Unified multiplicity scaling in electron-positron and muon-proton collisions

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A quantum-statistical approach involving the coherency phenomena is applied to electron-positron annihilations (in the c.m.-system energy range $4 \lesssim W \lesssim 34$ GeV) and muon-proton collisions ($4 \lesssim W \lesssim 20$ GeV). It is found that in both cases a new (energy-independent) multiplicity scaling distribution (with the same value of the only one parameter in its functional form) is sufficiently accurate for the data on charged multiplicities and on the corresponding statistical moments. At higher energies, the multiplicity distributions as well as the values of several statistical moments are predicted.

I. INTRODUCTION

The experimental data on the multiplicity distributions of charged secondaries in lepton-lepton, lepton-hadron, and hadron-hadron collisions, taken with the data on statistical moments, reveal important information about the scaling properties of the corresponding cross sections.

For instance, the work of Koba, Nielsen, and Olesen[1] (KNO) leads to the conclusion that the second dispersion D_2 is proportional to the average multiplicity $\langle n \rangle$. The plot of D_2 versus $\langle n \rangle$ for the e^+e^- data[2-7] is seen in Fig. 1(a) and for the μ^+p data[8] in Fig. 1(b). Both figures [in Fig. 1(a) see mainly the TASSO points] suggest with sufficient accuracy that the KNO scaling[1] is not satisfied at the presently available energies. On the other hand, the KNO scaling is not obeyed by the CERN collider data[9,10] even if until the highest CERN ISR energies the KNO scaling was satisfied. The multiplicity distributions arising in electron-positron annihilations have been compared with those arising in proton-proton and antiproton-proton collisions from different points of view just recently.[11-13] In this paper several aspects on the e^+e^- and pp together with $\bar{p}p$ multiplicity distributions are completed by considerations involving an interlink, the lepton-hadron case, especially the muon-proton collisions. The pieces of knowledge which follow in that way allow us to pick out the similarities as well as the discrepancies between lepton- and hadron-induced elementary reactions.

In Sec. II a quantum-statistical approach is briefly outlined which incorporates, aside from the stochastic phenomena, the influence of the coherent processes. It also takes into account the increasing number of secondaries produced in jets. As a result, a relatively simple scaling function is derived which involves only one free parameter in its functional form. This scaling function is very well fitted for the description of the e^+e^- as well as μ^+p data on multiplicities and corresponding statistical moments, as is seen in Sec. III. The conclusions about the applicability of the procedure developed in this paper can be found in the last section.

II. QUANTUM-STATISTICAL APPROACH

A. Stochastic and coherent fields

In this paper a quantum-statistical approach is applied which involves M phase-space cells with superposition of stochastic and coherent fields. Moreover, there is also a reservoir where pure coherent fields are generated and where after the production of particles several holes might appear. Let the coherent part of the superposition produce $\langle n_C \rangle \kappa^2$ charged secondaries while the pure coherent field (reservoir) produces

$$\langle n_C \rangle - \langle n_C \rangle \kappa^2 = \langle n_C \rangle (1-\kappa^2) \equiv \alpha , \tag{1}$$

of them (κ^2 is a convenient parameter). It holds $\langle n \rangle = \langle n_T \rangle + \langle n_C \rangle$ where $\langle n_T \rangle$ ($\langle n_C \rangle$) represents the average number of stochastically (coherently) produced charged secondaries. In this case the full probability to observe n charged particles (as derived by Peřina and Horák[14]) has the form

$$P(n) = \left[1+\frac{\langle n_T \rangle}{M}\right]^{-M} \exp\left[-\frac{M\langle n_C \rangle + \alpha \langle n_T \rangle}{\langle n_T \rangle + M}\right] \sum_{j=0}^{n} \frac{\alpha^{n-j}}{(n-j)!\Gamma(m+j)} \left[1+\frac{M}{\langle n_T \rangle}\right]^{-j} \times L_j^{M-1}\left[-\frac{\kappa^2 M^2 \langle n_C \rangle}{\langle n_T \rangle(\langle n_T \rangle + M)}\right] . \tag{2}$$

In Eq. (2), $L_a^b(x)$ are the Laguerre polynomials (in the normalization of Ref. 15). In the KNO limit, $n \to \infty$, $\langle n \rangle \to \infty$ (plus additional assumptions), and Eq. (2) gives[16]

$$\phi_\alpha = \psi(z_\alpha) , \tag{3}$$

where

$$\phi_\alpha \equiv (\langle n \rangle - \alpha) P(n) \tag{4}$$

and

$$\psi(z) = M(1+R^2)(\xi/R)^{M-1} e^{-M(\xi^2+R^2)} I_{M-1}(2MR\xi) \tag{5}$$

with

$$\xi = [z(1+R^2)]^{1/2} \equiv \xi(z) ; \tag{6}$$

$I_P(x)$ represents the modified Bessel function.[15]

In Eq. (3)

$$z_\alpha = (n-\alpha)/(\langle n \rangle - \alpha) , \tag{7}$$

α being defined by Eq. (1). The genuine KNO case is obtained if $\alpha = 0$; then Eq. (2) reduces to the Peřina-McGill distribution[17] which was considered in several other papers.[11,18]

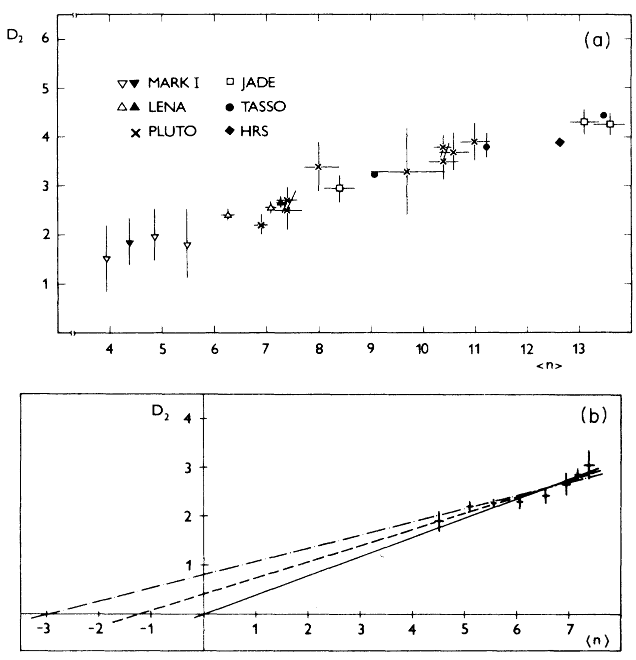

FIG. 1. (a) The dependence of the second dispersion $D_2 = \langle (n - \langle n \rangle)^2 \rangle^{1/2}$ on the average charged multiplicity $\langle n \rangle$ in e^+e^- annihilations. The data are taken from the following sources: Mark I (Ref. 2), LENA (Ref. 3), PLUTO (Ref. 4), JADE (Ref. 5), TASSO (Ref. 6), and HRS (Ref. 7). The points with filled-in symbols are used in Fig. 3(a). (b) The dependence of the second dispersion D_2 on the average charged multiplicity $\langle n \rangle$, in $\mu^+ p$ collisions. The data are taken from Ref. 8. The solid line corresponds to the KNO scaling while the data suggest an intercept $\langle n \rangle = \langle n \rangle_0$ with $-3 \lesssim \langle n \rangle_0 \lesssim -1.25$, as it is indicated by the dashed and dot-dashed lines.

In the present case the effective signal-to-noise ratio is expressed in the form

$$R^2 = (\langle n_C \rangle - \alpha)/\langle n_T \rangle$$
$$= \langle n_C \rangle \kappa^2 / \langle n_T \rangle . \tag{8}$$

The ratio on the right-hand side (RHS) of Eq. (8) is influenced by the quantities characterizing the cells with the aforementioned superposition of fields. We consider the quantities

$$\langle n \rangle, \; R, \; \alpha , \tag{9}$$

as primary ones and the others (namely, $\langle n_T \rangle$, $\langle n_C \rangle$, κ^2) can be expressed by means of them:

$$\langle n_T \rangle = (\langle n \rangle - \alpha)/(1+R^2) ,$$
$$\langle n_C \rangle = (\langle n \rangle R^2 + \alpha)/(1+R^2) ,$$
$$\kappa^2 = R^2(\langle n \rangle - \alpha)/(\langle n \rangle R^2 + \alpha) .$$

B. Statistical moments

The statistical moments C_q and D_q of the scaling distribution $\psi(z_\alpha)$, ψ given by (5) and z_α by (7), are expressed as

$$C_q = \langle n^q \rangle / \langle n \rangle^q$$
$$= \sum_{j=0}^{q} \begin{bmatrix} q \\ j \end{bmatrix} \alpha^{q-j} (\langle n \rangle - \alpha)^j V_j / \langle n \rangle^q \tag{10}$$

(with $C_0 = C_1 = 1$) and

$$D_q = \langle (n - \langle n \rangle)^q \rangle^{1/q}$$
$$= (\langle n \rangle - \alpha) \left[\sum_{j=0}^{q} (-1)^{q+j} \begin{bmatrix} q \\ j \end{bmatrix} V_j \right]^{1/q} , \tag{11}$$

where

$$V_j = [\Gamma(M+j)/\Gamma(M)]$$
$$\times F(-j, M, -MR^2)/[M(1+R^2)]^j \tag{12}$$

(with $V_0 = V_1 = 1$); F is the confluent hypergeometric function.[15] In the pure KNO case ($\alpha = 0$) we obtain $C_q = V_q$. The procedure adopted in this paper takes the quantities V_j as energy independent. Then a KNO-type result, $D_q = \text{const} \times \langle n \rangle$ follows from (11) in the energy or multiplicity regions where the dependence $\alpha = \alpha(\langle n \rangle)$ can be approximated by the proportionality $\alpha = \text{const} \times \langle n \rangle$; the validity of such a result cannot be excluded also at still higher energies.

The uncertainties of those moments are obtained in the form

$$\Delta C_q = -[q \langle n \rangle (C_q - C_{q-1})/(\langle n \rangle - \alpha)] \Delta(\alpha / \langle n \rangle) \tag{13}$$

and

$$\Delta D_q = [D_q / (\langle n \rangle - \alpha)] \Delta(\langle n \rangle - \alpha) . \tag{14}$$

C. Asymptotics

For our next purposes two asymptotic cases of the density distribution $\psi(z_\alpha)$ are important.

(i) If the effective signal-to-noise parameter is very small, $R^2 \to 0$, R^2 given by (8), then the scaling function $\psi(z_\alpha)$ leads to the gamma distribution

$$P_{\text{gamma}} = [M(Mz)^{M-1}/(M-1)!]e^{-Mz} \equiv [\psi(z)]_{R^2\to 0} \tag{15}$$

with $z = z_\alpha$, z_α being given by (7). In this case the statistical moments C_q and D_q are given by (10) and (11) where

$$V_j = \Gamma(M+j)/[M^j\Gamma(M)] \equiv (V_j)_{R^2\to 0} \,. \tag{16}$$

It is well known that just the Bose-Einstein distribution (with M cells) represents the full distribution corresponding to the asymptotics (15).

(ii) With respect to the increasing number of charged secondaries produced (coherently) in jets, let us assume that with the increasing energy of the collision, the relative number of stochastically produced secondaries decreases so that the effective signal-to-noise parameter $R^2 \to \infty$. Moreover, let us assume that when approaching this asymptotics, the number M of sources effectively seems to be very small, $M \to 0$, but the product

$$MR^2 \to K \,, \tag{17}$$

where K is a constant. (The asymptotics $M \to 0$ with an increasing c.m.-system energy is suggested by the existing data from the CERN collider[19] as well as from the TASSO group[20] as far as the Bose-Einstein distribution with the c.m.-system energy-dependent number of cells is applied.) In this case the function $\psi(z_\alpha)$ leads to a one-parametric scaling distribution

$$\lim_{\substack{M\to 0\\ R^2\to\infty}} [\psi(z_\alpha)]_{MR^2\to K} = Kz^{-1/2}e^{-K(z+1)}I_1(2Kz^{1/2}) \equiv [\psi(z)]R^2 \to \infty \tag{18}$$

with $z = z_\alpha$, z_α given by (7). For large z, the asymptotic tail of (18) is proportional to $z^{-3/4}e^{-Kz}$. The QCD calculations give for that tail similar expressions, e.g., $z^{-3/2}e^{-2z}$ in Ref. 21 or $ze^{-2.6z}$ in Ref. 22.

With respect to the distribution (18) the quantity V_j, Eq. (12), involved in the statistical moments C_q and D_q has the form

$$V_j = \sum_{a=1}^{j} \frac{[\Gamma(j+1)/\Gamma(j-a+1)]^2}{j(j-a+1)\Gamma(a)} \frac{1}{K^{a-1}} \equiv (V_j)_{R^2\to\infty} \,. \tag{19}$$

Using Mandel's relation,[23] the full distribution corresponding to the asymptotic form (18) is found to be

$$P(n) = \frac{K^2 e^{-K}}{n(n!)} \left[\frac{\langle n\rangle}{\langle n\rangle + K}\right]^{n+1} \times e^{-K^2/(\langle n\rangle+K)} L^1_{n-1}\left[-\frac{K^2}{\langle n\rangle + K}\right] . \tag{20}$$

We note that Eq. (20) does not follow simply as a special case of Eq. (2).

In the next section we show that the relations (18) and (19) are suited well for e^+e^- and μ^+p collisions.

III. APPLICATION TO THE e^+e^- AND μ^+p MULTIPLICITY DATA

A. Parameters α and K

In this paper the parameter K, Eq. (17), is considered as a c.m.-system energy-independent constant. It will be seen later that this parameter gets the same value for the e^+e^- as well as μ^+p data. On the other hand, the parameter α, Eq. (1), is allowed to be c.m.-system energy dependent. In this case the variables ϕ_α and z_α, Eqs. (4) and (7), change with the c.m.-system energy but the functional form of the scaling distribution (18) is c.m.-system energy independent. This point of view represents a reali-

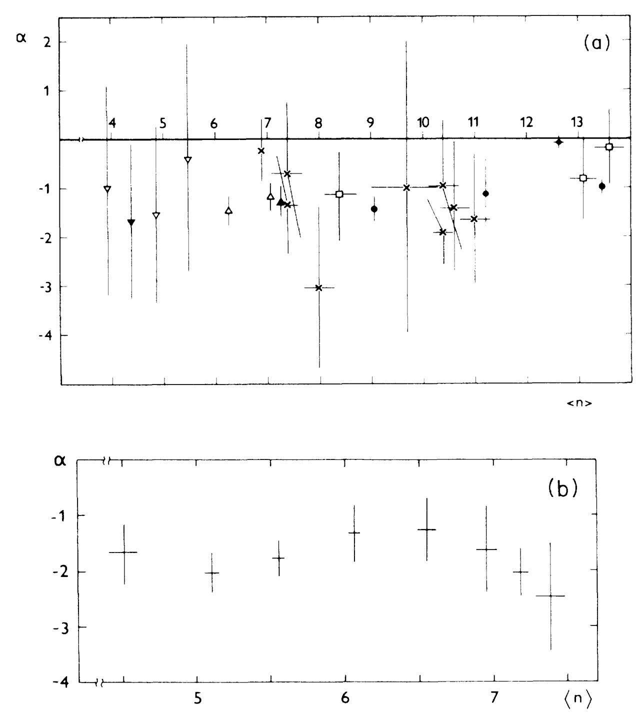

FIG. 2. (a) The dependence of the parameter α characterizing the number of charged secondaries produced by a reservoir, on the average charged multiplicity $\langle n\rangle$ for the e^+e^- annihilations. The notation of the data points is the same as in Fig. 1(a). The location of all points is fixed by the condition $\alpha + \Delta\alpha \approx 2$ (at the PLUTO c.m.-system energy 22 GeV where $\langle n\rangle = 9.7$). (b) The μ^+p collisions. The data points refer to those of Fig. 1(b). Otherwise the same as in (a).

TABLE I. The data on e^+e^- annihilations. The average charged multiplicities $\langle n \rangle$ and the second dispersions D_2 are taken from the sources quoted in the last column. The values of the parameter α are determined by the condition $(D_2)_{\text{theor}} = (D_2)_{\text{exper}}$, Eq. (21). The values of D_3 and D_4 represent the predictions involving the one-parametric scaling distribution (18).

1 W (GeV)	2 $\langle n \rangle$	3 D_2	4 α	5 D_3	6 D_4	7 Ref.
4.0	4.37±0.04	1.87±0.49	−1.69±1.59	1.45±0.38	2.52±0.66	2
9.3	7.28±0.11	2.64±0.09	−1.28±0.31	2.04±0.08	3.55±0.14	3
14.0	9.08±0.05	3.24±0.08	−1.43±0.26	2.51±0.06	4.36±0.11	6
22.0	11.22±0.07	3.81±0.25	−1.14±0.81	2.95±0.19	5.13±0.34	6
29.0	12.62±0.03	3.92±0.03	−0.09±0.10	3.03±0.02	5.28±0.04	7
34.5	13.48±0.03	4.46±0.05	−0.98±0.16	3.45±0.04	6.00±0.07	6
50.0	15.5±0.1	5.2±0.1	−1.36±0.34	4.0±0.1	7.0±0.2	Predictions
100.0	21.3±0.2	6.6±0.3	−0.10±0.99	5.1±0.3	8.9±0.4	

zation of the genuine KNO idea.[1]

With respect to the even (odd) values of multiplicities in the e^+e^- (μ^+p) collisions we allow the parameter $\alpha \le 2$ ($\alpha \le 1$) and, moreover, it may be negative.

B. Statistical moments

The basic ingredient in our procedure consists in identification of the theoretical second dispersion $D_2 = \langle (n - \langle n \rangle)^2 \rangle^{1/2}$ with the experimental one:

$$(D_2)_{\text{theor}} = (D_2)_{\text{exper}} . \tag{21}$$

With $(D_2)_{\text{theor}}$ from (11) we get

$$\alpha = \langle n \rangle - \sqrt{K/2}\, D_2 , \tag{22}$$

where, on the RHS, D_2 as well as $\langle n \rangle$ is taken from the experimental data.

The value of the parameter K is fixed as follows: The e^+e^- collisions reveal the simplest structure (from l-l, l-h, and h-h collisions); i.e., in this case the minimal number of particles is taken from the reservoir. We allow $\alpha + \Delta\alpha$ to be saturated, i.e.,

$$\alpha + \Delta\alpha \approx 2 \tag{23}$$

and we look for such an energy where (23) is satisfied while at all other energies $\alpha + \Delta\alpha < 2$. Using the same data as in Fig. 1(a) we observe that condition (23) is satisfied by the PLUTO data at the c.m.-system energy $W = 22$ GeV. Then

$$K = 21 . \tag{24}$$

Fixing the parameter K at that value, we obtain the parameter α (22) at every energy under consideration (together with the uncertainty $\Delta\alpha$ taking into account the experimental—only statistical—uncertainties $\Delta\langle n \rangle$ and ΔD_2). The result is seen in Fig. 2(a) and for some energies in the fourth column of Table I.

With the same value of the parameter K, Eq. (24), the corresponding picture for the μ^+p data is seen in Fig. 2(b) and in the fourth column of Table II. The more involved structure of the μ^+p collisions as compared with the e^+e^- annihilations is exhibited in that direction of our considerations which we have just described. Namely, this procedure cannot be performed in a reversed way: if the value of the parameter K is fixed by the requirement $\alpha + \Delta\alpha \approx 1$ in the μ^+p collisions (then $K = 12.4$ at the energy $12 \lesssim W \lesssim 14$ GeV; at other energies $\alpha + \Delta\alpha < 1$), and if the same value ($K = 12.4$) is applied also in the case of the e^+e^- annihilations, then at some energies (in the e^+e^- case) the value $\alpha + \Delta\alpha > 2$ is obtained giving rise to the complex values of the probabilities for lower values of the multiplicities [compare Eqs. (7) and (18)]. This fact means that the number of secondaries produced in the reservoir is now too small for the e^+e^- case.

TABLE II. The data on the μ^+p collisions. The average charged multiplicities $\langle n \rangle$ and the second dispersions D_2 are taken from Ref. 8. Otherwise the same as in Table I.

1 W (GeV)	2 $\langle n \rangle$	3 D_2	4 α	5 D_3	6 D_4
$4 \lesssim W \lesssim 6$	4.51±0.09	1.90±0.18	−1.65±0.59	1.47±0.14	2.56±0.25
$6 \lesssim W \lesssim 8$	5.10±0.05	2.20±0.11	−2.03±0.36	1.70±0.09	2.96±0.15
$8 \lesssim W \lesssim 10$	5.55±0.04	2.36±0.10	−1.78±0.33	1.75±0.08	3.04±0.14
$10 \lesssim W \lesssim 12$	6.06±0.04	2.28±0.16	−1.33±0.52	1.76±0.12	3.07±0.22
$12 \lesssim W \lesssim 14$	6.55±0.06	2.41±0.18	−1.26±0.59	1.86±0.14	3.24±0.25
$14 \lesssim W \lesssim 16$	6.95±0.08	2.65±0.24	−1.64±0.78	2.05±0.19	3.57±0.33
$16 \lesssim W \lesssim 18$	7.18±0.05	2.84±0.13	−2.03±0.42	2.20±0.10	3.82±0.18
$18 \lesssim W \lesssim 20$	7.38±0.10	3.04±0.30	−2.48±0.98	2.35±0.23	4.09±0.41
40	9.24±0.34	3.71±0.45	−2.79±1.50	2.86±0.37	4.99±0.64

The fifth and sixth columns of Tables I and II contain our predictions obtained by means of the corresponding values of α's quoted there in the fourth columns and using Eqs. (11) and (14). In the last lines of both tables (for the c.m.-system energies $W=\sqrt{s}\geq 40$ GeV) the quoted values of $\langle n \rangle$ and D_2 have been obtained by extrapolations (with relations of Refs. 6 and 8).

Because of the independency of the parameter K on the c.m.-system energy also the quantities V_j, Eq. (19), do not change with that energy. However, the experimentally observed energy variation of the C_q moments, Eq. (10), is in our approach achieved by the energy-dependent parameter α (besides the energy-dependent average multiplicity $\langle n \rangle$). We note that as far as the approximation $\alpha=\text{const}\times\langle n \rangle$ is used in some energy ranges then Eq. (13) leads to the constant C_q moments ($\Delta C_q=0$).

C. Multiplicity plots

We use the values of the parameter α as they are given in Table I and the multiplicity data from references quoted there in the last column (the multiplicities at other energies involved in Fig. 1(a) are left out with respect to a better insight into the following figure). The plot of ϕ_α, Eq. (4), versus z_α, Eq. (7), for the e^+e^- annihilations is seen in Fig. 3(a). Similarly, the plot for the μ^+p collisions (using the values of α from Table I and multiplicities from Ref. 8) is seen in Fig. 3(b).

The following applies to both Figs. 3(a) and 3(b). (i) The solid curves represent the asymptotic scaling function $[\psi(z_\alpha)]_{R^2\to\infty}$, Eq. (18), with the same value of the parameter K, $K=21$, Eq. (24). The dashed lines are obtained by means of the (asymptotic) gamma distribution $[\psi(z_\alpha)]_{R^2\to 0}$, Eq. (15), corresponding to the case when $R^2\to 0$ and $M=K/2=10.52$. Even if there is no big difference between those two kinds of curves, the scaling function $[\psi(z_\alpha)]_{R^2\to\infty}$ gives rise to a better fit. (ii) In between those two kinds of asymptotic curves the other curves lie which are obtained by means of the scaling function (5) with the parameters (M,R) specified by condition (21). (iii) The probability $P(n)$ (on the perpendicular axis) is multiplied by two, as is usual due to the charge conservation.[24] (iv) The solid lines allow the prediction of the values of the e^+e^- and μ^+p charged multiplicities at still higher energies as far as a convenient extrapolation for the c.m.-system energy dependence of the average charged multiplicity and of the parameter α [related to the second dispersion, e.g., via Eqs. (21) or (22)] is applied as the input information.

Retaining the value (24) of the parameter K one can continue the considerations in that direction which was followed in the preceding point (b) when passing from the e^+e^- to the μ^+p collisions: now one can try to extend this procedure also to the $pp+\bar{p}p$ case. However, the data of the last-mentioned collisions (involved in Refs. 9 and 10) exhibit a systematic deviation from the theoretical dependence as obtained by means of Eqs. (18) and (24). On the other hand, let us assume that the condition on the saturation, Eq. (23), is not satisfied, namely, in such a sense that the value of K is higher than that given in Eq. (24). Now, if condition (21) or (22) still represents the

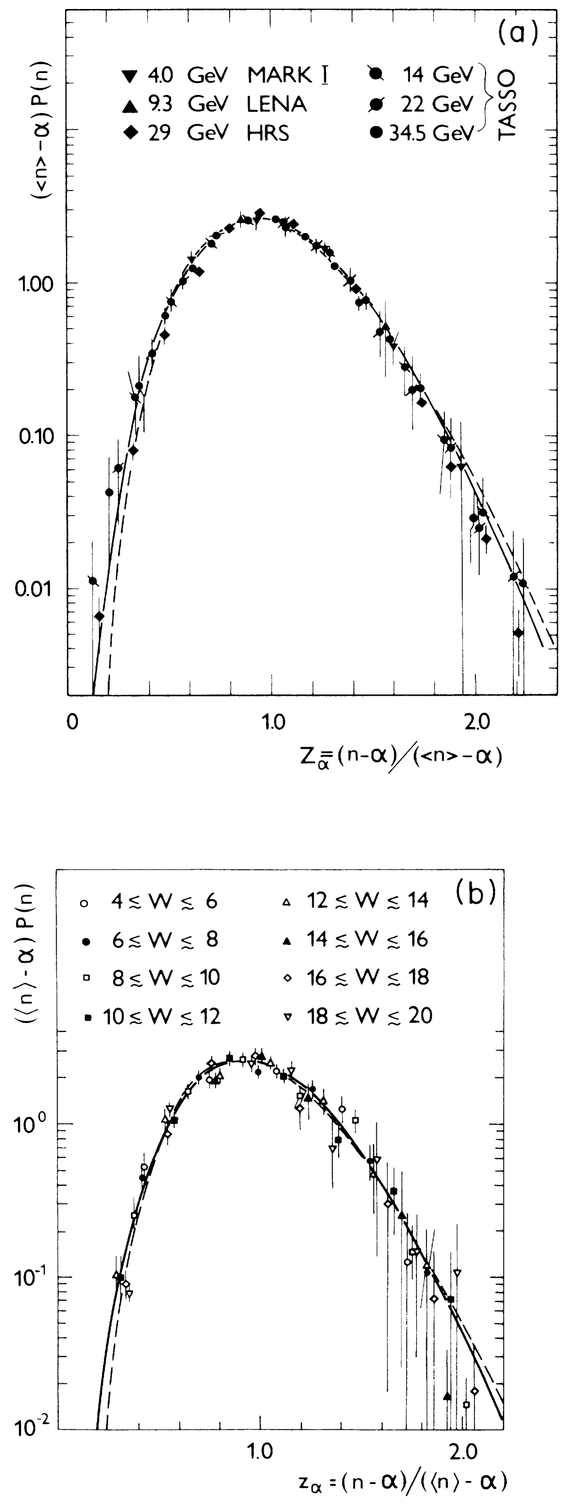

FIG. 3. The plot of the multiplicity ϕ_α, Eq. (4), vs the scaling variable z_α, Eq. (7), in the case of (a) the e^+e^- annihilations and (b) the μ^+p collisions. The values of α are taken from Tables I and II, respectively. The solid lines correspond to the scaling function (18) with $K=21$ and the dashed lines to the gamma distribution (15) with $M=K/2=10.52$.

starting point, an accurate description of the e^+e^- as well as μ^+p multiplicity data is again obtained (with the same value of the parameter K); however, the aforementioned systematic deviation in the $pp+\bar{p}p$ case still persists (even if the trends of the data in all three kinds of collisions are very similar). This fact is due probably to the insufficiently decreasing role of stochastically produced secondaries and therefore the approach involving Eq. (5) should be applied to the $pp+\bar{p}p$ case (some results are published in Ref. 25).

IV. CONCLUSIONS

The quantum-statistical approach which includes the emitting cells with the superposition of stochastic and coherent fields is completed by a reservoir containing pure coherent fields. With respect to the increasing influence of the secondaries produced (coherently) in jets, a new one-parametric multiplicity scaling distribution (18) is derived. While it fits well the e^+e^- and μ^+p data, a systematic deviation occurs when it is compared with the existing high-energy $pp+\bar{p}p$ data. The last fact may be related to the persisting considerable role of the stochastically (incoherently) produced secondaries. The one-parametric scaling function mentioned above is applicable first of all in the cases where the stochastically produced secondaries influence much less the results than the coherently produced ones.[26]

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