Adiabatic resonant oscillations of solar neutrinos in three generations

C. W. Kim and W. K. Sze
Department of Physics and Astronomy, Johns Hopkins University, Baltimore, Maryland 21218
(Received 6 October 1986)

The Mikheyev-Smirnov-Wolfenstein model of resonant solar-neutrino oscillations is discussed for three generations of leptons. Assuming adiabatic transitions, bounds for the μ- and e-neutrinos mass-squared difference $\Delta_{21,0}$ are obtained as a function of the e-μ mixing angle θ_1. The allowed region in the $\Delta_{21,0}$-θ_1 plot that would solve the solar-neutrino problem is shown to be substantially larger than that of the two-generation case. In particular, the difference between the two- and three-generation cases becomes significant for θ_1 larger than $\sim 20°$.

I. INTRODUCTION

The discovery of the Mikheyev-Smirnov-Wolfenstein[1,2] (MSW) mechanism as a possible solution to the solar-neutrino problem[3] has aroused much interest lately. In this proposed mechanism, within the Sun there is an effective potential visible only to the electron neutrinos. This gives them an effective mass and mixing angles different from their vacuum values, the additional contribution to the mass-squared value being proportional to the electron density in the environment. The variation of this effective mass as the neutrinos traverse the Sun results in a resonance oscillation of the high-energy ones to muon neutrinos. This supposedly explains the solar-neutrino depletion. The ^{71}Ga detector now under construction and future experiments based on neutrino neutral-current interactions will eventually put a test on the validity of this theory. In the meantime the original idea has been refined and improved.[4-8]

Most of these analyses were done in the two-generation model. According to the prevailing theoretical prejudice which is based on the see-saw mechanism[9] of neutrino mass generation and on the experience gained from the quark sector, lepton-mixing angles (in particular the e-μ mixing angle θ_1) are expected to be small[10] suggesting that the two-generation calculations are indeed a good approximation to the more realistic three-generation case. This prejudice, however, has not yet been substantiated by experiments so that it is worthwhile to explore the possibility of large mixing angles in the leptonic sector and its effects on the resonance oscillation of solar neutrinos.

For two generations, a detailed analysis[6] shows that in order to explain the experimental data θ_1 cannot exceed $\sim 35°$. However, it has been known for some time that for three generations the solar-neutrino problem can be explained by the assumption of maximal mixing of the three types of neutrinos, which implies, e.g., θ_1 value of $\sim 55°$ (Ref. 11). This suggests that in the large-angle case the two-generation analysis is clearly inadequate. Even for the small-angle case when one expects the two-generation calculation to be reasonably valid, it would be useful to examine explicitly how good the approximation actually is.

In this paper a three-generation calculation for the MSW mechanism is presented. For the sake of simplicity and due to the lack of experimental observation of CP violation for leptons, we assume that CP is conserved in the leptonic sector. In order to further simplify our calculations, only the case of adiabatic approximation, as in the work of Bethe,[3] is discussed as an example to demonstrate explicitly how the addition of the third-generation neutrino would affect the analysis of the two-generation case.

We outline the general procedure in Sec. II. In Sec. III it is shown, in order to verify the results in Sec. II for three generations, that they reduce to the well-known two-generation ones when $\theta_2=0$. Here θ_2 is another angle parameter characterizing the mixing of the third neutrino with the first two. Section IV contains discussion of the numerical results.

II. THREE-GENERATION CALCULATION

In order for the proposed mechanism to work, the neutrino weak eigenstate ψ cannot coincide with the mass eigenstate ψ_m. With the assumption of CP conservation in the leptonic sector, they are connected by an orthogonal matrix U_0, i.e., $\psi_m=U_0\psi$, with $U_0^T=U_0{}^{-1}$. Written out in full with the Kobayashi-Maskawa parametrization,[12] we have

$$\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} = \begin{bmatrix} C_1 & -S_1C_3 & -S_1S_3 \\ S_1C_2 & C_1C_2C_3-S_2S_3 & C_1C_2S_3+S_2C_3 \\ S_1S_2 & C_1S_2C_3+C_2S_3 & C_1S_2S_3-C_2C_3 \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} , \tag{1}$$

where $C_i=\cos\theta_i$, $S_i=\sin\theta_i$. Denoting the mass-squared matrices in the mass and weak bases as M_0^D and M_0, respectively, we have

$$\begin{aligned} M_0^D &= \mathrm{diag}(m_1{}^2, m_2{}^2, m_3{}^3) , \\ M_0 &= (M_{ij}), \quad i,j=1,2,3 \\ &= U_0^T M_0^D U_0 , \end{aligned} \tag{2}$$

where M_0 is a real symmetric matrix.

In the interior of the Sun the electron neutrino "sees" an effective potential $V=\sqrt{2}G_F N_e$ due to its weak interaction with the electrons present. Here G_F and N_e are the Fermi's constant and the electron density, respectively. As a result the mass-squared matrix will be modified to (in the weak basis)

$$M=M_0+\mathrm{diag}(A,0,0) = \begin{bmatrix} M_{11}+A & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{bmatrix} , \quad (3)$$

where $A=2EV$, E being the energy of the neutrino. Correspondingly the matrices M_0^D and U_0 will be changed to M^D and U, respectively, which are given as

$$M=U^T M^D U ,$$
$$M^D=\mathrm{diag}(\mu_1{}^2,\mu_2{}^2,\mu_3{}^2) , \quad (4)$$
$$U=U_0(\theta_i \to \phi_i) ,$$

and in the following we shall use the notation $c_i=\cos\phi_i$, $s_i=\sin\phi_i$. For given initial values $m_i{}^2$ and θ_i, the effective values $\mu_i{}^2$ and ϕ_i will be functions of A. The quantities of interest to us are ϕ_1 and $\Delta_{21}=\mu_2{}^2-\mu_1{}^2$. As will be shown below, they enter a coupled set of equations with ϕ_2 and $\Delta_{32}=\mu_3{}^2-\mu_2{}^2$.

Equation (4) gives the elements of M in terms of the ϕ_i's and μ_i's. In order to eliminate the variables which will be irrelevant to our discussion, we take the following linear combinations of elements of M (Ref. 13). We find

$$\tfrac{1}{2}(M_{22}+M_{33})-(M_{11}+A)=(1-\tfrac{3}{2}s_1{}^2)\Delta_{21} + \tfrac{1}{2}(1-3s_1{}^2s_2{}^2)\Delta_{32} , \quad (5)$$

$$M_{33}-M_{22}=B\cos2\phi_3-c_1 D\sin2\phi_3 ,$$
$$2M_{23}=-B\sin2\phi_3-c_1 D\cos2\phi_3 , \quad (6)$$

$$2M_{12}=F\cos\phi_3+s_1 D\sin\phi_3 ,$$
$$2M_{13}=F\sin\phi_3-s_1 D\cos\phi_3 , \quad (7)$$

where

$$B=s_1{}^2\Delta_{21}^e+\Delta_{32}\cos2\phi_2 ,$$
$$F=\Delta_{21}^e\sin2\phi_1 , \quad (8)$$
$$D=\Delta_{32}\sin2\phi_2 .$$

Here Δ_{21}^e is defined as

$$\Delta_{21}^e \equiv \Delta_{21}+s_2{}^2\Delta_{32} . \quad (9)$$

Now if we write, to define B_c, F_c, α, and β,

$$B=B_c\cos\beta, \quad c_1 D=B_c\sin\beta ,$$
$$F=F_c\cos\alpha, \quad s_1 D=F_c\sin\alpha , \quad (10)$$

Eqs. (6) and (7) can be rewritten as

$$M_{33}-M_{22}=B_c\cos(\beta+2\phi_3) ,$$
$$2M_{23}=-B_c\sin(\beta+2\phi_3) , \quad (11)$$

$$2M_{12}=F_c\cos(\alpha-\phi_3) ,$$
$$2M_{13}=-F_c\sin(\alpha-\phi_3) . \quad (12)$$

Keeping in mind that the M_{ij}'s assume the same values whether in matter or vacuum, Eqs. (11) and (12) mean that B_c, F_c, $\beta+2\phi_3$, and $\alpha-\phi_3$ are all constants, i.e., independent of A. The same is true for $\beta+2\alpha$.

If we denote by α' the departure of α from its vacuum value, we can write

$$\alpha=\alpha_0+\alpha', \quad \beta=\beta_0-2\alpha' , \quad (13)$$

where α_0 and β_0 are the vacuum values of α and β, respectively. The functional dependence of α' on A is determined from Eq. (5), which can be rewritten as

$$\tfrac{1}{2}(M_{22}+M_{33})-M_{11}=\Delta_{21}^e\cos2\phi_1+\frac{B}{2}+A . \quad (14)$$

Taking $A=0$ yields

$$\tfrac{1}{2}(M_{22}+M_{33})-M_{11}=\Delta_{21,0}^e\cos2\theta_1+\frac{B_0}{2} , \quad (15)$$

where variables with subscript 0 denote vacuum values, as above. Hence

$$A=\Delta_{21,0}^e\cos2\theta_1-\Delta_{21}^e\cos2\phi_1-\frac{B-B_0}{2} . \quad (16)$$

From Eq. (10) and its vacuum counterpart, i.e., $B_0=B_c\cos\beta_0$, etc., Eq. (16) can be rewritten in a form where the interdependence of A and α' can be made more explicit:

$$A=\Delta_{21,0}^e\cos2\theta_1-F_c\cos(\alpha_0+\alpha')\cot2\phi_1 - B_c\sin\alpha'\sin(\beta_0-\alpha') . \quad (17)$$

Again from Eq. (10),

$$\tan\phi_1\frac{\sin\beta}{\sin\alpha}=\frac{F_c}{B_c}=\tan\theta_1\frac{\sin\beta_0}{\sin\alpha_0} ,$$

i.e.,

$$\cot\phi_1=\cot\theta_1\frac{\sin(\beta_0-2\alpha')}{\sin(\alpha_0+\alpha')} \Big/ \frac{\sin\beta_0}{\sin\alpha_0} . \quad (18)$$

Equations (17) and (18) furnish two equations with two unknowns α' and ϕ_1 for any given A. Their values are then substituted back into Eq. (10) to yield values for B, F, and D. Then values of Δ_{21}, Δ_{32}, and ϕ_2 follow easily from Eq. (8). A more transparent way of displaying the system of equations is to rewrite them as

$$\Delta_{21}^e\cos2\phi_1+\frac{B}{2}+A=\Delta_{21,0}^e\cos2\theta_1+\frac{B_0}{2} ,$$
$$B^2+c_1{}^2D^2=B_0{}^2+C_1{}^2D_0{}^2 ,$$
$$F^2+s_1{}^2D^2=F_0{}^2+S_1{}^2D_0{}^2 , \quad (19)$$
$$\arctan\frac{c_1 D}{B}+2\arctan\frac{s_1 D}{F}=\arctan\frac{C_1 D_0}{B_0} + 2\arctan\frac{S_1 D_0}{F_0} .$$

The right-hand sides of the four equations above are all constants calculable from the vacuum values θ_1, etc. Thus they constitute a set of equations from which ϕ_1, ϕ_2, Δ_{21}, and Δ_{32} can be solved for given values of A. The values of ϕ_1 and ϕ_2 solved *numerically* from Eq. (19) are then substituted into the following expression for the probability for an emerging electron neutrino to maintain its identity:

$$P_a(\nu_e \to \nu_e) = \sum_i |U_{0,ie}|^2 |U_{ie}|^2 = C_1{}^2 c_1{}^2 + S_1{}^2 s_1{}^2 (C_2{}^2 c_2{}^2 + S_2{}^2 s_2{}^2) \quad (20)$$

which is valid when the source and the detector are far apart so that the oscillatory terms are averaged to zero. The neutrino capture rate is then given by

$$\int \sigma(E)\Phi(E)P_a(\nu_e \to \nu_e)dE \ .$$

Here $\sigma(E)$ and $\Phi(E)$ are the detection cross section of the detector and the solar-neutrino flux, respectively. In this section, to avoid undue complication, we will restrict ourselves to the case when the transition is adiabatic. This is indicated explicitly by the subscript a of P in Eq. (20).

The adiabatic condition can be formulated by considering the evolution of the neutrino mass eigenstates. From Eq. (1), we have

$$i\frac{d\psi_m}{dt} = i\frac{dU}{dt}\psi + iU\frac{d\psi}{dt} \ . \quad (21)$$

The equation of motion for ψ gives

$$i\frac{d\psi}{dt} = \frac{1}{2E}M\psi = \frac{1}{2E}MU^T\psi_m \ . \quad (22)$$

Substitution into Eq. (21) gives

$$i\frac{d\psi_m}{dt} = \left[\frac{1}{2E}M_D + i\frac{dU}{dt}U^T\right]\psi_m \ . \quad (23)$$

The orthogonality of U means that $(dU/dt)U^T$ is an antisymmetric matrix. Explicit evaluation yields

$$i\frac{d}{dt}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} = \begin{bmatrix}\mu_1{}^2/2E & -i(c_2\dot{\phi}_1 + s_1 s_2\dot{\phi}_3) & -i(s_2\dot{\phi}_1 - s_1 c_2\dot{\phi}_3)\\ & \mu_2{}^2/2E & -i(\dot{\phi}_2 + c_1\dot{\phi}_3)\\ & & \mu_3{}^2/2E\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} , \quad (24)$$

where $\dot{\phi}_i = d\phi_i/dt$, and the off-diagonal part of the square matrix is antisymmetric. For the problem at hand, the condition for the resonance transition[6,7] to be adiabatic can be stated as $|Q_{21}| \gg 1$ at resonance,[14] where Q_{21} is defined to be

$$Q_{21} = \frac{\Delta_{21}}{4E}(c_2\dot{\phi}_1 + s_1 s_2\dot{\phi}_3)^{-1} \ . \quad (25)$$

The constancy of $\alpha - \phi_3$ implies that $\alpha' = \phi_3 - \theta_3$ and hence $\dot{\phi}_3 = \dot{\alpha}'$. From Eqs. (17) and (18), it is straightforward to express $\dot{\alpha}'$ and $\dot{\phi}_1$ in terms of $\dot{A}$. Putting everything together, we get

$$Q_{21} = \frac{\Delta_{21}}{8c_1 s_1{}^2}\frac{(\cot\alpha + 2\cot\beta)F - s_1 D}{c_1 c_2(\cot\alpha + 2\cot\beta) + s_2}\frac{1}{E}\left|\frac{dA}{dr}\right|^{-1} . \quad (26)$$

Here we have replaced dA/dt by dA/dr, r being the distance from the center of the Sun, since the neutrinos are essentially traveling at the speed of light. In principle the above equation allows one to determine the value of Q_{21} at resonance in a general situation.

III. TWO-GENERATION APPROXIMATION

In order to check the validity of the formulas derived in the previous section, we demonstrate that they reduce to the by-now well-known two-generation formulas in the appropriate limit.[3–8]

When $\theta_2 = 0$, Eq. (19) can be easily solved. From Eq. (8) we see that $\theta_2 = 0$ means that $D_0 = 0$. Since D does not contain ϕ_1 or Δ_{21} explicitly, a solution to Eq. (19) can be obtained by inspection to be

$$\Delta_{21}^e\cos2\phi_1 + A = \Delta_{21,0}^e\cos2\theta_1, \quad B = B_0 \ , \quad F = F_0, \quad D = D_0 \equiv 0 \ . \quad (27)$$

$D = 0$ means $\phi_2 = 0$, which in turn means that $\Delta_{21}^e = \Delta_{21}$. The fact that B, F, and D stay constant implies $\alpha = \alpha_0$, i.e., $\alpha' \equiv 0$. Thus $\phi_3 \equiv \theta_3$. All these amount to

$$\phi_2 \equiv \theta_2 = 0, \quad \phi_3 \equiv \theta_3 \ , \quad (28a)$$

$$\Delta_{21}\sin2\phi_1 = \Delta_{21,0}\sin2\theta_1 \ , \quad (28b)$$

$$\Delta_{21}\cos2\phi_1 = \Delta_{21,0}\cos2\theta_1 - A \ . \quad (28c)$$

From Eqs. (28b) and (28c) one gets

$$\tan2\phi_1 = \frac{\tan2\theta_1}{1 - (A/A_R)} \ , \quad (29)$$

where A_R is the value of A at resonance,[11] defined to be the point at which Δ_{21} attains its minimum value $\Delta_{21,R}$:

$$A_R = \Delta_{21,0}\cos2\theta_1, \quad \Delta_{21,R} = \Delta_{21,0}\sin2\theta_1 \ , \quad (30)$$

which are the expressions obtained by Bethe.[3] Note that in order to recover the two-generation formulas, it is necessary that θ_2 equals 0 but θ_3 can be arbitrary. (This is true in the Kobayashi-Maskawa[12] parametrization.) One can see the reason for that by noting that given Eq. (28a), U can be written in the form

$$\begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_3 & S_3 \\ 0 & S_3 & -C_3 \end{bmatrix} .$$

Recalling that $M^D = UMU^T$, we see that what happens here is that M is always transformed first to the block-diagonal form

$$\begin{bmatrix} M_{11}+A & M'_{12} & 0 \\ M'_{21} & M'_{22} & 0 \\ 0 & 0 & m_3{}^2 \end{bmatrix}$$

by a rotation through an angle θ_3 about the ν_e axis. What remains is then just a two-dimensional problem.

When $\theta_2 = \phi_2 = 0$, Eq. (20) reduces to its two-generation counterpart:

$$P_a(\nu_e \to \nu_e) = C_1{}^2 c_1{}^2 + S_1{}^2 s_1{}^2 . \tag{31}$$

As noted in the previous section, the above formula is valid only when the transition is adiabatic. By putting $s_2 = D = 0$ in Eq. (26) and making use of Eq. (28b), the expression for Q_{21} simplifies to

$$Q_{21} = \frac{(\Delta_{21,0}\sin 2\theta_1)^2}{\sin^3 2\phi_1} \frac{1}{2E} \left[\frac{dA}{dt} \right]^{-1} .$$

The adiabatic parameter Q is the resonance value of $|Q_{21}|$. It is obtained, by putting $\phi_1 = \pi/4$ in the above, as

$$Q = \frac{(\Delta_{21,0}\sin 2\theta_1)^2}{2E} \left| \frac{dA}{dt} \right|_R^{-1} \tag{32}$$

which is the well-known expression for the adiabatic parameter in the two-generation case.[3–8]

To make the two-generation discussion complete, we discuss the probability $P(\nu_e \to \nu_e)$ for an electron to emerge unscathed in a more general case when Q is not necessarily large. With the assumption that the change in electron density in the transition region is linear, $P(\nu_e \to \nu_e)$ is given by[5,7,8]

$$\begin{aligned} P(\nu_e \to \nu_e) &= (1 - e^{-(\pi/2)Q}) P_a(\nu_e \to \nu_e) \\ &\quad + e^{-(\pi/2)Q}[1 - P_a(\nu_e \to \nu_e)] \\ &= P_a(\nu_e \to \nu_e) \\ &\quad + e^{-(\pi/2)Q}[1 - 2P_a(\nu_e \to \nu_e)] . \end{aligned} \tag{33}$$

When Q is much greater than 1, which is the adiabatic case, $P(\nu_e \to \nu_e)$ is equal to $P_a(\nu_e \to \nu_e)$. In the other extreme when the transition is abrupt, i.e., $Q \ll 1$, we have

$$\begin{aligned} P(\nu_e \to \nu_e) &= 1 - P_a(\nu_e \to \nu_e) \\ &= C_1{}^2 s_1{}^2 + S_1{}^2 c_1{}^2 \end{aligned} \tag{34}$$

which is essentially the second solution described by Rosen and Gelb.[4]

We emphasize here that an expression similar to $P(\nu_e \to \nu_e)$ in Eq. (33) for the three-generation case is presently not available.

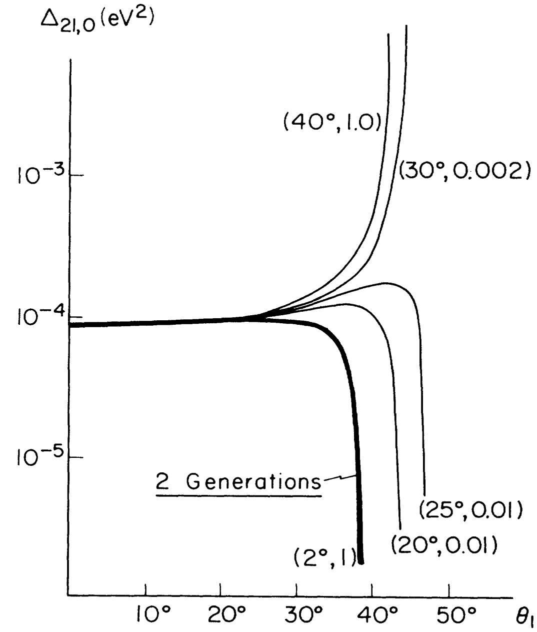

FIG. 1. Calculated 2.1-SNU contour plots for the ^{37}Cl detector on the θ_1-$\Delta_{21,0}$ plane, with different values of $(\theta_2, \Delta_{32,0})$ shown beside the curves. Values of $\Delta_{32,0}$ are in units of eV^2. The adiabatic condition is assumed.

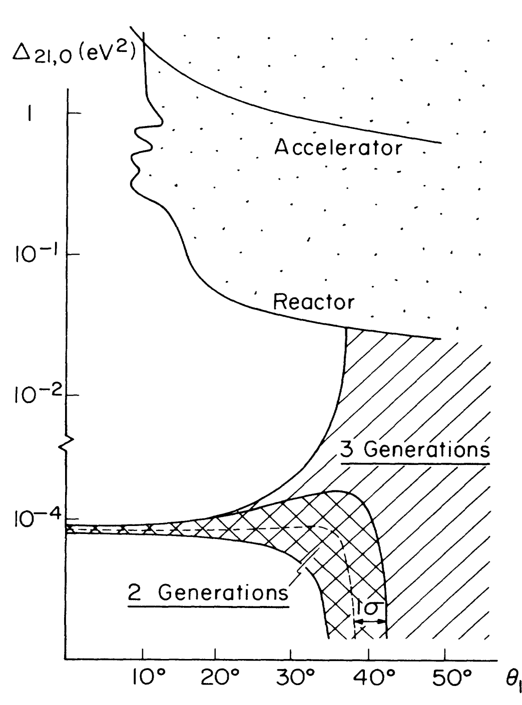

FIG. 2. Allowed regions on the θ_1-$\Delta_{21,0}$ plane consistent with the experimental ^{37}Cl data at 68.3% confidence level, for the three-generation (shaded) and the two-generation (cross-hatched) cases. The adiabatic condition is assumed. Also shown are excluded region (dotted) from some of the more recent terrestrial neutrino-oscillation experiments.

Equations (29) and (31) are the two-generation counterparts of Eqs. (19) and (20). Together with Eqs. (32) and (33) they form the main equations for the two-generation case which enable one to calculate the neutrino capture rate on the Earth. They are, in a sense, exact when $\theta_2=0$. One would expect that they are good approximations for small θ_2 values also.

IV. RESULTS

By following the procedure outlined in Sec. II, values for the expected neutrino capture rate can be obtained for given values of θ_1, θ_2, $\Delta_{21,0}$ and $\Delta_{32,0}$. The results are as follows. Figure 1 shows the 2.1-SNU contours on the θ_1-$\Delta_{21,0}$ plane for the ^{37}Cl detector, with different values of θ_2 and $\Delta_{32,0}$. The solar model in Ref. 15 is employed. Only contributions from the dominant ^{8}B and ^{7}Be reactions are included. It can be seen that the calculated neutrino capture rate is practically independent of θ_2 and $\Delta_{32,0}$ and there is no difference between the two- and three-generation cases if θ_1 is no larger than $\sim 20°$. However, the part of the contours beyond $\theta_1=20°$ shows a rapid up-shifting (from the two-generation curve) behavior as θ_2 gets larger, i.e., when there is substantial mixing of the third neutrino with the first two. In essence what would have been line contours in the two-generation treatment (dark solid curve) smears into broad bands towards larger θ_1 values, when θ_2 and $\Delta_{32,0}$ are taken into account. Beyond $\sim 30°$ the two-generation results merely serve as the lower bounds for the allowed region in the three-generation case.

Presently the experimental data on the ^{37}Cl detector capture rate is 2.1 ± 0.3 SNU. Assuming the adiabatic condition, the region on the θ_1-$\Delta_{21,0}$ plane compatible with the experimental data within one standard deviation, i.e., at a 68.3% confidence level, is shown in Fig. 2. It can be seen that even at this confidence level the allowed region in the three-generation case is markedly more extensive than that predicted by two-generation calculations, which is shown on the same graph together with upper bounds from previous accelerator and reactor neutrino oscillation experiments.[16]

Summing up, the observed solar-neutrino depletion can be accounted for by either the long-proposed maximal (large) mixing among the three species of neutrinos or the more recently discovered MSW mechanism, or any interplay between them. Three-generation calculations for the nonadiabatic case are in progress and will be reported elsewhere.[17]

ACKNOWLEDGMENTS

The authors would like to thank J. Kim, L. Madansky, and S. Nussinov for discussions. This work was supported in part by the National Science Foundation.

[1] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); **20**, 2634 (1979).

[2] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Nuovo Cimento **9C**, 17 (1986); in *'86 Massive Neutrinos Astrophysics and Particle Physics,* edited by O. Fackler and J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, 1986), p. 355.

[3] H. A. Bethe, Phys. Rev. Lett. **56**, 1305 (1986).

[4] S. P. Rosen and J. M. Gelb, Phys. Rev. D **34**, 969 (1986).

[5] W. C. Haxton, Phys. Rev. Lett. **57**, 1271 (1986).

[6] V. Barger, R. J. N. Phillips, and K. Whisnant, Phys. Rev. D **34**, 980 (1986).

[7] S. J. Parke, Phys. Rev. Lett. **57**, 1275 (1986); S. J. Parke and T. P. Walker, *ibid.* **57**, 2322 (1986).

[8] A. Dar, A. Mann, Y. Melina, and D. Zajfman, Report No. TECHNION-PH-86-30, 1986 (unpublished).

[9] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity,* proceedings of the Stony Brook Workshop, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 1979,* edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979).

[10] A. Bottino, C. W. Kim, H. Nishiura, and W. K. Sze, Phys. Rev. D **34**, 862 (1986).

[11] L. Wolfenstein, Phys. Rev. D **18**, 958 (1978).

[12] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[13] A straightforward diagonalization of M to obtain μ_i^2 has been carried out by V. Barger, K. Whisnant, S. Pakvas, and R. J. N. Phillips, Phys. Rev. D **22**, 2718 (1980).

[14] Here we are only concerned with the first resonance, at which ν_e and ν_μ cross.

[15] J. N. Bahcall *et al.*, Rev. Mod. Phys. **54**, 767 (1982).

[16] For a review see, e.g., V. Faminio and B. Saitta, Report No. INFN-PI/AE-85/86, 1985 (unpublished).

[17] C. W. Kim, S. Nussinov, and W. K. Sze, Phys. Lett. (to be published).