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New bound on the anomalous magnetic moment of the W boson

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The effect of an anomalous magnetic moment $\Delta\kappa$ of the W boson on the photon structure is calculated. The result depends quadratically on the cutoff Λ. Comparison with data from the DESY storage ring PETRA gives a limit $|\Delta\kappa(\Lambda/M_W)| \lesssim 33$. This bound is compared with the constraint from the anomalous magnetic moment of the muon. Also corrections to the ρ parameter are discussed.

Now that their existence is established, the next subject in vector-boson physics is the study of their self-interactions. Within the standard model these are completely determined by the gauge structure of the theory. Alternative models exist, however, in which the vector bosons are not fundamental particles, but are composite objects. If the vector bosons are really composite, their self-couplings will, in general, be different from the standard-model predictions. However, they are not completely arbitrary, since present experiments are sometimes sensitive to deviations from a gauge structure and can therefore constrain the magnitude of the coupling constants.

The particular coupling to be studied in this Rapid Communication is the magnetic moment μ_W of the charged W boson. For a gauge theory one has $\mu_W = e/M_W$. A deviation from this relation is described by adding to the standard model an interaction

$$\mathcal{L}_{\text{int}} = ie\Delta\kappa F^{\mu\nu} W_\mu^+ W_\nu^- \ , \qquad (1)$$

where $F_{\mu\nu}$ is the electromagnetic field tensor. The possibility of $\Delta\kappa \neq 0$ has been studied before in Refs. 1–4, where the correction to the $(g-2)$ factor of the muon was calculated. Using the experimental constraints the following bound was found:

$$|\Delta\kappa \ln(\Lambda/M_W)| \lesssim 2.2 \ . \qquad (2)$$

In this expression Λ is a cutoff that is needed to regularize a divergent loop integral. It presumably corresponds to the energy scale where the structure of the W boson becomes manifest.

However, the magnetic moment of the muon is not the only quantity in present-day physics that is sensitive to the W-boson magnetic moment. Also the photon propagator is affected by the interaction Lagrangian (1) through the intermediate-vector-boson loop of Fig. 1. Since this graph is naively quartically divergent one cannot use dimensional regularization as in Refs. 2 and 3, since dimensional regularization automatically puts quadratic and higher divergencies equal to zero. It is therefore necessary to introduce a structure in the vector-boson propagator. A simple way to do this is to write the W-boson propagator as

$$W_\mu^+(k)W_\nu^-(-k) = \frac{\delta_{\mu\nu}}{f(k^2)k^2+M_W^2} + \frac{k_\mu k_\nu}{k^2}\left[\frac{1}{g(k^2)k^2+M_W^2} - \frac{1}{f(k^2)k^2+M_W^2}\right] , \qquad (3)$$

where $f(k^2)$ and $g(k^2)$ are structure functions that depend on the cutoff Λ. For a pointlike W boson one has $f(k^2)=1$ and $g(k^2)=0$. Deviations of these values are assumed to be of order M_W^2/Λ^2. Keeping only quadratically and more divergent terms we find the following contribution to the photon two-point function:

$$A_\mu(k)A_\nu(-k) = \frac{e^2(\Delta\kappa)^2}{4(2\pi)^4 i} k^2(k^2\delta_{\mu\nu} - k_\mu k_\nu) \int \frac{d^4p}{p^2(g(p^2)p^2+M_W^2)^2} + (k^2\delta_{\mu\nu} - k_\mu k_\nu) O(\Lambda^2) + O(\ln(\Lambda)) \ . \qquad (4)$$

We notice that the naively present quartic divergence disappears upon integration and contraction with the vertices. The second part of this correction (quadratic in the momentum) is just a wave-function renormalization of the photon and therefore has no experimental consequences. The first part, however, modifies the form of the photon propagator and will lead to a deviation from quantum-electrodynamics predictions in experiments. For the simplest case of $g(p^2) = M_W^2/\Lambda^2$ it is given by

$$A_\mu(k)A_\nu(-k) = \frac{e^2(\Delta\kappa)^2\Lambda^2}{64\pi^2 M_W^4} k^2(k^2\delta_{\mu\nu} - k_\mu k_\nu) \ . \qquad (5)$$

With the contribution (5) the photon propagator

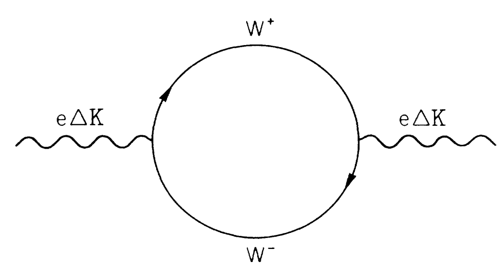

FIG. 1. Contribution to photon structure due to the magnetic moment of the W boson.

$W_{\mu\nu}(k)$ becomes in the Feynman gauge

$$W_{\mu\nu}(k)=\frac{1}{k^2}\left[\frac{\Lambda_{\rm expt}{}^2\delta_{\mu\nu}-k_\mu k_\nu}{\Lambda_{\rm expt}{}^2-k^2}\right] , \tag{6}$$

where

$$\Lambda_{\rm expt}=\frac{8\pi M_W{}^2}{e\Delta\kappa\Lambda} . \tag{7}$$

Because of current conservation the $k_\mu k_\nu$ piece of the propagator does not contribute to cross sections and we are left with an overall form factor $\Lambda_{\rm expt}{}^2/(\Lambda_{\rm expt}{}^2-k^2)$. This form factor can be determined by a precise measurement of the cross sections for $e^+e^-\to l^+l^-$ or $\bar{q}q$, with l any lepton and q any quark. In the experiments at DESY PETRA these measurements have been done[5-7] and a bound $\Lambda_{\rm expt}\gtrsim 200$ GeV has been found. Combining this limit with (7) one has

$$|\Delta\kappa(\Lambda/M_W)|\lesssim 33 . \tag{8}$$

Implicit in the derivation of the bound (8) is the assumption that it is sensible to introduce an intrinsic structure to the W boson, but not to the photon. This assumption is justified in composite models where the W boson, being massive, is composite but the photon is still a fundamental gauge particle. Without this assumption there is no reason to exclude a "bare" term in the Lagrangian, due to the compositeness of the photon, that can partially or completely cancel the contribution of formula (4). In that case the bound (8) is not rigorous and one can only say that large deviations of (8) are unlikely, because they imply an unnatural cancellation of terms. Similar assumptions also have to be made in order that formula (2) be rigorously valid.

With the previous caveat, formula (8) provides a useful limit on $\Delta\kappa$ as a function of the compositeness scale Λ. For large cutoff scales $\Delta\kappa\to 0$ and essentially no deviation of a gauge structure is allowed. This is in keeping with the so-called Veltman theorem,[8] which states that the effective low-energy theory of composite states with a mass much smaller than their binding scale has to be renormalizable, if they are to be described by perturbation theory. Because of the stronger cutoff dependence the new bound (8) becomes stronger than the bound from $(g-2)_\mu$ for a scale $\Lambda\gtrsim 5$ TeV (see Table I). Finally, one could argue from the fact that no substructure of the W boson has been seen so far that $\Lambda\gtrsim 100$ GeV, giving a rather useless bound of $|\Delta\kappa|\lesssim 26$. This argument is fallacious, however, because formula (4) shows that the cutoff dependence appears only in the longitudinal structure $g(k^2)$, for which present experiments are insensitive.

TABLE I. Limits on $|\Delta\kappa|$ from different experimental quantities.

Λ (TeV)	$(g-2)_\mu$	γ structure
1	0.87	2.6
3	0.61	0.88
5	0.53	0.53
10	0.46	0.26
20	0.40	0.13

Besides the photon structure and the $(g-2)_\mu$ factor one can consider other quantities that depend on the anomalous magnetic moment of the W boson. One of these is the ρ parameter $\rho=M_W{}^2/\cos^2\theta_w M_Z{}^2$. Suzuki[4] finds a quartically divergent correction to ρ, resulting in a much stricter limit than either (2) or (8). However, this bound is unreliable because the ρ parameter is sensitive to other deviations from a gauge structure in the couplings of the vector bosons. In particular, adding, in addition to (1), a contribution of the form

$$\mathcal{L}_{\rm int}=ig\Delta\kappa\{\cos(\theta_w)(\partial_\mu Z_\nu-\partial_\nu Z_\mu)W_\mu^+W_\nu^- + 1/\cos(\theta_w)[(\partial_\mu W_\nu^+-\partial_\nu W_\mu^+)W_\mu^-Z_\nu+\text{c.c.}]\} , \tag{9}$$

where g is the weak coupling constant and θ_w the electroweak mixing angle, removes the quartic divergence in ρ. The combination (1) and (9) gives a contribution to the ρ parameter:

$$\delta\rho=-\frac{3g^2\tan^2(\theta_w)(\Delta\kappa)^2\Lambda^2}{64\pi^2M_W{}^2} . \tag{10}$$

Since $\rho=1$ within 3% one finds a limit:

$$|\Delta\kappa(\Lambda/M_W)|\lesssim 7 . \tag{11}$$

This bound is better than (8), but it cannot be trusted because also 4-W vertices contribute to ρ.

The reason for taking the combination (1) and (9) rather than just (1) is best discussed in the Stuckelberg formalism for massive gauge fields. In the Stuckelberg formalism one introduces besides the usual gauge fields of the weak interactions an SU(2)-valued field U. The SU(2) $\times$U(1) gauge-covariant derivative of U is given by

$$D_\mu U=\partial_\mu U+i\frac{g}{2}W_\mu\cdot\tau U+i\frac{g}{2}\tan(\theta_w)U\tau_3B_\mu , \tag{12}$$

with W_μ the SU(2) gauge field and B_μ the hypercharge field. We also define $V_\mu=(D_\mu U)U^\dagger$ and $T=U\tau_3U^\dagger$. In terms of these fields, the standard model without the Higgs field is given by

$$\mathcal{L}=\tfrac{1}{4}f^2\mathrm{Tr}(V_\mu V^\mu)-\tfrac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})-\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} . \tag{13}$$

$F_{\mu\nu}$ and $B_{\mu\nu}$ are the SU(2) and hypercharge field strengths; f is the Fermi scale 250 GeV. This Lagrangian is invariant for local $SU_L(2)$ and U(1) gauge transformations. In the unitary gauge $U=1$ it describes massive vector bosons. Without the hypercharge coupling there would be a global $SU_R(2)$ invariance $U\to UA$, with $A\in SU(2)$. In the standard model (13) this symmetry is broken minimally through the covariant derivative coupling of B_μ to U. As a result one has $\rho=1$; a deviation of $\rho=1$ is described by a term $\mathrm{Tr}(TV_\mu)^2$, which is not of the minimal type. The reason for taking (1) and (9) is that they correspond to a term of the form $\mathrm{Tr}F_{\mu\nu}[V^\mu,V^\nu]$, where all breaking of $SU_R(2)$ is of the minimal type. For a composite model of the weak interactions to make sense, one actually has to assume such a coupling in order to ensure the restoration of the $SU_R(2)$ symmetry in the limit $\theta_w\to 0$. Without this assumption there is no reason for ρ to be even close to one and an artificial fine-tuning of parameters is needed to achieve this. The introduction of the anomalous magnetic moment (1) without the term (9) violates this condition. It is therefore no surprise that Suzuki finds a quartic divergence by keeping only (1). His bound should, however, be interpreted as a limit on the allowed $SU_R(2)$ breaking of the composite model, rather than as a bound on the anomalous magnetic moment of the W boson.

Finally, I want to make some remarks on the 4-W vertices that might be present beyond the standard-model terms. The terms satisfying the condition of minimal coupling to hypercharge are given by

$$\mathcal{L}=\frac{\alpha}{g^2}(\mathrm{Tr}V_\mu V^\mu)^2+\frac{\beta}{g^2}(\mathrm{Tr}V_\mu V^\nu)^2 \ . \quad (14)$$

I have not performed a complete analysis in terms of structure functions of the W-boson propagators in this case, but just assumed that one can substitute

$$\int \frac{p_\mu p_\nu}{p^4}d^4p=\frac{\delta_{\mu\nu}}{4}\int\frac{d^4p}{p^2}=\frac{i\pi^2\Lambda^2}{4}\delta_{\mu\nu} \ . \quad (15)$$

This prescription is not entirely satisfactory of course; however, I checked that all relevant Ward identities due to the $SU(2)\times U(1)$ gauge invariance are satisfied for the leading divergences. With this prescription one finds the contribution to ρ:

$$\delta\rho=\frac{3g^2\tan^2(\theta_w)\Lambda^2}{64\pi^2 M_W{}^2}(2\alpha+5\beta) \ . \quad (16)$$

Combining with (10) this gives a limit:

$$\left|\frac{\Lambda^2}{M_W{}^2}(-\Delta\kappa^2+2\alpha+5\beta)\right|\lesssim 53 \ . \quad (17)$$

We see, therefore, that the ρ parameter gives no limit on $\Delta\kappa$ per se. However (17) is only a rough estimate; for instance, the cutoff of (10) does not have to be the same as the one in (16), because in (10) there are two virtual W's involved and in (16) only one. The complete analysis is, however, rather complicated and beyond the scope of this discussion.

Finally, I wish to mention that the experimental limits will be improved in the near future. The planned BNL $g-2$ experiment is expected to improve the bound (2) by a factor of order 20. Experiments at the Stanford Linear Collider and CERN will improve the bound (8) because of the higher value of k^2 probed.

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