Incoherent mixing of neutrinos and νe scattering

G. V. Dass
Physics Department, Indian Institute of Technology, Powai, Bombay 400 076, India

K. V. L. Sarma
Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005, India
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It is pointed out that precise data on scattering of reactor antineutrinos by electrons can decide whether the state $\bar{\nu}_e$ is a mixture of mass eigenstates with well-separated masses. Analyzing the existing νe data by accommodating the incoherent mixing of two neutrino states in the framework of the standard model, we find the mixing angle $\theta_{\mu e}=0.12\pm0.09$.

I. INTRODUCTION

The question of whether the emission of neutrinos of a given flavor is indeed an incoherent emission of several neutrinos has been of considerable interest in the past few years.[1–7] An experimental claim[6] that the neutrino emitted in tritium beta decay could be a mixture of two states—one with a mass of at most a few eV/c^2 and the other of about 17 keV/c^2 with roughly 3% mixing probability—has not been confirmed in ^{35}S decay.[7] Clearly, other combinations of mixing parameters (e.g., with smaller mass differences) cannot be excluded by the existing data. On the other hand, experiments dedicated to search for neutrino oscillations, due to their limited sensitivities, cannot exclude large values of Δm^2 with appropriately small values for the mixing angle θ; for instance, the recent results from the Gösgen reactor experiment[8] admit any value of $\Delta m^2 > 5$ eV^2 provided $\theta < 0.2$.

Here we would like to examine the information to be extracted from a study of the reactions $\nu e \to \nu e$ assuming that the neutrino beams at reactors and accelerators are incoherent mixtures of eigenstates of well-separated masses. We shall show that a precise measurement of the recoil-electron energy spectrum in $\bar{\nu}_e e \to \bar{\nu}_e e$ is sufficient to decide whether $\bar{\nu}_e$ is a mixture of mass eigenstates [see Eqs. (8) and (15) below]. On the other hand, from all the available data on νe scattering (which consists mainly of $\nu_\mu e$ and $\bar{\nu}_\mu e$) we find that the ν_μ-ν_e mixing angle $\theta_{\mu e}=0.12\pm0.09$ rad. This analysis in Sec. III is carried out in the standard model, which is suitably extended to include the possibility for neutrino mixing.

In the last section we comment on the relation of the present study to the standard oscillation picture. We also remark on the previously proposed tests of neutrino identity in neutral-current interactions.

II. MIXING EFFECTS IN νe SCATTERING

We shall start with the effective interaction for the scattering of the two neutrino flavors ν_e and ν_μ, by electrons. Fierz transforming the charged-current interaction, we obtain

$$L=-\frac{G_F}{\sqrt{2}}[\bar{\nu}_e\Gamma^\lambda\nu_e\bar{e}\Gamma_\lambda e + (\bar{\nu}_e\Gamma^\lambda\nu_e+\bar{\nu}_\mu\Gamma^\lambda\nu_\mu)\bar{e}\gamma_\lambda(g_V-g_A\gamma_5)e]\ ,$$
$$\Gamma^\lambda\equiv\gamma^\lambda(1-\gamma_5)\ . \quad (1)$$

In the standard model of electroweak interactions,

$$g_V=-\tfrac{1}{2}+2\sin^2\theta_W,\ \ g_A=-\tfrac{1}{2}\ .$$

If we assume, for the moment, that ν_e and ν_μ are orthogonal combinations of two mass eigenstates ν_1 and ν_2 with real constants c_1 and c_2,

$$|\nu_e\rangle=c_1|\nu_1\rangle+c_2|\nu_2\rangle,\ \ |\nu_\mu\rangle=-c_2|\nu_1\rangle+c_1|\nu_2\rangle\ ,$$
$$c_2{}^2+c_2{}^2=1\ , \quad (2)$$

the above interaction can be rewritten as

$$L=-\frac{G_F}{\sqrt{2}}[(\bar{\nu}_1\Gamma^\lambda\nu_1)\bar{e}\gamma_\lambda(g_{V1}-g_{A1}\gamma_5)e + (\bar{\nu}_2\Gamma^\lambda\nu_2)\bar{e}\gamma_\lambda(g_{V2}-g_{A2}\gamma_5)e + (\bar{\nu}_1\Gamma^\lambda\nu_2+\bar{\nu}_2\Gamma^\lambda\nu_1)\bar{e}\gamma_\lambda h(1-\gamma_5)e]\ , \quad (3)$$

where we have used the abbreviations

$$g_{Vk}=g_V+c_k{}^2,\ \ g_{Ak}=g_A+c_k{}^2\ \ (k=1,2)\ ,$$
$$h=c_1c_2\ . \quad (4)$$

Notice that although the neutrino flavor-changing neutral currents are absent in Eq. (1), a mass-changing neutral current with strength h is present in Eq. (3) as a consequence of purely the charged-current contribution. This is to be expected because even if the decay $Z^0\to\bar{\nu}_e\nu_\mu$ is forbidden, we can still have the charged-current coupling $\bar{\nu}_1 e\to W^-\to\bar{\nu}_2 e$ due to mixing.

By regarding the reactor antineutrinos to be an incoherent mixture of $\bar{\nu}_1$ and $\bar{\nu}_2$ with relative populations $c_1{}^2$ and $c_2{}^2$, we have, for the y distribution (denoting $d\sigma/dy$ by just $d\sigma$),

$$d\sigma(\bar{\nu}e)=c_1{}^2[d\sigma(\bar{\nu}_1 e\to\bar{\nu}_1 e)+d\sigma(\bar{\nu}_1 e\to\bar{\nu}_2 e)] +c_2{}^2[d\sigma(\bar{\nu}_2 e\to\bar{\nu}_2 e)+d\sigma(\bar{\nu}_2 e\to\bar{\nu}_1 e)] \quad (5)$$

$$=\frac{G_F{}^2 m_e E}{2\pi}\left[A^{(-)}+(1-y)^2A^{(+)}+\frac{m_e}{E}yC\right] , \quad (6)$$

$$A^{(+)}=\sum_{k=1}^{2} c_k{}^2(g_{Ak}+g_{Vk})^2+4h^2 , \quad (7a)$$

$$A^{(-)}=(g_A-g_V)^2 , \quad (7b)$$

$$C=\sum_{k=1}^{2} c_k{}^2(g_{Ak}{}^2-g_{Vk}{}^2) . \quad (7c)$$

Here, m_e is the electron mass, E is the laboratory energy of $\bar{\nu}_e$, (yE) is the laboratory kinetic energy of the recoil electron, and neutrino masses are assumed to be too small to affect the kinematics.

Using Eqs. (4) and (7), we obtain

$$r\equiv\frac{A^{(-)}A^{(+)}-C^2}{A^{(-)}}=4\alpha(1-\alpha) , \quad (8)$$

where

$$\alpha=\sum_{k=1}^{2} c_k{}^4\le 1 . \quad (9)$$

By setting $c_1=\cos\theta_{\mu e}$ and $c_2=\sin\theta_{\mu e}$, we can write Eq. (8) as

$$\frac{A^{(-)}A^{(+)}-C^2}{A^{(-)}}=[2-\sin^2(2\theta_{\mu e})]\sin^2(2\theta_{\mu e}) . \quad (10)$$

Hence a nonvanishing value of r in Eqs. (8) and (10) signals the possibility of neutrino mixing.

Relation (8) is easily generalized to the case of an arbitrary number of mass states $|\nu_i\rangle$ coupled to the flavor state $|\nu_f\rangle$:

$$|\nu_f\rangle=\sum_i c_{fi}|\nu_i\rangle , \qquad \sum_i |c_{fi}|^2=1=\sum_f |c_{fi}|^2 ; \quad (11)$$

$$d\sigma(\bar{\nu}_f e)=\sum_i\sum_j |c_{fi}|^2 d\sigma(\bar{\nu}_i e\to\bar{\nu}_j e) . \quad (12)$$

The three constants of Eqs. (7) in this case of incident $\bar{\nu}_f$ are given by

$$A_f^{(-)}=(g_A-g_V)^2 , \quad (13a)$$

$$A_f^{(+)}=(g_A+g_V)^2+4\alpha_{fe}(1+g_A+g_V) , \quad (13b)$$

$$C_f=(g_A{}^2-g_V{}^2)+2\alpha_{fe}(g_A-g_V) , \quad (13c)$$

where

$$\alpha_{fe}\equiv\sum_i |c_{fi}|^2|c_{ei}|^2 . \quad (14)$$

These relations have also been given by Divakaran and Ramachandran.[3] Now we note that relation (8) generalizes to

$$r_f\equiv\frac{A_f^{(-)}A_f^{(+)}-C_f{}^2}{A_f^{(-)}}=4\alpha_{fe}(1-\alpha_{fe}) . \quad (15)$$

This relation is of practical use only for the flavor $f=e$ because it is difficult to get ν_f or $\bar{\nu}_f$ beams (for $f\neq e$) at low energies where one is sensitive to the mass term C_f.

Noticing that Eq. (13a) is independent of the mixing parameters, an interesting test of the flavor universality of neutral currents in (V,A) theory is

$$A_e^{(-)}=A_\mu^{(-)}=(g_A-g_V)^2 . \quad (16)$$

This relation, which is valid even in the presence of $\nu_e-\nu_\mu$ mixing, can be verifed by determining $A_e^{(-)}$ from reactor data or data from meson factories, and $A_\mu^{(-)}$ from high-energy $\nu_\mu e$ and $\bar{\nu}_\mu e$ data.[9]

III. MIXING PARAMETERS FROM PRESENT DATA

We shall now turn to the available experimental information on νe scattering. As far as the determination of α_{ee} using data from reactors or meson factories is concerned, accurate information on the left-hand side of Eq. (15) [or, equivalently, Eqs. (13)] is not available. We consider $\alpha_{\mu e}$ next. A recent analysis[10] of the existing νe data (under the customary assumption that ν_μ and ν_e are mass eigenstates) yields

$$g_V=0.011\pm0.052,\quad g_A=-0.529\pm0.035 . \quad (17)$$

In obtaining these couplings the usual $V\leftrightarrow A$ ambiguity is taken to be resolved in favor of the standard model by appealing to the data on charge asymmetry in $e\bar{e}\to\mu\bar{\mu}$. We shall use the above values of g_V and g_A to reconstruct the experimental values of the constants

$$A_{\mu,\text{expt}}^{(\pm)}=(g_V\pm g_A)^2 , \quad (18)$$

since in getting Eqs. (17) neutrino mixing was not invoked and $\overset{(-)}{\nu}_\mu e$ data had the greater weight. These "experimental" values of $A_\mu^{(\pm)}$ can be used for the left-hand sides of Eqs. (13a) and (13b); for the parameters g_V and g_A we shall use the standard model:

$$A_{\mu,\text{expt}}^{(-)}=(2\sin^2\theta_W)^2 , \quad (19)$$

$$A_{\mu,\text{expt}}^{(+)}=(1-2\sin^2\theta_W)^2+8\alpha_{\mu e}\sin^2\theta_W . \quad (20)$$

From these two equations we determine

$$\alpha_{\mu e}=0.03\pm0.04 , \quad (21)$$

$$\sin^2\theta_W=0.27\pm0.03 . \quad (22)$$

Because of the large error this value of $\sin^2\theta_W$ is consistent with the currently accepted determinations (from νe data without taking into account neutrino mixing), which have lower central values. This error is, in fact, a reflection of our having relied upon the reconstructed $A_{\mu,\text{expt}}^{(-)}$. Needless to say, it would be desirable to do the analysis directly with the experimental quantities.

For the simple case of two-level mixing,

$$\alpha_{\mu e}=\tfrac{1}{2}\sin^2(2\theta_{\mu e}) , \quad (23)$$

and hence we obtain

$$\theta_{\mu e}=0.12\pm0.09 , \quad (24)$$

i.e., $\theta_{\mu e}<0.26$ with a 90% confidence level (C.L.). This value of $\theta_{\mu e}$ is comparable to the limit obtained at the

Gösgen reactor,[8] $\theta < 0.2$ (90% C.L.), although it should be remarked that the reactor result, being based on searches for inclusive $\bar{\nu}_e$ oscillations, is a more stringent limit.

When accurate data on $\bar{\nu}_e e$ scattering at reactors become available, one can obtain α_{ee} and check for the consistency of the two-level mixing through the relation

$$\alpha_{ee} + \alpha_{\mu e} = 1 \ . \tag{25}$$

A determination of the parameter $\alpha_{\mu\mu}$ (analogous to α_{ee}) is, on the other hand, more difficult as it requires measurements of ν_μ scattering on μ targets. Instead, one can, following the proposal of Divakaran and Ramachandran,[3] use the charged-current (CC) and neutral-current (NC) data on $\nu_\mu N$ and $\bar{\nu}_\mu N$ scattering cross sections. Since the value $\sin^2\theta_W = 0.220 \pm 0.014$ coming from the SLAC *ed* experiment after radiative corrections[11] is independent of neutrino mixing, it can be substituted in the Paschos-Wolfenstein relation to extract $\alpha_{\mu\mu}$:

$$\frac{\sigma_{\rm NC}(\nu_\mu N) - \sigma_{\rm NC}(\bar{\nu}_\mu N)}{\sigma_{\rm CC}(\nu_\mu N) - \sigma_{\rm CC}(\bar{\nu}_\mu N)} = \frac{1}{\alpha_{\mu\mu}}\left(\tfrac{1}{2} - \sin^2\theta_W\right) . \tag{26}$$

To evaluate the left-hand side of Eq. (26), we take the one-parameter ($\rho = 1$) fit[12] to the Chicago-Caltech-Columbia-Fermilab-Rochester-Rockefeller (CCCFRR) data, to which radiative corrections have been applied; we get the value 0.258 ± 0.012. This leads to

$$\alpha_{\mu\mu} = 1.09 \pm 0.07 \ . \tag{27}$$

The parameters $\alpha_{\mu\mu}$ and $\alpha_{e\mu}$ ($=\alpha_{\mu e}$) should satisfy $\alpha_{\mu\mu} + \alpha_{e\mu} = 1$ in the $\nu_\mu - \nu_e$ mixing model; using Eqs. (21) and (27), the present data are clearly consistent with such a model.

Radiative corrections.

For the purpose of extracting neutrino-mixing parameters, we next consider the importance of applying the standard-model radiative corrections to the data. It turns out that our estimate $\alpha_{\mu e} = 0.03 \pm 0.04$ is hardly affected as a result of applying radiative corrections (up to first order in QED and electroweak processes). To be specific, let us consider the reaction $\nu_\mu e \to \nu_\mu e$ and follow closely the study of Sarantakos *et al*;[13] we shall replace their $\epsilon_\mp{}^2(q^2)$ by $\frac{1}{4}A^{(\pm)}(q^2)$ to conform to our notation and write

$$\begin{aligned}\frac{d\sigma}{dy}(\nu_\mu e) &= \frac{d\sigma_{\rm eff}}{dy} + \left[\frac{d\sigma}{dy}\right]_{\rm QED} \\ &= \frac{G_\mu{}^2 m_e E}{2\pi}\rho_{\rm NC}{}^2\left[A^{(+)}(q^2)\left[1+\frac{\alpha}{\pi}f_-(y)\right] + A^{(-)}(q^2)\left[1+\frac{\alpha}{\pi}f_+(y)\right](1-y)^2 + (A^{(+)}A^{(-)})^{1/2}\frac{m_e}{E}y\right] .\end{aligned} \tag{28}$$

Here, G_μ is the Fermi coupling constant obtained from the free muon decay after applying first-order QED corrections, and α is the fine-structure constant.

For a typical ν_μ incident energy $E = 10$ GeV [e.g., CERN-Hamburg-Amsterdam-Rome-Moscow (CHARM) Collaboration] the last term containing m_e/E in the above formula can be omitted and one can make use of the numerical tables of Ref. 13. First, we note that the q^2-independent factor $\rho_{\rm NC}{}^2$ lies in the range 1.00—1.01 (if the mass of the Higgs scalar lies in the range $m_\phi = 30$—300 GeV, then $\rho_{\rm NC} = 1.004$—1.005). As for the q^2 dependence, we have

$$\begin{aligned} &A^{(-)}(q^2) = [2\sin^2\theta_W(q^2)]^2 , \\ &A^{(+)}(q^2) = [1 - 2\sin^2\theta_W(q^2)]^2 , \\ &\sin^2\theta_W(q^2) = \hat{\kappa}(q^2)\sin^2\hat{\theta}_W(m_W) . \end{aligned} \tag{29}$$

The function $\hat{\kappa}(q^2)$ corresponds to the so-called modified minimal-subtraction ($\overline{\rm MS}$) scheme and is independent of the Higgs-boson mass m_ϕ and very insensitive to the top-quark mass. The experimental quantities of Eqs. (18) now become

$$A^{(-)}_{\mu,\rm expt} = \rho_{\rm NC}{}^2\left[1 + \frac{\alpha}{\pi}f_+(y)\right][2\hat{\kappa}(q^2)\sin^2\hat{\theta}_W(m_W)]^2 , \tag{30}$$

$$\begin{aligned} A^{(+)}_{\mu,\rm expt} = \rho^2_{\rm NC}\left[1 + \frac{\alpha}{\pi}f_-(y)\right][1 - 2\hat{\kappa}(q^2)\sin^2\hat{\theta}_W(m_W)]^2 \\ + 8\alpha_{\mu e}\sin^2\hat{\theta}_W(m_W) , \end{aligned} \tag{31}$$

wherein the term containing $\alpha_{\mu e}$ arises from the possibility of ν_μ-ν_e mixing. We shall ignore radiative corrections to this term because of the (presumed) smallness of the mixing angle $\theta_{\mu e}$. We have therefore, for definiteness, identified the $\sin^2\theta_W$ of this term with the $\sin^2\hat{\theta}_W(m_W)$.

To get a feeling for the magnitude of radiative corrections at a typical value of y, we note that at $y \simeq 0.5$ ($q^2 = -5 \times 10^{-3}$ GeV^2) we have to deal with the quantities

$$\hat{\kappa} = 1.00435, \quad 1 + \frac{\alpha}{\pi}f_-(y) = 0.9872 ,$$

$$1 + \frac{\alpha}{\pi}f_+(y) = 0.9632 .$$

Substituting these quantities, $\rho_{\rm NC}{}^2 = 1.01$, and also the "data" [from Eqs. (17) and (18)] in Eqs. (30) and (31), we obtain

$$\alpha_{\mu e} = 0.03 \pm 0.04 , \tag{32}$$

$$\sin^2\hat{\theta}_W(m_W) = 0.27 \pm 0.03 . \tag{33}$$

Since these values are identical with the corresponding ones in Eqs. (21) and (22), we conclude that the effects of radiative corrections are not noticeable because of the limited accuracy of the present data. One should also note that our Eqs. (30) and (31) for radiative corrections pertain only to $\nu_\mu e$ scattering, while the experimental numbers in Eqs. (17) have been extracted from data which include $\bar\nu_\mu e$, $\nu_e e$, and $\bar\nu_e e$ scattering.

IV. COMMENTS AND SUMMARY

How is relation (5) to be viewed in the conventional oscillation picture wherein the rates vary with distance L of the electron target from the reactor core? The coherence between the states ν_1 and ν_2 would be reflected in the probability for the production of $\bar\nu_\mu$ at distance L:

$$P(L)=\sin^2(2\theta)\sin^2\left[\frac{\pi L}{\lambda}\right],\quad \lambda\equiv 4\pi E/|m_1{}^2-m_2{}^2| \ . \tag{34}$$

If $L \gg \lambda$, then we are allowed to replace P by its average value

$$\langle P(L)\rangle=\tfrac{1}{2}\sin^2(2\theta)=1-\alpha_{ee}=\alpha_{\mu e} \ , \tag{35}$$

in the notation of Eqs. (9) and (10). To ensure the smallness of the oscillation length λ compared to the flight path L, we require $|m_1{}^2-m_2{}^2|\gtrsim$ a few eV^2 for reactor experiments, and $|m_1{}^2-m_2{}^2|\gtrsim$ a few hundred eV^2 for accelerator experiments. In this paper we have implicitly assumed the validity of such limits of large Δm^2, evidence for which may appear in an accurate Kurie plot of tritium decay. In addition, the incoherent limit or the averaging implied in Eq. (35) could arise due to the averaging over the energy spectrum of the ν beam or the averaging over the uncertain distances between the point of production of ν and the position of the e target. Thus relation (5) corresponds to averaging over the L dependence in the usual analysis of oscillations in νe scattering.

Sensitivity to neutrino mixing

It may be mentioned that a list of the νe total cross sections in order of decreasing sensitivity to ν_e-ν_μ mixing is $\sigma(\nu_\mu e)$, $\sigma(\bar\nu_\mu e)$, $\sigma(\nu_e e)$, and $\sigma(\bar\nu_e e)$. In the case of ν_e or $\bar\nu_e$ projectiles, the contributions of $W^\pm$ exchanges are larger than those due to Z exchanges [in the absence of oscillations, e.g., $\sigma(\nu_e e)\simeq 6.1\sigma(\nu_\mu e)$ for $\sin^2\theta_W=0.23$]; thus the measured $\nu_\mu e$ cross section, for instance, will be significantly increased when even a small ν_e component gets developed due to the ν_μ-ν_e oscillation during propagation.[2] While some of the earlier analyses[4,5] to test for neutrino flavor mixings had essentially depended on the $\sigma(\bar\nu_e e)$ data from the reactor, our present analysis starts with the values of g_V and g_A of Eqs. (17) as extracted from a comprehensive analysis of the entire νe scattering data base, which is dominated by the $\nu_\mu e$ and $\bar\nu_\mu e$ accelerator data accumulated over the years.

Secondly, it should be noted that the y distributions are more sensitive to the small parameter $\alpha_{\mu e}$ than the total cross sections. For example, in $\nu_\mu e$ scattering at high energies the y distribution has the form $A_\mu^{(+)}+(1-y)^2A_\mu^{(-)}$. Here the parameter $\alpha_{\mu e}$ occurs in the coefficient $A_\mu^{(+)}$ [see Eq. (13b)] with a relative weight

$$4(1+g_A+g_V)/(g_A+g_V)^2 \ ,$$

while in the y-integrated cross section $\sigma(\nu_\mu e)$ this parameter occurs with a relative weight

$$4(1+g_A+g_V)/[(g_A+g_V)^2+\tfrac{1}{3}(g_A-g_V)^2] \ ;$$

for $\sin^2\theta_W=0.25$ these two weights are in the ratio 4:3.

In view of the above comments the most sensitive place to look for ν_μ-ν_e mixing would be the angular distribution of $\nu_\mu e$ scattering.

It should also be remarked that the parameter α_{ee} can be obtained directly from relation (8) using data from the single-$\bar\nu_e e$ experiment, without having to take $\sin^2\theta_W$ from some other experimental source (in fact, the relation follows for an arbitrary mixture of V and A for neutral-current structure). However, because of the absence of precise data with reactor antineutrinos this relation cannot be used at present. On the other hand, extraction of α_{ee} from y-integrated $\sigma(\bar\nu_e e)$ requires a knowledge of $\sin^2\theta_W$ from a source which is independent of neutrino mixing, such as, e.g., the $\vec{e}$D asymmetry experiment.

Neutrino identity

If one assumes that there are no extra neutrinos to be associated with the existing flavors (such as $\nu'_e, \nu'_\mu, \ldots$), the hypothesis of "neutrino identity" (i.e., the outgoing antineutrino in a scattering experiment at the reactor is identical to the incident $\bar\nu_e$) is equivalent to the absence of flavor-changing neutral currents. We shall now see that the tests previously claimed to provide signals for such flavor-changing neutral currents in the leptonic sector do not have a unique interpretation; this is because the corresponding relations get violated due to neutrino mixing even in the standard model, wherein such nondiagonal neutral currents are absent.

Referring to the y distribution (6) it has earlier been suggested that the relation[14,15]

$$A^{(-)}A^{(+)}=C^2 \tag{36}$$

is a test of neutrino identity. However, this can be violated if $\bar\nu_e$ is a mixture of mass eigenstates, even though neutral currents are flavor diagonal; see Eq. (8).

Furthermore, from Eq. (13c) we can obtain

$$|C_e-C_\mu|=2z(A_e^{(-)})^{1/2} \ ,$$

$$z=|\alpha_{ee}-\alpha_{\mu e}|\le 1 \ .$$

Previously it was emphasized[15] that if experiment revealed that $z\neq 1$, it would indicate the existence of nondiagonal neutral currents. Such a conclusion would not be compelling because, as shown above, z could be less than unity also due to neutrino mixing. Similar remarks apply to the relation[15]

$$\tfrac{1}{16}(A_e^{(+)}-A_\mu^{(+)}-4)^2=xA_\mu^{(+)} \ .$$

In summary, using the standard model, we have examined the observable consequences of neutrino mixing in νe and $\bar\nu e$ scattering. It is assumed that the mass eigenstates have well-separated masses [$|m_1{}^2-m_2{}^2|\gtrsim 100$ eV^2, keeping the accelerator experiments in view]. Relation (8) can be used to test whether reactors emit antineutrinos which have a unique mass. Secondly, a test of the ν_e-ν_μ universality in the standard model is provided by Eq. (16). To get a numerical estimate of the mixing parameter $\alpha_{\mu e}$, we have reconstructed the experimental quantities $A^{(\pm)}_{\mu,\mathrm{expt}}$ by using a typical analysis[10] of νe data. When analyzed within the framework of the standard model extended to incorporate two-level incoherent mixing of neutrinos, these $A^{(\pm)}_{\mu,\mathrm{expt}}$ give $\frac{1}{2}\sin^2(2\theta_{\mu e})=0.03\pm0.04$ and $\sin^2\theta_W=0.27\pm0.03$; the modifications due to radiative corrections are well within the quoted errors.

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[9]It should be remarked that in relation (8) the ratio $r\leq 1$ (equality occurs for $\alpha=\frac{1}{2}$, i.e., the case of maximal mixing $c_1=c_2=1/\sqrt{2}$). If future experiments clearly show that $r>1$ even after radiative corrections, then it would be a serious violation of the standard model. One possible way out in that case would be to introduce some kind of nonuniversal couplings. For instance, we may assume that the Z^0 coupling to ν_e is an overall factor ξ times the Z^0 coupling to ν_μ, $(\xi\bar{\nu}_e\Gamma^\lambda\nu_e+\bar{\nu}_\mu\Gamma^\lambda\nu_\mu)Z_\lambda$, and thereby obtain $r=4\alpha(1-\alpha)/[1+(\xi^2-1)\alpha]$. Now r can exceed unity for sufficiently small values of ξ^2. Obviously, these departures from universality would also affect relation (16): $A_f^{(-)}-A_{f'}^{(-)}=(g_A-g_V)^2(\xi^2-1)(\alpha_{fe}-\alpha_{f'e})$. Other interesting consequences of such nonuniversality were discussed earlier by us [G. V. Dass and K. V. L. Sarma, Phys. Rev. D **28**, 49 (1983)].

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