$	$[\bar{K}K^*(1420)+\text{c.c.}]$	250
	$(\bar{K}Q_2+\text{c.c})$	200
$x_1^{-+}(2000)$	πB	150
	πD	60
$y_0^{+-}(2200)$	πB	500
	ηH	200
	$(\bar{K}Q_1+\text{c.c})$	400

[a] This assumes no suppression of the $\mathbf{L}\cdot\mathbf{S}$ splitting in the hybrids. See text.
[b] From Ref. 8 with corrections to phase space where appropriate.

too broad and/or possess decay channels that are too difficult to be easily observable. The only one of these four states for which the conclusions of Ref. 8 may be substantially altered is the y_0^{+-}, whose mass, and hence decay width, is raised considerably by the Thomas effect. Table II summarizes the masses and decay modes of these four states. Their estimated masses are obtained by applying the spin splittings of Table I(b) to our best estimates of the hybrid masses in the absence of spin effects,[11] also shown in Table I(b). The main effect of any suppression of the Thomas interaction analogous to that found in ordinary quark model mesons would be to lower somewhat the mass and reduce the width of the y_0^{+-}.

It still seems that identification of one of these exotic states, with masses around 2 GeV, appearing predominantly in the decay channels πB, $[\bar{K}K^*(1420)+\text{c.c.}]$, or $(\bar{K}Q_2+\text{c.c.})$, would afford the most convincing demonstration of the need for the gluonic degree of freedom in spectroscopy. The likely background in these decay channels due to orbitally and radially excited ordinary quark model states and nonexotic hybrids is at present under investigation.[14]

Note added in proof. C. Michael [Phys. Rev. Lett. **56**, 1219 (1986)] gives numerical evidence from lattice gauge theory for the Lorentz scalar nature of the long-range potential in quark model states.

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