# Helicity amplitudes of the process $J/\psi \to \gamma\theta$ in the glueball picture of $\theta(1700)$

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It is shown that in order to explain the ratios $x$ and $y$ of the helicity amplitudes of the process $J/\psi \to \gamma + \theta$ in the glueball interpretation of $\theta$, a $d$-wave component has to be present in addition to the $s$-wave glueball wave function of $\theta$.

Quantum chromodynamics predicts the existence of glueballs.[1] Light glueballs in the mass range of 1–2 GeV have been predicted in the MIT bag model[2] and in lattice Monte Carlo calculations.[3] $J/\psi$ radiative decay is expected to be an ideal process to search for glueballs. Compared to pure $q\bar{q}$ states, glueballs are expected to be copiously produced in $J/\psi$ radiative decays. A $2^{++}$ meson $\theta(1690)$ has been observed in $J/\psi \to \gamma\eta\eta, \gamma K\bar{K}, \gamma\pi\pi$ by the Mark II, Crystal Ball,[4] Mark III (Refs. 5 and 6), and DM2 (Ref. 7) groups. It is perceived to be a candidate for the $2^{++}$ glueball. Recently, the helicity ratios

$$x = \frac{T_1}{T_0}, \quad y = \frac{T_2}{T_0} , \tag{1}$$

where $T_2, T_1, T_0$ are the three independent helicity amplitudes of the process $J/\psi \to \gamma\theta$ have been measured to be

$$\begin{aligned} x &= -1.07 \pm 0.20 , \\ y &= -1.09 \pm 0.25 \quad \text{(Ref. 5)} , \\ y &= -1.47 \pm 0.21 , \\ y &= -1.44 \pm 0.20 \quad \text{(Ref. 6)} . \end{aligned} \tag{2}$$

In contrast with this, the corresponding ratios in the processes $J/\psi \to \gamma f$ and $\gamma f'$ are very different with $x > 0$ and $y \approx 0$ (Ref. 6). Considering the glueball interpretation of $\theta$, can we explain the observed values of $x$ and $y$ in (2)? We shall address this question in this Brief Report.

In perturbative QCD the process of the $J/\psi$ radiative decay is described as[8] $J/\psi \to \gamma + g + g$. Considering $\theta$ as a $2^{++}$ glueball, the diagrams for the process $J/\psi \to \gamma\theta$ are shown in Fig. 1. The $S$-matrix element corresponding to these diagrams can be written as

$$\begin{aligned} \langle \theta_{\lambda_2} \gamma_{\lambda_1} | S | J_\lambda \rangle = {} & (2\pi)^4 \delta^4(p_J - p_\gamma - p_\theta) \frac{eg^2}{3\sqrt{6\omega_\gamma}} \delta_{ab} e_\mu^{\lambda_1 *}(p_\gamma) \\ & \times \int d^4x_1 d^4x_2 \, \mathrm{Tr}[\chi_\lambda(x_1,x_2)\gamma^\beta s_F(x_2)\gamma^\mu s_F(-x_1)\gamma^\alpha + \chi_\lambda(x_2,0)\gamma^\mu s_F(-x_1)\gamma^\alpha s_F(x_1-x_2)\gamma^\beta \\ & + \chi_\lambda(0,x_1)\gamma^\alpha s_F(x_1-x_2)\gamma^\beta s_F(x_2)\gamma^\mu] G^{ab}_{\alpha\beta}(x_1,x_2)_{\lambda_2} , \end{aligned} \tag{3}$$

where $\chi_\lambda(x_1,x_2)$ is the wave function of the $J/\psi$ particle which is

$$\chi_\lambda(x_1,x_2) = \frac{\sqrt{m_J}}{2\sqrt{2E_J}} \psi_J(x) e^{-ip_J X} \left[ 1 + \frac{\not{p}_J}{m_J} \right] \not{e}^{\lambda}(p_J), \quad X = \tfrac{1}{2}(x_1+x_2), \quad x = x_1 - x_2 . \tag{4}$$

$\psi_J(x)$ is the internal wave function of $J/\psi$ particle. In view of the fact that the charm quarks are heavy, it is a good approximation to consider that they annihilate at one point; thus, we take $\psi_J(x)$ to be $\psi_J(0)$. $G^{ab}_{\alpha\beta}(x_1,x_2)_{\lambda_2}$ is the wave function of the $2^{++}$ glueball $\theta$ which is defined in the lowest order as

$$G^{ab}_{\alpha\beta}(x_1,x_2)_{\lambda_2} = \langle \theta_{\lambda_2} | T[A^a_\alpha(x_1) A^b_\beta(x_2)] | 0 \rangle . \tag{5}$$

$A^a_\alpha(x)$ is the gluon field. We shall work in the rest frame of the $\theta$ particle. The helicity amplitude is defined as

$$\langle \theta_{\lambda_2} \gamma_{\lambda_1} | S | J_\lambda \rangle = (2\pi)^4 \delta(p_J - p_\gamma - p_\theta) \frac{e}{(8\omega_\gamma E_\theta E_J)^{1/2}} T_{\lambda_2} . \tag{6}$$

 

Because of helicity conservation we have

$$\lambda_2=\pm 2,\ \lambda_1=\mp 1,\ \lambda=\pm 1;\ \lambda_2=\pm 1,\ \lambda_1=\mp 1,\ \lambda=0;\ \lambda_2=0,\ \lambda_1=\pm 1,\ \lambda=\pm 1\ . \tag{7}$$

Because of the invariance of space reflection there is further an identity

$$T_{\lambda_2}=T_{-\lambda_2}\ . \tag{8}$$

The kinematics of the amplitudes $T_{\lambda_2}$ can be found in Ref. 7. From Eqs. (3) and (6) it is known that if $G^{ab}_{\alpha\beta}(x_1,x_2)_{\lambda_2}$ is given the ratios $x$ and $y$, (1) can be computed. Since the gluon is a vector meson, for a $2^{++}$ glueball the relative orbital angular momentum can be $l=0,2,4$. For simplicity, here, we only consider $s$ and $d$ waves. There is one wave function in the $s$ wave and two in the $d$ waves:

$$s\text{ wave: } G_s(x)_{\lambda_2}=\frac{1}{\sqrt{2m_\theta}}\delta_{ab}G_s(x)e^{ip_\theta X}\sum_{m_1m_2}C^{2\lambda_2}_{1m_1 1m_2}e_\alpha^{m_1 *}e_\beta^{m_2 *}\ , \tag{9}$$

$$d\text{ wave: } G_d(x)_{\lambda_2}=\frac{1}{\sqrt{2m_\theta}}\delta_{ab}G_d(x)e^{ip_\theta X}\sum_{m_1m_2}C^{2\lambda_2}_{1m_1 1m_2}x\cdot e^{m_1 *}x\cdot e^{m_2 *}g_{\alpha\beta}\ , \tag{10}$$

$$G_{d'}(x)_{\lambda_2}=\frac{1}{\sqrt{2m_\theta}}\delta_{ab}G_{d'}(x)e^{ip_\theta X}\sum_{m_1m_2,n_1n_2,M_1M_2}C^{2\lambda_2}_{2m_1 2m_2}C^{2m_1}_{1n_1 1n_2}C^{2m_2}_{1M_1 1M_2}e_\alpha^{n_1 *}e_\beta^{n_2 *}\times x\cdot e^{M_1 *}x\cdot e^{M_2 *}\ . \tag{11}$$

The wave function (10) is the combination of $s=0$ and $l=2$, (11) is the combination of $s=2$ and $l=2$.

According to Ref. 9 the helicity amplitude $T_{\lambda_2}$ can be written as

$$T_{\lambda_2}=\sum_{m_1m_2}C_{1m_1 1m_2}{}^{2\lambda_2}e_\alpha^{m_1 *}e_\beta^{m_2 *}e_\mu^{\lambda_1 *}(p_\gamma)e_\nu^{\lambda}(p_J)[A_1g^{\mu\nu}p_J^\alpha p_J^\beta+A_2(g^{\alpha\mu}g^{\beta\nu}+g^{\alpha\nu}g^{\beta\mu})+A_3(g^{\alpha\mu}p_J^\beta+g^{\beta\mu}p_J^\alpha)p_\theta^\nu]\ , \tag{12}$$

so that

$$T_2=-2A_2,\ \ T_1=\frac{\sqrt{2}}{m_J}(E_J+m_\theta p_J{}^2A_3/A_2)A_2,\ \ T_0=-\frac{2}{\sqrt{6}}(1+p_J{}^2A_1/A_2)A_2\ , \tag{13}$$

where

$$E_J=\frac{1}{2m_\theta}(m_J{}^2+m_\theta{}^2),\ \ p_J=\frac{1}{2m_\theta}(m_J{}^2-m_\theta{}^2)\ . \tag{14}$$

For the glueball wave function in $s$ wave the expressions for $A_1$, $A_2$, and $A_3$ are given in Ref. 9. If the glueball wave function $G_d(x)_{\lambda_2}$ in Eq. (10) is taken, the expressions for the corresponding $A_1$, $A_2$, and $A_3$ are

$$\begin{aligned}A_{1d}&=\frac{64}{3\sqrt{3}}g^2G_d(0)\psi_J(0)\frac{\sqrt{m_J}}{m_c{}^6}\left[\frac{4m_c{}^2}{m_J{}^2+4m_c{}^2-2m_G{}^2}\left[3-\frac{2m_c}{m_J}+\frac{1}{4m_c{}^2}(m_J{}^2-m_G{}^2)-\frac{1}{2m_Jm_c}(m_J{}^2+3m_G{}^2)\right]-1\right]\ ,\\ A_{2d}&=-\frac{64}{3\sqrt{3}}g^2G_d(0)\psi_J(0)\frac{\sqrt{m_J}}{m_c{}^4},\ \ A_{3d}=\frac{64}{3\sqrt{3}}g^2G_d(0)\psi_J(0)\frac{4\sqrt{m_J}}{m_c{}^4(m_J{}^2+4m_c{}^2-2m_G{}^2)}\ .\end{aligned} \tag{15}$$

If the glueball wave function $G_{d'}(x)_{\lambda_2}$ in Eq. (11) is taken, we have

$$\begin{aligned}A_{1d'}&=\frac{32}{3\sqrt{3}}g^2G_d'(0)\psi_J(0)\sqrt{m_J}\left[\frac{8p_J{}^2g(\lambda)}{m_c{}^6(m_J{}^2+4m_c{}^2-2m_G{}^2)}\left[\frac{1}{2}+\frac{m_c}{m_J}\right]\right.\\&\qquad\left.+\frac{16f(\lambda)}{m_c{}^4(m_J{}^2+4m_c{}^2-2m_G{}^2)}\left[\frac{1}{2}+\frac{m_c}{m_J}\right]-\frac{2p_J{}^2}{m_Jm_c{}^7}g(\lambda)\right]\ ,\\ A_{2d'}&=\frac{16}{3\sqrt{3}}g^2G_d'(0)\psi_J(0)\sqrt{m_J}\left[\frac{2p_J{}^2}{m_c{}^8}g(\lambda)\left[m_c{}^2-\frac{m_c}{2m_J}(m_J{}^2-m_G{}^2)\right]+\frac{4}{m_c{}^4}f(\lambda)\right]\ ,\\ A_{3d'}&=\frac{32}{3\sqrt{3}}g^2G_d'(0)\psi_J(0)\sqrt{m_J}\left[\frac{4p_J{}^2g(\lambda)}{m_c{}^6(m_J{}^2+4m_c{}^2-2m_G{}^2)}+\frac{8f(\lambda)}{m_c{}^4(m_J{}^2+4m_c{}^2-2m_G{}^2)}-\frac{p_J{}^2}{m_Jm_c{}^7}g(\lambda)\right]\ ,\end{aligned} \tag{16}$$

where $f(\lambda)$ and $g(\lambda)$ are functions of the glueball's helicity and they are

$$f(0)=f(\pm 1)=f(\pm 2)=\frac{\sqrt{7}}{2\sqrt{3}},\ \ g(\pm 2)=\frac{2}{\sqrt{21}},\ \ g(\pm 1)=-\frac{1}{\sqrt{21}},\ \ g(0)=-\frac{2}{\sqrt{21}}\ . \tag{17}$$

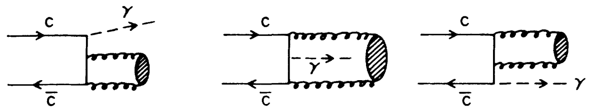


FIG. 1. Diagrams for the reaction $J/\psi \to \gamma\theta$, where $\theta$ is a glueball.

In the formulas above, instead of the internal wave functions of the glueballs we shall take $G_s(0)$, $G_d(0)$, and $G_{d'}(0)$, respectively.[9] Using formulas (13), (14), (15), and (16), the corresponding $x$ and $y$ in Eq. (1) can be computed. In formulas (15) and (16), $m_c$ is the charm-quark mass. In perturbative QCD, $m_c$ is the running charm-quark mass, similar to the running coupling constant $\alpha_s$. For each glueball wave function in Eqs. (9), (10), and (11), the amplitudes $G(0)$ are scaled out in the ratios $x$ and $y$; therefore, the only parameter in $x$ and $y$ is $m_c$. Our numerical calculation shows that in the $m_c$ range of 1.1—1.8 GeV $x$ and $y$ are all positive for each separate glueball wave function. Only when $m_c < 1.09$ GeV can we obtain $y < 0$ for the glueball wave function in the $s$ wave but $x$ is still positive. These results are in disagreement with the data in Eq. (2). However, it is quite natural to perceive that there is a mixture between the $s$ wave and $d$ wave in the wave function of the glueball $\theta$. This is enlightened by the fact that in the multipole expansion of the gluon field, the gluons, like the photons, do not have the orbital angular momentum as a good quantum number. From the wave functions in (9) and (10) the mixed wave function can be taken to be

$$\frac{\delta_{ab}}{\sqrt{2m_\theta}} G(0) e^{ip_\theta X} \left[ \sum_{m_1 m_2} C^{2\lambda_2}_{1m_1 1m_2} e_\alpha^{m_1 *} e_\beta^{m_2 *} + a g_{\alpha\beta} \sum_{m_1, m_2} C^{2\lambda_2}_{1m_1 1m_2} x \cdot e^{m_1 *} x \cdot e^{m_2 *} \right] , \tag{18}$$

where $a$ is the mixing parameter which can be determined by fitting the data (2). Using the helicity amplitudes for the wave functions in $s$ wave given in Ref. 9 and the amplitudes in Eq. (15) for the wave function in $d$ wave ($s=0$, $l=2$) we can obtain the $x$ and $y$ for the wave function in Eq. (18). From the numerical calculation of $x$ and $y$ it is learned that in a reasonable range of $m_c$ (1.2—1.8 GeV), we can fit the data (2) by choosing a corresponding mixing parameter $a$. For example, taking $m_c = 1.4$ GeV we obtain

$$\begin{aligned} a &= -0.18, \quad x = -1.07, \quad y = -1.09 , \\ a &= -0.19, \quad x = -1.47, \quad y = -1.43 , \end{aligned} \tag{19}$$

which agree with the data in Eq. (2) rather well. The ratios $x$ and $y$ determined this way are reasonably sensitive to the mixing parameter $a$. If taking $m_c = m_J/2$ and choosing $a = -0.25$ we obtain

$$x = -1.29, \quad y = -0.99 . \tag{20}$$

It is worthwhile to point out that we cannot fit the data by mixing the wave functions (9) and (11) or the wave functions (10) and (11).

To conclude, we find that in the glueball picture of $\theta$ the ratios of the helicity amplitudes $x$ and $y$ for the process $J/\psi \to \gamma\theta$ cannot be explained by taking the glueball wave function in the $s$ wave or the $d$ wave alone. We can fit the data (2), however, by mixing the wave functions with $s=2$, $l=0$ and $s=0$, $l=2$. We emphasize that in this fit the value of the charm-quark mass is about the same as that determined from the charmonium spectrum. It is also learned that we cannot fit the data by mixing the wave functions with $s=2$, $l=0$ and $s=2$, $l=2$ or with $s=0$, $l=2$ and $s=2$, $l=2$. This fit suggests that in the glueball wave function of $\theta$ there is a small $d$-wave component in addition to the major $s$-wave component, which is quite natural for a glueball.

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