Quark distribution amplitudes for the nucleon from perturbative QCD and QCD sum rules

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We discuss quark distribution amplitudes for the nucleon obtained from perturbative QCD in connection with QCD sum rules on the light cone. Because of nonperturbative contributions these amplitudes have a complex nonsmooth structure and are dominated by strong correlations at the edges of the phase space. The antisymmetric part of the mixed symmetry amplitudes is found to be enhanced.

The currently accepted theory describing the hadronic strong interactions is quantum chromodynamics (QCD). Considering exclusive processes of hadrons at large momentum transfer,[1] it is possible to separate the short-distance dynamics, governed by QCD perturbation theory, from the nonperturbative contributions responsible for quark and gluon confinement at large distances.[2] All binding effects involving low-momentum interactions are associated with wave functions for the hadrons. In the light-cone frame, which is used for convenience, these wave functions

$$\Phi(x_i,Q)=\int^{k_{\perp i}{}^2\leq Q^2}[d^2k_\perp]\Psi(x_i,\mathbf{k}_{\perp i})$$

are process-independent "distribution amplitudes" for finding the valence quarks with light-cone fractions of the hadron's momentum, $x_i=k_i^+/p^+=(k^0+k^3)_i/(p^0+p^3)$, integrated over transverse momenta $k_{\perp i}\leq Q$, where $p^\mu=(p^+,p^-,\mathbf{p}_\perp)$ is the momentum of the hadron. In the infinite-momentum frame $(p^3\rightarrow\infty)$ x_i denotes the longitudinal momentum of the ith quark. Since the amplitudes $\Phi(x_i,Q)$ are universal functions, they provide the link between different exclusive reactions. Therefore, their estimation is of fundamental importance.

A complete determination of the hadronic wave functions would clearly imply the solution of the nonperturbative bound-state problem in QCD. Since this is still beyond our capabilities, some effective treatment has to be applied. One way to reach the resonance region, where nonperturbative effects become important, is the method of QCD sum rules.[3] Recently, Chernyak and Zhitnitsky[4] (CZ) developed an approach which provides the possibility to determine nucleon wave functions incorporating nonperturbative effects via QCD sum rules. The underlying idea is quite simple and appealing: They reconstruct the wave functions from the first ten moments, which they attempt to fix using QCD sum rules. However, neither the structure of the wave functions they derive, nor the conclusions and predictions they draw from them, seem convincing to us. The reasons are mainly the following: In the extreme asymptotic region, i.e., for $Q^2\rightarrow\infty$, the quarks inside the nucleon are effectively free. Thus the wave function which describes the valence-quark distribution in this region is given by a positive smooth and equally weighted function centered at $x_1=x_2=x_3=\frac{1}{3}$: $\Phi_{\rm as}(x_i)=120x_1x_2x_3$. Actually this function is the solution of the evolution equation in the formal limit $Q^2\rightarrow\infty$ (Ref. 2). Since the dependence of the quark distribution amplitude on the renormalization point (some typical quark virtuality) is only weak,[2] the rate at which the "true" nucleon wave function $\Phi_{\rm true}(x_i,Q)$ approaches its asymptotic form depends strongly on its initial value: $\Phi_{\rm true}(x_i,Q_0)$. Thus, if $\Phi_{\rm true}(x_i,Q_0)$ deviates crucially from $\Phi_{\rm as}$, then the asymptotic form is reached only for tremendously large values of Q^2. Regarding $\Phi_{\rm as}$ as an essential ingredient of the "true" nucleon wave function, we argue that nonperturbative effects at finite Q^2 should modify the structure of the perturbative amplitude predominantly near the phase-space boundaries. Furthermore, we show that a quark distribution amplitude determined solely via its moments obtained from the two-point sum-rule approach of CZ is not unique. This problem of nonuniqueness can be traced back to several uncertainties entering the calculation of the higher moments.[4,5] While the lowest moments can be derived independently using, e.g., uncorrected vertex functions, this is not possible for the higher-order moments.[5,6] Here, more improved techniques are needed in order to check the validity of the sum rules of CZ (Ref. 7). This fact might suggest that one should relax the requirement that the sum rules for the higher moments should be treated as a main input in the determination of realistic wave functions. Indeed, performing a complete analysis to determine the nucleon distribution amplitude in terms of all twist-3 Appell polynomials,[2] we found that there is actually an infinite number of possible solutions which satisfy exactly the sum rules of Ref. 4, but differ dramatically in their shape, while the predicted absolute values for the electromagnetic (EM) form factors vary by a factor 2—3 (Table I). Therefore, the predictions derived from such wave functions, as these proposed by CZ, are quite arbitrary,[8] and in addition they are in contradiction with experiment which shows $|G_M^n/G_M^p|\lesssim\frac{1}{6}$ already at $Q^2\approx 20\ {\rm GeV}^2/c^2$.

In order to discriminate between different choices of possible wave functions, we rely on a recent analysis[9] of the EM form factors of the nucleon which is consistent with the latest high-Q^2 SLAC data[10] showing $F_1^n/F_1^p\sim 0$, $G_M^n\sim 0$ and look for that solution which yields the small-

TABLE I. Moments of the wave function $\Phi_{(-)}$ derived from (i) sum rules (SR's), (ii) model wave function of CZ (CZ), and (iii) this work in comparison with the asymptotic wave function Φ_{as}. The numbers in parentheses are the values we obtained using the CZ model wave function. The expansion coefficients B_n and the strength coefficients A_i are defined in the text. $\alpha_s=0.3$ and $f_N=5.2\times10^{-3}$ GeV^2 (Ref. 4).

$n_1n_2n_3$	$2\Phi_{(-)}^{(n_1n_2n_3)}$ (SR)	$2\Phi_{(-)}^{(n_1n_2n_3)}$ (CZ)		$2\Phi_{(-)}^{(n_1n_2n_3)}$ (This work)		$\Phi_{as}^{(n_1n_2n_3)}$
		$A_1=0$	Example 1	Example 2	Favorite	
		$A_2=-6.72$	$B_0=1$, $B_1=3.9$	$B_0=1$, $B_1=4.0$	$B_0=1$, $B_1=4.105$	$B_0=1$
			$B_2=1.9$, $B_3=1.3$	$B_2=2.0$, $B_3=1.95$	$B_2=2.06$, $B_3=-4.72$	$B_i=0$
			$B_4=9.0$, $B_5=1.8$	$B_4=-5.0, B_5=2.0$	$B_4=5.0$, $B_5=9.3$	$A_i\equiv0$
			$A_1=-13.7, A_2=11.8$	$A_1=-6.6, A_2=2.1$	$A_1=-47.3, A_2=56.2$	
000	1	1	1	1	1	1
100	0.60–0.75	0.63	0.61	0.62	0.63	$\frac{1}{3}=0.333$
010	0.09–0.16	0.15	0.15	0.14	0.14	$\frac{1}{3}=0.333$
001	0.18–0.24	0.22	0.238	0.238	0.236	$\frac{1}{3}=0.333$
200	0.25–0.40	0.4	0.35	0.38	0.29	$\frac{1}{7}=0.143$
020	0.03–0.08	0.025(0.024)	0.032	0.031	0.032	$\frac{1}{7}=0.143$
002	0.08–0.12	0.08	0.1	0.085	0.008	$\frac{1}{7}=0.143$
110	0.07–0.12	0.11	0.12	0.1	0.11	$\frac{2}{21}=0.095$
101	0.09–0.14	0.123	0.14	0.14	0.23	$\frac{2}{21}=0.095$
011	−0.03–0.03	0.027(0.017)	0.002	0.015	−0.003	$\frac{2}{21}=0.095$
$Q^4F_1^p$ (GeV^4)		1.17(0.89)	0.3	0.69	0.89	0
$Q^4F_1^n$ (GeV^4)		−0.57(−0.43)	−0.124	−0.335	−0.086	+0.013

est possible value of the ratio F_1^n/F_1^p. In correspondence with the sum rules of CZ for the lower-order moments, and regarding their sum rules for the higher moments merely as a guide, this additional criterion enables us to reduce considerably the variation of the expansion coefficients in the representation for the quark distribution amplitude.

Our starting point is the helicity-conserving color-singlet proton Fock state to leading twist 3 in the infinite-momentum frame:[2,4]

$$|p^\uparrow\rangle=\text{const}\times\int_0^1[dx]\{\tfrac{1}{2}V(x_i)[\,|u^\uparrow(x_1)u^\downarrow(x_2)d^\uparrow(x_3)\rangle+(1\leftrightarrow2)]-\tfrac{1}{2}A(x_i)[\,|u^\uparrow(x_1)u^\downarrow(x_2)d^\uparrow(x_3)\rangle+(1\leftrightarrow2)]$$
$$-T(x_i)\,|u^\uparrow(x_1)u^\uparrow(x_2)d^\downarrow(x_3)\rangle\} \tag{1}$$

[$(u\leftrightarrow d)$ for the neutron] with

$$[dx]=\delta\left[1-\sum_{i=1}^{3}x_i\right]\prod_{i=1}^{3}dx_i\ .$$

The distribution amplitudes $V(x_i)$, $A(x_i)$, and $T(x_i)$ are scalar wave functions in the light-cone frame controlling the valence-quark distributions in the nucleon at fixed scale Q^2. In the limit of strict collinear symmetry, combination of spin and flavor leads to the symmetry properties: $V(1,2,3)=V(2,1,3)$, $A(1,2,3)=-A(2,1,3)$, and $T(1,2,3)=T(2,1,3)$. The total isospin $\frac{1}{2}$ of the three-quark bound system requires $2T(1,2,3)=V(1,3,2)-A(1,3,2)+V(2,3,1)-A(2,3,1)$.

To determine the quark distribution amplitudes we make the ansatz

$$\Phi_{\text{nucleon}}(x_i)=\Phi_{\text{as}}(x_i)\Phi_{\text{nonpert}}(x_i) \tag{2}$$

at fixed scale $\mu\geq1$ GeV. In Ref. 2 it was shown that the general solution of the evolution equation which controls the dynamics of the nucleon distribution amplitude can be expressed as an expansion in terms of Appell polynomials (these constitute orthogonal polynomials on the triangle). Therefore, we look for a solution for $\Phi_{\text{nonpert}}(x_i)$ in the form

$$\Phi_{\text{nonpert}}(x_i)=\sum_n B_n\tilde{\Phi}_n(x_i)\ , \tag{3}$$

where $\tilde{\Phi}_n(x_i)$ are *all* Appell polynomials contributing to twist 3. Thus this is a complete representation of the nucleon distribution amplitude in leading twist. It takes into account terms of the type $x_i^2, x_i x_j$ $(i,j=1,2,3)$, and thus connects to the idea that the nontrivial structure of the QCD physical vacuum may induce correlations of the longitudinal momenta of the quarks inside the nucleon.

In terms of the expansion coefficients B_n the amplitudes are given by

$$V(x_i)=\Phi_{\rm as}(x_i)[(B_0+B_2-5B_3-5B_5)+\tfrac{1}{2}(B_1-3B_2+11B_3+B_4+21B_5)(x_1+x_2)-(B_1+B_4)x_3 + \tfrac{1}{6}(12B_3-4B_4-28B_5)(x_1^2+x_2^2)+\tfrac{1}{3}(24B_3+4B_4+14B_5)x_3^2-(4B_3+14B_5)x_1x_2] \,, \tag{4}$$

$$A(x_i)=\Phi_{\rm as}(x_i)[\tfrac{1}{2}(-B_1-3B_2+3B_3-B_4-7B_5)(x_1-x_2)+\tfrac{1}{6}(-12B_3+4B_4+28B_5)(x_1^2-x_2^2)] \equiv \Phi_{\rm as}(x_i)[A_1(x_1-x_2)+A_2(x_1^2-x_2^2)] \,, \tag{5}$$

$$T(x_i)=\Phi_{\rm as}(x_i)[(B_0+B_2-5B_3-5B_5)+(-3B_2+7B_3+7B_5)x_3+(8B_3+\tfrac{14}{3}B_5)(x_1^2+x_2^2)+(4B_3+14B_5)x_1x_2] \,. \tag{6}$$

Considering the computation of nucleon form factors it is more convenient to use the mixed symmetry wave functions

$$\Phi_{(\pm)}(x_i)=\tfrac{1}{2}[V(x_i)\pm A(x_i)] \tag{7}$$

with the property $\Phi_{(+)}(1,2,3)=\Phi_{(-)}(2,1,3)$.

Following now the procedure described above, we determine the coefficients B_n using the moments of the wave functions defined by

$$\Phi^{(n_1 n_2 n_3)}=\int_0^1 [dx]\, x_1^{\,n_1} x_2^{\,n_2} x_3^{\,n_3}\Phi(x_i) \,. \tag{8}$$

The results for the coefficients B_n, the wave-function moments, and the predictions for the EM form factors of the nucleon are presented in Table I.

The wave functions we propose (denoted "favorite" in Table I), have a complex "bump-dip" structure, which is illustrated in Figs. 1(b)—1(f). Comparison with the asymptotic wave function $\Phi_{\rm as}$ [Fig. 1(a)] reveals that at the edges of the phase space, correlation effects become significant. They are accentuated by nodes and are certainly to be traced to nonperturbative interactions of the quarks inside the nucleon with the vacuum fields. Note that because the antisymmetric amplitude A is enhanced [Fig. 1(d)] compared to the symmetric amplitude V [Fig. 1(b)], it strongly specifies the structure of the mixed symmetry amplitudes $\Phi_{(\pm)}$. Thus a parametrization of the form $\Phi_{\rm nonpert}(x_i)\sim(x_1x_2x_3)^\eta$ with η some constant[2] seems to be ruled out. This was first pointed out by CZ (Ref. 4).

In order to clarify the physical content of our wave functions, we show their pattern on the Mandelstam plane for the variables x_1,x_2,x_3 constrained by $x_1+x_2+x_3=1$ (Fig. 2). In the asymptotic limit $Q^2\to\infty$, when A becomes negligible and V becomes totally symmetric under particle exchange, $\Phi_{(\pm)}\to V\to\Phi_{\rm as}$ and $T\to\Phi_{\rm as}$, as required by perturbative QCD (Refs. 1 and 2). It is worth noting that the maximum of the symmetric amplitude V coincides exactly with that of $\Phi_{\rm as}$ at $x_1=x_2=x_3=\frac{1}{3}$ [Fig. 2(a)].

We emphasize that a small ratio F_1^n/F_1^p corresponds to an enhancement of the antisymmetric amplitude A (Table I). Since A plays a crucial role in the determination of the mixed symmetry amplitudes $\Phi_{(\pm)}$ the strength coefficients A_1 and A_2, defined in Eq. (5), are listed in Table I.

It is clear from what has been said above that more precise experiments on electron-neutron scattering at $Q^2>10$ GeV$^2/c^2$ are desirable. Especially the separation of electric and magnetic form factors of the neutron would be of

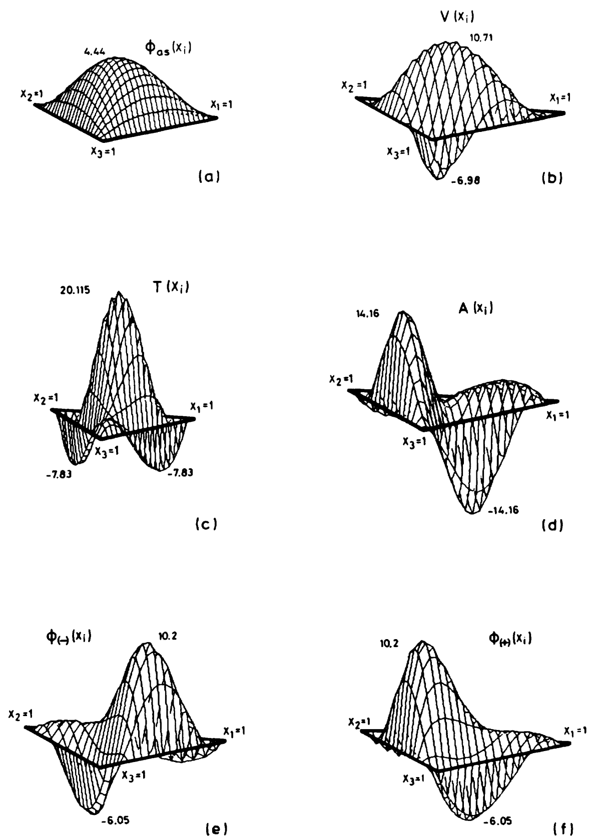

FIG. 1. Illustration of the quark distribution amplitudes proposed in this work in comparison with the asymptotic form. As a scale the maximal and minimal values of the functions are displayed.

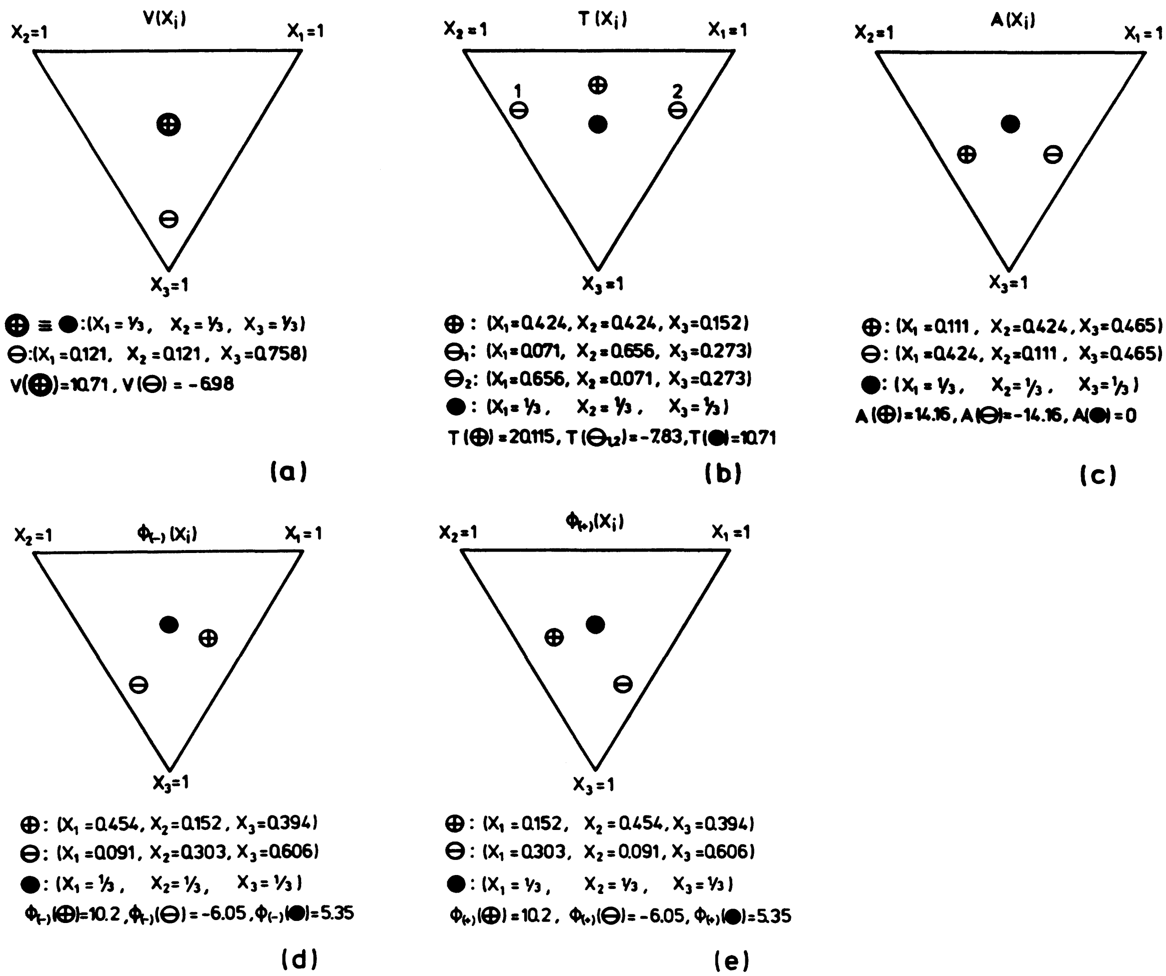

FIG. 2. Pattern of our quark distribution amplitudes on the Mandelstam plane (triangle) for the variables x_1, x_2, x_3 with $x_1+x_2+x_3=1$. The main maxima (⊕) and minima (○) are indicated. The black dot marks the central point ($x_1=x_2=x_3=\frac{1}{3}$) of the triangle where Φ_{as} has its maximal value.

great value.[11] In addition, further improvement of the sum-rule techniques could help to narrow down the uncertainties in the higher-order moments of the wave functions.

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[7]This however lies outside the scope of the present work.

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