Brief Reports

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Leading-logarithmic and α_s corrections to the decay $B \rightarrow \psi X$

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Leading-logarithmic corrections to all first-order radiative processes are used to calculate the branching ratio for $B \rightarrow \psi X$ to a higher precision than has been previously achieved. The calculated ratio is sensitive to α_s and is in reasonable agreement with the experimental value.

INTRODUCTION

In a previous paper[1] we presented a calculation of the branching ratio for $B \rightarrow \psi x$, including all first-order (in α_s) radiative corrections, and summing the leading-logarithmic corrections to the Born term. As this process is of continuing importance in the study of heavy-quark interactions and in understanding perturbative QCD, refinements to this calculation are of interest. We noted in Ref. 1 that a potentially large correction to our result would come from the leading-logarithm corrections to the bremsstrahlung rate, which is also of order α_s. In fact, it is possible to sum the leading logarithms in *all* terms with relatively little effort. We present here the results of such a calculation. With the improvements described here, the predicted branching ratio is rather sensitive to α_s and is in reasonable agreement with experiment.

Analysis of first-order radiative corrections to the basic weak decay amplitude begins with a classification of the three types of diagrams: vertex corrections, box diagrams, and bremsstrahlung contributions. As is well known, the box diagrams contain terms of order $\ln(m_W/m)$, where m_W is the W mass, and m is a typical quark mass. These large logarithms can be summed using a renormalization-group approach, to yield a "QCD-correction" factor Z. The QCD-corrected amplitude is then the basic Fermi interaction amplitude M_0, multiplied by the correction factor Z, which we will discuss below in more detail.

In addition to these large logarithms, we have chosen to keep all the nonleading terms of order α_s (the logarithms are summed to *all* orders in α_s), in all three types of diagrams. In the present calculation we combine vertex corrections to the QCD-corrected Born term, nonleading box-diagram corrections, and bremsstrahlung contributions. If the QCD-corrected Born term is represented by a black box as in Fig. 1(a), then typical nonleading contributions come from the diagrams of Figs. 1(b) and 1(c). Our decay amplitude will then have the form

$$M = M_{\mathrm{LLA}}(1 + b\alpha_s) ,$$

where M_{LLA} is the amplitude in the leading-logarithm approximation.

Our calculation resembles a next-to-leading-logarithm calculation, in the sense that we calculate corrections to the leading-logarithm amplitude. However, we have performed no two-loop calculations. In particular, our anomalous dimensions are computed from one-loop diagrams. A complete next-to-leading-logarithm calculation has been carried out by Altarelli *et al.*[2] for the massless

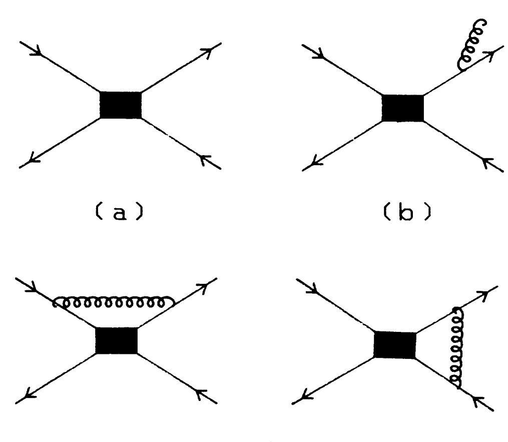

FIG. 1. (a) Diagrammatic representation of the QCD-corrected 4-point weak interaction. (b) Typical bremsstrahlung process. (c) Typical virtual radiative corrections to the QCD-corrected 4-point interaction.

fermion case. We have found that fermion masses play a significant role in B decay, so we cannot use their results.

We present the details of our calculation in the following section. Our result is presented as a graph of the predicted branching ration as a function of α_s. This and its interpretation are discussed in the final section.

DETAILS OF THE CALCULATION

We look first at the logarithmic terms which appear. We find that the four box diagrams contribute to a given amplitude M_i the quantity

$$M_i^{(\mathrm{logs})} = -12 C_i \frac{\alpha_s}{4\pi} \ln(m_W/\mu) M_i^0 \, , \tag{1}$$

where M_i^0 is the uncorrected amplitude for a given process (vertex, box, or bremsstrahlung). μ is the quark-mass scale, taken as m_c in our calculation. C_i is a color factor which takes on one of two forms and deserves further discussion.

We have adopted the point of view that the color factor should be determined by the two-loop diagram which leads to Eq. (1). In other words, we imagine expanding the black boxes of Figs. 1(b) and 1(c) (or Z) in powers of α_s, and insist that the order $(\alpha_s)^2$ term be that obtained from the corresponding two-loop diagram. This will not always give us the same correction factor Z as is obtained in Fig. 1(a). The authors of Ref. 2 consider nonleading corrections to M_{LLA} without insisting on this consistency at the two-loop level. This is an ambiguity in the leading-logarithm approximation which one can resolve by requiring agreement at some particular level in the perturbation expansion. By choosing to renormalize virtual diagrams separately, we have made the choice that the leading-logarithm correction to these diagrams must give the correct lowest-order term. Similar arguments apply to the bremsstrahlung amplitudes.

Two examples of second-order virtual corrections are shown in Fig. 2. Note that the inner box is going to contribute a logarithmic term, and the other gluon, being a correction to this, must attach to quark legs *outside* the box. This limits the number of two-loop diagrams we must consider. The color matrices in the two diagrams have different structures, namely,

$$T_1 = T^a T^b T^b T^a \, , \tag{2a}$$

$$T_2 = T^a T^b T^a T^b \, . \tag{2b}$$

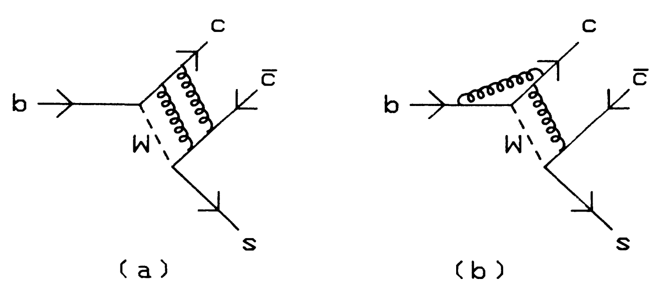

FIG. 2. Two representative virtual radiative corrections at the two-loop level.

The corresponding color factors are

$$\frac{1}{N}\,\mathrm{Tr}(T_1) = \left[\frac{N^2-1}{2N}\right]^2 \equiv \left[\frac{N^2-1}{2N}\right] C_1 \, , \tag{3a}$$

$$\frac{1}{N}\,\mathrm{Tr}(T_2) = \frac{1-N^2}{4N^2} \equiv \left[\frac{N^2-1}{2N}\right] C_2 \, . \tag{3b}$$

Here $N=3$ is the number of the quark colors. The color factors differ in large part because the two charmed quarks are assumed to form a color singlet. It is in this respect that our calculation differs (in Ref. 1 as here) from that of Kuhn *et al.*[3]

Because we have two types of diagram at the two-loop level, we will have separate renormalization factors. Z_1 (corresponding to C_1) is the same Z as in Ref. 1:

$$Z_1 = \left[\frac{\alpha_s(m_t)}{\alpha_s(m_W)}\right]^{-d/2b_6} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_t)}\right]^{-d/2b_5} \times \left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{-d/2b_4} , \tag{4}$$

with $d=16$, $b_n=(33-2n)/3$. The second factor Z_2 is of the same form with $d \to d_2$, where

$$d_2 = 12\left[-\frac{1}{2N}\right] = -2 \, . \tag{5}$$

We now calculate the decay rate to order $Z_i{}^2$ and α_s according to the following prescription:

$$\Gamma_{\mathrm{direct}} = F\,|\,Z_1(M_0+M_{c\bar c}^0+M_{bs}^0) + Z_2(M_{\bar c s}^0+M_{bc}^0+M_{b\bar c}^0+M_{cs}^0)\,|^2 + F'\int |\,Z_1(M_b^0+M_s^0)+Z_2(M_c^0+M_{\bar c}^0)\,|^2 d\Phi \, . \tag{6}$$

The notation here is as follows: M_0 is the Born-term amplitude, M_{ij}^0 the amplitude due to virtual gluon connecting quark lines i-j, M_i^0 the bremsstrahlung amplitude for gluon emission from quark line i, and F and F' are the appropriate kinematic factors. Details regarding these amplitudes are presented in Ref. 1.

Equation (6) incorporates the leading-logarithm corrections to all the diagrams to order α_s, and as such constitutes a substantial improvement over the calculation of Ref. 1. Numerically it was only necessary to sort out the different combinations which occurred in our earlier calculation and multiply by the appropriate factor of $Z_1{}^2$, $Z_2{}^2$, or $Z_1 Z_2$.

The branching ratio is obtained by adjusting Γ_{direct} for cascade decays and dividing by the theoretical total decay rate (the theoretical rate is used so that unknown mixing angle factors cancel). In the present calculation we use the familiar QCD-corrected rate[3,4]

$$\Gamma = \Gamma_0[\tfrac{1}{3}(2C_+{}^2 + C_-{}^2)] \, , \tag{7}$$

where

$$C_+ = \left[\frac{\alpha_s(\mu)}{\alpha_s(m_W)} \right]^{-6/b} , \tag{8a}$$

$$C_+ = \left[\frac{\alpha_s(\mu)}{\alpha_s(m_W)} \right]^{12/b} . \tag{8b}$$

In Ref. 1 we used a result due to Guberina *et al.*[5] which is, in fact, an order-α_s calculation. To order α_s, the leading logarithms cancel, as can be seen by expanding Eqs. (7) and (8) in powers of α_s. However, the authors of Ref. 4 were not suggesting that their result is a more accurate estimation of the total rate. Equation (7) is, in fact, more reliable.

RESULTS AND DISCUSSION

The results of our calculation are presented in Fig. 3 as a plot of the branching ratio versus $\alpha_s(\mu)$. Shown also are the most recent experimental data[6] which give $B = (1.09 \pm 0.16 \pm 0.21)\%$. We note that the ratio is a rather strong function of α_s, and that agreement with the data only occurs within a relatively short range in α_s. The commonly quoted value $\alpha_s \approx 0.2$ is within the acceptable range given the uncertainties of our calculation, which we discuss below. As it happens, the two major improvements of this calculation tend to cancel one another, so that our results are similar to those of Ref. 1.

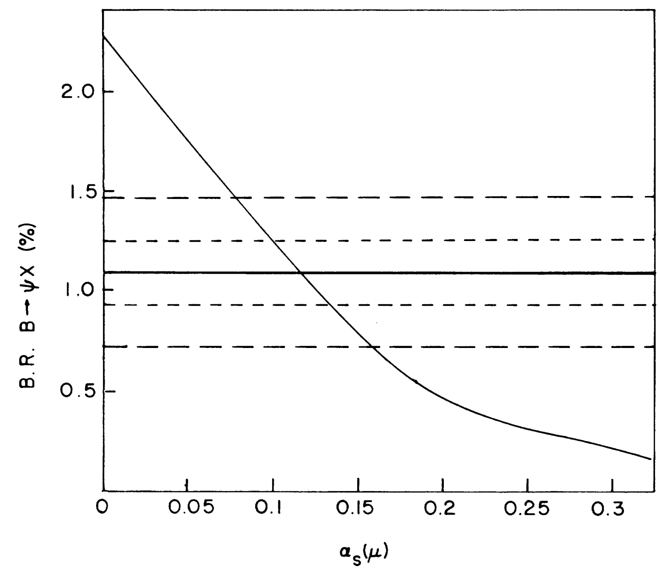

FIG. 3. The branching ratio for $B \to \psi x$ as a function of the QCD coupling $\alpha_s(\mu)$. The experimental value is shown with its statistical and systematic errors added.

Although our treatment of the leading logarithms has substantially improved the reliability of our calculation, there remain a number of uncertainties. The most significant of these are the following.

(1) The width $\Gamma_{\psi \to e^+ e^-}$, which determines the normalization of the ψ wave function, is known only to within $\pm 16\%$, experimentally. This is also the major systematic error in the data to which we have compared our prediction in Fig. 3.

(2) The radiative QCD corrections to the ψ wave function are strongly dependent on α_s. Specifically, an order-α_s calculation gives[7]

$$\psi^2(0) = \frac{\psi^2(\alpha_s = 0)}{1 - 16\alpha_s / 3\pi} . \tag{9}$$

For $\alpha_s = 0.2$, the denominator is $(1 - 0.34)$. Clearly, the $(\alpha_s)^2$ correction is important, perhaps on the order of 10%.

(3) QCD corrections of order $\alpha_s{}^2$ might be expected to change our results by another 5—10%, based on our experience with the lowest-order terms.

In summary we note that our analysis confirms, again, the importance of QCD corrections in bringing the branching ratio down from around 2% (Ref. 8) nearer to the observed value of 1.0%. The non-Abelian nature of the QCD corrections introduces some ambiguity when the renormalization-group technique is applied. We have resolved this ambiguity by insisting that two-loop diagrams be treated properly. As the effects only appear at the level of $\alpha_s{}^2 \ln(m_W/\mu)$, the ambiguity is not significant numerically. Unfortunately, our result is still uncertain by at least 20%, so that it cannot be used as an accurate determinant of α_s.

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