Zeros in the nucleon form factors and the quark model

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We find no evidence in a simple quark model for zeros of the nucleon form factors in the spacelike region.

There is a great deal of interest in the question of zeros of the electromagnetic form factors of the nucleon at large momentum transfers. Such zeros have been predicted from perturbative QCD (Ref. 1), and also in a phenomenological approach based on the eikonal model.[2] Here we look at this question from the viewpoint of low-energy quark models.[3,4] In the absence of hyperfine interactions the orbital wave function of the quarks in the proton is totally symmetric[5] under permutations and rotations, belonging to the multiplet $(56,0^+)$ of SU(6). The charge density and form factor of the proton are then monotonically decreasing and nodeless in many potential models.[6] In the same approximation the neutron charge density is identically zero.[7,8] In the next approximation, hyperfine interactions induce some mixing with the nearest allowed excited states, the symmetric $(56',0^+)$ and the mixed permutational symmetry $(70,0^+)$. This mixing is known to improve our understanding of nucleon properties:[4] in particular, the charge radius of the neutron receives a contribution of the correct sign and size.[4,8,9] We want to consider here the nucleon form factors in the same approximation.

In the nonrelativistic approximation, before hyperfine interactions are switched on, the Hamiltonian of the relative coordinates

$$\boldsymbol{\rho}=\frac{1}{\sqrt{2}}(\mathbf{r}_1-\mathbf{r}_2) , \tag{1}$$

$$\boldsymbol{\lambda}=\frac{1}{\sqrt{6}}(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3) , \tag{2}$$

of the three nucleonic quarks is

$$H=\frac{p_\rho{}^2}{2m}+\frac{p_\lambda{}^2}{2m}+\sum_{i<j}\left[-\frac{2\alpha_s}{3r_{ij}}+\tfrac{1}{2}br_{ij}\right] \tag{3}$$

up to an additive constant. Here b is the mesonic string tension[10] of 0.18 GeV^2 and α_s, which may also be taken from meson spectroscopy, is approximately 0.6. When solved in a harmonic-oscillator variational space up to $2\hbar\omega$, this Hamiltonian produces a ground state that is an essentially pure harmonic-oscillator $0\hbar\omega$ ground state:

$$\psi^S\simeq\frac{\alpha^3}{\pi^{3/2}}\exp[-\tfrac{1}{2}\alpha^2(\rho^2+\lambda^2)] , \tag{4}$$

with $\alpha=0.40$ GeV.

The hyperfine interactions perturb the eigenstates of (3) to mix $(56',0^+)$ and $(70,0^+)$ into the ground state $(56,0^+)$ built out of the wave function (4). In this same $2\hbar\omega$ approximation this will lead to a new ground state which is essentially the one given in Ref. 4 (see also Ref. 11):

$$\begin{aligned}|N\rangle\simeq a\,|{}^2N(56,0^+)\rangle+b\,|{}^2N(56',0^+)\rangle\\+c\,|{}^2N(70,0^+)\rangle+\cdots\end{aligned} \tag{5}$$

with $a\simeq0.90$, $b\simeq-0.34$, and $c\simeq-0.27$. Here

$$|{}^2N(56,0^+)\rangle=\frac{1}{\sqrt{2}}(\chi^\rho\phi^\rho+\chi^\lambda\phi^\lambda)\psi^S , \tag{6}$$

$$|{}^2N(56',0^+)\rangle=\frac{1}{\sqrt{2}}(\chi^\rho\phi^\rho+\chi^\lambda\phi^\lambda)\psi^{S'} , \tag{7}$$

$$|{}^2N(70,0^+)\rangle=\tfrac{1}{2}(\chi^\rho\phi^\rho\psi^\lambda+\chi^\rho\phi^\lambda\psi^\rho+\chi^\lambda\phi^\rho\psi^\rho-\chi^\lambda\phi^\lambda\psi^\lambda) , \tag{8}$$

where ψ^S is given in (4) and

$$\psi^{S'}\simeq\frac{\alpha^2}{\sqrt{3}}(\rho^2+\lambda^2-3\alpha^{-2})\psi^S , \tag{9}$$

$$\psi^\lambda\simeq\frac{\alpha^2}{\sqrt{3}}(\rho^2-\lambda^2)\psi^S , \tag{10}$$

$$\psi^\rho\simeq\frac{\alpha^2}{\sqrt{3}}2\boldsymbol{\rho}\cdot\boldsymbol{\lambda}\psi^S . \tag{11}$$

χ^ρ and ϕ^ρ are spin and flavor wave functions given in Refs. 3 and 4.

In the nonrelativistic limit the electric form factor of the proton is then given by

$$G_E^p(Q^2)=\left\langle p\left|\sum_i e_i e^{-i\mathbf{Q}\cdot\mathbf{r}_i}\right|p\right\rangle \tag{12}$$

$$=3\langle p\,|\,e_3 e^{-i\mathbf{Q}\cdot\mathbf{r}_3}\,|\,p\,\rangle \tag{13}$$

$$=a^2\langle\psi^S|\,e^{-i\mathbf{Q}\cdot\mathbf{r}_3}\,|\,\psi^S\rangle+b^2\langle\psi^{S'}|\,e^{-i\mathbf{Q}\cdot\mathbf{r}_3}\,|\,\psi^{S'}\rangle+\tfrac{1}{2}c^2[\langle\psi^\lambda|\,e^{-i\mathbf{Q}\cdot\mathbf{r}_3}\,|\,\psi^\lambda\rangle+\langle\psi^\rho|\,e^{-i\mathbf{Q}\cdot\mathbf{r}_3}\,|\,\psi^\rho\rangle]$$

$$+2ab\langle\psi^S|\,e^{-i\mathbf{Q}\cdot\mathbf{r}_3}\,|\,\psi^{S'}\rangle+\sqrt{2}\,ac\langle\psi^S|\,e^{-i\mathbf{Q}\cdot\mathbf{r}_3}\,|\,\psi^\lambda\rangle+\sqrt{2}\,bc\langle\psi^{S'}|\,e^{-i\mathbf{Q}\cdot\mathbf{r}_3}\,|\,\psi^\lambda\rangle \tag{14}$$

$$=\left[1+\left[\frac{Q^2}{6\alpha^2}\right]\left[-\tfrac{2}{3}b^2-\tfrac{2}{3}c^2-\frac{2}{\sqrt{3}}ab+(\tfrac{2}{3})^{1/2}ac+\frac{2\sqrt{2}}{3}bc\right]+\left[\frac{Q^2}{6\alpha^2}\right]^2\left[\frac{b^2}{3}+\frac{1}{6}c^2-\frac{\sqrt{2}}{3}bc\right]\right]e^{-Q^2/6\alpha^2} \tag{15}$$

$$\simeq\left[1+0.12\left[\frac{Q^2}{6\alpha^2}\right]+0.007\left[\frac{Q^2}{6\alpha^2}\right]^2\right]e^{-Q^2/6\alpha^2}\,, \tag{16}$$

where $Q^2\equiv|\mathbf{Q}|^2>0$. Thus no zero is predicted in $G_E^p(Q^2)$ by this simple low-energy model. The model also predicts that

$$G_M^p(Q^2)=\left\langle p\left|\sum_i \mu_i\sigma_{iz}e^{-i\mathbf{Q}\cdot\mathbf{r}_i}\right|p\right\rangle \tag{17}$$

$$=\frac{e}{2m}\left[(1-\tfrac{2}{3}c^2)+\left[\frac{Q^2}{6\alpha^2}\right]\left[-\tfrac{2}{3}b^2-\tfrac{2}{9}c^2-\frac{2ab}{\sqrt{3}}+(\tfrac{2}{3})^{1/2}ac+\frac{2\sqrt{2}}{3}bc\right]\right.$$

$$\left.+\left[\frac{Q^2}{6\alpha^2}\right]^2\left[\tfrac{1}{3}b^2+\tfrac{1}{6}c^2-\frac{\sqrt{2}}{3}bc\right]\right]e^{-Q^2/6\alpha^2} \tag{18}$$

$$\simeq\mu_p\left[1+0.16\left[\frac{Q^2}{6\alpha^2}\right]+0.008\left[\frac{Q^2}{6\alpha^2}\right]^2\right]e^{-Q^2/6\alpha^2}\,, \tag{19}$$

$$G_M^n(Q^2)=-\frac{e}{3m}\left[(1-c^2)+\left[\frac{Q^2}{6\alpha^2}\right]\left[-\tfrac{2}{3}b^2-\frac{2}{\sqrt{3}}ab+\frac{1}{\sqrt{6}}ac+\frac{\sqrt{2}}{3}bc\right]+\left[\frac{Q^2}{6\alpha^2}\right]^2\left[\tfrac{1}{3}b^2+\tfrac{1}{6}{}^2-\frac{1}{3\sqrt{2}}bc\right]\right]e^{-Q^2/6\alpha^2} \tag{20}$$

$$\simeq\mu_n\left[1+0.24\left[\frac{Q^2}{6\alpha^2}\right]+0.03\left[\frac{Q^2}{6\alpha^2}\right]^2\right]e^{-Q^2/6\alpha^2} \tag{21}$$

and, as previously given,[9]

$$G_E^n(Q^2)=-\left[\left[\frac{Q^2}{6\alpha^2}\right]\left[(\tfrac{2}{3})^{1/2}ac+\frac{2\sqrt{2}}{3}bc\right]\right.$$

$$\left.+\left[\frac{Q^2}{6\alpha^2}\right]^2\left[-\frac{\sqrt{2}}{3}bc\right]\right]e^{-Q^2/6\alpha^2} \tag{22}$$

$$\simeq\left[0.11\left[\frac{Q^2}{6\alpha^2}\right]+0.04\left[\frac{Q^2}{6\alpha^2}\right]^2\right]e^{-Q^2/6\alpha^2}\,. \tag{23}$$

Although none of these formulas give zeros for the form factors for any $Q^2>0$, this can only be considered a prediction of the quark model over the formulas' limited range of validity. Their range is limited, first of all, since for Q^2 greater than about $10(6\alpha^2)\simeq 10$ GeV^2 they are simply incomplete: beyond such momentum transfers the $(Q^2/6\alpha^2)^2$ terms have become important, but the $4\hbar\omega$ contributions to such terms have been neglected. This limitation is purely calculational and would be easy to correct if physics considerations did not place a more stringent limitation on the maximum reliable value of $Q^2/6\alpha^2$. There are at least two such considerations.

(1) To compute a form factor at Q^2 requires that the nucleon state vector be boosted to a three-momentum of

at least the magnitude $\frac{1}{2}Q$. Effects of the order $Q/2m_q$ which the nonrelativistic formalism sets to zero can then lead to very significant effects for $Q^2 \sim 4m_q{}^2$ (such terms can even modify charge radii by terms of order $1/m_q{}^2$) (Ref. 12).

(2) The constituent-quark model corresponds to an effective field theory which is cut off at a scale of the order of 1 GeV. Probing at higher Q^2 will decompose the valence wave function into higher Fock-space components.[13]

We conclude that formulas (16), (19), (21), and (23) should represent the dominant physics of the nucleon electromagnetic form factors up to $Q^2 \sim 1$ GeV^2, but that by Q^2 of 10 GeV^2 they will be invalid.

In the region below $Q^2 \sim 1$ GeV^2 they are indeed in reasonable agreement with the data.[14] One feature of these calculations in this region is that, in contrast with expectations without $(70,0^+)$ mixing, these formulas predict small violations of form-factor scaling. Such violations have been observed in the ratio G_M^n/G_M^p for $Q^2 \sim 10$ GeV^2, but the observed deviations are reversed from those we predict at low Q^2. Since these effects are themselves relativistic corrections, this discrepancy is, in our opinion, not meaningful: from these simple considerations one can only properly draw the weaker but still interesting conclusions that effects of roughly the observed magnitude should exist. By Q^2 of 10 GeV^2, as expected, these formulas are not at all good representation of the data. Therefore our low-Q^2 model does not overlap with the domain of validity of Refs. 1 and 2, and so the zeros predicted in those approaches are not excluded by our model. Our formulas might be extended to higher Q^2 by using our rest-frame wave functions as a basis for guessing some reasonable infinite-momentum frame (or light-cone) wave functions[13] but we can nevertheless already conclude from these results that zeros in the nucleon form factors, if they exist, must occur for Q^2 of the order of 10 GeV^2 or greater.

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