Radiation of Goldstone bosons from cosmic strings

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It is shown that the interaction of global strings with Goldstone bosons can be described by a model of strings coupled to an antisymmetric tensor field with a particular choice of the coupling constant. This formalism is applied to calculate the rate of Goldstone-boson radiation from a class of closed-loop trajectories. The lifetime of a typical loop is found to be very short, about twenty oscillation periods, in agreement with an earlier estimate by Davis.

Cosmic strings could arise as a random network of linelike defects at a phase transition in the early Universe. Strings formed as a result of gauge- or global-symmetry breaking are called gauge or global strings, respectively. Both types of strings have condensed-matter analogues. Quantized tubes of magnetic flux in superconductors and vortex lines in liquid helium are closely analogous to gauge and global strings, respectively. The cosmological evolution of strings has been studied extensively, especially in relation to the string scenario of galaxy formation. In this scenario galaxies and clusters of galaxies condense around oscillating loops of string, while the loops gradually radiate away their energy. For a recent review of strings see Refs. 1 and 2. The main mechanism of energy loss for gauge string loops is the gravitational radiation, while for global strings it is the radiation of Goldstone bosons. The radiation rate, which determines the lifetime of the loops, is of crucial importance for cosmological scenarios. For the gravitational radiation it was calculated in Refs. 3–5. In this paper we shall develop a theory describing the interaction of global strings with a Goldstone field and apply this theory to a calculation of the radiation rate from various loop configurations. Our results are in agreement with an order-of-magnitude estimate by Davis.[6]

The prototypical model that gives rise to global strings is the Goldstone model of a self-interacting scalar field ϕ:

$$L=\partial_\mu\phi^\dagger\partial^\mu\phi-\tfrac{1}{2}\lambda(\phi^\dagger\phi-\eta^2)^2 \ . \tag{1}$$

The U(1) symmetry of this model is spontaneously broken, and ϕ acquires a vacuum expectation value with $|\phi|=\eta$. Strings are described by solutions of the classical field equations in which the phase of ϕ changes by 2π around a string. The magnitude of ϕ is substantially different from η only in the string core of radius

$$\delta\sim\lambda^{-1/2}\eta^{-1} \ . \tag{2}$$

The energy per unit length of a straight string is logarithmically divergent:[7]

$$\mu=\int |\nabla\phi|^2 2\pi r\, dr\approx 2\pi\eta^2\ln(R/\delta) \ , \tag{3}$$

where R is the cutoff radius. For a closed loop, the cutoff is given by the loop size.

Outside the string core, ϕ can be represented as $\phi\approx\eta\exp[i\theta(x)]$; the effective Lagrangian for θ,

$$L=\eta^2\partial_\mu\theta\partial^\mu\theta \ , \tag{4}$$

describes a massless Goldstone field. Variations of $|\phi|$ correspond to a scalar particle of mass $m=\lambda^{1/2}\eta$. In general, massive and massless scalar fields and the strings are all described by the same complex field ϕ with the Lagrangian (1). However, in the low-energy limit, when the curvature radius of the string is much greater than δ, the massive excitations are not produced. Then the Goldstone field is coupled to strings only through the requirement that θ changes by 2π around a string. Now it will be shown that this very unusual interaction is equivalent to the interaction of strings with an antisymmetric tensor field:[8,9] $A_{\mu\nu}=-A_{\nu\mu}$. The corresponding action is

$$S=\tfrac{1}{6}\int F_{\mu\nu\sigma}F^{\mu\nu\sigma}d^4x+g\int A_{\mu\nu}d\sigma^{\mu\nu}-\mu_0\int d\sigma \ , \tag{5}$$

where

$$F_{\mu\nu\sigma}=\partial_\mu A_{\nu\sigma}+\partial_\nu A_{\sigma\mu}+\partial_\sigma A_{\mu\nu} \ , \tag{6}$$

$$d\sigma^{\mu\nu}=(\dot x^{\mu}x'^{\nu}-\dot x^{\nu}x'^{\mu})d\zeta\, d\tau \ , \tag{7}$$

is the surface element on the string world sheet $x^\mu(\zeta,\tau)$; overdots and primes stand for derivatives with respect to τ and ζ, respectively, and

$$d\sigma=(-\tfrac{1}{2}d\sigma_{\mu\nu}d\sigma^{\mu\nu})^{1/2} \ .$$

The last term in Eq. (5) is the Nambu action for a string with a bare mass per unit length μ_0. The logarithmic contribution (3) is due to the interaction. The field equations for the tensor field are obtained by varying (5) with respect to $A_{\mu\nu}$:

$$\partial_\sigma F^{\mu\nu\sigma}=g\int \delta^{(4)}[x-x(\zeta,\tau)]d\sigma^{\mu\nu}\equiv 4\pi j^{\mu\nu} \ . \tag{8}$$

To establish the relation between the tensor field and the Goldstone field θ, we define[10]

$$F_\mu=\tfrac{1}{6}\epsilon_{\mu\nu\sigma\tau}F^{\nu\sigma\tau}, \quad F^{\mu\nu\sigma}=\epsilon^{\mu\nu\sigma\tau}F_\tau \ . \tag{9}$$

Using Eq. (8) it is easily seen that, outside the strings,

$$\partial_\mu F_\nu - \partial_\nu F_\mu = 0 \ . \tag{10}$$

Hence, F_μ is a gradient of a scalar,[8,9]

$$F_\mu = \eta \partial_\mu \theta \ , \tag{11}$$

where the coefficient η is chosen so that the energy-momentum tensor of the field $F_{\mu\nu\sigma}$,

$$T_\mu^\nu = F_{\mu\alpha\beta} F^{\nu\alpha\beta} - \tfrac{1}{6}\delta_\mu^\nu F_{\alpha\beta\gamma} F^{\alpha\beta\gamma} = 2\eta^2(\partial_\mu\theta\partial^\nu\theta - \tfrac{1}{2}\delta_\mu^\nu \partial_\alpha\theta\partial^\alpha\theta) \ ,$$

is the same as the energy-momentum tensor one would obtain from Eq. (4). The coupling constant g is determined by the requirement that θ changes by 2π around the string. Using the Stokes theorem, we have

$$\oint_C dx^\mu \partial_\mu \theta = \eta^{-1} \int_\Sigma d\Sigma^{\mu\nu} \partial_\mu F_\nu = -\pi\eta^{-1} \int_\Sigma d\Sigma^{\mu\nu} \epsilon_{\mu\nu\sigma\tau} j^{\sigma\tau} \ , \tag{12}$$

where the surface Σ is bounded by the curve C. For any curve enclosing the string, the surface Σ can be chosen so that it crosses the string world sheet in the perpendicular direction. Then it is easily shown that the last integral in (12) is equal to g/π, and thus

$$g = 2\pi\eta \ . \tag{13}$$

The action (5) is invariant under arbitrary reparametrization of the world sheet and under gauge transformations

$$A_{\mu\nu} \to A_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \ . \tag{14}$$

A convenient choice of gauge is[9]

$$\partial_\nu A^{\mu\nu} = 0, \quad \dot{x}\cdot x' = 0, \quad \dot{x}^2 + x'^2 = 0, \quad \tau = t \ . \tag{15}$$

With this choice, the string trajectory can be written as $\mathbf{x}(\zeta,t)$ and the field equations and string equations of motion take the form

$$\partial_\sigma \partial^\sigma A^{\mu\nu} = 4\pi j^{\mu\nu} \ , \tag{16}$$

$$j^{\mu\nu}(\mathbf{x},t) = \tfrac{1}{2}\eta \int d\zeta\, \delta^{(3)}[\mathbf{x} - \mathbf{x}(\zeta,t)](\dot{x}^\mu x'^\nu - \dot{x}^\nu x'^\mu) \ , \tag{17}$$

$$\mu_0(\ddot{x}_\mu - x''_\mu) = 4\pi\eta F_{\mu\nu\sigma} \dot{x}^\nu x'^\sigma \ . \tag{18}$$

In the last equation $F_{\mu\nu\sigma}$ includes the external field, as well as the field produced by the string element itself. The effect of the latter is, in particular, to renormalize the bare mass density μ_0.

We shall calculate the radiation rate from oscillating loops assuming that the back reaction of the radiation on the loop is small and using noninteracting loop trajectories for $j^{\mu\nu}$ in Eq. (17). This assumption is justified by our final results. Neglecting interaction, it can be shown[11] that a loop of mass M oscillates with a period $T = L/2$, where $L = M/\mu$ is called the invariant length of the loop (in fact, the period can be smaller than $L/2$ for some special loop trajectories). The total power P and the angular distribution of radiation from a periodic source can be found from the following equations:

$$P = \sum_n P_n = \sum_n \int \frac{dP_n}{d\Omega} d\Omega \ , \tag{19}$$

$$dP_n/d\Omega = 2\omega_n^2 j^*_{\mu\nu}(\mathbf{k},\omega_n) j^{\mu\nu}(\mathbf{k},\omega_n) \ , \tag{20}$$

$$j^{\mu\nu}(\mathbf{k},\omega_n) = \frac{2}{L} \int_0^{L/2} dt\, e^{i\omega_n t} \int d^3x e^{-i\mathbf{k}\cdot\mathbf{x}} j^{\mu\nu}(\mathbf{x},t) \ . \tag{21}$$

Here, $dP_n/d\Omega$ is the power radiated per unit solid angle at frequency $\omega_n = 4\pi n/L$ in the direction of $\mathbf{k}$; $|\mathbf{k}| = \omega_n$.

We have calculated the radiation power for several loops from a simple family of trajectories found by Burden:[5]

$$\mathbf{x}(\zeta,t) = \tfrac{1}{2}[\mathbf{a}(\zeta - t) + \mathbf{b}(\zeta + t)] \ , \tag{22}$$

$$\mathbf{a}(\zeta) = \alpha^{-1}(\hat{\mathbf{e}}_1 \sin\alpha\zeta + \hat{\mathbf{e}}_3 \cos\alpha\zeta) \ , \tag{23}$$

$$\mathbf{b}(\zeta) = \beta^{-1}[(\hat{\mathbf{e}}_1\cos\psi + \hat{\mathbf{e}}_2\sin\psi)\sin\beta\zeta + \hat{\mathbf{e}}_3\cos\beta\zeta] \ . \tag{24}$$

Here, $\alpha = 2\pi N_1/L$, $\beta = 2\pi N_2/L$, N_1, and N_2 are relatively prime integers, $\hat{\mathbf{e}}_i$ is a unit vector along x^i axis, and the period of the loops is $T = L/2N_1N_2$. We used the same combined analytic and numerical technique as in Refs. 4 and 5. The result for the total power is

$$P = \kappa\eta^2 \ , \tag{25}$$

where κ is a numerical coefficient which depends on the loop's trajectory, but not on its size.[12] The values of κ for several loops of the family (22)—(24) are given in Fig. 1. Note that the power vanishes for $N_1 = N_2 = 1$, $\psi = \pi$. A loop with such values of parameters is simply a rotating double line. The currents $j^{\mu\nu}$ are equal and opposite for the two coinciding lines, and so there is no radiation. Apart from this very special case, κ is typically ~ 50.

The large values of κ are due partly to the large-n contributions to Eq. (19). For "normal" sources of radiation, P_n decreases exponentially in the large-n limit, while for the loops an asymptotic analysis shows that (for $N_1 = N_2 = 1$)

$$P_n = \text{const} \times n^{-4/3} \ . \tag{26}$$

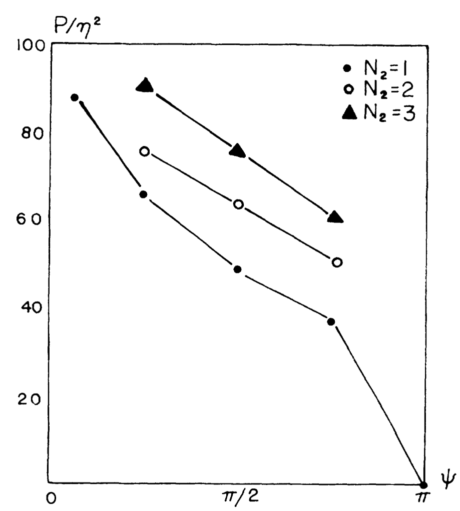

FIG. 1. Radiation power for the family of loops (22)—(24) with $N_1 = 1, N_2 = 1,2,3$ for several values of ψ.

The difference can be attributed to the singular behavior of the string near the points of luminal motion.[13] Large amounts of radiation are beamed from those points in the direction of luminal velocity. Analysis similar to that in Ref. 4 shows that near the beam

$$dP/d\Omega = \sum_n dP_n/d\Omega \propto \chi^{-1} , \tag{27}$$

where χ is the angle between the beam and the wave vector $\mathbf{k}$. The radiation intensity diverges as $\chi \to 0$. However, the divergence is integrable and the total power is finite, except in a few degenerate cases, such as the circular loop ($N_1 = N_2 = 1, \psi = 0$) (Ref. 14).

The lifetime of a loop is, using Eq. (3),

$$\tau \sim M/P \sim 2\pi\kappa^{-1} L \ln(L/\delta) = KL , \tag{28}$$

where $K \sim 10$. The fraction of the total energy radiated in one period is $(2K)^{-1} \sim 0.05 \ll 1$. The smallness of this number justifies the use of noninteracting loop trajectories in our calculation of P.

Radiation emitted by individual loops adds up to a stochastic Goldstone-boson background. Loops decaying at time $\sim t$ during the radiation era have length $L \sim t/K$ and number density[1] $n_L \sim (Lt)^{-3/2}$. They produce waves of a typical frequency $\sim L^{-1}$ and energy density $\rho_b \sim \mu L n_L \sim K^{1/2} \mu t^{-2}$. Noticing that ρ_b red-shifts in the same way as the electromagnetic radiation density, ρ_γ, we obtain

$$\Omega_b(\omega) \sim 30 K^{1/2} G\mu \Omega_\gamma \sim 100 G\mu \Omega_\gamma , \tag{29}$$

where G is Newton's constant, $\Omega_b(\omega) = (\omega/\rho_c) d\rho_b/d\omega$ is the energy density in units of the critical density ρ_c per logarithmic frequency interval, $\Omega_\gamma = \rho_\gamma/\rho_c = 2\times 10^{-5} h^{-2}$, and we assume that the total density of the Universe is $\rho = \rho_c$. Equation (29) applies for waves of frequency greater than $\omega_{\min} \sim 10^{-11}$ sec^{-1}. The upper bound depends on the mass density of strings,[1,4] μ. The string scenario of galaxy formation requires[1,15] $G\mu \sim 10^{-6}$; then $\omega_{\max} \sim 10^5$ sec^{-1}.

The most stringent constraint on the value of μ comes from the isotropy of the cosmic microwave background:[16] $G\mu \lesssim 10^{-5}$. Using Eq. (29) it can be shown that global strings with such values of $G\mu$ are consistent with nucleosynthesis. (The nucleosynthesis constraint for gauge strings has been discussed in Refs. 17 and 18.)

The main difference in the evolution of gauge and global strings is the different lifetime of closed loops. For gauge strings $\tau \sim L/\gamma G\mu$, where $\gamma \sim 100$. With $G\mu \sim 10^{-6}$ this gives $\tau \sim 10^4 L$. The lifetime of global string loops is much shorter, $\tau \sim 10L$. In the string scenario of galaxy formation with a baryon- or neutrino-dominated universe, density fluctuations on scales down to $\sim 10^{12} M_\odot$ can be preserved due to the long lifetime of gauge string loops. Baryons and neutrinos erase their own fluctuations by photon viscosity and free streaming, respectively, but then fall in the potential wells of surviving loops.[19,1] With global strings, however, this mechanism is not sufficient to preserve galactic scale density fluctuations. In a universe dominated by cold dark matter, the mass accreted by loop seeds is practically the same for gauge and global strings, but the mass distribution and the shapes of the resulting objects may be different. For example, we do not expect the formation of tightly bound nuclei in clusters of galaxies seeded by global string loops.

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