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Cartan's contortion as a pair of massless spin-2 fields

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Cartan's contortion is either considered as a nonpropagating field hardly having any physical significance or as a field determined by exotic very massive particles called tordions. In this paper an essentially different alternative is proposed: the contortion is determined, among other extraordinary vector fields, by a pair of standard massless spin-2 fields (V and W fields). It is conjectured that the extraordinary fields are vanishing. The forms of the interactions and self-interactions of the V and W fields are also briefly discussed.

I. INTRODUCTION

The Einstein-Cartan theory of gravitation assumes that space-time is described by a U_4 manifold.[1] At any point of this manifold there exists an independent metric tensor field $g_{ij}=g_{ji}$, and the connection is given by

$$\tilde{\Gamma}^i{}_{jk}=\Gamma^i{}_{jk}+\frac{f}{2}K^i{}_{jk}, \quad f=\sqrt{32\pi G} \ , \tag{1.1}$$

where $\Gamma^i{}_{jk}$ are the Christoffel symbols defined by g_{ij} in the usual manner and $K^{ijk}=-K^{jik}$ are the components of contortion tensor with respect to a coordinate basis.[2] G is the gravitational constant having the dimension (length)^2, because I use the natural system $\hbar=c=1$. Einstein's Lagrangian takes the form[3]

$$L=-\frac{2g^{ik}}{f^2}(\tilde{\Gamma}^j{}_{i[j,k]}+\tilde{\Gamma}^j{}_{mk}\tilde{\Gamma}^m{}_{ij}-\tilde{\Gamma}^m{}_{ik}\tilde{\Gamma}^j{}_{mj}) \ . \tag{1.2}$$

Today it is widely accepted that the relevant gauge fields of the Poincaré group are described by Einstein-Cartan theory. In this paper I *a priori* accept this point of view.[4] In fact the puzzle arises not from the gauge behavior of contortion but from its physical significance. There exist two points of view: contortion is either not propagated or is determined by very massive exotic particles called tordions.[5]

The purpose of this paper is to propose a third, essentially different physical significance of contortion.

The paper is organized as follows. In Sec. II the Hodge theorem is discussed. This mathematical part is necessary for further considerations. In Sec. III the decomposition of contortion is presented, and the so-called V and W fields are introduced and studied. In Sec. IV some remaining open questions are reviewed. Section V summarizes the main ideas of the paper.

II. GENERALIZATION OF THE HODGE THEOREM

Let $F^{ij}=-F^{ji}$ be the components of a tensor. Then the Hodge decomposition theorem defines the relation

$$\begin{aligned}&F^{ij}=V^{[i;j]}+\mu^{ijkm}W_{k;m}+G^{ij} \ ,\\&G^{ij}=-G^{ji}, \quad G^{ij}{}_{;j}=0 \ , \quad (\mu^{ijkm}G_{km})_{;j}=0 \ ,\end{aligned} \tag{2.1}$$

where μ^{ijkm} is the totally antisymmetric tensor.[6]

One requires that this decomposition be held also for a U_4 manifold. This generalization may be done easily; in addition to it, without loss of generality G^{ij} may identically be vanishing. In order to show this one may proceed as follows.

Let (2.1) with $G^{ij}=0$ be considered as a system of six first-order differential equations; F^{ij} are assumed to be known; V^i and W^i are unknown functions of coordinates. One has to show that V^i and W^i are determined by F^{ij}. Let $V^0=W^0=0$ be chosen. Then (2.1) is a typical Cauchy problem for six unknown functions. Let $\phi(x)=0$ be a Cauchy surface [for example, $\phi(x)=x^0-\text{const}=0$], and let $V^\alpha\,|_\phi, W^\alpha\,|_\phi$ ($\alpha=1,2,3$) be given (i.e., V^α and W^α are known on $\phi=0$). Then (2.1) is solvable and V^α, W^α are determined unambiguously on the whole manifold. Oppositely, any two vector fields V^i and W^i define unambiguously the components F^{ij} in accordance with (2.1), where $G^{ij}=0$.

We note the following.

(1) Without loss of generality the initial values $V^\alpha\,|_\phi$ and $W^\alpha\,|_\phi$ may be vanishing.

(2) One may substitute the conditions $V^0=W^0=0$ also with other ones (for example, $V^i{}_{;i}=W^i{}_{;i}=0$ are convenient), because there is a freedom in the choice of V^i and W^i:

$$\bar{V}^i=V^i+A^{;i}+B^i, \quad \bar{W}^i=W^i+C^{;i}+D^i \ , \tag{2.2}$$

$$\begin{aligned}&A^{;[ij]}=M^{[i;j]}+\mu^{ijkm}N_{k;m} \ ,\\&C^{;[ij]}=R^{[i;j]}+\mu^{ijkm}Q_{k;m} \ ,\end{aligned} \tag{2.3}$$

$$M^i+Q^i+B^i=0, \quad N^i+R^i+D^i=0 \ . \tag{2.4}$$

(3) Another procedure may also lead to vanishing G^{ij}. If (2.1) holds, then the harmonic two-form G_{ij} satisfies the relations

$$\begin{aligned}&[\mu_{iprs}(\mu^{ijkm}G_{km})_{;j}]^{;s}=0 \ ,\\&2G_{pr;j}{}^{;j}+2G_{j[p;r]}{}^{;j}+(\mu_{iprs}\mu^{ijkm}{}_{;j}G_{km})^{;s}=0 \ .\end{aligned} \tag{2.5}$$

Only the term $G_{pr,j}{}^{,j}=0$ contains the second derivative of G_{pr}. Let $\phi(x)=0$ be a Cauchy surface, and $G_{ij}\,|_\phi$ and $G_{ij,k}\,|_\phi$ be given. Then G_{ij} are given on the whole manifold. If the initial values are vanishing, $G_{ij}=0$. Because these vanishing initial values may always be chosen, in what follows I shall use the truncated Hodge theorem with $G^{ij}=0$.

Given an antisymmetric tensor $F^{ijk}=-F^{jik}$. Then the decomposition ("generalization of the Hodge theorem")

$$F^{ijk}=V^{k[i;j]}+\mu^{ijpr}W^{k}{}_{p;r} \tag{2.6}$$

may also be done. In order to show this one may proceed identically to the case (2.1). [Let (2.6) be considered as a system of 24 first-order differential equations, etc.] Nevertheless, this consideration does not prove the uniqueness of this decomposition. In other words, it is not certain that (2.6) is the only possible decomposition. In addition, it is not certain that (2.6) must always be done. In fact only the following result is obvious: (2.6) may always be done. For the purpose of this paper this is fully enough.

III. V AND W FIELDS

First, I will consider a highly special case of contortion when the following restrictions hold.

(a) The Riemannian part of the curvature is zero; the metric tensor satisfies the conditions $g_{ij}=g^{ij}=\eta_{ij}=\eta^{ij}$, and the Christoffel symbols are vanishing ("manifold with Minkowskian metric tensor").[7]

(b) fK_{ijk} are infinitesimally small on the whole flat background.

Applying (2.6) one obtains

$$K^{ijk}=V^{k[i;j]}+\mu^{ijmn}W^{k}{}_{m;n}\ , \tag{3.1}$$

where V^{ij} and W^{ij} are necessarily infinitesimal here. In detail,

$$V_{ki;j}=V_{ki,j}-\frac{f}{2}K^{m}{}_{kj}V_{mi}-\frac{f}{2}K^{m}{}_{ij}V_{km}\ , \tag{3.2}$$

and the analogous relations for $W_{ki;j}$ are obvious. Substituting K^{ijk} from (3.1) into (3.2) one obtains the products of form $V\cdot V$, $V\cdot W$. These products are negligible here. Thus here covariant derivatives may simply be substituted by partial derivatives.

In the general case V^{ij} and W^{ij} are not symmetrical.

(c) Let us assume here the fulfillment of a third restriction also:

$$V^{ij}=V^{ji},\quad W^{ij}=W^{ji}\ . \tag{3.3}$$

Note that these relations, and the relations

$$\mu^{ijkm}K_{jkm}=0,\quad K^{ij}{}_{j}=0 \tag{3.4}$$

are in our special case equivalent.[8]

The Lagrangian (1.2) takes the form

$$\begin{aligned}L_{(0)}&=-\tfrac{1}{2}(K^{ijk}K_{jki}+K^{ij}{}_{j}K_{i}{}^{k}{}_{k})\\&=\tfrac{1}{2}(V^{ij,k}V_{ij,k}+2V^{,i}V^{k}_{i,k}-2V^{ij,k}V_{ik,j}-V^{,i}V_{,i})+\tfrac{1}{2}(W^{ij,k}W_{ij,k}+2W^{,i}V^{k}_{i,k}-2W^{ij,k}W_{ik,j}-W^{,i}W_{,i})\\&=L_{(0)}(V)+L_{(0)}(W)\ ,\end{aligned} \tag{3.5}$$

where I introduced the term ($V^{i}_{i}=V, W^{i}_{i}=W$)

$$L_{(0)}(V)=\tfrac{1}{2}[V^{ij,k}V_{ij,k}+2(V^{,i}V^{k}_{i,k}-V^{ij,k}V_{ik,j})-V^{,i}V_{,i}]\ , \tag{3.6}$$

and simply omitted some four-divergences. The Lagrangian (3.6) is identical to the standard one of the free massless spin-2 field.[9] Thus (3.5) defines a pair of massless spin-2 free fields.

Any standard massless spin-2 free field should change under gauge transformations as

$$\overline{V}^{ij}=V^{ij}+a^{(i,j)},\quad \overline{W}^{ij}=W^{ij}+b^{(i,j)}\ , \tag{3.7}$$

where a^{i} and b^{i} are infinitesimal components of arbitrary four-vectors.[10] Therefore, the contortion should change under the gauge transformations as

$$\begin{aligned}\overline{K}^{ijk}&=K^{ijk}+a^{[i,j]k}+\mu^{ijmn}b_{m,n}{}^{k}\\&=K^{ijk}+p^{ij,k}\ ,\end{aligned} \tag{3.8}$$

where $p^{ij}=-p^{ji}$ are infinitesimal components of an arbitrary tensor [a^{i} and b^{i} define p^{ij} via (2.1)]. As is well known, under the local infinitesimal Lorentz rotation of vierbein basis the components of contortion change in accordance with (3.8) (see the Appendix). Thus in the special case when restrictions (a), (b), and (c) hold, contortion may be given by two standard massless spin-2 free fields ("V field," "W field") by the gauge fields of the Lorentz group.

Second, I will consider a more general case of contortion when the following restrictions hold.

(a) U_4 is again a manifold with Minkowskian metric tensor. Applying (2.1) one expects that (3.1) again holds. Nevertheless, here the formula (3.1) is in fact an infinite series of form

$$\sum_{n=0}^{\infty}f^{n}(\cdots)=K^{ijk}\ . \tag{3.9}$$

In order to show this it is enough to substitute K^{ijk} from (3.1) into (3.2). Then new terms of the form $f^{2}K^{ijk}$ arise; substituting (3.1) into it new terms of form $f^{3}K^{ijk}$ arise, etc. The same procedure is to be done also for $W_{ij;k}$.

The emerging V^{ij} and W^{ij} in the general case are not symmetric.

(b) Therefore, let us assume that also a second restriction holds:

$$V^{ij}=V^{ji},\quad W^{ij}=W^{ji}\ . \tag{3.10}$$

Note that these restrictions, as it seems, in the general case, are not identical to (3.4).

Here the Lagrangian (1.2) takes the form

$$L=\sum_{n=0}^{\infty} f^n L_{(n)} , \tag{3.11}$$

where $L_{(0)}$ is given by (3.5). Terms $L_{(n)}$, $n \geq 1$, have obvious physical significance. They define the self-interactions and interactions of the V and W fields.

In order to illustrate this self-interaction consider the case when $W^{ij}=0$ and $L_{(n)}$, $n \geq 2$, are negligible. Omitting the four-divergences one obtains, after a long but straightforward calculation,

$$\begin{aligned} L &= \tfrac{1}{2} V^{ik;j} V_{k[i;j]} - \tfrac{1}{2} V_j^{[i;j]} V^m_{[i;m]} \\ &= L_{(0)}(V) + f L_{(1)}(V) \\ &= L_{(0)}(V) + \frac{f}{2} V^{i[j,k]} (V_{j[m,k]} V_i^m + V_{j[m,i]} V_k^m) \\ &\quad + \frac{f}{2} V_j^{[i,j]} (V_m^{[p,m]} V_{ip} + V^{mp} V_{m[p,i]}) . \end{aligned} \tag{3.12}$$

The self-interaction of the W field may be described analogously. In the general case $L_{(1)}$ contains, aside from the two terms $L_{(1)}(V)$ and $L_{(1)}(W)$, the term $L_{(1)}(V,W)$. This term describes the coupling of the V field to the W field at the lowest order of f. [In order to write down $L_{(1)}(V,W)$ one has to use (3.2) and the similar relation for $W_{ij;k}$, and to separate the terms $f \cdot V \cdot V \cdot W$ and $f \cdot V \cdot W \cdot W$. This is a trivial but long procedure.]

Because of the dimensional character of f, neither the self-interaction of the V field (W field) nor the interaction of the V field with the W field is renormalizable. The self-interactions are highly similar to the self-interaction of the graviton.[11]

Third, I will consider the most general case. Let the potential describing the graviton be $U^{ij}=U^{ji}$, i.e., the metric tensor be given by the relations[12]

$$\begin{aligned} g^{ij} &= \eta^{ij} + f U^{ij} , \\ g_{ij} &= \eta_{ij} - f U_{ij} + f^2(\cdots) + \cdots . \end{aligned} \tag{3.13}$$

Applying (2.6) one obtains again (3.1). Denoting

$$\begin{aligned} & v^{ij} = \tfrac{1}{2} V^{(ij)}, \quad w^{ij} = \tfrac{1}{2} W^{(ij)}, \quad Q^{ij} = V^{[ij]} , \\ & R^{ij} = W^{[ij]}, \quad Q^{ij} = M^{[i;j]} + \mu^{ijmn} N_{m;n} , \\ & R^{ij} = P^{[i;j]} + \mu^{ijmn} S_{m;n} ; \end{aligned} \tag{3.14}$$

it is obvious that in the most general case Lagrangian (1.2) again takes the form (3.11), and defines seven interacting and self-interacting fields: three standard massless spin-2 fields (graviton, V field, W field; the relevant potentials are U^{ij}, v^{ij}, w^{ij}) and four extraordinary vector fields (relevant potentials are M^i, N^i, P^i, and S^i). These four vector fields hardly can be interpreted as standard spin-1 fields. This follows from the fact that $L_{(0)}$ here contains the second derivatives of M^i, N^i, P^i, and S^i; the relevant field equations are fourth-order differential equations.

I think that any discussion of the properties of these four extraordinary vector fields is not needed, because they hardly can be fields with physical significance (see the next section). Therefore, in this section I shall assume the fulfillment of

$$M^i = N^i = P^i = S^i = 0 . \tag{3.15}$$

In order to describe the interactions of our three massless spin-2 particles one obviously has to begin with Lagrangian (1.2) and use (1.1), (3.1), (3.2) (of course, also the similar relation for $W_{ij;k}$), (3.13), and (3.15). I would like to discuss the interaction of graviton and contortion particles separately. I would like to remark only that the term

$$-\frac{g^{ik}}{f} K^j{}_{i[j,k]} \tag{3.16}$$

gives no interaction between Einstein's gravity and contortion.[13] Obviously, the coupling of gravity to the contortion particles is again a badly nonrenormalizable interaction.

In order to illustrate the coupling of contortion to matter fields consider the following case. Let Einstein's gravity be vanishing, and let a Proca field (standard massive spin-1 field; $m \neq 0$) be given which is interacting with contortion. In accordance with the general procedure, "introduce the contortion into covariant derivative, if it is possible."[14] Then one obtains the Lagrangian

$$\begin{aligned} & L(P,V,W) = -\tfrac{1}{2} X^{i;j} X_{i;j} + \frac{m^2}{2} X^i X_i + L = L_P + L , \\ & X^i{}_{,i} = 0 , \end{aligned} \tag{3.17}$$

where X^i is the vector potential, and L is given by (1.2). The interaction of contortion with the Proca field is determined by

$$\begin{aligned} L_P &= \frac{m^2}{2} X^i X_i - \tfrac{1}{2} X^{i,j} X_{i,j} + \frac{f}{2} X^{i,j} X^m K_{mij} \\ &\quad - \frac{f^2}{8} X^m X_r K_{mij} K^{rij} . \end{aligned} \tag{3.18}$$

The last two terms determine the interaction; it is again badly nonrenormalizable. A more detailed study of coupling of contortion to matter will again be given separately.[15]

IV. PROBLEMS OF INTERPRETATION

In the previous section a new interpretation of contortion was proposed. Of course, I do not allege that the presented physical significance is correct in any sense. Here I would like to survey the remaining open questions.

(1) Decomposition (3.1) is based on the key relation (2.6). This relation need not hold in any case. Nevertheless, it is highly reasonable that applying this "natural" decomposition contortion may be defined by standard fields.[16]

(2) The convergency of Lagrangian (3.11) is in the general case an open question. Nevertheless, Einstein's Lagrangian itself has the form (3.11); its convergency is also an open question. Thus the situation is not worse than in quantum gravity.

(3) Introduction of constant f in (1.1) leads to nonrenormalizable interactions and self-interactions. Nevertheless, the situation is again not worse than in quantum gravity. [Constant f is necessary from dimensional reasoning; $L_{(0)}$

cannot contain constants with the dimension of length. The Lagrangian describing a standard free field is given by the potentials only; compare (3.5).]

(4) If the contortion is defined by the pair of massless spin-2 fields, is the proposed decomposition in any sense unique? Indeed, it seems to be "natural." (For example, it can be done in any coordinate or vierbein system; self-interaction is highly similar to the case of graviton, etc.) Nevertheless, on the other hand, there is no exact proof of uniqueness.

(5) What about the extraordinary vector fields? In my opinion they do not exist; in (3.1) V^{ij} and W^{ij} are always symmetric. Some theoretical arguments can be given in order to support this conjecture. (a) Any known free fields are determined by the Klein-Gordon equation. (b) How do we define the dynamical invariants for these vector fields? (c) Contortion may well be the gauge field of the Lorentz group also for symmetrical V^{ij} and W^{ij}.

(6) If one accepts the nonexistence of these vector fields, the contortion must have some additional constraints. [In the special case of infinitesimal K^{ijk}, when the metric tensor is given by Minkowskian tensor, we know them; see (3.4). But what about the general case?]

V. CONCLUSION

Today it is not clear whether gravitation is described by the Einstein-Cartan theory. This question was not studied here. I *a priori* assumed that the contortion—the gauge field of the Lorentz group—was nonvanishing. Not the gauge behavior, but the physical significance of contortion was investigated.

The main result of this paper is that contortion may be determined by a massless spin-2 field twin. These particles seem to be highly similar to the graviton. If this alternative physical significance is accepted, contortion never has strange and exotic properties.

In my opinion there are three possible areas for further study.

(i) One should try to clarify the open problems listed in the previous section.

(ii) How can we detect the V field and W field empirically? Obviously, this question has an essential importance.[17]

(iii) One should attempt to study the impact of the existence of contortion on supergravity. If contortion does exist in nature, then there may exist three different spin-2 particles; the present supergravity could drastically be changed.

I hope that this will encourage others to continue in these studies.

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APPENDIX: LOCAL LORENTZ ROTATIONS

This appendix contains no new results. It is added for the reader's convenience only.

Let e_a^i be the components of the ath vierbein vector. Components of connection with respect to this vierbein basis are given by

$$\tilde{\Gamma}_{abc} = -\gamma_{abc} + \frac{f}{2} K_{abc} , \tag{A1}$$

where

$$\gamma_{abc} = \tfrac{1}{2}(\lambda_{abc} - \lambda_{bac} - \lambda_{cab}) , \tag{A2}$$

$$\lambda_{abc} = e_{a[i,j]} e_b^i e_c^j \tag{A3}$$

are well-known quantities. Let a new vierbein basis be introduced by local Lorentz rotations

$$\bar{e}^{\,i}_{\ a} = \Lambda^b{}_a e_b^i, \quad \eta_{ab} = \Lambda^c{}_a \Lambda^d{}_b \eta_{cd} . \tag{A4}$$

In this basis the connection is given by

$$\begin{aligned}\bar{\tilde{\Gamma}}_{abc} &= -\bar{\gamma}_{abc} + \frac{f}{2} \bar{K}_{abc} \\ &= -\Lambda_{da,g} \Lambda^d{}_b \Lambda^g{}_c \\ &\quad + \left[-\gamma_{dgh} + \frac{f}{2} K_{dgh} \right] \Lambda^d{}_a \Lambda^g{}_b \Lambda^h{}_c .\end{aligned} \tag{A5}$$

Especially if $g^{ij} = \eta^{ij}$ (i.e. $e_a^i = \delta_a^i$ may be chosen) and K_{abc} are infinitesimal, then after the infinitesimal local Lorentz rotations

$$\Lambda^a{}_b = \delta_b^a + p^a{}_b, \quad p^{ab} = -p^{ba} , \tag{A6}$$

where p^{ab} are infinitesimal, one obtains

$$\bar{\tilde{\Gamma}}_{abc} = \frac{f}{2} K_{abc} + p_{ab,c} . \tag{A7}$$

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[1]For a review, see, e.g., F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, Rev. Mod. Phys. **48**, 393 (1976). Some new points of view may be found in W. Szczyrba, Ann. Phys. (N.Y.) **158**, 320 (1984); D. Rapoport and S. Sternberg, *ibid.* **158**, 447 (1984); W. Drechsler, Fortschr. Phys. **32**, 449 (1984); P. Baekler and E. M. Mielke, Phys. Lett. **A113**, 471 (1986); W. M. Baker, Class. Quant. Gravit. 3, L19 (1986).

[2]In this paper indices $i,j,k,\ldots$ denote holonomic coordinates; $a,b,\ldots,h$ denote vierbein indices. Any index takes the values 0,1,2,3, where 0 is reserved for the time coordinate.

[3]In this paper parentheses (square brackets) denote symmetrization (antisymmetrization) without the factor $\frac{1}{2}$; a semicolon denotes a covariant derivative with respect to (1.1); a comma denotes a partial derivative.

[4]A review of gauge treatment may be found in D. Ivanenko and G. Sardanashvily, Phys. Rep. **94**, 1 (1983); see, also, Ref. 1. I think that the relevant gauge fields of the Poincaré group are doubtlessly described by the Einstein-Cartan theory. In addi-

tion to it, I also think that the Einsteinian part itself may be considered as the gauge field of the translation group [A. Mészáros, Acta Phys. Hung. **59**, 379 (1986)] and the contortion itself as the gauge field of the Lorentz group [A. Mészáros, Ann. Phys. (Leipzig) (to be published)].

[5]F. W. Hehl, Y. Ne'eman, J. Nitsch, and P. von der Heyde, in *Einstein Commemorative Volume,* edited by A. Held (Plenum, New York, 1980); see, also, Ref. 1. Tordions are discussed, e.g., in M. Fukui and J. Masukawa, Prog. Theor. Phys. **73**, 75 (1985).

[6]F. W. Warner, *Foundations of Differentiable Manifolds and Lie Groups* (Scott Foresman, Glenview-London, 1971); T. Eguchi, P. B. Gilkey, and A. J. Hanson, Phys. Rep. **66**, 213 (1980). In Warner's book the Hodge theorem is proven for compact, positive-definite Riemannian manifolds only.

[7]A great inconsistency exists concerning this special case. The special case, when the metric tensor is identical to the Minkowskian tensor $\eta^{ij}=\eta_{ij}=\mathrm{diag}(1,-1,-1,-1)$ but the contortion is nonvanishing, is either called Weitzenböck space [W. Drechsler, Ann. Inst. Henri Poincaré **37**, 155 (1982)] or teleparallelism space [D. Ivanenko and G. Sardanashvily, Phys. Rep. **94**, 1 (1983)]. On the other hand, in W. Kopczynski, J. Phys. A **15**, 493 (1982), "the teleparallelism space" is the special manifold, when the whole curvature is zero. For the sake of preciseness, I will call the special case of the manifold with $g^{ij}=\eta^{ij}$ and $K^{ijk}\neq 0$ as "manifold with Minkowskian metric tensor."

[8]Equivalence of (3.3) and (3.4) can be shown as follows. For $V^{[ij]}$ use (2.1); $V^{[ij]}=M^{[i,j]}+\mu^{ijkm}N_{k,m}$. Because $M^{[k,i]j}-M^{[k,j]i}=-M^{(k,i)j}+M^{(k,j)i}$ holds, this vector can be added to the symmetric part of V^{ij}. The same trick has to be done for W^{ij} also. The remaining two vector fields are vanishing for (3.4).

[9]W. Pauli and M. Fierz, Proc. R. Soc. London **A173**, 594 (1939).

[10]W. E. Thirring, Ann. Phys. (N.Y.) **16**, 96 (1961).

[11]S. N. Gupta, Proc. Phys. Soc. London **A65**, 161 (1952).

[12]S. N. Gupta, Proc. Phys. Soc. London **A65**, 608 (1952); M. J. Duff, in *Quantum Gravity,* proceedings of the Oxford Symposium, edited by C. J. Isham, R. Penrose, and D. W. Sciama (Clarendon, Oxford, 1975).

[13]A. Mészáros, Astrophys. Space Sci. (to be published).

[14]Interaction of contortion with other standard fields is surveyed in the paper of Hehl, von der Heyde, Kerlick, and Nester (Ref. 1).

[15]A. Mészáros, Astrophys. Space Sci. **125**, 405 (1986). This paper studies also the macroscopic behavior of contortion. It is also to be noted that the probably detected fifth force [E. Fischbach, D. Sudarsky, A. Szafer, C. Talmadge, and S. H. Aronson, Phys. Rev. Lett. **56**, 3 (1986)] may be mediated by the spin-2 field twin of contortion.

[16]Several papers are discussing the properties of antisymmetric tensors of rank 2 and higher, and of spin-2 fields; see, e.g., S. Weinberg, Phys. Rev. **138**, B988 (1965); L. P. S. Singh and C. R. Hagen, Phys. Rev. D **9**, 898 (1974); D. E. Nevill, *ibid.* **18**, 3535 (1978); C. Fronsdal, *ibid.* **18**, 3624 (1978); T. L. Curtright and G. O. Freund, Nucl. Phys. **B172**, 413 (1980); N. A. Batakis, Phys. Lett. **128B**, 165 (1983); A. S. Schwarz and Yu. S. Tyupkin, Nucl. Phys. **B242**, 436 (1984); A. Mészáros, Astrophys. Space Sci. **111**, 399 (1985). But, as far as is known, the Hodge theorem has never been applied in the Einstein-Cartan theory, and contortion has never been interpreted by standard massless spin-2 fields.

[17]If contortion is determined by massless spin-2 particles, the detection of contortion should be more hopeful than in the case of nonpropagating contortion. A detection of contortion, when it is not propagated, is practically excluded. The introduction of tordions hardly leads to a better chance of detection; see, e.g., W. R. Stoeger, Gen. Relativ. Gravit. **17**, 981 (1985). On the other hand, it is even possible that the contortion—mediated by spin-2 particles—was already detected; see Ref. 15.