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Effective-Lagrangian study of three generations of supersymmetric composite quarks and leptons

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An effective Lagrangian for a supersymmetric composite model due to Greenberg, Mohapatra, and Yasué is constructed. In particular the four-fermion contact interactions for three generations of composite quarks and leptons are examined. A geometric approach is used to couple one quasi-Goldstone-fermion (QGF) generation to two matter generations. A heuristic argument leads to a particularly simple form for the bilinear matter coupling terms. Isoscalar four-fermion terms containing QGF's may be eliminated by a special choice of certain parameters. However, the residual interactions are found not to reproduce weak interactions due to the presence of flavor-changing processes and the failure of quark-lepton and e-μ-τ universality. Introducing fundamental weak forces leads to a lower bound of a few tens of TeV's for the hypercolor scale parameter.

I. INTRODUCTION

Composite models of quarks and leptons have been studied over the last few years as extensions of the highly successful standard model of particle interactions. The aims of composite models include an elucidation of why certain arbitrary parameters in the standard model (e.g., Yukawa couplings) have the values that they do, and why there exist at least three generations of quarks and leptons. Our understanding of confining gauge theories with massless fundamental fields has grown because of these efforts, but no truly realistic and compelling model has yet emerged.

Because experiment has not yet revealed the existence of "preons" (i.e., the fundamental constituent particles) an upper bound can be put on the size of quarks and leptons. This in turn implies a lower bound on the energy scale of the strongly interacting gauge theory of preons ($\Lambda_{\text{preon}} > 1$ TeV approximately). Quark and lepton masses are significantly smaller than 1 TeV or so and thus the fundamental problem in composite models arises: how do massless composite states emerge from a gauge theory that confines at a high-energy scale (the small quark and lepton masses then ascribed to perturbations). Two main solutions have emerged: (i) unbroken chiral symmetry (via the 't Hooft anomaly-matching equations; two-hypercolor-representation models are studied by the authors in Ref. 1, first paper, and by Davidson *et al.*;[1] one-hypercolor-representation models are studied in the remaining papers[1]) and (ii) quarks and leptons as quasi-Goldstone fermions.[2–6]

In both of the above approaches the gauge theory of massless preons possesses a chiral-symmetry group G. If G is to remain unbroken then the anomalies pertaining to currents associated with G must be the same whether calculated by using massless preons or massless spin-$\frac{1}{2}$ bound states. Thus if some preonic anomalies are nonzero then the assumption of unbroken chiral symmetry implies the existence of massless spin-$\frac{1}{2}$ bound states. If there cannot be found representations of G for bound spin-$\frac{1}{2}$ states which anomaly match with the preons, then one must conclude that G is dynamically broken.

If G is dynamically broken then Goldstone's theorem requires there to exist massless composite Goldstone bosons. If the preon theory is supersymmetric then there will exist massless composite spin-$\frac{1}{2}$ particles as superpartners of the Goldstone bosons (quasi-Goldstone fermions or QGF's).

It is necessary to combine these two approaches if one postulates that G is broken to a subgroup H which itself gives rise to anomalies. The QGF's contribute to the anomaly-matching equations for H since they are massless spin-$\frac{1}{2}$ particles belonging to H representations. Three possibilities arise in this case: (i) the anomalies due to the QGF's match precisely with the preonic anomalies; (ii) other massless composite spin-$\frac{1}{2}$ particles must be added to the spectrum with the QGF's to achieve anomaly matching;[2] and (iii) the QGF anomalies do not match and no other representations can be found which together with the QGF's would ensure matching. Cases (i) and (ii) yield consistent schemes while the situation in (iii) implies that the assumed breakdown $G \to H$ is not allowed.

There are two ways of constructing a supersymmetric Goldstone multiplet.[3] The fundamental supersymmetric matter multiplet (chiral superfield) has a complex spin-0 field and a Weyl spin-$\frac{1}{2}$ field. The "minimal case" has both spin-0 degrees of freedom as true Goldstone bosons. The "total doubling case" has only one spin-0 degree of freedom as a true Goldstone boson. The other is referred to as a "quasi-Goldstone boson" (QGB) and together with the Weyl QGF is kept massless by supersymmetry. In general a theory will have both types of Goldstone superfields present as determined by its dynamics.

In the QGF scheme one must find groups G and H such that the breakdown $G \to H$ yields QGF's with the group-theoretic properties of quarks and leptons. Since the standard-model gauge group $SU(3)\times SU(2)\times U(1)$ should be contained in G (and possibly also in H) one is led to consider $G=\mathrm{SU}(5)$ and larger groups. Indeed the breakdown $\mathrm{SU}(5)\rightarrow\mathrm{SU}(3)_{\mathrm{color}}\times\mathrm{U}(1)_{\mathrm{em}}$ yields exactly one generation of quarks and leptons as QGF's in the *total doubling* situation.[3] In Ref. 3 the four-fermion interactions of these QGF's were studied using an effective-Lagrangian approach. It was found that the known low-energy four-fermion interactions of quarks and leptons could be approximately reproduced for special choices of certain parameters in the effective Lagrangian. In particular it was possible to arrange quark-lepton universality and approximate $\mathrm{SU}(2)_L$ invariance. However since these parameters are in principle calculable from the fundamental dynamical theory it would require the dynamics to yield these special relationships "by accident." One generation also emerges in the *minimal* case for $\mathrm{SU}(5)_L\times\mathrm{SU}(5)_R\rightarrow\mathrm{SU}(3)_L\times\mathrm{U}(1)_L\times\mathrm{SU}(3)_R\times\mathrm{U}(1)_R$. Only the neutrino multiplet doubles, and so the lack of arbitrariness together with broken $\mathrm{SU}(2)_L$ probably renders the effective four-fermion interactions nonstandard.

Enlarging G to SU(6) or $\mathrm{SU}(6)_L\times\mathrm{SU}(6)_R$ solves the above problem. In the total doubling case[3] one generation emerges from $\mathrm{SU}(6)\rightarrow\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{U}(1)$, and in the minimal case[5] one generation is obtained from

$$\mathrm{SU}(6)_L\times\mathrm{SU}(6)_R\rightarrow[\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{U}(1)]_L \times[\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{U}(1)]_R\ .$$

Quark-lepton universality is guaranteed by the SU(4) Pati-Salam symmetry and there remains an unbroken $\mathrm{SU}(2)_L$ subgroup. [Of course in the minimal case one is dealing with $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$, that is, a left-right-symmetric model.]

Thus SU(6) plays a special role in composite models with one generation of quarks and leptons as QGF's. (It is also possible to have three or more generations as QGF's by considering large exceptional groups.[6])

One remaining problem in SU(6) models is the existence of an isoscalar four-fermion term in the effective theory.[3,4] In the total doubling case this may be suppressed by the special choice of a parameter. In the minimal case the isoscalar term is of the same strength as the phenomenologically relevent isovector term and thus the known low-energy interactions are not reproduced. Buchmüller *et al.*[4] have shown that the isoscalar term is suppressed in a very natural way in a slightly extended theory based on $\mathrm{U}(6)\rightarrow\mathrm{U}(4)\times\mathrm{SU}(2)$ or more realistically

$$\mathrm{U}(6)_L\times\mathrm{U}(6)_R\rightarrow[\mathrm{U}(4)\times\mathrm{SU}(2)]_L\times[\mathrm{U}(4)\times\mathrm{SU}(2)]_R\ .$$

Because of a broken U(1) a neutral totally doubled Goldstone multiplet joins one minimal QGF generation. This "novino" multiplet alters the effective interactions in such a way that the isoscalar contribution is small or zero.

In this paper we study a model proposed by Greenberg *et al.*[2] which features the breakdown

$$\mathrm{U}(6)_L\times\mathrm{U}(6)_R\times\mathrm{U}_R(1)\rightarrow\mathrm{SU}(4)_L\times\mathrm{SU}(2)_L\times\mathrm{SU}(4)_R \times\mathrm{SU}(2)_R\ ,$$

where $\mathrm{U}_R(1)$ is the supersymmetric R symmetry. The massless composite spectrum contains one minimal QGF generation and two other generations required by anomaly matching. There also exist four totally doubled neutral Goldstone multiplets which hopefully induce a novinolike mechanism. It is of considerable interest to see what the effective four-fermion interactions of a three-generation model are.

The plan of this paper is as follows. In Sec. II we review the Greenberg-Mohapatra-Yasué (GMY) model. In Sec. III we discuss the topic of effective Lagrangians, and in particular, of supersymmetric σ models coupled to matter. In Sec. IV we apply the techniques of Sec. III to calculate the effective Lagrangian to fourth order. Section V is devoted to a phenomenological discussion of our results and Sec. VI is a conclusion.

II. THE GREENBERG-MOHAPATRA-YASUÉ MODEL

This model[2] features an $\mathrm{SU}(6)_{\mathrm{HC}}$-hypercolor gauge interaction which is assumed to confine at a phenomenologically acceptable scale. The preon degrees of freedom consist of six chiral superfields and six antichiral superfields in the fundamental representation of $\mathrm{SU}(6)_{\mathrm{HC}}$. There are no mass or other superpotential terms. Thus the chiral-symmetry group G is

$$G=\mathrm{U}(6)_L\times\mathrm{U}(6)_R\times\mathrm{U}_R(1)\ , \tag{2.1}$$

where $\mathrm{U}_R(1)$ is a supersymmetric R symmetry. $\mathrm{U}_R(1)$ and the axial group $\mathrm{U}(1)_A\equiv\mathrm{U}(1)_{L\text{-}R}$ suffer from gauge-gauge-current anomalies. However one linear combination of $Q_{R\,\mathrm{sym}}$ and A generates an anomaly-free group $\mathrm{U}(1)_X$. Thus anomalies break G to G', where

$$G'=\mathrm{SU}(6)_L\times\mathrm{SU}(6)_R\times\mathrm{U}(1)_V\times\mathrm{U}(1)_X \tag{2.2}$$

and $\mathrm{U}(1)_V\equiv\mathrm{U}(1)_{L+R}$ is the preon-number group.

It is now assumed that the hypercolor interaction causes the formation of certain condensates which dynamically break G' to H, where

$$H=\mathrm{SU}(4)_L\times\mathrm{SU}(2)_L\times\mathrm{SU}(4)_R\times\mathrm{SU}(2)_R\ . \tag{2.3}$$

This breakdown implies the existence of Goldstone bosons (GB's) which belong to the following representation R_{GB} of H:

$$R_{\mathrm{GB}}=(4,2,1,1)+(1,1,\bar{4},2)+(\bar{4},2,1,1) +(1,1,4,2)+4(1,1,1,1)\ . \tag{2.4}$$

The four neutral Goldstone bosons are associated with the following broken U(1)'s: $\mathrm{U}(1)_L$, $\mathrm{U}(1)_R$, $\mathrm{U}(1)_V$, and $\mathrm{U}(1)_X$, where $\mathrm{U}(1)_{L,R}$ comes from the breakdown

$$\mathrm{SU}(6)_{L,R}\rightarrow\mathrm{SU}(4)_{L,R}\times\mathrm{SU}(2)_{L,R}\times\mathrm{U}(1)_{L,R}\ .$$

There is no Goldstone boson associated with the anomalous U(1).

It is assumed that the nonsinglet Goldstone bosons are arranged in minimal Goldstone supermultiplets while the neutral Goldstone bosons totally double. Thus the left-handed QGF spectrum is given by

$$R_{\mathrm{QGF}}=(4,2,1,1)_L+(1,1,\bar{4},2)_L+4(1,1,1,1)_L\ . \tag{2.5}$$

$(4,2,1,1)_L$ contains one generation of left-handed quarks and leptons while $(1,1,\bar{4},2)$ yields the left-handed antiquarks and antileptons.

The anomaly-matching equations are now used to yield information on which other fermions remain massless with the QGF's. Since H contains no U(1) factors and all representations of SU(2) are anomaly free, there is only one anomaly-matching equation, which arises from triangle graphs containing three SU(4) currents:

$$[\mathrm{SU}(4)_{L,R}]^3:\ 2(l_1-l_2+l_3)=6\ . \tag{2.6}$$

Here l_1, l_2, and l_3 are the 't Hooft indices for (4,2,1,1) and $(1,1,\bar{4},2)$, $(\bar{4},2,1,1)$ and (1,1,4,2), and (4,1,1,2) and $(1,\bar{4},2,1)$, respectively. Clearly $l_1=3$, $l_2=l_3=0$ is a solution for (2.6) which yields precisely three generations of quarks and leptons. Obviously other solutions also exist and thus the choice made is a dynamical assumption which is in principle falsifiable. However it is consistent to assume on the basis of present knowledge that this model yields one QGF generation and two generations which are kept massless by chiral symmetry alone. The fact that one generation is kept massless by two separate mechanisms while the other two are only protected by one may be a useful feature for obtaining a generational mass hierarchy.

It is clear that this model has some very interesting properties which all require further study (e.g., inter- and intragenerational effective interactions, mass hierarchy, left-right-symmetry breaking, and supersymmetry breaking). In this work we focus attention on the effective interactions using techniques which we now discuss.

III. EFFECTIVE LAGRANGIANS FOR GOLDSTONE FIELDS COUPLED TO MATTER

Consider a theory which features the spontaneous (or dynamical) breakdown $G\to H$. The Goldstone boson fields A^α form coordinates for a manifold endowed with a metric, and the effective Lagrangian is the invariant line element. For a supersymmetric theory the appropriate class of manifolds to consider are the Kähler manifolds, with the A^α and their complex conjugates $\bar{A}^{\bar{\alpha}}$ being the complex coordinates for the manifold.[7] A manifold is Kähler if it satisfies the conditions

$$g_{\alpha\bar{\beta},\gamma}=g_{\gamma\bar{\beta},\alpha},\quad g_{\alpha\bar{\beta},\bar{\gamma}}=g_{\alpha\bar{\gamma},\bar{\beta}}\ , \tag{3.1}$$

where $g_{\alpha\bar{\beta}}$ are the components of the metric tensor. Equations (3.1) imply that the metric may be derived from a function K, called the Kähler potential, in the following way:

$$g_{\alpha\bar{\beta}}(\bar{A},A)=\frac{\partial^2 K(\bar{A},A)}{\partial A^\alpha \partial \bar{A}^{\bar{\beta}}}\ . \tag{3.2}$$

The effective Lagrangian is just the D term of K as a function of the Goldstone superfields ϕ and $\bar{\phi}$:

$$L_{\mathrm{eff}}=K(\bar{\phi},\phi)\,|_{\theta\theta\bar{\theta}\bar{\theta}}\ . \tag{3.3}$$

In terms of component Goldstone fields defined by

$$\phi^\alpha=A^\alpha+\sqrt{2}\theta\psi^\alpha+\theta\theta F^\alpha\ , \tag{3.4a}$$

$$\bar{\phi}^{\bar{\alpha}}=\bar{A}^{\bar{\alpha}}+\sqrt{2}\bar{\theta}\bar{\psi}^{\bar{\alpha}}+\bar{\theta}\bar{\theta}\bar{F}^{\bar{\alpha}} \tag{3.4b}$$

in the chiral representation of superspace, L_{eff} may be written as

$$\begin{aligned}L_{\mathrm{eff}}&=\tfrac{1}{16}\,\mathbb{D}^\alpha\bar{\mathbb{D}}^2\mathbb{D}_\alpha K\,|_{\theta=\bar{\theta}=0}\\&=-g_{\alpha\bar{\beta}}(\partial_\mu\bar{A}^{\bar{\beta}}\partial^\mu A^\alpha+\tfrac{1}{2}i\bar{\psi}^{\bar{\beta}}\overleftrightarrow{\not{D}}\psi^\alpha)\\&\quad+\tfrac{1}{4}R_{\alpha\bar{\beta}\gamma\bar{\delta}}(\bar{\psi}^{\bar{\beta}}\bar{\psi}^{\bar{\delta}})(\psi^\alpha\psi^\gamma)\ ,\end{aligned} \tag{3.5}$$

where the covariant derivative D_μ is defined by

$$D_\mu\psi^\alpha\equiv\partial_\mu\psi^\alpha+\partial_\mu A^\beta\Gamma^\alpha_{\beta\gamma}\psi^\gamma\ , \tag{3.6a}$$

$$D_\mu\bar{\psi}^{\bar{\alpha}}\equiv\partial_\mu\bar{\psi}^{\bar{\alpha}}+\partial_\mu\bar{A}^{\bar{\beta}}\bar{\Gamma}^{\bar{\alpha}}_{\bar{\beta}\bar{\gamma}}\bar{\psi}^{\bar{\gamma}}\ , \tag{3.6b}$$

with the connection coefficients obeying

$$\Gamma^\alpha_{\beta\gamma}=g^{\alpha\bar{\delta}}g_{\beta\bar{\delta},\gamma}\ , \tag{3.7a}$$

$$\bar{\Gamma}^{\bar{\alpha}}_{\bar{\beta}\bar{\gamma}}=g^{\delta\bar{\alpha}}g_{\delta\bar{\beta},\bar{\gamma}}\ . \tag{3.7b}$$

$\mathbb{D}$ and $\bar{\mathbb{D}}$ are the usual supersymmetric covariant derivatives. $R_{\alpha\bar{\beta}\gamma\bar{\delta}}$ are the components of the curvature tensor and they obey the equation

$$\begin{aligned}R_{\alpha\bar{\beta}\gamma\bar{\delta}}&=g_{\gamma\bar{\gamma}}\bar{\Gamma}^{\bar{\gamma}}_{\bar{\beta}\bar{\delta},\alpha}\\&=g_{\alpha\bar{\beta},\gamma\bar{\delta}}-g^{\sigma\bar{\tau}}g_{\alpha\bar{\tau},\gamma}g_{\sigma\bar{\beta},\bar{\delta}}\ .\end{aligned} \tag{3.8}$$

The manifold is actually a symmetric space due to the existence of G invariance in the fundamental theory. The generators T^A of G generate infinitesimal isometry transformations:

$$\delta A^\alpha=-i\epsilon_A[T^A,A^\alpha]=\epsilon_A R^{A\alpha}(A)\ , \tag{3.9}$$

where the ϵ_A are infinitesimal group parameters and the $R^{A\alpha}$ are Killing vectors. The Killing vectors form a Lie algebra, that is,

$$[R^A,R^B]^\alpha=R^{A\beta}R^{B\alpha}{}_{,\beta}-R^{B\beta}R^{A\alpha}{}_{,\beta}=f^{ABC}R^{C\alpha}\ , \tag{3.10}$$

where the f^{ABC} are the structure constants of G: $[T^A,T^B]=if^{ABC}T^C$. When the isometry transformations are restricted to lie in H then the Killing vectors are simply the adjoint representation matrices of H acting on the A's. When the isometry transformations lie in G/H then the Killing vectors are nonlinear functions of the Goldstone fields and by virtue of Eq. (3.10) yield a nonlinear realization of G/H.

For the effective Lagrangian to be invariant under infinitesimal isometry transformations we must demand that

$$K(\bar{\phi},\phi)\to K(\bar{\phi},\phi)+F(\phi)+\bar{F}(\bar{\phi})\ , \tag{3.11}$$

where $F(\phi)$ is a chiral superfield formed from the ϕ's. This powerful restriction on K may be implemented through the Killing equation,[3]

$$\frac{\partial^2}{\partial A^\alpha\partial\bar{A}^{\bar{\beta}}}\left[R^{A\gamma}(A)\frac{\partial}{\partial A^\gamma}+\bar{R}^{A\bar{\gamma}}(\bar{A})\frac{\partial}{\partial\bar{A}^{\bar{\gamma}}}\right]K(\bar{A},A)=0\ . \tag{3.12}$$

We will use this equation to determine the Kähler potential to order four for the GMY model.

We now wish to couple other chiral and antichiral matter superfields to the Goldstone superfields in such a way that the symmetry structure of the original supersymmetric (SUSY) σ model is preserved. In particular general coordinate invariance should be retained. This problem has been discussed by van Holten[8] and we now review and extend the relevant sections of his analysis.

The starting point is to notice that in order to have L_{eff} invariant under general coordinate transformations $A^{\alpha} \to A'^{\alpha}(A)$ we must require that the QGF's transform as a contravariant coordinate vector:

$$\psi'^{\alpha} = \frac{\partial A'^{\alpha}}{\partial A^{\beta}} \psi^{\beta} . \tag{3.13}$$

It can easily be checked that the supersymmetric partner of A'^{α} is the ψ'^{β} given by Eq. (3.13), so this interpretation is consistent with SUSY. Thus the theory defined by Eq. (3.5) can be considered as the coupling of a bosonic σ model to fermionic matter. One may similarly couple other matter fields by demanding that they transform as coordinate tensors and then forming invariant interaction terms by contracting with coordinate tensors formed from the metric and the curvature.

Of particular relevance to us will be chiral and antichiral matter superfields transforming as contravariant vectors:

$$\begin{aligned} &W^{\alpha} \sim (B^{\alpha}, \chi^{\alpha}, G^{\alpha}) , \\ &W'^{\alpha} = \frac{\partial \phi'^{\alpha}}{\partial \phi^{\beta}} W^{\beta} . \end{aligned} \tag{3.14}$$

Equation (3.14) implies that B^{α} is a contravariant vector though χ^{α} is not:

$$\chi'^{\alpha} = \frac{1}{\sqrt{2}} \frac{\partial A'^{\alpha}}{\partial A^{\beta}} \chi^{\beta} + \frac{1}{\sqrt{2}} \frac{\partial^2 A'^{\alpha}}{\partial A^{\beta} \partial A^{\gamma}} B^{\beta} \psi^{\gamma} . \tag{3.15}$$

However a contravariant spinor field $\hat{\chi}$ may be defined as

$$\hat{\chi}^{\alpha} \equiv \chi^{\alpha} + \Gamma^{\alpha}_{\beta\gamma} B^{\beta} \psi^{\gamma} . \tag{3.16}$$

General coordinate invariance is preserved if we can write the matter-coupled supersymmetric Lagrangian in terms of B^{α}, $\hat{\chi}^{\alpha}$, covariant derivatives and tensors like $g_{\alpha\bar{\beta}}$, $R_{\alpha\bar{\beta}\gamma\bar{\delta}}$, and so on. Also, the new Lagrangian must yield the original SUSY σ model when the matter fields are set to zero.

Of course, to maintain supersymmetry the new Kähler potential must be written in terms of superfields. If the matter coupling terms are also written as coordinate *scalars* in superfield form, e.g.,

$$\tilde{K}(\bar{\phi}, \phi, \bar{w}, w) = K(\bar{\phi}, \phi) + C g_{\alpha\bar{\beta}} \bar{w}^{\bar{\beta}} w^{\alpha}$$

then it is guaranteed that the new Lagrangian (which is the D term of $\tilde{K}$) will also be a coordinate scalar. Thus the D term will automatically be expressible as a function of purely tensorial objects. This situation is to be contrasted with the case where a superfield tensor is not a scalar, for instance w^{α}. Then, as Eq. (3.15) shows, it is not true to say that all the component fields are also tensors.

To form bilinear matter couplings a new Kähler potential is introduced:

$$\begin{aligned} \tilde{K}_2(\bar{\phi}, \phi, \bar{w}, w) = K(\bar{\phi}, \phi) &+ C_1 g_{\alpha\bar{\beta}} \bar{w}^{\bar{\beta}} w^{\alpha} \\ &+ C_2 R_{\alpha\bar{\beta}} \bar{w}^{\bar{\beta}} w^{\alpha} + \cdots , \end{aligned} \tag{3.17}$$

where $R_{\alpha\bar{\beta}}$ is the Ricci tensor

$$R_{\alpha\bar{\beta}} \equiv g^{\gamma\bar{\delta}} R_{\gamma\bar{\delta}\alpha\bar{\beta}} \tag{3.18}$$

and $C_1, C_2, \ldots$ are arbitrary constants. The ellipsis indicates that all independent rank-two covariant tensors are *a priori* allowed to implement bilinear coupling, of which there are only two explicitly displayed. Among these are contributions like

$$R_{;\gamma\delta} W^{\gamma} W^{\delta} + R_{;\bar{\gamma}\bar{\delta}} \overline{W}^{\bar{\gamma}} \overline{W}^{\bar{\delta}}$$

which are pairs of non-Hermitian terms, where R is the curvature scalar: $R \equiv R^{\alpha}_{\alpha}$. These types of terms must feature covariant derivatives of the curvature so that tensors with unequal numbers of barred and unbarred indices are obtained.

Cubic couplings may be introduced through covariant derivatives of rank-two tensors (e.g., $R_{\alpha\bar{\beta};\gamma}$), and through rank-three tensors formed by contracting a covariant rank-$(n+3)$ tensor with a contravariant rank-n tensor. Note that the simplest such terms, $g_{\alpha\bar{\beta};\gamma}$ and $g_{\alpha\bar{\beta};\bar{\gamma}}$, actually vanish.

Linear matter coupling terms may be implemented through covariant vectors formed by contracting rank-$(n+1)$ covariant tensors with rank-n contravariant tensors. The highest-order couplings to be discussed are those of order four. The simplest examples of such couplings are

$$C_1'' g_{\alpha\bar{\beta}} g_{\gamma\bar{\delta}} \bar{w}^{\bar{\beta}} \bar{w}^{\bar{\delta}} w^{\alpha} w^{\gamma} + C_2'' R_{\alpha\bar{\beta}\gamma\bar{\delta}} \bar{w}^{\bar{\beta}} \bar{w}^{\bar{\delta}} w^{\alpha} w^{\gamma} .$$

In the GMY model there exist two generations of matter fields. To preserve covariance it is necessary to consider two contravariant vectors of supersymmetric matter. An immediate consequence of this is that a new set of four neutral massless supersymmetric multiplets (and their charge conjugates) need to be introduced with each matter generation in order to form a contravariant vector. Thus the number of novinolike particles in the model increases from four to twelve. Clearly their existence is *not* in conflict with the anomaly-matching equations because they are totally neutral with respect to H [see Eqs. (2.3) and (2.4)]. Also, with two independent contravariant matter vectors in the model, cross terms [e.g., $g_{\alpha\bar{\beta}}(\bar{w}^{\bar{\beta}}_1 w^{\alpha}_2 + \bar{w}^{\bar{\beta}}_2 w^{\alpha}_1)$] are *a priori* possible.

The above considerations make it clear that many possible interaction terms exist *a priori*, coupling via many arbitrary parameters. ("Arbitrary" in this context means that they cannot be determined by symmetry arguments alone. Of course, the GMY model dynamics is principle yields definite values for all these numbers.)

However, a physically motivated argument may be presented which indicates that most of the couplings *a priori* present are actually small, i.e., that most of the constants (e.g., C_2) are small. To illustrate the argument consider bilinear coupling through the Ricci tensor:

$$\tilde{K}(\bar{\phi},\phi,\bar{w},w)=K(\bar{\phi},\phi)+C_2 R_{\alpha\bar{\beta}}\bar{w}^{\bar{\beta}}w^{\alpha}+\cdots \ . \quad (3.19)$$

To calculate $R_{\alpha\bar{\beta}}$ the inverse metric tensor $g^{\alpha\bar{\beta}}$ is needed:

$$g^{\alpha\bar{\beta}}g_{\gamma\bar{\beta}}=\delta^{\alpha}_{\gamma},\ \ g^{\alpha\bar{\beta}}g_{\alpha\bar{\gamma}}=\delta^{\bar{\beta}}_{\bar{\gamma}} \ .$$

In general any coupling tensor will be formed from $g_{\alpha\bar{\beta}}$, $g^{\alpha\bar{\beta}}$, $R_{\alpha\bar{\beta}\gamma\bar{\delta}}$, $R_{\alpha\bar{\beta}\gamma\bar{\delta};\gamma}$, $R_{\alpha\bar{\beta}\gamma\bar{\delta};\bar{\gamma}}$, and higher covariant derivatives of the curvature. A schematic argument illustrates the basic idea. Consider the Kähler potential as a power series in the Goldstone field ϕ:

$$K(\phi)\sim v^2\phi^2+f_3\phi^3+f_4\phi^4+f_5\phi^5+f_6\phi^6+\cdots \ . \quad (3.20)$$

The metric tensor is schematically the second derivative of K:

$$g(\phi)\sim 2v^2+3f_3\phi+4f_4\phi^2+\cdots \ . \quad (3.21)$$

The inverse metric tensor is then

$$g^{-1}(\phi)\sim\frac{1}{2v^2}-\frac{3f_3}{4v^4}\phi-\left[\frac{f_4}{v^4}-\frac{9f_3{}^2}{16v^6}\right]\phi^2+\cdots \ . \quad (3.22)$$

The curvature is of the form

$$R(\phi)\sim\frac{\partial^2 g}{\partial\phi^2}-g^{-1}\left[\frac{\partial g}{\partial\phi}\right]^2$$
$$\sim r_0(v^2,f_3,f_4)+r_1(v^2,f_3,f_4,f_5)\phi$$
$$+r_2(v^2,f_3,f_4,f_5,f_6)\phi^2+\cdots \ , \quad (3.23)$$

where r_0,r_1,r_2 are the coefficients of each order in the power series. So the Ricci tensor is schematically

$$(\phi)\sim R(\phi)g^{-1}(\phi)$$
$$\sim r_0'(v^2,f_3,f_4)+r_1'(v^2,f_3,f_4,f_5)\phi$$
$$+r_2'(v^2,f_3,f_4,f_5,f_6)\phi^2+\cdots \ . \quad (3.24)$$

Thus the fourth-order coupling term in Eq. (3.19) is of the form

$$\delta K\sim C_2 r_2'(v^2,f_3,f_4,f_5,f_6)\phi^2 w^2 \ . \quad (3.25)$$

Hence a fourth-order term depends explicitly on f_5 and f_6 which are coefficients relating to higher-order processes among the Goldstone fields, namely, fifth- and sixth-order processes.

The point of this analysis is that from a physical viewpoint it is difficult to see how the strength of fifth- and sixth-order processes among Goldstone fields should be strongly related to fourth-order Goldstone-field—matter interactions. Consider Figs. 1–6. It is physically reasonable to suppose that two of the QGF's in Fig. 1 may be replaced by matter fermions (whose quantum numbers are identical to the QGF's) to obtain a fourth-order coupling of similar strength (Fig. 2). The contact interaction in Fig. 3 may yield a fourth-order interaction only if a loop is created. However this procedure is disallowed because all possible processes that contribute to

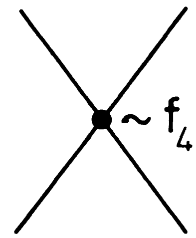

FIG. 1. Feynman graph of four-QGF contact interaction of strength f_4.

fourth-order interactions have already been accounted for in Fig. 1. Another sixth-order term generates four-fermion—two-scalar coupling (Fig. 5). A four-fermion term can be envisaged to follow from this interaction only if the matter fermion is a bound state of a QGF and a Goldstone boson (Fig. 6). This idea would imply, for example, that the muon in the GMY model is a bound state of an electron and a scalar neutrino. This possibility must be considered to be unlikely. It is more probable that the matter generations in the GMY model are true multipreon composites or bound states of two preons with one or two hypergluons.[9] In these cases it is far less clear that interactions such as Fig. 5 should be related to interactions like Fig. 2. These issues can only be clarified by a more detailed study of the hypercolor force. However, it seems reasonable to assume that higher-order Goldstone terms do not contribute strongly to lower-order Goldstone-field—matter terms.

The cause of the trouble with the Ricci tensor is that it is related to the fourth derivative of the Kähler potential, while the matter fields are bilinear. Coupling tensors can thus be classified according to what order derivative of K they are related to: (i) $g_{\alpha\bar{\beta}}\rightarrow$second derivative of K (unique); (ii) $R_{\alpha\bar{\beta}\gamma\bar{\delta}}\rightarrow$fourth derivative of K; (iii) $R_{\alpha\bar{\beta}\gamma\bar{\delta};\lambda}$ and $R_{\alpha\bar{\beta}\gamma\bar{\delta};\bar{\lambda}}\rightarrow$fifth derivative of K, and so on. As the order of the derivatives increases so does the order of the process to which they refer in the Goldstone sector. It seems physically reasonable that at some point for a given power of matter fields the higher-derivative coupling tensors yield negligible contributions. Here we have argued that the knowledge of the Kähler potential to fourth order should suffice to calculate all the possible order-four terms in the matter-coupled system.

This restriction has the remarkable consequence that linear and cubic matter couplings should be negligible.

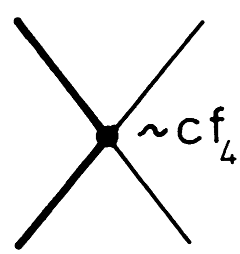

FIG. 2. Contact interaction of two QGF's with two matter fermions (heavy lines).

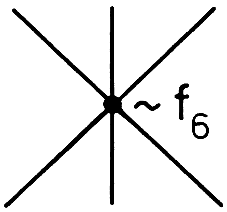

FIG. 3. Sixth-order contact interaction of QGF's with strength f_6.

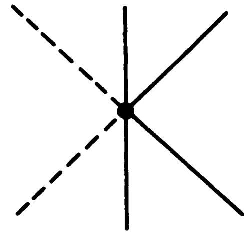

FIG. 5. Sixth-order contact interaction between four QGF's and two GB's or QGB's.

The only way to obtain a rank-three tensor from the metric is to take the covariant derivative; however, this vanishes. Similarly, linear couplings vanish. Also, all non-Hermitian terms are zero.

Thus to *third* order in the matter fields the most general form for the new Kähler potential is

$$\begin{aligned}\tilde K_3(\bar\phi,\phi,\bar w_1,w_1,\bar w_2,w_2) = {} & K(\bar\phi,\phi)+c_1 g_{\alpha\bar\beta}(\bar\phi,\phi)\bar w_1^{\beta} w_1^{\alpha} \\ & + c_2 g_{\alpha\bar\beta}(\bar\phi,\phi)\bar w_2^{\beta} w_2^{\alpha} \\ & + g_{\alpha\bar\beta}(\bar\phi,\phi)(c_3 \bar w_2^{\beta} w_1^{\alpha} \\ & + c_3^* \bar w_1^{\beta} w_2^{\alpha}) \; . \end{aligned} \tag{3.26}$$

An important observation now eliminates the remaining arbitrariness in Eq. (3.26). When the metric tensor is expanded as a power series in the fields, the zeroth-order coefficient yields a purely bilinear vector superfield in the w's:

$$T = g_{\alpha\bar\beta}(0,0)(c_1 \bar w_1^{\beta} w_1^{\alpha} + c_2 \bar w_2^{\beta} w_2^{\alpha} + c_3 \bar w_2^{\beta} w_1^{\alpha} + c_3^* \bar w_1^{\beta} w_2^{\alpha}) \; , \tag{3.27}$$

where $g_{\alpha\bar\beta}(0,0)$ is a numerical quantity. T must therefore be identified with the kinetic energy terms for the matter fields. Since kinetic energy terms must be diagonal in the fields and appropriately normalized, certain relations hold for c_1, c_2, c_3, and c_3^*. The diagonal requirement implies that $c_3 = c_3^* = 0$, while the normalization condition means that $c_1 = c_2 = 1$. Another way to view it is that the true physical fields are appropriately rescaled linear combinations of the fields used in Eq. (3.27).

The revised Kähler potential is, therefore,

$$\begin{aligned}\tilde K_3(\bar\phi,\phi,\bar w_1,w_1,\bar w_2,w_2) = {} & K(\bar\phi,\phi)+ g_{\alpha\bar\beta}(\bar\phi,\phi)\bar w_1^{\beta} w_1^{\alpha} \\ & + g_{\alpha\bar\beta}(\bar\phi,\phi)\bar w_2^{\beta} w_2^{\alpha} \; . \end{aligned} \tag{3.28}$$

The relations among the "arbitrary" parameters have important consequences for the four-fermion terms. An obvious observation is that there are no cross terms. If the effective interactions are to be identified with the weak interactions then this means that certain family lepton-number-changing processes are absent (e.g., $g_{\alpha\bar\beta}\bar w_1^{\beta} w_2^{\alpha}$ contains $\Delta L_\mu, \Delta L_\tau \neq 0$ processes). As well as the form also the strength of Goldstone-field—matter interactions is not arbitrary.

To discover how strong they are we need to calculate the curvature tensor for the expanded Kähler manifold defined by coordinates $Z^\alpha \sim (A^\alpha, B_1^\beta, B_2^\gamma)$:

$$\tilde R_{a\bar b c\bar d} = \tilde g_{a\bar b,c\bar d} - \tilde g^{m\bar n}\tilde g_{m\bar b,\bar d}\tilde g_{a\bar n,c} \; , \tag{3.29}$$

where

$$\tilde g_{a\bar b} \equiv \begin{bmatrix} \dfrac{\partial^2\tilde K}{\partial\bar A^{\beta}\partial A^{\alpha}} & \dfrac{\partial^2\tilde K}{\partial\bar A^{\beta}\partial B_1^{\alpha}} & \dfrac{\partial^2\tilde K}{\partial\bar A^{\beta}\partial B_2^{\alpha}} \\ \dfrac{\partial^2\tilde K}{\partial\bar B_1^{\beta}\partial A^{\alpha}} & \dfrac{\partial^2\tilde K}{\partial\bar B_1^{\beta}\partial B_1^{\alpha}} & \dfrac{\partial^2\tilde K}{\partial\bar B_1^{\beta}\partial B_2^{\alpha}} \\ \dfrac{\partial^2\tilde K}{\partial\bar B_2^{\beta}\partial A^{\alpha}} & \dfrac{\partial^2\tilde K}{\partial\bar B_2^{\beta}\partial B_1^{\alpha}} & \dfrac{\partial^2\tilde K}{\partial\bar B_2^{\beta}\partial B_2^{\alpha}} \end{bmatrix}$$

$$= \begin{bmatrix} G_{\alpha\bar\beta} & g_{\alpha\bar\beta,\bar\gamma}\bar B_1^{\gamma} & g_{\alpha\bar\beta,\bar\gamma}\bar B_2^{\gamma} \\ g_{\alpha\bar\beta,\gamma}B_1^{\gamma} & g_{\alpha\bar\beta} & 0 \\ g_{\alpha\bar\beta,\gamma}B_2^{\gamma} & 0 & g_{\alpha\bar\beta} \end{bmatrix} . \tag{3.30}$$

Here,

$$G_{\alpha\bar\beta} \equiv g_{\alpha\bar\beta} + g_{\alpha\bar\beta,\gamma\bar\delta}(\bar B_1^{\delta} B_1^{\gamma} + \bar B_2^{\delta} B_2^{\gamma}) \; . \tag{3.31}$$

The inverse metric may be verified to be

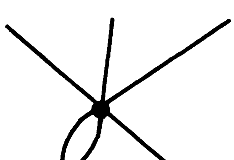

FIG. 4. Sixth-order contact interaction with one loop, thus seemingly yielding a fourth-order interaction.

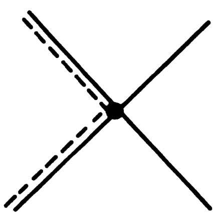

FIG. 6. Fourth-order contact interaction generated from a sixth-order interaction, where two of the fermions are now composites of a QGF and a scalar boson.

$$\tilde{g}^{a\bar{b}}=\begin{bmatrix} \tilde{g}^{\alpha\bar{\beta}} & -\tilde{g}^{\alpha\bar{\gamma}}\bar{\Gamma}^{\bar{\beta}}_{\bar{\gamma}\bar{\delta}}\bar{B}^{\bar{\delta}}_{1} & -\tilde{g}^{\alpha\bar{\gamma}}\bar{\Gamma}^{\bar{\beta}}_{\bar{\gamma}\bar{\delta}}\bar{B}^{\bar{\delta}}_{2} \\ -\tilde{g}^{\gamma\bar{\beta}}\Gamma^{\alpha}_{\gamma\delta}B^{\delta}_{1} & g^{\alpha\bar{\beta}}+\tilde{g}^{\gamma\bar{\delta}}\Gamma^{\alpha}_{\gamma\kappa}\bar{\Gamma}^{\bar{\beta}}_{\bar{\delta}\bar{\lambda}}\bar{B}^{\bar{\lambda}}_{1}B^{\kappa}_{1} & 0 \\ -\tilde{g}^{\gamma\bar{\beta}}\Gamma^{\alpha}_{\gamma\delta}B^{\delta}_{2} & 0 & g^{\alpha\bar{\beta}}+\tilde{g}^{\gamma\bar{\delta}}\Gamma^{\alpha}_{\gamma\kappa}\bar{\Gamma}^{\bar{\beta}}_{\bar{\delta}\bar{\lambda}}\bar{B}^{\bar{\lambda}}_{2}B^{\kappa}_{2} \end{bmatrix}, \tag{3.32}$$

where

$$\begin{aligned} &\tilde{g}_{\alpha\bar{\beta}}\equiv g_{\alpha\bar{\beta}}+R_{\alpha\bar{\beta}\gamma\bar{\delta}}(\bar{B}^{\bar{\delta}}_{1}B^{\gamma}_{1}+\bar{B}^{\bar{\delta}}_{2}B^{\gamma}_{2})\ , \\ &\tilde{g}^{\alpha\bar{\beta}}\tilde{g}_{\gamma\bar{\beta}}\equiv\delta^{\alpha}_{\gamma},\ \ \tilde{g}^{\alpha\bar{\beta}}\tilde{g}_{\alpha\bar{\gamma}}\equiv\delta^{\bar{\beta}}_{\bar{\gamma}}\ . \end{aligned} \tag{3.33}$$

It can be readily shown that the four-fermion terms arising from Eq. (3.28) by use of Eqs. (3.29)—(3.33) are

$$\begin{aligned} L^{4\ \mathrm{fermion}}_{\mathrm{eff}}=\tfrac{1}{4}R_{\alpha\bar{\beta}\gamma\bar{\delta}}[&(\bar{\psi}^{\bar{\beta}}\bar{\psi}^{\bar{\delta}})(\psi^{\alpha}\psi^{\gamma}) \\ &+4(\bar{\psi}^{\bar{\beta}}\bar{\chi}^{\bar{\delta}}_{1})(\psi^{\alpha}\chi^{\gamma}_{1}) \\ &+4(\bar{\psi}^{\bar{\beta}}\bar{\chi}^{\bar{\delta}}_{2})(\psi^{\alpha}\chi^{\gamma}_{2})]\ . \end{aligned} \tag{3.34}$$

The factors of 4 in the last two terms on the right-hand side of Eq. (3.34) are related to the symmetries of the curvature tensor ($R_{\alpha\bar{\beta}\gamma\bar{\delta}}=R_{\gamma\bar{\beta}\alpha\bar{\delta}}=R_{\alpha\bar{\delta}\gamma\bar{\beta}}=R_{\gamma\bar{\delta}\alpha\bar{\beta}}$) and define the relative strength of the purely Goldstone and mixed Goldstone-field—matter interactions. We shall see in subsequent sections of this paper that in the GMY model this form of QGF-matter interactions contains flavor-changing processes as well as conventional processes. This will preclude an identification of the residual force with the weak force.

Even though Eq. (3.34) will turn out to admit exotic processes there is still a high degree of similarity between the four-QGF terms and QGF-matter terms. In particular if a novinolike mechanism exists for the first generation then it also exists for second- and third-generation interactions with the first. Of course the importance of this result is diminished if the residual interactions are not to be identified with the weak force. It would recover its interest in attempts to modify the GMY model to suppress flavor-changing processes.

It is now pertinent to comment on mass generation within this scheme. As discussed by GMY, a generational mass hierarchy may be produced because the masslessness of the QGF family is guaranteed by two separate mechanisms, while the other families are only protected by chiral symmetry. To achieve this result the theory must be extended to include supersymmetry breaking and additional chiral-symmetry breaking. Such extensions are permitted with the framework discussed in this section The breaking of H in the exact supersymmetry limit may be phenomenologically described by an F term involving the matter superfields. Supersymmetry breaking may be implemented in a number of ways, including explicit scalar and gaugino masses, explicit gauging of a flavor subgroup without the inclusion of gauginos, and breaking via supergravity from a hidden sector. A detailed examination of these possibilities is beyond the scope of this paper.

Another effect the mass generation mechanism might have would be to reintroduce the cross terms parametrized by c_3 and c_3^* in Eq. (3.26). This would occur if the mass eigenstates did not coincide with the interaction eigenstates defined by Eq. (3.28). The analysis in this paper would then need to be modified to include mixing angles.

The specification of the new Kähler potential is completed by defining the quartic terms. These are far more arbitrary than the quadratic terms because the curvature tensor may now be used and because they can have arbitrary strengths. (Note that covariant derivatives of the curvature should still not be used because then fifth- and higher-order derivatives of K would multiply quartic matter products.) The two simplest possibilities are

$$\begin{aligned} (\delta K)_4=g_{\alpha\bar{\beta}}g_{\gamma\bar{\delta}}&\sum_{i,j,k,l=1,2}C^{(1)}_{ijkl}\bar{W}^{\bar{\beta}}_{i}\bar{w}^{\bar{\delta}}_{j}w^{\alpha}_{k}w^{\gamma}_{l} \\ &+R_{\alpha\bar{\beta}\gamma\bar{\delta}}\sum_{i,j,k,l=1,2}C^{(2)}_{ijkl}\bar{w}^{\bar{\beta}}_{i}\bar{w}^{\bar{\delta}}_{j}w^{\alpha}_{k}w^{\gamma}_{l}\ . \end{aligned} \tag{3.35}$$

Cross terms cannot be eliminated in this sector. Quartic coupling through $R_{\alpha\bar{\beta}\gamma\bar{\delta}}$ yields the novino mechanism structure; however, other terms such as $g_{\alpha\bar{\beta}}g_{\gamma\bar{\delta}}$ do not in general display the novino mechanism. The techniques discussed in this section will now be used to analyze the GMY model explicitly.

IV. AN EFFECTIVE LAGRANGIAN FOR THE GMY MODEL

The first stage of the calculation involves finding a nonlinear realization[3,4,10] on the Goldstone fields for infinitesimal transformations in the coset space

$$\frac{G}{H}=\frac{\mathrm{U}(6)_L\times\mathrm{U}(6)_R\times\mathrm{U}_R(1)}{\mathrm{SU}(4)_L\times\mathrm{SU}(2)_L\times\mathrm{SU}(4)_R\times\mathrm{SU}(2)_R}\ . \tag{4.1}$$

This is done by demanding that the Killing vectors satisfy Eq. (3.10) or equivalently that the Jacobi identity

$$0=[[X_A,X_B],\phi_C]-[[X_A,\phi_C],X_B]+[[X_B,\phi_C],X_A] \tag{4.2}$$

is satisfied, where X_A,X_B are any broken generators and ϕ_C is a Goldstone field.

To clarify the presentation, we will consider first just the left-handed sector

$$\left[\frac{G}{H}\right]_L=\frac{\mathrm{U}(6)_L\times\mathrm{U}_R(1)}{\mathrm{SU}(4)_L\times\mathrm{SU}(2)_L} \tag{4.3}$$

and generalize to the left-right-symmetric case at the end.

In order to implement Eq. (4.2) the algebra of the broken and unbroken generators for the model is needed. The Lie algebra of $\mathrm{U}(6)_L\times\mathrm{U}_R(1)$ is given by

$$\begin{aligned} &[T^A_B,T^C_D]=\delta^A_D T^C_B-\delta^C_B T^A_D\ , \\ &[T^A_B,Q_R]=0\ , \end{aligned} \tag{4.4}$$

where $A,B,C,D=1,\ldots,6$, L^A_B is a generator of $\mathrm{U}(6)_L$, and Q_R is the generator of $U_R(1)$.

The generators of $SU(4)_L$ and $SU(2)_L$ are given by

$$U^\alpha_\beta \equiv T^\alpha_\beta - \tfrac{1}{4}\delta^\alpha_\beta T^\gamma_\gamma \tag{4.5a}$$

and

$$U^i_j \equiv T^i_j - \tfrac{1}{2}\delta^i_j T^k_k \, , \tag{4.5b}$$

respectively. The greek indices refer to SU(4) and run from 1 to 4, while the latin indices refer to SU(2) and run from 1 to 2. Denote the broken generators by

$$X^\alpha_i \equiv T^\alpha_i, \quad X^i_\alpha \equiv T^i_\alpha, \quad X \equiv T^\gamma_\gamma, \quad X' \equiv T^k_k \, . \tag{4.6}$$

The coset space algebra now follows from Eq. (4.4):

$$\begin{aligned}
&[X^\alpha_i, X^j_\beta] = \delta^\alpha_\beta U^j_i - \delta^j_i U^\alpha_\beta + \tfrac{1}{2}\delta^j_i \delta^\alpha_\beta (X' - \tfrac{1}{2}X) \, , \\
&[X^\alpha_i, X^\beta_j] = [X^i_\alpha, X^j_\beta] = 0 \, , \\
&[X^\alpha_i, X] = X^\alpha_i, \quad [X^i_\alpha, X] = -X^i_\alpha \, , \\
&[X^\alpha_i, X'] = -X^\alpha_i, \quad [X^i_\alpha, X'] = X^i_\alpha \, , \\
&[X, X'] = 0 \, , \\
&[Q_R, X^\alpha_i] = [Q_R, X^i_\alpha] = [Q_R, X] = [Q_R, X'] = 0 \, .
\end{aligned} \tag{4.7}$$

The existence of a right-handed sector is employed to rewrite the algebra in terms of V, A, and X_L which generate $U(1)_V$, $U(1)_A$, and $U(1)_L$, respectively:

$$\begin{aligned}
&[X^\alpha_i, X^j_\beta] = \delta^\alpha_\beta U^j_i - \delta^j_i U^\alpha_\beta - \tfrac{1}{4}\delta^j_i \delta^\alpha_\beta X_L \, , \\
&[X^\alpha_i, X_L] = 3X^\alpha_i, \quad [X^i_\alpha, X_L] = -3X^i_\alpha \, , \\
&[X^\alpha_i, V] = [X^\alpha_i, A] = [X^\alpha_i, Q_R] = 0 \, , \\
&[X^i_\alpha, V] = [X^i_\alpha, A] = [X^i_\alpha, Q_R] = 0 \, , \\
&[V, X_L] = [A, X_L] = [A, V] = [V, Q_R] \\
&\qquad = [A, Q_R] = [X_L, Q_R] = 0 \, .
\end{aligned} \tag{4.8}$$

This is convenient because X_L, V, and Q_R (with R-character zero) are free of strong anomalies, while A is anomalous. A further convenient redefinition involves taking linear combinations of X_L, V, and Q_R so that a more symmetric algebra is obtained:

$$\begin{aligned}
&[X^\alpha_i, X^j_\beta] = \delta^\alpha_\beta U^j_i - \delta^j_i U^\alpha_\beta - \tfrac{1}{4}\delta^j_i \delta^\alpha_\beta (X_1 + X_2 + X_3) \, , \\
&[X^\alpha_i, X_1] = [X^\alpha_i, X_2] = [X^\alpha_i, X_3] = X^\alpha_i \, , \\
&[X^i_\alpha, X_1] = [X^i_\alpha, X_2] = [X^i_\alpha, X_3] = -X^i_\alpha \, ,
\end{aligned} \tag{4.9}$$

where all other commutators vanish. Here,

$$\begin{aligned}
&X_1 \equiv \tfrac{1}{3}X_L + V + Q_R \, , \\
&X_2 \equiv \tfrac{1}{3}X_L + V - Q_R \, , \\
&X_3 \equiv \tfrac{1}{3}X_L - 2V \, .
\end{aligned} \tag{4.10}$$

Denote the nonsinglet left-sector Goldstone fields by L^α_i and the singlet fields by L_1, L_2, and L_3. The transformation laws for these fields under the unbroken generators are

$$\begin{aligned}
&[U^\alpha_\beta, L^\gamma_i] = -\delta^\gamma_\beta L^\alpha_i + \tfrac{1}{4}\delta^\alpha_\beta L^\gamma_i \, , \\
&[U^i_j, L^\alpha_k] = \delta^i_k L^\alpha_j - \tfrac{1}{2}\delta^i_j L^\alpha_k \, , \\
&[U^\alpha_\beta, \text{singlet}] = [U^i_j, \text{singlet}] = 0 \, .
\end{aligned} \tag{4.11}$$

It may now be verified that the following is a nonlinear realization in the coset space:

$$\begin{aligned}
&-i[X^\alpha_i, L^\beta_j] = L^\alpha_j L^\beta_i \, , \\
&-i[X^i_\alpha, L^\beta_j] = \delta^i_j \delta^\beta_\alpha \, , \\
&-i[X_a, L^\beta_j] = iL^\beta_j \, , \\
&-i[X^\alpha_i, L_a] = -\tfrac{3}{4} i L^\alpha_i \, , \\
&-i[X^i_\alpha, L_a] = 0 \, , \\
&-i[X_a, L_b] = 1 \, ,
\end{aligned} \tag{4.12}$$

where $a,b = 1,2,3$.

It is consistent with the Jacobi identities to take the Goldstone fields as singlets under the anomalous $U(1)_A$. The contribution which the $U(1)_A$ anomaly makes to the effective Lagrangian is beyond the scope of this paper.[11]

The right-handed sector obeys analogous relations to Eqs. (4.12). However the technique employed has introduced a linearly dependent set of broken singlet generators $X_1, X_2, X_3, Y_1, Y_2, Y_3$, where the latter three refer to the right-handed sector. X_1 and Y_2 may be eliminated by use of

$$\begin{aligned}
&X_1 = Y_1 + X_3 - Y_3 \, , \\
&Y_2 = X_2 - X_3 + Y_3 \, .
\end{aligned} \tag{4.13}$$

Omitting one singlet Goldstone field from each sector, Eqs. (4.12) and (4.13) imply that the following is a left-right-symmetric nonlinear realization:

$$\begin{aligned}
&-i[X^\alpha_i, L^\beta_j] = L^\alpha_j L^\beta_i \, , \\
&-i[X^\alpha_i, L_1] = -i[X^\alpha_i, L_2] = -\tfrac{3}{4} i L^\alpha_i \, , \\
&-i[X^\alpha_i, R^{\bar{j}}_{\bar{\beta}}] = i[X^\alpha_i, R_1] = -i[X^\alpha_i, R_2] = 0 \, ;
\end{aligned} \tag{4.14a}$$

$$\begin{aligned}
&-i[X^i_\alpha, L^\beta_j] = \delta^i_j \delta^\beta_\alpha \, , \\
&-i[X^i_\alpha, L_1] = -i[X^i_\alpha, L_2] = 0 \, , \\
&-i[X^i_\alpha, R^{\bar{j}}_{\bar{\beta}}] = -i[X^i_\alpha, R_1] = -i[X^i_\alpha, R_2] = 0 \, ;
\end{aligned} \tag{4.14b}$$

$$\begin{aligned}
&-i[X_2, L^\beta_j] = iL^\beta_j \, , \\
&-i[X_2, L_1] = -i[X_2, L_2] = 1 \, , \\
&-i[X_2, R^{\bar{j}}_{\bar{\beta}}] = i[X_2, R_1] = -i[X_2, R_2] = 0 \, .
\end{aligned} \tag{4.14c}$$

Similar relations hold for X_3. $R^{\bar{j}}_{\bar{\beta}}$ is the right-sector nonsinglet Goldstone field, while R_1 and R_2 are the singlet fields. Analogous relations obtain for the right-sector generators and nonlinear realizations for charge-conjugate fields may be obtained by Hermitian conjugation.

The Killing vectors defined by Eqs. (4.14a)–(4.14c) are now used in Eq. (3.12) to determine the Kähler potential to fourth order.[3] The full result is presented in the Appendix. Here we shall display only the order-four terms involving the quark-lepton Goldstone fields:

$$K_4(\overline{L},L,\overline{R},R)=\tfrac{9}{16}(v_{11}{}^2+u_{12})\overline{L}\,{}^i_{\alpha}\overline{L}\,{}^j_{\beta}L_i^{\alpha}L_j^{\beta}-\tfrac{1}{2}v_1{}^2\overline{L}\,{}^i_{\alpha}\overline{L}\,{}^j_{\beta}L_j^{\alpha}L_i^{\beta}$$
$$+\tfrac{9}{16}(v_{33}{}^2+u_{34})\overline{R}\,{}^{\overline{\alpha}}_{\overline{i}}\overline{R}\,{}^{\overline{\beta}}_{\overline{j}}R^{\overline{i}}_{\overline{\alpha}}R^{\overline{j}}_{\overline{\beta}}-\tfrac{1}{2}v_2{}^2\overline{R}\,{}^{\overline{\alpha}}_{\overline{i}}\overline{R}\,{}^{\overline{\beta}}_{\overline{j}}R^{\overline{i}}_{\overline{\beta}}R^{\overline{j}}_{\overline{\alpha}}+\tfrac{9}{4}u_{13}\overline{L}\,{}^i_{\alpha}L_i^{\alpha}\overline{R}\,{}^{\overline{\alpha}}_{\overline{i}}R^{\overline{i}}_{\overline{\alpha}}\ . \tag{4.15}$$

The four-fermion terms which follow from Eq. (4.15) may be obtained by use of Eqs. (3.5) and (3.8). Written in terms of left-handed Dirac spinors they are

$$L^4_{\text{eff, Goldstone}}=-\frac{9}{32}\left[\frac{v_{11}{}^2+u_{12}}{v_1{}^4}\right]\overline{\psi}\,{}^i_{\alpha L}\gamma^{\mu}\psi^{\alpha}_{iL}\overline{\psi}\,{}^j_{\beta L}\gamma_{\mu}\psi^{\beta}_{jL}+\frac{1}{4v_1{}^2}\overline{\psi}\,{}^i_{\alpha L}\gamma^{\mu}\psi^{\alpha}_{jL}\overline{\psi}\,{}^j_{\beta L}\gamma_{\mu}\psi^{\beta}_{iL}$$
$$-\frac{9}{32}\left[\frac{v_{33}{}^2+u_{34}}{v_2{}^4}\right]\overline{\Lambda}\,{}^{\overline{\alpha}}_{\overline{i}L}\gamma^{\mu}\Lambda^{\overline{i}}_{\overline{\alpha}L}\overline{\Lambda}\,{}^{\overline{\beta}}_{\overline{j}L}\gamma_{\mu}\Lambda^{\overline{j}}_{\overline{\beta}L}$$
$$+\frac{1}{4v_2{}^2}\overline{\Lambda}\,{}^{\overline{\alpha}}_{\overline{i}L}\gamma^{\mu}\Lambda^{\overline{i}}_{\overline{\beta}L}\overline{\Lambda}\,{}^{\overline{\beta}}_{\overline{j}L}\gamma_{\mu}\Lambda^{\overline{j}}_{\overline{\alpha}L}-\frac{9}{8}\frac{u_{13}}{v_1{}^2v_2{}^2}\overline{\psi}\,{}^i_{\alpha L}\gamma^{\mu}\psi^{\alpha}_{iL}\overline{\Lambda}\,{}^{\overline{\alpha}}_{\overline{i}L}\gamma_{\mu}\Lambda^{\overline{i}}_{\overline{\alpha}L}\ . \tag{4.16}$$

$\Lambda^{\overline{i}}_{\overline{\alpha}L}$ is the right-sector left-handed Dirac spinor. Using the Fierz identity,

$$\delta^i_l\delta^k_j=\tfrac{1}{2}\delta^i_j\delta^k_l+\tfrac{1}{2}(\tau)^i_j\cdot(\tau)^k_l\ , \tag{4.17}$$

the isoscalar and isovector terms in Eq. (4.16) may be isolated. The left-sector contribution then becomes

$$L^{(4)\,\text{left}}_{\text{eff, Goldstone}}=\frac{1}{8v_1{}^2}\overline{\psi}\,{}^p_L\gamma^{\mu}\tau\psi^p_L\cdot\overline{\psi}\,{}^q_L\gamma_{\mu}\tau\psi^q_L+\frac{1}{8v_1{}^4}(v_1{}^2-\tfrac{9}{8}v_{11}{}^2-\tfrac{9}{8}u_{12})\overline{\psi}\,{}^p_L\gamma^{\mu}1\psi^p_L\cdot\overline{\psi}\,{}^q_L\gamma_{\mu}1\psi^q_L\ . \tag{4.18}$$

Here $p,q=1,\dots,4$ refer to a sum over the SU(4) indices. The right-sector contribution has an analogous form, while the left-right cross term is purely isoscalar.

The isoscalar piece of Eq. (4.18) displays a novinolike mechanism[4] since it vanishes if

$$v_1{}^2=\tfrac{9}{8}(v_{11}{}^2+u_{12})\ . \tag{4.19}$$

Exactly how small the coefficient is depends on the detailed dynamics of the GMY model. Note that this scheme differs from the novino model in that there are two novinos per sector, and that isoscalar suppression depends on two different parameters (the scale $v_{11}{}^2$ and the mixing parameter u_{12}) rather than one.

The matter coupling terms may be obtained directly from Eqs. (3.8) and (3.34):

$$L^{(4)\,\text{left}}_{\text{eff, Goldstone matter}}=\frac{1}{8v_1{}^2}\overline{\psi}\,{}^p_L\gamma^{\mu}\tau\psi^p_L\cdot(\overline{\psi}\,{}^q_L\gamma_{\mu}\tau\psi^q_L+2\overline{\chi}\,{}^q_{1L}\gamma_{\mu}\tau\chi^q_{1L}+2\overline{\chi}\,{}^q_{2L}\gamma_{\mu}\tau\chi^q_{2L})$$
$$+\frac{1}{4v_1{}^2}(\overline{\psi}\,{}^p_L\gamma^{\mu}\tau\chi^p_{1L}\cdot\overline{\chi}\,{}^q_{1L}\gamma_{\mu}\tau\psi^q_L+\overline{\psi}\,{}^p_L\gamma^{\mu}\tau\chi^p_{2L}\cdot\overline{\chi}\,{}^q_{2L}\gamma_{\mu}\tau\psi^q_L)$$
$$+\frac{1}{8v_1{}^4}(v_1{}^2-\tfrac{9}{8}v_{11}{}^2-\tfrac{9}{8}u_{12})\overline{\psi}\,{}^p_L\gamma^{\mu}1\psi^p_L\cdot(\overline{\psi}\,{}^q_L\gamma_{\mu}1\psi^q_L+2\overline{\chi}\,{}^q_{1L}\gamma_{\mu}1\chi^q_{1L}+2\overline{\chi}\,{}^q_{2L}\gamma_{\mu}1\chi^q_{2L})$$
$$+\frac{1}{4v_1{}^4}(v_1{}^2-\tfrac{9}{8}v_{11}{}^2-\tfrac{9}{8}u_{12})(\overline{\psi}\,{}^p_L\gamma^{\mu}1\chi^p_{1L}\cdot\overline{\chi}\,{}^q_{1L}\gamma_{\mu}1\psi^q_L+\overline{\psi}\,{}^p_L\gamma^{\mu}1\chi^p_{2L}\cdot\overline{\chi}\,{}^q_{2L}\gamma_{\mu}1\psi^q_L)$$
$$+\text{matter self-interaction terms}\ . \tag{4.20}$$

The matter self-interaction terms are more arbitrary due to the fact that the curvature tensor may be used. However, only two types of four-fermion terms are generated: isoscalar and isovector terms. While couplings through the curvature tensor yield similar structures to Eq. (4.20), couplings such as $g_{\alpha\overline{\beta}}g_{\gamma\overline{\delta}}\overline{w}\,{}^{\overline{\beta}}_1\overline{w}\,{}^{\overline{\delta}}_1w^{\alpha}_1w^{\gamma}_1$ yield purely isoscalar terms. As explained previously, flavor-changing vertices are in general possible. Hence the four-fermion matter self-couplings may be parametrized as

$$L^{(4)\,\text{left}}_{\text{eff, matter}}=\frac{1}{v_1{}^2}(c'_1\overline{\chi}\,{}^p_{1L}\gamma^{\mu}\tau\chi^p_{1L}\cdot\overline{\chi}\,{}^q_{1L}\gamma_{\mu}\tau\chi^q_{1L}+c'_2\overline{\chi}\,{}^p_{2L}\gamma^{\mu}\tau\chi^p_{2L}\cdot\overline{\chi}\,{}^q_{2L}\gamma_{\mu}\tau\chi^q_{2L}+c'_3\overline{\chi}\,{}^p_{1L}\gamma^{\mu}\tau\chi^p_{1L}\cdot\overline{\chi}\,{}^q_{2L}\gamma_{\mu}\tau\chi^q_{2L}$$
$$+c_4\overline{\chi}\,{}^p_{1L}\gamma^{\mu}\tau\chi^p_{2L}\cdot\overline{\chi}\,{}^q_{2L}\gamma_{\mu}\tau\chi^q_{1L}+c_5\overline{\chi}\,{}^p_{2L}\gamma^{\mu}\tau\chi^p_{1L}\cdot\overline{\chi}\,{}^q_{2L}\gamma_{\mu}\tau\chi^q_{1L}+c_5^*\overline{\chi}\,{}^p_{1L}\gamma^{\mu}\tau\chi^p_{2L}\cdot\overline{\chi}\,{}^q_{1L}\gamma_{\mu}\tau\chi^q_{2L}$$
$$+c_6\overline{\chi}\,{}^p_{1L}\gamma^{\mu}\tau\chi^p_{1L}\cdot\overline{\chi}\,{}^q_{1L}\gamma_{\mu}\tau\chi^q_{2L}+c_6^*\overline{\chi}\,{}^p_{1L}\gamma^{\mu}\tau\chi^p_{1L}\cdot\overline{\chi}\,{}^q_{2L}\gamma_{\mu}\tau\chi^q_{1L}+c_7\overline{\chi}\,{}^p_{2L}\gamma^{\mu}\tau\chi^p_{2L}\cdot\overline{\chi}\,{}^q_{2L}\gamma_{\mu}\tau\chi^q_{1L}$$
$$+c_7^*\overline{\chi}\,{}^p_{2L}\gamma^{\mu}\tau\chi^p_{2L}\cdot\overline{\chi}\,{}^q_{1L}\gamma_{\mu}\tau\chi^q_{2L})$$

$$+\frac{1}{v_1{}^2}(s_1\bar{\chi}^{p}_{1L}\gamma^{\mu}1\chi^{p}_{1L}\cdot\bar{\chi}^{q}_{1L}\gamma_{\mu}1\chi^{q}_{1L}+s_2\bar{\chi}^{p}_{2L}\gamma^{\mu}1\chi^{p}_{2L}\cdot\bar{\chi}^{q}_{2L}\gamma_{\mu}1\chi^{q}_{2L}+s_3\bar{\chi}^{p}_{1L}\gamma^{\mu}1\chi^{p}_{1L}\cdot\bar{\chi}^{q}_{2L}\gamma_{\mu}1\chi^{q}_{2L}$$
$$+s_4\bar{\chi}^{p}_{1L}\gamma^{\mu}1\chi^{p}_{2L}\cdot\bar{\chi}^{q}_{2L}\gamma_{\mu}1\chi^{q}_{1L}+s_5\bar{\chi}^{p}_{2L}\gamma^{\mu}1\chi^{p}_{1L}\cdot\bar{\chi}^{q}_{2L}\gamma_{\mu}1\chi^{q}_{1L}+s_5^*\bar{\chi}^{p}_{1L}\gamma^{\mu}1\chi^{p}_{2L}\cdot\bar{\chi}^{q}_{1L}\gamma_{\mu}1\chi^{q}_{2L}$$
$$+s_6\bar{\chi}^{p}_{1L}\gamma^{\mu}1\chi^{p}_{1L}\cdot\bar{\chi}^{q}_{1L}\gamma_{\mu}1\chi^{q}_{2L}+s_6^*\bar{\chi}^{p}_{1L}\gamma^{\mu}1\chi^{p}_{1L}\cdot\bar{\chi}^{q}_{2L}\gamma_{\mu}1\chi^{q}_{1L}+s_7\bar{\chi}^{p}_{2L}\gamma^{\mu}1\chi^{p}_{2L}\cdot\bar{\chi}^{q}_{2L}\gamma_{\mu}1\chi^{q}_{1L}$$
$$+s_7^*\bar{\chi}^{p}_{2L}\gamma^{\mu}1\chi^{p}_{2L}\cdot\bar{\chi}^{q}_{1L}\gamma_{\mu}1\chi^{q}_{2L})\ . \tag{4.21}$$

Equations (4.20) and (4.21) contain the complete effective Lagrangian. Phenomenological issues arising from this analysis will be considered in the next section.

V. PHENOMENOLOGY OF THE GMY MODEL

The effective four-fermion terms that arise in the standard model before $W^0-\gamma$ and Cabibbo mixing are

$$L^{\rm st}_{\rm eff}(G_F)=\frac{G_F}{\sqrt{2}}\mathbf{J}^{\mu}\cdot\mathbf{J}_{\mu}\ , \tag{5.1}$$

where the weak current $\mathbf{J}^{\mu}$ is given by

$$\mathbf{J}^{\mu}=\sum_{i=1}^{3}\bar{\psi}^{p}_{iL}\gamma^{\mu}\tau\psi^{p}_{iL}\ . \tag{5.2}$$

i labels generations and $p=1,2,3,4$. $P=1$ yields the leptonic weak doublet $\binom{\nu_e}{e}$ while $p=2,3,4$ and gives three colors of quark doublets $\binom{u_c}{d_c}$, $c=R,B,G$. G_F is the Fermi constant.

Consider first the effective Lagrangian describing QGF-matter interactions in Eq. (4.20). For simplicity assume the validity of the novino mechanism [Eq. (4.19)] so that all the isoscalar terms are zero. Then if we make the identification

$$v_1{}^2=\frac{\sqrt{2}}{8G_F'}\ , \tag{5.3}$$

the effective Lagrangian has the form

$$L_{\rm eff}=L^{\rm st}_{\rm eff}(G_F')+\frac{2G_F'}{\sqrt{2}}(\bar{\psi}^{p}_{L}\gamma^{\mu}\tau\chi^{p}_{1L}\cdot\bar{\chi}^{q}_{1L}\gamma_{\mu}\tau\psi^{q}_{L}+\bar{\psi}^{p}_{L}\gamma^{\mu}\tau\chi^{p}_{2L}\cdot\bar{\chi}^{q}_{2L}\gamma_{\mu}\tau\psi^{q}_{L})\ . \tag{5.4}$$

Thus Eq. (4.20) does *not* reproduce standard phenomenology because of a strong contribution from terms with the SU(4) indices p and q contracted between ψ's and χ's. G_F' is therefore not identifiable with the Fermi constant G_F.

In view of the importance of this conclusion it is worthwhile emphasizing upon which properties and assumptions it depends: (i) the fundamental assumption that the higher-generation quarks and leptons form a contravariant-vector superfield; (ii) the heuristic argument which indicated that bilinear coupling should only be implemented through the metric tensor; (iii) the observation that since the bilinear terms contained the kinetic energy terms for the matter fields, the strength as well as the form of QGF-matter interactions was precisely determined.

If the above conclusion is incorrect then it is probably due to the failure of point (ii). Let $\{T^{i}_{\alpha\bar\beta}(\bar\phi,\phi)\}$ be the set of linearly independent rank-two Hermitian covariant tensors formed from the metric, curvature, and covariant derivatives. Then, the bilinear terms are, in general,

$$K_2=\sum_i[C_i^{(1)}T^{i}_{\alpha\bar\beta}(\bar\phi,\phi)\bar{w}^{\beta}_{1}w^{\alpha}_{1}+C_i^{(2)}T^{i}_{\alpha\bar\beta}(\bar\phi,\phi)\bar{w}^{\beta}_{2}w^{\alpha}_{2}+C_i^{(3)}T^{i}_{\alpha\bar\beta}(\bar\phi,\phi)\bar{w}^{\beta}_{2}w^{\alpha}_{1}+C_i^{(3)*}T^{i}_{\alpha\bar\beta}(\bar\phi,\phi)\bar{w}^{\beta}_{1}w^{\alpha}_{2}]\ . \tag{5.5}$$

The kinetic energy terms are

$$T=\sum_i T^{i}_{\alpha\bar\beta}(0,0)(C_i^{(1)}\bar{w}^{\beta}_{1}w^{\alpha}_{1}+C_i^{(2)}\bar{w}^{\beta}_{2}w^{\alpha}_{2}+C_i^{(3)}\bar{w}^{\beta}_{2}w^{\alpha}_{1}+C_i^{(3)*}\bar{w}^{\beta}_{1}w^{\alpha}_{2})\ . \tag{5.6}$$

Diagonality and correct normalization require

$$\sum_i T^{i}_{\alpha\bar\beta}(0,0)C_i^{(1)}=\sum_i T^{i}_{\alpha\bar\beta}(0,0)C_i^{(2)}=g_{\alpha\bar\beta}(0,0)\ ,$$
$$\sum_i T^{i}_{\alpha\bar\beta}(0,0)C_i^{(3)}=0\ . \tag{5.7}$$

By expanding $T^{i}_{\alpha\bar\beta}(\bar\phi,\phi)$ in a Taylor series we find that the quartic terms are

$$K_2^{(4)}=\sum_i T^{i}_{\alpha\bar\beta,\gamma\bar\delta}(0,0)\times(C_i^{(1)}\bar\phi^{\bar\delta}\bar{w}^{\beta}_{1}\phi^{\gamma}w^{\alpha}_{1}+C_i^{(2)}\bar\phi^{\bar\delta}\bar{w}^{\beta}_{2}\phi^{\gamma}w^{\alpha}_{2}+C_i^{(3)}\bar\phi^{\bar\delta}\bar{w}^{\beta}_{2}\phi^{\gamma}w^{\alpha}_{1}+C_i^{(3)*}\bar\phi^{\bar\delta}\bar{w}^{\beta}_{1}\phi^{\gamma}w^{\alpha}_{2})\ . \tag{5.8}$$

The form of the $T^{i}_{\alpha\bar\beta,\gamma\bar\delta}(0,0)$ will determine whether or not isoscalar or isovector terms are produced. It is conceivable that Eqs. (5.7) can be maintained while simultaneously reproducing $L^{\rm st}_{\rm eff}$ from Eq. (5.8). However it is not clear that this procedure is "natural" or simple.

A further complication if assumption (ii) is relaxed even further is the possibility of non-Hermitian tensor couplings. These would yield additional flavor-changing terms at the four-fermion level. Linear couplings would then be allowed and these also have the potential of creating mixed Goldstone-field—matter kinetic terms. Further constraints such as Eq. (5.7) would then need to be imposed, or alternatively linear combinations of the Goldstone and matter fields would have to be taken so as to achieve diagonalization.

If it is conceded that the residual interactions of the GMY model do not yield weak interactions then consistency with experiment demands that v_1 be sufficiently large. Since v_1 is related to the hypercolor scale $\Lambda_{\rm HC}$ this implies an upper bound on the sizes of quarks and leptons.

It is now of interest to know explicitly what processes are admitted in Eq. (5.4). To this end we compare the terms

$$\bar{\psi}^p_L \gamma^\mu \tau \psi^p_L \cdot \bar{\chi}^q_{1L} \gamma_\mu \tau \chi^q_{1L}$$

and

$$\bar{\psi}^p_L \gamma^\mu \tau \chi^p_{1L} \cdot \bar{\chi}^q_{1L} \gamma_\mu \tau \psi^q_L \ .$$

Using the condensed notation $\bar{\psi}_{1L}\gamma^\mu\psi_{2L} \equiv \bar{\psi}_1\psi_2$ we obtain

$$(\bar{\psi}\tau\psi)\cdot(\bar{\chi}_1\tau\chi_1) = 2[(\bar{e}\nu_e)(\bar{\nu}_\mu\mu) + (\bar{e}\nu_e)(\bar{c}s) + (\bar{d}u)(\bar{\nu}_\mu\mu) + (\bar{d}u)(\bar{c}s) + \text{H.c.}] + (\bar{\nu}_e\nu_e - \bar{e}e + \bar{u}u - \bar{d}d)(\bar{\nu}_\mu\nu_\mu - \bar{\mu}\mu + \bar{c}c - \bar{s}s) \tag{5.9}$$

which of course has the form of the weak interactions. The charge-changing terms of $(\bar{\psi}\tau\chi_1)\cdot(\bar{\chi}_1\tau\psi)$ are displayed in Table I and the neutral terms are displayed in Table II.

The nonstandard piece has a number of different effects.

(i) The neutral-current processes $\bar{\nu}_e\nu_e \to \nu_\mu\bar{\nu}_\mu$, $e^+e^- \to \mu^+\mu^-$, $\bar{c}c \to \bar{u}u$, $\bar{s}s \to \bar{d}d$ are forbidden because a cancellation occurs via a Fierz transform between the standard and cross term.

(ii) There are flavor-changing charged currents which induce processes like $K^+ \to \mu^+\nu_e$. Note that $L_\mu + L_e$ is conserved.

(iii) There are flavor-changing neutral currents which cause $K^0_L \to \nu_e\bar{\nu}_\mu$, $K^0_L \to e^-\mu^+$, and other such $L_\mu + L_e$ conserving processes.

(iv) Certain standard processes are enhanced. One effect of this is a violation of quark-lepton universality. For example, the standard neutral-current terms $-(\bar{e}e)(\bar{\nu}_\mu\nu_\mu) - (\bar{e}e)(\bar{c}c)$ alter to $-3(\bar{e}e)(\bar{\nu}_\mu\nu_\mu) - (\bar{e}e)(\bar{c}c)$. Another consequence is the breakdown of e-μ universality, e.g., $-(\bar{e}e)(\bar{\nu}_e\nu_e) - (\bar{e}e)(\bar{\nu}_\mu\nu_\mu)$ is modified to $-(\bar{e}e)(\bar{\nu}_e\nu_e) - 3(\bar{e}e)(\bar{\nu}_\mu\nu_\mu)$.

The basic structure of the nonstandard term is that flavor-changing neutral currents (FCNC's) either enhance standard charged-current processes or cancel standard neutral-current processes or yield exotic effects. The flavor-changing charged currents (FCCC's) either enhance standard neutral-current effects or yield new generation-changing processes.

TABLE I. The charge-changing terms in $(\bar{\psi}\tau\chi)\cdot(\bar{\chi}\tau\psi)$. The full contribution is the sum of the terms in the left-hand column. The Fierz transform of these terms is displayed when they correspond to pieces of $(\bar{\psi}\tau\psi)\cdot(\bar{\chi}\tau\chi)$. The "comment" column either indicates a flavor-changing process which is induced or asserts that a particular term enhances a standard process in $(\bar{\psi}\tau\psi)\cdot(\bar{\chi}\tau\chi)$.

Four-fermion term	Fierz transform	Comment
$2(\bar{e}\nu_\mu)(\bar{\nu}_\mu e)$	$-2(\bar{e}e)(\bar{\nu}_\mu\nu_\mu)$	Enhances
$2(\bar{e}\nu_\mu)(\bar{c}d) + \text{H.c.}$		
$2(\bar{d}c)(\bar{c}d)$	$-2(\bar{d}d)(\bar{c}c)$	Enhances
$2(\bar{\nu}_e\mu)(\bar{\mu}\nu_e)$	$-2(\bar{\nu}_e\nu_e)(\bar{\mu}\mu)$	Enhances
$2(\bar{\nu}_e\mu)(\bar{s}u) + \text{H.c.}$		$K^+ \to \mu^+\nu_e$
$2(\bar{u}s)(\bar{s}u)$	$-2(\bar{u}u)(\bar{s}s)$	Enhances

A bound can be obtained for v_1 by comparing the decay $K^0_L \to e^-\mu^+$ with $K^+ \to \mu^+\nu_\mu$. The effective Lagrangians describing these processes are

$$L_{\rm eff} = 2\sqrt{2}G'_F(\bar{e}\mu)(\bar{s}d) \tag{5.10}$$

and

$$L^{\rm st}_{\rm eff} = 2\sqrt{2}G_F \sin\theta_c(\bar{\nu}_\mu\mu)(\bar{s}u) \ , \tag{5.11}$$

respectively. From this it follows that

$$\frac{\Gamma(K^0_L \to e^-\mu^+)}{\Gamma(K^+ \to \mu^+\nu_\mu)} = \left[\frac{G'_F}{G_F}\right]^2 \frac{1}{\sin^2\theta_c} \ . \tag{5.12}$$

Using the experimentally determined limit[12]

$$\frac{\Gamma(K^0_L \to e^-\mu^+)}{\Gamma(K^+ \to \mu^+\nu_\mu)} < 2.3\times 10^{-6} \tag{5.13}$$

we find that

$$G'_F < 3.3\times 10^{-4} G_F \ , \tag{5.14a}$$

$$v_1 > 7 \text{ TeV} \ . \tag{5.14b}$$

$\Lambda_{\rm HC}$ would be related to v_1 by a factor of order one.[13] Λ_{HC} would then have a lower bound of about 30 TeV.

The process which generally leads to the most severe bounds on $\Lambda_{\rm HC}$ is $K^0 - \bar{K}^0$ mixing.[13,14] This $\Delta S = 2$ process would be induced by a four-fermion term such as $(\bar{s}d)^2$. Note that such a term in our formalism can only arise from a non-Hermitian coupling such as $R_{;\bar{\gamma}\bar{\delta}}\bar{\omega}^{\bar{\gamma}}_1\bar{\omega}^{\bar{\delta}}_1$ which we have argued is surpressed or absent. This is an illustration of the phenomenological insight that the geometric approach provides.

Turning to the quartic matter terms displayed in Eq. (4.21) we note again the presence of family lepton-number-violating processes. There is an experimental bound on one such process:[15]

TABLE II. The neutral terms in $(\bar{\psi}\tau\chi)\cdot(\bar{\chi}\tau\psi)$. The layout is the same as Table I. The word "cancels" in the right-hand column means that this term cancels with a term in $(\bar{\psi}\tau\psi)\cdot(\bar{\chi}\tau\chi)$.

Four-fermion term	Fierz transform	Comment
$(\bar{\nu}_e\nu_\mu)(\bar{\nu}_\mu\nu_e)$	$-(\bar{\nu}_e\nu_e)(\bar{\nu}_\mu\nu_\mu)$	Cancels
$-(\bar{\nu}_e\nu_\mu)(\bar{\mu}e)$ + H.c.	$(\bar{\nu}_e e)(\bar{\mu}\nu_\mu)$ + H.c.	Enhances
$(\bar{\nu}_e\nu_\mu)(\bar{c}u)$ + H.c.		
$-(\bar{\nu}_e\nu_\mu)(\bar{s}d)$ + H.c.		$K_L^0\to\nu_e\bar{\nu}_\mu$
$(\bar{e}\mu)(\bar{\mu}e)$	$-(\bar{e}e)(\bar{\mu}\mu)$	Cancels
$-(\bar{e}\mu)(\bar{c}u)$ + H.c.		
$(\bar{e}\mu)(\bar{s}d)$ + H.c.		$K_L^0\to e^-\mu^+$, $K^+\to\pi^+\mu e$
$(\bar{u}c)(\bar{c}u)$	$-(\bar{u}u)(\bar{c}c)$	Cancels
$-(\bar{u}c)(\bar{s}d)$ + H.c.	$(\bar{u}d)(\bar{s}c)$ + H.c.	Enhances
$(\bar{d}s)(\bar{s}d)$	$-(\bar{d}d)(\bar{s}s)$	Cancels

$$\frac{\Gamma(\tau^+\to\mu^+\mu^+\mu^-)}{\Gamma(\tau^+\to\text{all})}<4.9\times10^{-4}\,. \tag{5.15}$$

The strength of this process is governed by the value of $(c_5^*+s_5^*)/v_1^2$. Neglecting the muon mass gives the following expression for the decay width:

$$\Gamma(\tau^+\to\mu^+\mu^+\mu^-)=\frac{m_\tau^5}{6\pi^3}\frac{|s_5+c_5|^2}{v_1^4} \tag{5.16}$$

the τ lifetime is

$$\tau_\tau=(3.3\pm0.4)\times10^{-13}\text{ sec}\,, \tag{5.17}$$

which yields a total decay width of

$$\Gamma_\tau=\frac{\hbar}{\tau}=(1.9\pm0.2)\times10^{-12}\text{ GeV} \tag{5.18}$$

and so

$$|c_5+s_5|<10^{-7}v_1^2\,. \tag{5.19}$$

If we take $v_1\simeq10$ TeV then

$$|c_5+s_5|<10\,. \tag{5.20}$$

VI. CONCLUSION

An effective-Lagrangian study of a supersymmetric composite model which features one QGF generation and two matter generations has shown that under physically reasonable assumptions the residual four-fermion interactions do not reproduce weak coupling. The effective interactions display some of the characteristics of weak interactions due to the possibility of a novinolike mechanism, but they fail to suppress flavor-changing processes such as $K_L^0\to e^-\mu^+$.

Introducing fundamental weak interactions and using experimental data show that the hypercolor scale Λ_{HC} must be greater than a few tens of TeV's.

The effective Lagrangian constructed yields a definite pattern for the residual interactions as displayed in Tables I and II. Further tests of quark-lepton and e-μ-τ universality as well as further searches for flavor-changing processes are necessary to see if this pattern is realized in nature.

The geometric approach employed provided an insight into the structure of effective Lagrangians for all QGF-matter models. The observation is that the candidate coupling tensors may be classified according to the order of the derivative of the Kähler potential that they depend on. This is also connected with the order of processes in the Goldstone sector. It was argued that this provides a natural suppression of Goldstone-field—matter couplings which require knowledge of the Kähler potential to higher order than the matter coupling itself.

Finally, there are other major issues that have not been considered in this study, namely, mass generation, left-right-symmetry breaking, and supersymmetry breaking. These sorts of problems must be addressed with more and more seriousness so that the predictive power of supersymmetric composite models rises, in the end hopefully meeting the challenges originally set for subconstituent models.

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APPENDIX

The Kähler potential to order four for the GMY model is listed below in convenient groupings.

(i) Kinetic energy terms:

$$\begin{aligned}K_2=&v_1^2\bar{L}^i_\alpha L_i^\alpha+v_2^2\bar{R}^{\bar\alpha}_{\bar i}R^{\bar i}_{\bar\alpha}+v_{11}^2(\bar{L}_1L_1+\bar{L}_2L_2)+v_{33}^2(\bar{R}_1R_1+\bar{R}_2R_2)+u_{12}(\bar{L}_2L_1+\bar{L}_1L_2)\\&+u_{34}(\bar{R}_2R_1+\bar{R}_1R_2)+u_{31}(\bar{R}_1+\bar{R}_2)(L_1+L_2)+u_{31}(\bar{L}_1+\bar{L}_2)(R_1+R_2)\,.\end{aligned} \tag{A1}$$

The singlet sector is not diagonalized; the physical singlet fields correspond to linear combinations of L_1, L_2, R_1, and

R_2 which diagonalize the kinetic energy terms. All the fields used in this section are dimensionless. Since the Kähler potential must be of dimension two to make the Lagrangian given by Eq. (3.5) of dimension four, v_1, v_2, v_{11}, and v_{33} have the dimensions of mass and all other coefficients have dimension (mass)^2. If the scalar component of L_i^α is A_i^α then the canonical scalar field is actually $v_1 A_i^\alpha$.

(ii) Third-order terms involving nonsinglet fields:

$$K_3^{\rm NS}=\tfrac{3}{4}i(v_{11}{}^2+u_{12})\bar{L}{}^i{}_\alpha L_i^\alpha(L_1+L_2-\bar{L}_1-\bar{L}_2)-\tfrac{3}{2}iu_{13}\bar{L}{}^i{}_\alpha L_i^\alpha(R_1+R_2-\bar{R}_1-\bar{R}_2+L_1+L_2-\bar{L}_1-\bar{L}_2)$$
$$+\tfrac{3}{4}i(v_{33}{}^2+u_{34})\bar{R}{}^{\bar\alpha}_{\bar i}R^{\bar i}_{\bar\alpha}(R_1+R_2-\bar{R}_1-\bar{R}_2)\ . \tag{A2}$$

(iii) Third-order terms for singlet fields:

$$\begin{aligned}K_3^S={}&x_{111}(\bar{L}_2L_2{}^2+\bar{L}_1L_1{}^2)+\bar{x}_{111}(\bar{L}_2{}^2L_2+\bar{L}_1{}^2L_1)+x_{122}(\bar{L}_1L_2{}^2+\bar{L}_2L_1{}^2)+\bar{x}_{122}(\bar{L}_1{}^2L_2+\bar{L}_2{}^2L_1)\\&+x_{112}(\bar{L}_2+\bar{L}_1)L_1L_2+\bar{x}_{112}\bar{L}_1\bar{L}_2(L_1+L_2)+x_{123}(\bar{L}_1L_2+\bar{L}_2L_1)(R_1+R_2)\\&-x_{123}(\bar{R}_1+\bar{R}_2)(\bar{L}_1L_2+\bar{L}_2L_1)+x_{134}(\bar{L}_1+\bar{L}_2)R_1R_2+\bar{x}_{134}\bar{R}_1\bar{R}_2(L_1+L_2)\\&+x_{133}(\bar{L}_1+\bar{L}_2)(R_1{}^2+R_2{}^2)+\bar{x}_{133}(\bar{R}_1{}^2+\bar{R}_2{}^2)(L_1+L_2)\\&+x_{113}(\bar{L}_2L_2+\bar{L}_1L_1)(R_1+R_2)-x_{113}(\bar{R}_1+\bar{R}_2)(\bar{L}_2L_2+\bar{L}_1L_1)+(L_1\leftrightarrow R_1,L_2\leftrightarrow R_2,1\leftrightarrow 3,2\leftrightarrow 4)\ .\end{aligned}\tag{A3}$$

The last term indicates that the Kähler potential has left-right symmetry. For example,

$$x_{122}(\bar{L}_1L_2{}^2+\bar{L}_2L_1{}^2)\to x_{344}(\bar{R}_1R_2{}^2+\bar{R}_2R_1{}^2)\ .$$

Note also that the permutation symmetries $L_1\leftrightarrow L_2$ and $R_1\leftrightarrow R_2$ have been imposed due to the form of the algebra given by Eq. (4.19). In the equations that follow it will be understood that these symmetries imply additional equations. The coefficients x also obey

$$\begin{aligned}&0=(x_{112}+\bar{x}_{112})+2(x_{111}+\bar{x}_{111})\ ,\\&0=(x_{112}+\bar{x}_{112})+2(x_{122}+\bar{x}_{122})\ ,\\&0=x_{123}+x_{113}+\bar{x}_{312}+2\bar{x}_{311}\ ,\\&0=\bar{x}_{123}+\bar{x}_{113}+x_{312}+2x_{311}\ ,\\&x_{123},\ x_{113}\ \text{pure imaginary}\ .\end{aligned}\tag{A4}$$

(iv) Fourth-order terms involving nonsinglet fields only:

$$K_4^{\rm NS}=\tfrac{9}{16}(v_{11}{}^2+u_{12})\bar{L}{}^i{}_\alpha\bar{L}{}^j{}_\beta L_i^\alpha L_j^\beta-\tfrac{1}{2}v_1{}^2\bar{L}{}^i{}_\alpha\bar{L}{}^j{}_\beta L_j^\alpha L_i^\beta+\tfrac{9}{16}(v_{33}{}^2+u_{34})\bar{R}{}^{\bar\alpha}_{\bar i}\bar{R}{}^{\bar\beta}_{\bar j}R^{\bar i}_{\bar\alpha}R^{\bar j}_{\bar\beta}$$
$$-\tfrac{1}{2}v_2{}^2\bar{R}{}^{\bar\alpha}_{\bar i}\bar{R}{}^{\bar\beta}_{\bar j}R^{\bar j}_{\bar\alpha}R^{\bar i}_{\bar\beta}+\tfrac{9}{4}u_{13}\bar{L}{}^i{}_\alpha L_i^\alpha\bar{R}{}^{\bar\beta}_{\bar j}R^{\bar j}_{\bar\beta}\ . \tag{A5}$$

(v) Fourth-order terms involving singlet and nonsinglet fields:

$$\begin{aligned}K_4^{S\text{-}\rm NS}={}&\tfrac{3}{4}i(2x_{111}+x_{112})\bar{L}{}^i{}_\alpha L_i^\alpha(\bar{L}_1L_1+\bar{L}_2L_2)+\tfrac{3}{2}ix_{313}\bar{L}{}^i{}_\alpha L_i^\alpha(\bar{R}_1R_1+\bar{R}_2R_2)\\&+\tfrac{3}{4}i(2x_{122}+x_{112})\bar{L}{}^i{}_\alpha L_i^\alpha(\bar{L}_1L_2+\bar{L}_2L_1)+\tfrac{3}{2}ix_{314}\bar{L}{}^i{}_\alpha L_i^\alpha(\bar{R}_1R_2+\bar{R}_2R_1)\\&+\tfrac{3}{4}i(x_{123}+x_{113})\bar{L}{}^i{}_\alpha L_i^\alpha(\bar{L}_1+\bar{L}_2)(R_1+R_2)+\tfrac{3}{4}i(x_{123}+x_{113})\bar{L}{}^i{}_\alpha L_i^\alpha(\bar{R}_1+\bar{R}_2)(L_1+L_2)\\&-\tfrac{3}{4}i(x_{111}+x_{122})\bar{L}{}^i{}_\alpha L_i^\alpha(L_1{}^2+L_2{}^2)+\tfrac{3}{4}i(\bar{x}_{111}+\bar{x}_{122})\bar{L}{}^i{}_\alpha L_i^\alpha(\bar{L}_1{}^2+\bar{L}_2{}^2)\\&-\tfrac{3}{2}ix_{133}\bar{L}{}^i{}_\alpha L_i^\alpha(R_1{}^2+R_2{}^2)+\tfrac{3}{2}i\bar{x}_{133}\bar{L}{}^i{}_\alpha L_i^\alpha(\bar{R}_1{}^2+\bar{R}_2{}^2)-\tfrac{3}{2}ix_{112}\bar{L}{}^i{}_\alpha L_i^\alpha L_1L_2\\&+\tfrac{3}{2}i\bar{x}_{112}\bar{L}{}^i{}_\alpha L_i^\alpha\bar{L}_1\bar{L}_2-\tfrac{3}{2}ix_{134}\bar{L}{}^i{}_\alpha L_i^\alpha R_1R_2+\tfrac{3}{2}i\bar{x}_{134}\bar{L}{}^i{}_\alpha L_i^\alpha\bar{R}_1\bar{R}_2-\tfrac{3}{4}i(x_{123}+x_{113})\bar{L}{}^i{}_\alpha L_i^\alpha(L_1+L_2)(R_1+R_2)\\&+\tfrac{3}{4}i(\bar{x}_{123}+\bar{x}_{113})\bar{L}{}^i{}_\alpha L_i^\alpha(\bar{L}_1+\bar{L}_2)(\bar{R}_1+\bar{R}_2)+(L\leftrightarrow R)\ .\end{aligned}\tag{A6}$$

(vi) The fourth-order contribution involving the singlet fields only will not be listed as it is lengthy and not particularly informative. Suffice it to say that there are two types of terms (of the form $\bar\psi^2\psi^2$ and $\bar\psi\psi^3$) and the Killing equation (3.12) relates their coefficients among themselves.

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