Helicity projection techniques for evaluating cross sections of heavy-fermion production and decay via W bosons

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We discuss and illustrate the application of density-matrix techniques in the helicity basis for the tree-level production and decay of heavy fermions, via real or virtual W bosons. Both Drell-Yan and $O(\alpha_s)$ QCD production processes for W production are considered, with $W \to L\bar{\nu}_L$ and subsequent L and $\bar{\nu}_L$ decays, where L and $\bar{\nu}_L$ denote heavy fermions with no strong interactions (such as lepton and neutrino or W gaugino and photino). Compact expressions are given for the complete production/decay cross sections at the parton level, including possible multistage cascade decays.

I. INTRODUCTION

The tree-level squared matrix element for a heavy-lepton production and decay sequence

$$ab \to Lx_1, \ldots, x_n, \quad L \to \nu_L y_1 y_2 , \tag{1}$$

where x_i and y_i denote additional particles, can be evaluated directly by γ-matrix reduction routines. However, this can lead to enormously complicated algebraic expressions that are very hard to simplify; great simplification can be achieved if we factor the amplitude into L-production and L-decay parts. In this paper we show how this comes about, for more general (V,A) interactions than in previous work[1] and with applications to some heavy-lepton production processes of physical interest at present-day $p\bar{p}$ colliders.[2]

First let us decompose the amplitude $\mathscr{M}$ for the complete process of Eq. (1) into factors describing L production, L decay as

$$\mathscr{M} = \sum_{\lambda} A_\lambda(ab \to Lx_1, \ldots, x_n) B_\lambda(L \to \nu_L y_1 y_2) \Big/ D , \tag{2}$$

where $D = (L^2 - m_L{}^2 + i m_L \Gamma_L)$ is a propagator denominator, and we keep track explicitly of the helicity $\lambda = \pm\frac{1}{2}$ of the intermediate heavy lepton L. We consistently use particle labels to denote their four-momenta. The squared matrix element, averaged over initial and summed over final spins and colors, then takes the form

$$\begin{aligned} |\mathscr{M}|^2 &= \frac{1}{N_i} \sum_{\pi} \sum_{\delta} \sum_{\lambda,\mu} A_\lambda B_\lambda A_\mu^* B_\mu^* \Big/ |D|^2 \\ &= \frac{1}{N_i} \sum_{\lambda,\mu} \left[\sum_{\pi} A_\lambda A_\mu^* \right] \left[\sum_{\delta} B_\lambda B_\mu^* \right] \Big/ |D|^2 . \end{aligned} \tag{3}$$

Here N_i is the number of initial spin/color states being averaged, while symbols π and δ denote the spin/color states of the external particles in the production and decay processes, respectively. It is to be understood that the cross section is normalized as

$$d\sigma = \frac{1}{2s} |\mathscr{M}|^2 \prod_{k = \nu_L, y_1, y_2, x_1, \ldots, x_n} \left[\frac{d^3k}{(2\pi)^3 2E_k} \right] (2\pi)^4 \delta^4(a + b - \nu_L - y_1 - y_2 - x_1 - \cdots - x_n) . \tag{4}$$

The decomposition in Eq. (3) applies quite generally to *any* production and decay of L.

The individual terms $\sum_\pi A_\lambda A_\mu^*$ and $\sum_\delta B_\lambda B_\mu^*$ in Eq. (3) are algebraically much less complicated than the complete amplitude squared; they are somewhat like the squared matrix elements for production and decay separately (actually they are proportional to elments of the density matrix for L production and decay). They can be calculated using the helicity spinors

$$u(p,+) = (E+m)^{-1/2} (\gamma \cdot p + m) \begin{bmatrix} \cos\frac{1}{2}\theta \\ \exp(i\phi)\sin\frac{1}{2}\theta \\ 0 \\ 0 \end{bmatrix} , \tag{5}$$

$$u(p,-) = (E+m)^{-1/2} (\gamma \cdot p + m) \begin{bmatrix} -\exp[i\phi]\sin\frac{1}{2}\theta \\ \cos\frac{1}{2}\theta \\ 0 \\ 0 \end{bmatrix} ,$$

labeled $\pm$ for helicity $\pm\frac{1}{2}$, with the outer products

$$\begin{aligned} u(p,\pm)\bar{u}(p,\pm) &= \tfrac{1}{2}(p\cdot\gamma + m)(1 \pm \gamma_5 \gamma \cdot S) , \\ u(p,+)\bar{u}(p,-) &= \tfrac{1}{2}\exp(i\phi)(p\cdot\gamma + m)\gamma_5\gamma\cdot C , \\ u(p,-)\bar{u}(p,+) &= \tfrac{1}{2}\exp(-i\phi)(p\cdot\gamma + m)\gamma_5\gamma\cdot C^* . \end{aligned} \tag{6}$$

[For the corresponding antiparticle $v\bar{v}$-spinor outer products, set $m \to -m$ in Eq. (6).] Here S is the covariant spin vector

$$S_\mu = (|\mathbf{p}|/m, E\mathbf{p}/(m|\mathbf{p}|)) \tag{7}$$

and C is defined as

$$C_\mu = (0, \cos\theta\cos\phi - i\sin\phi, \cos\theta\sin\phi + i\cos\phi, -\sin\theta) , \tag{8}$$

where the polar angle θ and azimuthal angle ϕ define the direction of $\mathbf{p}$. The four-vectors p, S, C, C^* obey orthogonality relations

$$p\cdot S = p\cdot C = p\cdot C^* = S\cdot C = S\cdot C^* = 0 . \tag{9}$$

The summations over λ and μ in Eq. (3) take particularly simple forms in a number of cases, thanks to algebraic identities; we describe several cases in the following section in order of increasing complexity. The first cases are for pure $V\pm A$ couplings at the production and decay vertices of a heavy lepton L; these are described in Sec. II. The next case is general V, A couplings for L production and decay, assuming ν_L is massless; this is described in Sec. III A. Finally we address general V, A couplings with massive ν_L in Secs. III B—III C. Our considerations allow a sequence of decays to be strung together; for example, we include the possible decay of ν_L above.

Our discussion of production/decay helicity correlations applies formally to any spin-$\frac{1}{2}$ fermion L but is physically relevant only when L has no strong interactions: e.g., L is a lepton or W gaugino of supersymmetry and ν_L is a neutrino or Z gaugino or photino. For strongly interacting fermions such as quarks or gluinos, there is an additional hadronization process between production and decay. Although the eventual heavy-hadron decay may sometimes be approximated by a heavy-quark or gluino decay process, the hadronization inevitably introduces some depolarization (for spinless hadrons the depolarization is complete) which has to be specified and is not included in our treatment.

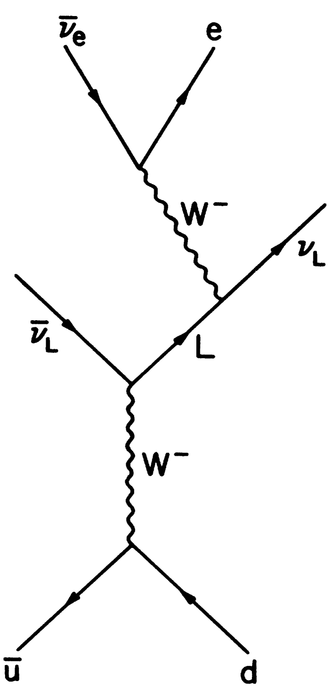

FIG. 1. Feynman diagram for lowest-order heavy-L production $\bar{u}d\to W\to L\bar{\nu}_L$ with subsequent decay $L\to\nu_L e\bar{\nu}_e$.

II. $V\pm A$ INTERACTIONS

If $V-A$ coupling is assumed for both production and decay, there are effectively additional projection operators $\frac{1}{2}(1-\gamma_5)$ on the left-hand side and $\frac{1}{2}(1+\gamma_5)$ on the right-hand side of Eq. (6). This greatly simplifies the algebra and Eq. (6) can be represented by the substitutions

$$\begin{aligned} u(p,\pm)\bar{u}(p,\pm) &\to \tfrac{1}{2}(p\cdot\gamma \mp mS\cdot\gamma) , \\ u(p,+)\bar{u}(p,-) &\to -\tfrac{1}{2}m\exp(i\phi)C\cdot\gamma , \\ u(p,-)\bar{u}(p,+) &\to -\tfrac{1}{2}m\exp(-i\phi)C^*\cdot\gamma . \end{aligned} \tag{10}$$

Since the spinor outer products are effectively of the form $u(\mu)\bar{u}(\lambda) = \gamma\cdot V_{\mu\lambda}$, where $V_{\mu\lambda} = \frac{1}{2}p \mp mS$, $-\frac{1}{2}mC\exp(i\phi)$, or $-\frac{1}{2}mC^*\exp(-i\phi)$ as determined by Eq. (10), it follows that the expressions for $\sum_\pi A_\lambda A_\mu^*$ are all of the scalar product form $X\cdot V_{\mu\lambda}$ where X is some four vector:

$$\begin{aligned} \sum_\pi A_\pm A_\pm^* &= \tfrac{1}{2}X\cdot(L\mp m_L S) , \\ \sum_\pi A_- A_+^* &= -\tfrac{1}{2}X\cdot C m_L \exp(i\phi) , \\ \sum_\pi A_+ A_-^* &= -\tfrac{1}{2}X\cdot C^* m_L \exp(-i\phi) . \end{aligned} \tag{11}$$

Here we explicitly specialize to L production, $p\to L, m\to m_L$; S, C, C^* are defined by the L kinematics. Similarly, the decay sums $\sum_\delta B_\lambda B_\mu^*$ are all described by the scalar products of a common four-vector Y and the same vectors $V_{\lambda\mu}$ (note that λ,μ are reversed here):

$$\begin{aligned} \sum_\delta B_\pm B_\pm^* &= \tfrac{1}{2}Y\cdot(L\mp m_L S) , \\ \sum_\delta B_+ B_-^* &= -\tfrac{1}{2}Y\cdot C m_L \exp(i\phi) , \\ \sum_\delta B_- B_+^* &= -\tfrac{1}{2}Y\cdot C^* m_L \exp(-i\phi) . \end{aligned} \tag{12}$$

The relevant vectors, X, Y can be determined by evaluating particular cases. For example, they appear separately in the spin-averaged production and decay of L:

$$\begin{aligned} |\mathcal{M}(\text{production})|^2 &= \frac{1}{N_i}\sum_{\pi,\lambda} A_\lambda A_\lambda^* = N_i^{-1} X\cdot L , \\ |\mathcal{M}(\text{decay})|^2 &= \sum_{\delta,\lambda} B_\lambda B_\lambda^* = Y\cdot L \end{aligned} \tag{13}$$

(note that the decay amplitude squared is *not* spin/color averaged in this definition). To extract X and Y from these simpler calculations, it is imperative to keep track throughout of the momentum vector L arising from the $u(L)\bar{u}(L)$ outer product and not to re-express it as a linear combination of other momenta.

The summation over λ and μ in Eq. (3) can now be performed:

$$|\mathcal{M}|^2 = N_i^{-1} \sum_{\lambda,\mu} (X\cdot V_{\mu\lambda})(Y\cdot V_{\lambda\mu})/|D|^2$$
$$= N_i^{-1}[(X\cdot L)(Y\cdot L) - \tfrac{1}{2}m_L^2(X\cdot Y)]/|D|^2 , \quad (14)$$

where we have used the identity

$$2(X\cdot S)(Y\cdot S) + (X\cdot C)(Y\cdot C^*) + (X\cdot C^*)(Y\cdot C)$$
$$= 2(X\cdot L)(Y\cdot L)/m_L^2 - 2(X\cdot Y) . \quad (15)$$

The complete result Eq. (14) can thus be inferred directly from the easy-to-compute average quantities in Eq. (13).

Had we calculated the spin-averaged production of L multipled by the spin-averaged (i.e., unpolarized) decay of L, we would have found

$$|\mathcal{M}|^2 = \tfrac{1}{2}N_i^{-1}(X\cdot L)(Y\cdot L)/|D|^2$$

instead. This gives the correct total cross section, since the decay width of L is independent of helicity, but not the correct details since it omits the effects of L polarization on the decay distributions.

Similar results follow with $V+A$ couplings; in this case S,C, and C^* appear with extra negative signs in Eq. (10). When both production and decay couplings are $V+A$, these negative signs occur squared in Eq. (14) and the result is unchanged. When one coupling is $V+A$ and one is $V-A$, the bilinear S,C,C^* terms in Eq. (14) change sign and the result becomes instead

$$|\mathcal{M}|^2 = N_i^{-1}[\tfrac{1}{2}m_L^2(X\cdot Y)]/|D|^2 . \quad (16)$$

Our discussion has referred to the production and decay of a particle, requiring positive-energy spinors $u,\bar{u}$. To describe antiparticles instead we use negative-energy spinors $v,\bar{v}$; the m_L terms in Eqs. (6) and (10)—(12) then change sign but the final results are unchanged. These techniques can be applied successively to add in the sequential decay correlations of a number of particles, provided their production and decay couplings are always $V\pm A$.

A. Application: L production via $d\bar{u}\to W\to L\bar{\nu}_L$ with $L\to\nu_L e\bar{\nu}_e$

For a simple example, consider L production by the lowest-order process $d\bar{u}\to L\bar{\nu}_L$ followed by $L\to\nu_L e\bar{\nu}_e$ decay, with $V-A$ couplings throughout (see Fig. 1). There are $N_i=36$ initial spin/color states to average over. The squared matrix elements for the spin/color-averaged production and decay of L corresponding to Eq. (13) are then

$$N_i^{-1}\sum_{\pi,\lambda} A_\lambda A_\lambda^* = N_i^{-1}384G_F^2M_W^4(d\cdot\bar{\nu}_L)(\bar{u}\cdot L)/[(s-M_W^2)^2+\Gamma_W^2M_W^2] , \quad (17a)$$

$$\sum_{\delta,\lambda} B_\lambda B_\lambda^* = 128G_F^2M_W^4(e\cdot\nu_L)(\bar{\nu}_e\cdot L)/[(s'-M_W^2)^2+\Gamma_W^2M_W^2] , \quad (17b)$$

where $s=(d+\bar{u})^2$ and $s'=(e+\bar{\nu}_e)^2$. We can immediately identify

$$X_\alpha = \{384G_F^2M_W^4(d\cdot\bar{\nu}_L)/[(s-M_W^2)^2+\Gamma_W^2M_W^2]\}\bar{u}_\alpha \equiv F\bar{u}_\alpha ,$$
$$Y_\alpha = \{128G_F^2M_W^4(e\cdot\nu_L)/[(s'-M_W^2)^2+\Gamma_W^2M_W^2]\}(\bar{\nu}_e)_\alpha \equiv G(\bar{\nu}_e)_\alpha , \quad (18)$$

and hence construct the spin/color-averaged matrix element for the complete process

$$|\mathcal{M}|^2 = N_i^{-1}FG[(\bar{u}\cdot L)(\bar{\nu}_e\cdot L) - \tfrac{1}{2}m_L^2(\bar{u}\cdot\bar{\nu}_e)]/|D|^2 . \quad (19)$$

B. Application: Subsequent $\bar{\nu}_L\to\bar{e}\mu\bar{\nu}_\mu$ decay

Continuing the previous application, if $\bar{\nu}_L$ is heavy and subsequently decays via $\nu_e-\nu_L$ mixing by $\bar{\nu}_L\to\bar{e}\mu\bar{\nu}_\mu$ (see Fig. 2) we can include this step in an analogous way, after identifying the appropriate vectors X' and Y' from spin-averaged production and decay of $\bar{\nu}_L$. From Eqs. (18) and (19) we extract

$$X'_\alpha = \{(d\cdot\bar{\nu}_L)^{-1}FG|D|^{-2}[(\bar{u}\cdot L)(\bar{\nu}_e\cdot L) - \tfrac{1}{2}m_L^2(\bar{u}\cdot\bar{\nu}_e)]\}d_\alpha$$
$$\equiv F'd_\alpha , \quad (20)$$

while a relation similar to Eq. (17b) gives

$$Y'_\alpha = \{128G_F^2|U_{Le}|^2M_W^4(\bar{e}\cdot\bar{\nu}_\mu)/[(s''-M_W^2)^2+\Gamma_W^2M_W^2]\}\mu_\alpha$$
$$\equiv G'\mu_\alpha , \quad (21)$$

where $s''=(\bar{\nu}L-\bar{e})^2$ and U_{Le} is the ν_L, ν_e mixing matrix element. Hence, the spin/color-averaged matrix element for the complete process is

$$|\mathcal{M}|^2 = N_i^{-1}F'G'[(d\cdot\bar{\nu}_L)(\mu\cdot\bar{\nu}_L) - \tfrac{1}{2}m_{\nu_L}^2(d\cdot\mu)]/|D'|^2 , \quad (22)$$

where

$$D' = (\bar{\nu}_L^2 - m_{\nu_L}^2 + im_{\nu_L}\Gamma_{\nu_L})$$

is the propagator denominator for $\bar{\nu}_L$.

In a similar way we can, if necessary, add in the decay

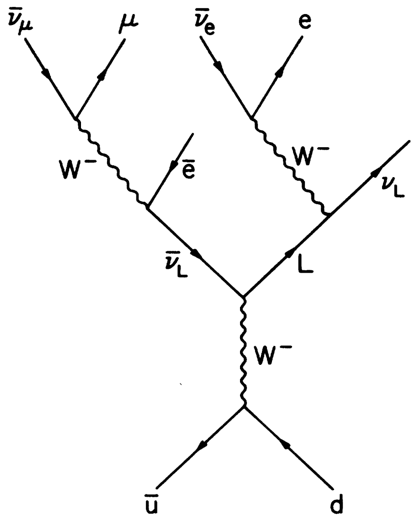

FIG. 2. Feynman diagram for lowest-order heavy-L production $\bar u d \to W \to L\bar\nu_L$ with subsequent decays of both $L\to\nu_L e\bar\nu_e$ and $\bar\nu_L\to\bar e\mu\bar\nu_\mu$.

of ν_L as well as $\bar\nu_L$, and indeed the decays of their leptonic decay products, provided that all decay couplings are $V\pm A$.

C. Application: L production via $d\bar u\to Wg\to L\bar\nu_L g$ with $L\to\nu_L q\bar q\,'$

The dominant p_T dependence in W production is due to gluon emission; the lowest-order heavy-lepton production/decay process of this type is shown in Fig. 3. The squared matrix element for the decay process is similar to that of Eq. (17b) but with an additional factor of 3 for quark color and a squared quark-mixing matrix element $|U_{qq'}|^2$. Hence, we conclude that the relevant vector Y is

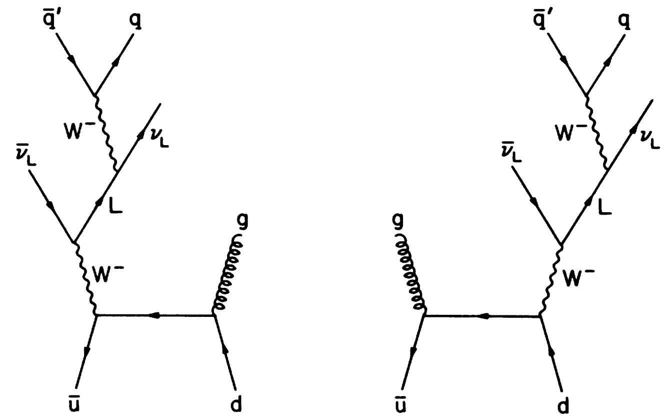

FIG. 3. Feynman diagrams for $O(\alpha_s)$ heavy-L production with $L\to\nu_L q\bar q\,'$ decay.

$$Y_\alpha=\{384G_F^2M_W^4\,|\,U_{qq'}\,|^2(q\cdot v)/[(y^2-M_W^2)^2+M_W^2\Gamma_W^2]\}\bar q\,'_\alpha\equiv G\bar q\,'_\alpha\,, \tag{23}$$

where v denotes ν_L and $y=q+\bar q\,'$.

The squared spin/color-averaged matrix element for the L production stage has the form[3]

$$|\mathcal{M}\,|^2=\frac{2\pi\alpha_s G_F^2M_W^4\,|\,U_{ud}\,|^2}{9[(x^2-M_W^2)^2+M_W^2\Gamma_W^2}L_{\alpha\beta}H^{\alpha\beta}\equiv KL_{\alpha\beta}H^{\alpha\beta}\ . \tag{24}$$

Here, $x=d+\bar u-g$ and $L_{\alpha\beta}H^{\alpha\beta}$ is the product of leptonic and hadronic spin tensors. The latter depends linearly on the L momentum vector through

$$L_{\alpha\beta}=8(L_\alpha\bar v_\beta+\bar v_\alpha L_\beta-g_{\alpha\beta}L\cdot\bar v-i\epsilon_{\alpha\beta\gamma\delta}L^\gamma\bar v^\delta)\ . \tag{25}$$

Keeping track of this L vector, the product can be written

$$L_{\alpha\beta}H^{\alpha\beta}=256(g\cdot d)^{-1}(g\cdot\bar u)^{-1}\{(L\cdot d)(\bar v\cdot d)(g\cdot\bar u)-(L\cdot g)(\bar v\cdot d)(x\cdot d)+(L\cdot\bar u)[(\bar v\cdot d)(x\cdot\bar u)-(\bar v\cdot g)(x\cdot\bar u)+(\bar v\cdot d)(x\cdot d)+(\bar v\cdot\bar u)(g\cdot d)]\}\ . \tag{26}$$

Hence, for $N_i=36$ initial spin/color states being averaged, we conclude that

$$X_\alpha=9216K(g\cdot d)^{-1}(g\cdot\bar u)^{-1}\{(\bar v\cdot d)(g\cdot\bar u)d_\alpha-(\bar v\cdot d)(x\cdot d)g_\alpha+[(\bar v\cdot d)(x\cdot\bar u)-(\bar v\cdot g)(x\cdot\bar u)+(\bar v\cdot d)(x\cdot d)+(\bar v\cdot\bar u)(g\cdot d)]\bar u_\alpha\}\ , \tag{27}$$

where $\bar v$ denotes $\bar\nu_L$.

Combining these results for X and Y, we find the spin/color-averaged matrix element for the overall production and decay sequence to be

$$\begin{aligned}|\mathcal{M}\,|^2=F(g\cdot d)^{-1}(g\cdot u)^{-1}\,|\,D'\,|^{-2}(q\cdot v)\{&[2(L\cdot d)(L\cdot\bar q\,')-m_L^2(d\cdot\bar q\,')](\bar v\cdot d)(g\cdot\bar u)\\&-[2(L\cdot g)(L\cdot\bar q\,')-m_L^2(g\cdot\bar q\,')](\bar v\cdot d)(x\cdot d)\\&+[2(L\cdot\bar u)(L\cdot\bar q\,')-m_L^2(\bar u\cdot\bar q\,')][(\bar v\cdot d)(x\cdot\bar u)-(\bar v\cdot g)(x\cdot\bar u)\\&+(\bar v\cdot d)(x\cdot d)+(\bar v\cdot\bar u)(g\cdot d)]\}\ .\end{aligned} \tag{28}$$

Here D' is a product of W and L propagator denominators

$$D'=(L^2-m_L{}^2+im_L\Gamma_L)(x^2-M_W{}^2+iM_W\Gamma_W)(y^2-M_W{}^2+iM_W\Gamma_W) \tag{29}$$

with $x=d+\bar{u}-g$ and $y=q+\bar{q}'$. F contains the remaining numerical factors

$$F=\left[\frac{2^{15}}{3}\right]\pi\alpha_s G_F{}^4 M_W{}^8 \,|\, U_{ud}\,|^2\,|\,U_{qq'}\,|^2 \,, \tag{30}$$

where U_{ij} is the Kobayashi-Maskawa quark mixing matrix.

D. Application: Further decays of ν_L and $\bar{\nu}_L$

If the neutrinos ν_L and $\bar{\nu}_L$ in Sec. II C are heavy, they may undergo charged-current decays via lepton mixing $\nu_L \to lA\bar{B}$, $\bar{\nu}_L \to \bar{l}'C\bar{D}$ where $l\bar{l}'$ are charged leptons and $A\bar{B}, C\bar{D}$ denote fermion pairs coupled to $W^{\pm}$. This would extend the production and decay chain of Sec. II C into an eleven-particle transition:

$$d\bar{u} \to W^- g \to l\bar{l}'A\bar{B}C\bar{D}\,q\bar{q}'g \ . \tag{31}$$

This process (illustrated in Fig. 4) is evaluated in Ref. 4, in a context where L and ν_L are exotic E_6 fermions, forming either a left- or right-handed weak-isospin doublet. It is interesting to show the power of the helicity projection technique by giving below the squared eleven-particle matrix element (averaged over spin and colors). Each time we add a neutrino decay, we read the vector X from the squared matrix element for the previous process; for the first added neutrino decay we read X directly from Eq. (27). The vector Y is defined in analogy with Eq. (21), assuming that (A,B), (D,C) are lepton isodoublets. For the case of $V-A$ couplings we obtain the spin/color-averaged matrix element

$$\begin{aligned}
|\mathscr{M}|^2 = {} & \frac{4\pi\alpha_s}{3}(8M_W{}^2G_F)^8 \frac{|U_{ud}|^2\,|U_{qq'}|^2\,|U_{Ll}|^2\,|U_{Ll'}|^2}{(g\cdot d)(g\cdot \bar{u})} \left[\prod_{\substack{j=L,\nu_L,\bar{\nu}_L,\\ W_1,\ldots,W_4}} [(j^2-m_j{}^2)^2+\Gamma_j{}^2 m_j{}^2]^{-2}\right] \\
& \times (l\cdot A)(\bar{l}'\cdot\bar{D})[2(\nu_L\cdot q)(\nu_L\cdot\bar{B})-m_{\nu_L}{}^2(q\cdot\bar{B})] \\
& \times \{(2\hat{s}+\hat{t}+\hat{u})[2(L\cdot\bar{u})(L\cdot\bar{q}')-m_L{}^2(\bar{u}\cdot\bar{q}')][2(\bar{\nu}_L\cdot d)(\bar{\nu}_L\cdot C)-m_{\bar{\nu}_L}{}^2(d\cdot C)] \\
& \quad -(\hat{s}+\hat{u})[2(L\cdot g)(L\cdot\bar{q}')-m_L{}^2(g\cdot\bar{q}')][2(\bar{\nu}_L\cdot d)(\bar{\nu}_L\cdot C)-m_{\bar{\nu}_L}{}^2(d\cdot C)] \\
& \quad -(\hat{s}+\hat{t})[2(L\cdot\bar{u})(L\cdot\bar{q}')-m_L{}^2(\bar{u}\cdot\bar{q}')][2(\bar{\nu}_L\cdot g)(\bar{\nu}_L\cdot C)-m_{\bar{\nu}_L}{}^2(g\cdot C)] \\
& \quad -\hat{t}[2(L\cdot d)(L\cdot\bar{q}')-m_L{}^2(d\cdot\bar{q}')][(2(\bar{\nu}_L\cdot d)(\bar{\nu}_L\cdot C)-m_{\bar{\nu}_L}{}^2(d\cdot C)] \\
& \quad -\hat{u}[2(L\cdot\bar{u})(L\cdot\bar{q}')-m_L{}^2(\bar{u}\cdot\bar{q}')][2(\bar{\nu}_L\cdot\bar{u})(\bar{\nu}_L\cdot C)-m_{\bar{\nu}_L}{}^2(\bar{u}\cdot C)]\} \ ,
\end{aligned} \tag{32}$$

where $W_1,\ldots,W_4$ denote the four participating W-boson momenta $d+\bar{u}-g, L-\nu_L, \nu_L-l, \bar{\nu}_L-\bar{l}'$ and U_{Ll}, U_{Ll}' are the $\nu_L-\nu_l, \nu_L-\nu_{l'}$ mixing matrix elements. The Mandelstam variables $\hat{s},\hat{t}$, and $\hat{u}$ are defined as follows: $\hat{s}=(\bar{u}+d)^2, \hat{t}=(\bar{u}-g)^2$, and $\hat{u}=(d-g)^2$. This formula assumes $A,\bar{B},C,\bar{D}$ are leptons with full-strength $V-A$ charged-current couplings. If either (both) pairs are quarks, multiply by 3(9) for color summation and also by the appropriate Kobayashi-Maskawa matrix elements squared.

If the $\nu_L \to l$ coupling is $V+A$ instead, the result is simply obtained by replacing factors as follows:

$$(l\cdot A) \to (l\cdot\bar{B}) \ , \tag{33a}$$

$$[2(\nu_L\cdot X)(\nu_L\cdot\bar{B})-m_{\nu_L}{}^2(X\cdot\bar{B})] \to m_{\nu_L}{}^2(X\cdot A) \ , \tag{33b}$$

where X is a generic vector. A similar rule applies to $\bar{\nu}_L \to \bar{l}'C\bar{D}$ decay, with $\nu_L, l, A, \bar{B}$ replaced by $\bar{\nu}_L, \bar{l}', \bar{D}, C$.

III. GENERAL V,A INTERACTIONS

With general V,A interactions at the L production and decay vertices, it is possible to adapt the previous techniques. For continuity we still use the notation L and ν_L of the preceding sections. In the special case of massless ν_L the changes are rather simple, but in general we need to use explicit forms for the production and decay amplitudes. We illustrate below three cases involving W bosons with general V,A couplings; the first has $m_{\nu_L}=0$ but the others have massive ν_L.

We assume that both the production and decay of L are governed by an interaction Lagrangian of the form

$$\mathscr{L}=\frac{g}{2\sqrt{2}}\bar{L}\gamma^{\mu}[C_L(1-\gamma_5)+C_R(1+\gamma_5)]\nu_L W_{\mu}+\text{H.c.} \ , \tag{34}$$

where g is the weak SU(2) coupling constant; for standard $V-A$ interactions $C_L=1$ and $C_R=0$.

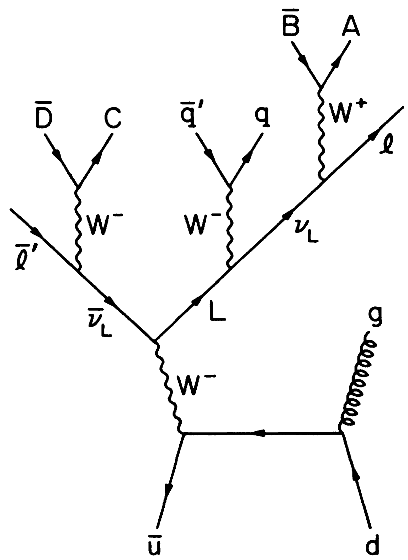

FIG. 4. Typical Feynman diagram for $O(\alpha_s)$ heavy-L production with subsequent decays $L\to\nu_L q\bar{q}'$, $\nu_L\to lA\bar{B}$, $\bar{\nu}_L\to\bar{l}'C\bar{D}$.

A. Application: L production via $d\bar{u}\to Wg\to L\bar{\nu}_L g$ with $L\to\nu_L q\bar{q}'$; $m_{\nu_L}=0$

The Feynman diagrams for this process are shown in Fig. 3. Examining the traces involved in $\sum A_\lambda A_\mu^*$ (and $\sum B_\lambda B_\mu^*$) we see that the $C_L{}^2$ and $C_R{}^2$ terms are simply the $V-A$ and $V+A$ production (decay) cases of Sec. II C. The $C_L C_R$ terms vanish because $m_{\nu_L}=0$. Hence the complete result for $|\mathcal{M}|^2$ is simply $(C_L{}^4+C_R{}^4)$ times the result of Eq. (28) plus $2C_L{}^2C_R{}^2$ times the corresponding result for $(V\pm A)$ production with $(V\mp A)$ decay using Eq. (16).

In other words, the result is obtained by replacing in Eq. (28) the quantities in square brackets as follows:

$$[2(L\cdot x)(L\cdot y)-m_L{}^2(x\cdot y)]\to 2(C_L{}^4+C_R{}^4)(L\cdot x)(L\cdot y) - m_L{}^2(C_L{}^2-C_R{}^2)^2(x\cdot y)\ . \tag{35}$$

B. Application: L production via $d\bar{u}\to W\to L\bar{\nu}_L$ with $L\to\nu_L e\bar{\nu}_e$; massive ν_L

The Feynman diagram for this production/decay chain has been shown in Fig. 1. Since $m_{\nu_L}\neq 0$, it is now necessary to evaluate explicitly the traces associated with the L in the production and decay density matrices. It is convenient to express the spinor outer products in the generic form

$$u(L,\mu)\bar{u}(L,\lambda)=(a\cdot\gamma+\gamma_5 b\cdot\gamma+A\cdot\gamma\gamma_5 B\cdot\gamma+T)_{\mu\lambda}\ ,$$
$$v(L,\mu)\bar{v}(L,\lambda)=(a\cdot\gamma-\gamma_5 b\cdot\gamma+A\cdot\gamma\gamma_5 B\cdot\gamma-T)_{\nu\lambda}\ . \tag{36}$$

From Eq. (6) we identify the coefficients above as

$$a_{\pm\pm}=\tfrac{1}{2}L,\quad a_{+-}=a_{-+}=0\ ,$$
$$b_{\pm\pm}=\pm\tfrac{1}{2}m_L S,\quad b_{+-}=b^*_{-+}=\tfrac{1}{2}m_L e^{i\phi}C\ ,$$
$$A\pm\pm=\pm\tfrac{1}{2}L,\quad A_{+-}=A_{-+}=\tfrac{1}{2}L\ , \tag{37}$$
$$B_{\pm\pm}=S,\quad B_{+-}=B^*_{-+}=e^{i\phi}C\ ,$$
$$T_{\pm\pm}=\tfrac{1}{2}m_L,\quad T_{+-}=T_{-+}=0\ .$$

For the $C_L{}^2$ term in such a trace, $u(\mu)\bar{u}(\lambda)$ effectively reduces to $(a-b)\cdot\gamma$; for the $C_R{}^2$ term it reduces to $(a+b)\cdot\gamma$; for the $C_L C_R$ terms it reduces to $(A\cdot\gamma)\gamma_5(B\cdot\gamma)+T$. In writing the results of the trace evaluations, it is convenient to introduce the combinations of coefficients

$$e_{\lambda\mu}=a_{\lambda\mu}-b_{\lambda\mu}\ ,$$
$$f_{\lambda\mu}=a_{\lambda\mu}+b_{\lambda\mu}\ , \tag{38}$$
$$E_{\lambda\mu}(x,y)=(x\cdot y)T_{\lambda\mu}-(x\cdot A_{\lambda\mu})(y\cdot B_{\lambda\mu})+(x\cdot B_{\lambda\mu})(y\cdot A_{\lambda\mu})\ .$$

The spin/color-averaged density matrix for the lowest-order production subprocess $d\bar{u}\to W\to L\bar{\nu}_L$ has the following form, in the L helicity basis, where $W_1=d+\bar{u}$:

$$N_i{}^{-1}A_\lambda A_\mu^*=\tfrac{1}{3}g^4[(W_1{}^2-M_W{}^2)^2+\Gamma_W{}^2M_W{}^2]^{-1}[C_L{}^2(d\cdot\bar{\nu}_L)(\bar{u}\cdot e_{\mu\lambda})+C_R{}^2(\bar{u}\cdot\bar{\nu}_L)(d\cdot f_{\mu\lambda})+m_{\nu_L}C_L C_R E_{\mu\lambda}(d,\bar{u})]$$
$$\equiv(X_L\cdot e_{\mu\lambda})+(X_R\cdot f_{\mu\lambda})+X_M E_{\mu\lambda}(d,\bar{u})\ . \tag{39}$$

Similarly the spin-averaged decay density matrix can be expressed as

$$B_\lambda B_\mu^*=4g^4[(W_2{}^2-M_W{}^2)^2+\Gamma_W{}^2M_W{}^2]^{-1}[C_L{}^2(e\cdot\nu_L)(\bar{\nu}_e\cdot e_{\lambda\mu})+C_R{}^2(\nu_L\cdot\bar{\nu}_e)(e\cdot f_{\lambda\mu})-m_{\nu_L}C_L C_R E_{\lambda\mu}(e,\bar{\nu}_e)]$$
$$\equiv(Y_L\cdot e_{\lambda\mu})+(Y_R\cdot f_{\lambda\mu})+Y_M E_{\lambda\mu}(e,\bar{\nu}_e)\ . \tag{40}$$

To evaluate the summations over λ and μ in the full matrix element squared, we use Eqs. (15) and (37) to derive the identities

$$\sum_{\lambda,\mu}(x\cdot e_{\mu\lambda})(y\cdot e_{\lambda\mu})=\sum_{\lambda,\mu}(x\cdot f_{\mu\lambda})(y\cdot f_{\lambda\mu})=(x\cdot L)(y\cdot L)-\tfrac{1}{2}m_L{}^2(x\cdot y)\ ,$$
$$\sum_{\lambda,\mu}(x\cdot e_{\mu\lambda})(y\cdot f_{\lambda\mu})=\tfrac{1}{2}m_L{}^2(x\cdot y)\ ,$$
$$\sum_{\lambda,\mu}(x\cdot e_{\mu\lambda})E_{\lambda\mu}(a,b)=\tfrac{1}{2}m_L[(x\cdot L)(a\cdot b)-(a\cdot L)(x\cdot b)+(b\cdot L)(x\cdot a)]\equiv J(x,a,b)\ , \tag{41}$$
$$\sum_{\lambda,\mu}(x\cdot f_{\mu\lambda})E_{\lambda\mu}(a,b)=J(x,b,a)\ ,$$
$$\sum_{\lambda,\mu}E_{\mu\lambda}(a,b)E_{\lambda\mu}(c,d)=\tfrac{1}{2}m_L{}^2(a\cdot b)(c\cdot d)+\tfrac{1}{2}[(a\cdot L)(d\cdot L)(b\cdot c)+(b\cdot L)(c\cdot L)(a\cdot d)-(a\cdot L)(c\cdot L)(b\cdot d)-(b\cdot L)(d\cdot L)(a\cdot c)]\equiv K(a,b,c,d)\ .$$

The spin/color-averaged matrix element squared for the complete process can then be written down as

$$|\mathcal{M}|^2=|D|^{-2}\{(X_L\cdot L)(Y_L\cdot L)-\tfrac{1}{2}m_L{}^2(X_L\cdot Y_L)+(X_R\cdot L)(Y_R\cdot L)-\tfrac{1}{2}m_L{}^2(X_R\cdot Y_R)+\tfrac{1}{2}m_L{}^2(X_L\cdot Y_R+X_R\cdot Y_L)+Y_M[J(X_L,e,\overline{\nu}_e)+J(X_R,\overline{\nu}_e,e)]+X_M[J(Y_L,d,\overline{u})+J(Y_R,\overline{u},d)]+X_MY_MK(d,\overline{u},e,\overline{\nu}_e)\}\ . \tag{42}$$

The contributions here can be recognized as left-left, right-right, right-left (as given in Sec. II for pure $V\pm A$ couplings), followed by three contributions due to $L-R$ interference that vanish when $m_{\nu_L}=0$.

C. Application: L production via $\overline{u}d\to Wg\to L\overline{\nu}_Lg$ with $L\to\nu_Le\overline{\nu}_e$; massive ν_L

The relevant Feynman diagrams are analogous to those in Fig. 3. In this case the spin/color-averaged production density matrix has the form

$$\frac{1}{N_i}A_\lambda A_\mu^*=\frac{8g^4g_s{}^2|U_{ud}|^2}{9\hat{u}\hat{t}}[(W_1{}^2-M_W{}^2)^2+(\Gamma_WM_W)^2]^{-1}$$
$$\times\{C_L{}^2[(2\hat{s}+\hat{t}+\hat{u})(d\cdot\overline{\nu}_L)(\overline{u}\cdot e_{\mu\lambda})-(\hat{s}+\hat{u})(d\cdot\overline{\nu}_L)(g\cdot e_{\mu\lambda})-(\hat{s}+\hat{t})(g\cdot\overline{\nu}_L)(\overline{u}\cdot e_{\mu\lambda})-\hat{t}(d\cdot\overline{\nu}_L)(d\cdot e_{\mu\lambda})-\hat{u}(\overline{u}\cdot\overline{\nu}_L)(\overline{u}\cdot e_{\mu\nu})]$$
$$+C_R{}^2[(2\hat{s}+\hat{t}+\hat{u})(\overline{u}\cdot\overline{\nu}_L)(d\cdot f_{\mu\lambda})-(\hat{s}+\hat{u})(g\cdot\overline{\nu}_L)(d\cdot f_{\mu\lambda})-(\hat{s}+\hat{t})(\overline{u}\cdot\overline{\nu}_L)(g\cdot f_{\mu\lambda})-\hat{t}(d\cdot\overline{\nu}_L)(d\cdot f_{\mu\lambda})-\hat{u}(\overline{u}\cdot\overline{\nu}_L)(\overline{u}\cdot f_{\mu\lambda})]$$
$$-m_{\nu_L}C_LC_R[(\hat{s}+\hat{u})E_{\mu\lambda}(d,g)+(\hat{s}+\hat{t})E_{\mu\lambda}(g,\overline{u})-(2\hat{s}+\hat{t}+\hat{u})E_{\mu\lambda}(d,\overline{u})]\}$$
$$\equiv(X_L\cdot e_{\mu\lambda})+(X_R\cdot f_{\mu\lambda})+\sum_{j=1}^{3}X_M^jE_{\mu\lambda}^j(p_j,q_j)\ , \tag{43}$$

where $(p_1,q_1)=(d,g),(p_2,q_2)=(g,\overline{u})$, and $(p_3,q_3)=(d,\overline{u})$. Here g_s is the strong-interaction coupling constant, $g_s{}^2=4\pi\alpha_s$, and the Mandelstam variables $\hat{s}$, $\hat{t}$, and $\hat{u}$ are as defined below Eq. (32). The decay amplitude squared is given by Eq. (40). The matrix element squared for the complete production/decay process is of the form in Eq. (42) with the X_M,Y_M, and X_MY_M terms replaced by

$$\sum_{j=1}^{3}X_M^j[J(Y_L,p_j,q_j)+J(Y_R,q_j,p_j)]+Y_M\sum_{j=1}^{3}X_M^jK(p_j,q_j,e,\overline{\nu}_e)\ . \tag{44}$$

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