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Pseudo-Dirac solar neutrinos?

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The compatibility of a recent possible explanation of the solar-neutrino puzzle and the result of Boris *et al.* on limits on neutrino mass is discussed. It is shown that if the electron and muon Majorana neutrinos combine to form a pseudo-Dirac particle of mass ~ 35 eV, then the radiative corrections will generate a mass-square difference 3×10^{-5} eV, which explains why Davis and his collaborators find fewer solar neutrinos than predicted. This idea can be naturally embedded in superstring theories.

One of the outstanding problems with neutrinos is the discrepancy between the experimentally measured solar neutrinos by Davis and his collaborators and the theoretical predictions. One possible explanation[1] of this solar-neutrino puzzle has recently been suggested following the idea[2] that ν_e above a certain minimum energy may all be converted into ν_μ on their way out through the Sun. For this mechanism to work[1] the mass of ν_μ (m_{ν_μ}) has to be greater than that of ν_e (m_{ν_e}), and $(m_{\nu_\mu}{}^2 - m_{\nu_e}{}^2) = 6\times 10^{-5}$ eV. It was then assumed that $m_{\nu_\mu} = 8\times 10^{-3}$ eV and $m_{\nu_e} < m_{\nu_\mu}$. This result is inconsistent with the lower bound[3] on the neutrino mass, $m_{\nu_e} > 20$ eV, obtained by Boris *et al.* by measuring the near-end-point shape of the β spectrum in tritium decay. Furthermore, if m_{ν_e} and m_{ν_μ} are less than 1 eV, they will be of no cosmological relevance. Finally, most of the grand unified theories,[4,5] theories,[4,5] particularly in the context of superstring theories,[6] predict a neutrino mass of the order of few eV.

In the present article, an attempt is made to accommodate all the above-mentioned results in a simple model. It will be shown that the results of Boris *et al.* on neutrino mass can be made compatible with the recent explanation of the solar-neutrino puzzle. All neutrinos in this theory are heavier than 1 eV and hence are equally relevant to cosmology. This model can be embedded naturally in superstring theories.

In grand unified theories, a Majorana mass for ν_e of the order of a few eV is quite natural,[4] although lighter Majorana neutrinos are also possible.[7] There are some models[6] which accommodate naturally the result of Boris *et al.* on the lower bound of neutrino mass, i.e., $m_{\nu_e} > 20$ eV along with the upper bound[8] on the Majorana mass of ν_e, i.e., $m_{\nu_e}(\mathrm{Maj}) < 10$ eV, set from the unobservability of neutrinoless double-β decay. However, superstring theories accommodate only a few eV Majorana neutrinos.[9]

Most of the above-mentioned theories do not discuss the mixing between neutrinos of different generations. There are some models where the freedom of this generational structure has been exploited to get phenomenologically consistent mass matrix.[10] A similar approach will be taken here. Since none of the previously discussed models with 20-eV Dirac electron neutrinos can accommodate the recent explanation of the solar-neutrino puzzle and the lighter ($m_{\nu_\mu} \ll 1$ eV) neutrino models cannot explain the result of Boris *et al.* and are difficult to accommodate in superstring theories, we shall start with Majorana neutrinos heavier than 1 eV for all the generations. To explain the experiment of Boris *et al.* we shall assume that the electron and the muon neutrinos are degenerate and have opposite *CP* properties. We shall then implement the idea of pseudo-Dirac neutrino,[11] which will then generate a very small mass difference radiatively to accommodate the recent possible explanation of the solar-neutrino puzzle.

We shall consider only two neutrino flavors: ν_e and ν_μ. Our present model does not constrain the mixing of the third flavor and for simplicity we shall assume that there is no mixing between the first two flavors with the other neutrino flavor states. Then, as considered previously,[10] we shall assume that the lepton mass matrices have a global symmetry: $aL_e + bL_\mu$. When $a = -b = 1$, the theory will predict a mass matrix with degenerate mass eigenstates with opposite *CP* properties. Although this is consistent with the result of Boris *et al.* and the result on neutrinoless double-β decay, since the mass difference vanishes, the solar-neutrino puzzle cannot be solved. We shall consider $a = \cos\theta$ and $b = -\sin\theta$. As we shall now notice, this choice will give us a pseudo-Dirac neutrino, which will be consistent with the result of Boris *et al.* neutrinoless double-β decay and the solar-neutrino experiment.

Let us now study the Majorana mass matrices. The neutrino mass matrix consistent with the symmetry $(\cos\theta L_e - \sin\theta L_\mu)$ can be written conveniently in the basis in which the first (second) row and column refers to the eigenstates:

$$\begin{aligned} |1\rangle &= \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta)\nu_e + \frac{1}{\sqrt{2}}(\cos\theta - \sin\theta)\nu_\mu , \\ |2\rangle &= \frac{1}{\sqrt{2}}(\sin\theta - \cos\theta)\nu_e + \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta)\nu_\mu . \end{aligned} \tag{1}$$

From now on we shall denote ν_e and ν_μ as the weak neutrino eigenstates and assume that the mass matrix of the charged leptons e and μ is diagonal. Now the global symmetry becomes $(|1\rangle - |2\rangle)$ and hence the mass matrix in the basis $(|1\rangle\, |2\rangle)$ will be

$$m_0 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} .$$

This mass matrix can be written in the basis $(\nu_e \nu_\mu)$ as

$$M = m_0 \begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} . \qquad (2)$$

From the global symmetry considered here it is obvious that this matrix will correspond to two degenerate mass eigenstates with opposite *CP* properties. So, even if we consider $m_0 = 35$ eV assuming that the results of Boris *et al.* will be confirmed in the future, there would not be any neutrinoless double-β decay. The weak eigenstates $(\nu_e \nu_\mu)$ are related to the mass eigenstates $(\nu_1 \nu_2)$ with masses $-m_0$ and m_0 by

$$\begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} . \qquad (3)$$

Although the weak eigenstates are mixtures of the mass eigenstates in this case, since the mass squared difference is zero, there would not be any oscillation and hence the solar-neutrino puzzle will be unaffected at the tree level.

We shall now consider the radiative correction in this model.[11] Because of the radiative correction the mass matrix (2) in the weak eigenstate will now become

$$M = M_0 + M_1 , \qquad (4)$$

where M_0 is given by (2) and

$$M_1 = \begin{bmatrix} -\beta_e^2 \cos 2\theta & 0 \\ 0 & \beta_\mu^2 \cos 2\theta \end{bmatrix} , \qquad (4a)$$

where

$$\beta_\sigma^2 = \alpha m_0 \frac{m_\sigma^2}{m_W^2} , \quad \sigma = e, \mu . \qquad (4b)$$

The induced mass difference

$$(\beta_\mu^2 - \beta_e^2)\cos 2\theta = \alpha m_0 \frac{m_\mu^2 - m_e^2}{m_W^2} \cos 2\theta$$
$$= 4 \times 10^{-7} \cos 2\theta \text{ eV} \qquad (5)$$

will now cause neutrinoless double-β decay, but this contribution is too small to be detected experimentally. For this small mass difference (5) the neutrino-oscillation experiments cannot set any limit on the mixing angle θ. However, we shall consider θ to be small for convenience.

Now to discuss how the present model can explain the solar-neutrino puzzle we write the mass-squared matrix in the representation $(\nu_e \nu_\mu)$,

$$M^2 = [m_0^2 + (\beta_e^2 + \beta_\mu^2) m_0 \cos 2\theta] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$+ (\beta_\mu^2 - \beta_e^2) m_0 \cos 2\theta \begin{bmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} . \qquad (6)$$

If we now consider the charge-current interactions of ν_e with matter, there will be an additional effective electron neutrino mass term[1,2] $A = 0.76 \times 10^{-7} \rho E$ eV^2, where the matter density ρ is in g/cm^3 and the neutrino energy E is in MeV. The mass-squared difference between the two eigenstates will be

$$\{[2m_0(\beta_\mu^2 - \beta_e^2)\cos^2 2\theta - A]^2$$
$$+ 4m_0^2(\beta_\mu^2 - \beta_e^2)^2 \sin^2 2\theta\}^{1/2} . \qquad (7)$$

From (4b) it can be seen that $\beta_\mu^2 > \beta_e^2$. Then, assuming $\cos 2\theta$ to be positive, a minimum of (7) is reached when

$$A = A_{\min} = 2m_0(\beta_\mu^2 - \beta_e^2)\cos^2 2\theta$$
$$= 2.8 \times 10^{-5} \cos^2 2\theta \text{ eV}^2 . \qquad (8)$$

Now consider electron neutrinos which are produced in the Sun at sufficiently high density ($A > A_{\min}$). As they will move outward A will decrease. According to Refs. 1 and 2, as A reaches $A_{\min}$, all these ν_e will be converted into ν_μ. Thus only those ν_e will come out as ν_e which started with energy E, less than a certain critical energy E_c, which corresponds to $A = A_{\min}$. If we choose $\rho = 130$ g/cm^3 (Ref. 1), then we get $E_c = 2.8$ MeV. Now neutrinos from all the nuclear species except ^{8}B have a maximum energy of 2.8 MeV or less. These neutrinos will escape from the Sun as ν_e, whereas only a fraction of the neutrinos from ^{8}B will emerge from the Sun as ν_e and the rest will be converted to ν_μ. Following Ref. 1, the present model predicts the total number of neutrinos in solar-neutrino units (SNU's) to be $1.6 + 0.02 \pm 0.2$, which is within experimentally observed limit of 2.1 ± 0.3.

Unlike Ref. 1 where the muon neutrino mass had to be adjusted to explain the solar-neutrino puzzle, in the present model the explanation of the solar-neutrino puzzle emerges naturally. The only input in this theory is the assumption of the existence of the global symmetry and the neutrino mass consistent with the experiments of Boris *et al.*

In summary, we started with Majorana neutrinos and postulated a certain global symmetry which may arise naturally in some theories depending on the generation structure. The electron and muon neutrino will combine to form a pseudo-Dirac particle, which is consistent with neutrinoless double-β-decay and neutrino-oscillation experiments. We then demand that the model is consistent with the results of Boris *et al.* on limits on neutrino mass. As a result of this the theory predicts the number of solar neutrinos consistent with the observations of Davis and his collaborators. This idea can be embedded naturally in superstring theories and other grand unified theories.

Note added. R. N. Mohapatra and J. W. F. Valle [Phys. Lett. **177B**, 47 (1986)] have also proposed a similar solution of the solar-neutrino puzzle in certain superstring theories. In their model the physical neutrino mixes with a sterile neutrino to form a Dirac particle and a much smaller mass difference is generated at the tree level after symmetry breaking.

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