Quark mass and spin effects in meson wave functions

Tao Huang, Xinnian Wang,* and Xiaodong Xiang†
Institute of High Energy Physics, Academia Sinica, Beijing, China
(Received 31 March 1986; revised manuscript received 18 August 1986)

We apply the background-field method to calculate the operator expansion of the two-point correlation function related to the moments of the meson distribution amplitude. It is shown that the nonperturbative effects depend in detail on the meson helicity and quark mass.

I. INTRODUCTION

Over the last decade, perturbative QCD theory has made considerable progress. It has been applied not only to many inclusive processes, but also to a series of exclusive processes.[1] The distribution amplitudes $\phi_H(x_i,Q^2)$ in exclusive processes satisfy the renormalization-group equation and QCD evolution equations at short distances. Their solutions depend on the initial condition $\phi_H(x_i,Q_0{}^2)$. Although the evolution with increasing resolution scale Q^2 is described by perturbative theory, the initial conditions are determined mainly by nonperturbative interactions at long distances, which cannot be solved at present.

Important progress has been made using the QCD sum-rule method.[2] The main idea of this method is to evaluate both perturbative and nonperturbative effects in the operator-product expansion (OPE) by means of introducing the nonvanishing expectation values of the field quantities, which are due to nonperturbative effects of the physical vacuum in QCD. Because it includes nonperturbative effects, the QCD sum-rule method is a useful tool at intermediate Q^2.

We have applied the background-field method for calculating the OPE of the two-point correlation function related to the moments of the meson distribution amplitude.[3] With the aid of a dispersion relation and its Borel transformation, we obtained the moments of the pion's distribution amplitude. The results show that the influence of the nonperturbative effects on the pion's distribution amplitude is strong at intermediate Q^2 region. The behavior of the distribution amplitude is quite different from its asymptotic form. The difference increases the branching ratios of $\chi^0\to\pi^+\pi^-$ and $\chi^2\to\pi^+\pi^-$ by 2 orders of magnitude from their asymptotic values. The experimental data[4] certainly supports this analysis.

In this paper we generalize our analysis to other mesons, e.g., the ρ meson and the η_c meson. We will examine the influence due to the different interactions which depend upon the spin of the meson with the nonperturbative condensates as well as the effect of heavy-quark mass.

II. QUARK AND GLUON PROPAGATOR

It should be emphasized that the QCD vacuum corresponds to a real physical state. The existence of nonvanishing expectation values in the vacuum is a special characteristic of QCD theory with nonperturbative effects. These condensates can be considered as classical average effects. Classical background fields can be used to account for all the nonvanishing expectation values. Therefore, one can make the following substitutions in the QCD Lagrangian and all the Green's functions,[5]

$$\begin{aligned} &A_\mu{}^a(x)\to A_\mu{}^a+\Phi_\mu{}^a(x)\ ,\\ &\Psi(x)\to\Psi(x)+\eta(x)\ , \end{aligned} \tag{1}$$

to obtain the effective Lagrangian $\mathscr{L}_{\rm eff}$. In Eq. (1), $A_\mu{}^a(x)$ and $\Psi(x)$ are gluon and quark background fields, respectively, and $\Phi_\mu{}^a(x)$, $\eta(x)$ are their quantum fluctuations.

We choose the "background gauge" for the quantum gluon field

$$\begin{aligned} &D_\mu{}^{ab}(A)\Phi^\mu{}_b=0\ ,\\ &D_\mu{}^{ab}(A)=\delta^{ab}\partial_\mu-gf^{abc}A_{\mu c}\ . \end{aligned} \tag{2}$$

Because the background fields satisfy the equations of motion,

$$\begin{aligned} &[iD_\mu(A)\gamma^\mu-m]\Psi=0\ ,\\ &D_\mu(A)=\partial_\mu+igA_\mu{}^aT_a\ ,\\ &D_\mu{}^{ab}(A)G^{\mu\nu}{}_b(A)=g\overline{\Psi}\gamma^\nu T^a\Psi\ ,\\ &G^{\mu\nu}{}_b(A)=\partial^\mu A^\nu{}_b-\partial^\nu A^\mu{}_b+gf_{bac}A^{\mu a}A^{\nu c}\ , \end{aligned} \tag{3}$$

only the terms which have at most one background field coupled with the quantum fields are left in the effective Lagrangian, such as $g\overline{\Psi}\phi_\mu{}^a\gamma^\mu T_a\eta$. The terms which have more than one background field such as $g\overline{\Psi}\phi_\mu{}^a\gamma^\mu\Psi$, are canceled. In Eq. (3), $T^a(a=1,\ldots,8)$ are Gell-Mann matrices.

Using the effective Lagrangian, one can derive the quark and gluon propagators in the background fields:

$$\begin{aligned} &S(x,0\mid A)=S_F(x)=i\,[i\gamma^\mu D_\mu(A)-m]^{-1}\ ,\\ &S_{\mu\nu}{}^{ab}(x,0\mid A)=S_{\mu\nu}{}^{ab}(x)=i[g_{\mu\nu}D_\sigma{}^a{}_c(A)D^{\sigma cb}(A)\\ &\qquad\qquad +2gf^{abc}G_{\mu\nu c}(A)]^{-1}\ . \end{aligned} \tag{4}$$

Obviously the propagators depend on the classical background field $A_\mu{}^a$. In order to express $A_\mu{}^a(x)$ in a series of gauge-invariant operators at some point x^μ, we choose the Schwinger gauge or "fixed-point gauge":

$$x_\mu A^{\mu}{}_{a}(x)=0 . \qquad (5)$$

Then one can express $A_\mu{}^a(x)$ in terms of $G_{\mu\nu}{}^a(x)$:

$$A_\mu{}^a(x)=\int_0^1 dt\, tx^\nu G_{\nu\mu}{}^a(tx) = x^\nu \sum_{n=0}^{\infty} \frac{1}{n!(n+2)} x^{\alpha_1}\cdots x^{\alpha_n} G_{\nu\mu}{}^{a}{}_{;\alpha_n\cdots\alpha_1}(0) , \qquad (6)$$

where

$$G_{\nu\mu}{}^{a}{}_{;\alpha_n\cdots\alpha_1}(0) = (D_{\alpha_1})^{a}{}_{a_1}(D_{\alpha_2})^{a_1}{}_{a_2}\cdots(D_{\alpha_n})^{a_{n-1}}{}_{b}G_{\nu\mu}{}^{b}(0) . \qquad (7)$$

The expression (6) enables us to write down the propagators for the quark and gluon as a perturbative series in a gauge-invariant form. If $\widetilde{S}(p)$ is the quark propagator in momentum space, it can be written as the following series expansion ($m\neq 0$):[6]

$$\widetilde{S}(p)=\sum_i \widetilde{S}_i(p) , \qquad (8)$$

where

$$\widetilde{S}_0(p)=\frac{m+p_\mu\gamma^\mu}{m^2-p^2} , \qquad (9)$$

$$\widetilde{S}_2(p)=\frac{i}{2}\frac{(-\gamma^\alpha p_\mu\gamma^\mu\gamma^\beta G_{\alpha\beta}+m\gamma^\alpha G_{\alpha\beta}\gamma^\beta)}{(m^2-p^2)^2} , \qquad (10)$$

$$\widetilde{S}_3(p)=\frac{2p^\alpha\gamma^\nu(m-p_\beta\gamma^\beta)\gamma^\mu}{3(m^2-p^2)^3}(G_{\nu\mu;\alpha}+G_{\alpha\mu;\nu})-\frac{2G_{\alpha\mu}{}^{;\alpha}\gamma^\mu}{3(m^2-p^2)^2} , \qquad (11)$$

$$\widetilde{S}_4(p)=\frac{p^\alpha\gamma^\beta G_{\alpha\beta}(\gamma^\mu G_{\mu\nu}\gamma^\nu)}{(m^2-p^2)^3}-\frac{m+p_\sigma\gamma^\sigma}{(m^2-p^2)^3}\left[\frac{(\gamma^\mu G_{\mu\nu}\gamma^\nu)^2}{4}+2b_{2\alpha}{}^{\alpha}+\frac{8p^\alpha p^\beta b_{2\alpha\beta}}{(m^2-p^2)}\right] + i\frac{\gamma^\mu G_{\nu\mu;\alpha\beta}}{(m^2-p^2)^4}[(g^{\alpha\beta}p^\nu+g^{\alpha\nu}p^\beta+g^{\beta\nu}p^\alpha)(m^2-p^2)+6p^\alpha p^\beta p^\nu] , \qquad (12)$$

with

$$G_{\alpha\beta}=\tfrac{1}{2}gT_aG_{\alpha\beta}{}^a(0) ,$$
$$b_{2\alpha\beta}=\tfrac{1}{4}\gamma^\nu\gamma^\mu G_{\mu\alpha}G_{\beta\nu}+\frac{i}{8}\gamma^\nu\gamma^\mu(G_{\nu\mu;\beta\alpha}+G_{\alpha\mu;\beta\nu}+G_{\alpha\mu;\nu\beta}) . \qquad (13)$$

Similarly, the gluon propagator in momentum space $\widetilde{S}_{\mu\nu}{}^{ab}(p)$ is

$$\widetilde{S}_{\mu\nu}(p)=-\frac{g_{\mu\nu}}{p^2}+\frac{2i\widetilde{G}_{\mu\nu}}{(-p^2)^2}+\frac{p^\alpha}{(-p^2)^3}(\tfrac{2}{3}g_{\mu\nu}\widetilde{G}_{\alpha\beta;}{}^{\beta}+4\widetilde{G}_{\mu\nu;\alpha})-\frac{1}{(-p^2)^3}(4\widetilde{G}_{\mu\beta}\widetilde{G}_\nu{}^\beta+2i\widetilde{G}_{\mu\nu;\alpha}{}^\alpha+\tfrac{1}{2}g_{\mu\nu}\widetilde{G}_{\alpha\beta}\widetilde{G}^{\alpha\beta}) - \frac{p^\alpha p^\beta}{(-p^2)^4}[2ig_{\mu\nu}(\widetilde{G}_{\alpha\sigma}{}^{;\sigma}{}_{\beta}+\widetilde{G}_{\alpha\sigma;\beta}{}^{\sigma})-8i\widetilde{G}_{\mu\nu;\alpha\beta}-8g_{\mu\nu}\widetilde{B}_{2\alpha\beta}] , \qquad (14)$$

where

$$\widetilde{G}_{\mu\nu}{}^{ab}=-igf^{abc}G_{\mu\nu c}(0) ,$$
$$\widetilde{G}_{\mu\nu;\alpha}{}^{ab}=-igf^{abc}G_{\mu\nu;\alpha c}(0) , \qquad (15)$$
$$\widetilde{B}_{2\alpha\beta}{}^{ab}=\frac{i}{8}(\widetilde{G}^{\mu}{}_{\alpha\mu;\beta}{}^{ab}+\widetilde{G}_{\alpha\mu;\beta}{}^{\mu ab})-\tfrac{1}{4}\widetilde{G}_{\alpha\mu}{}^{a}{}_{c}\widetilde{G}_\beta{}^{\mu cb} ,$$

and color indices as in Eq. (15) must be understood in Eq. (14).

III. THE TWO-POINT CORRELATION FUNCTION

Now with the quark and gluon propagators in the background fields, we can calculate the two-point correlation function

$$\Pi_{\Gamma_1\Gamma_2}(x)=\langle 0|T(j_{\Gamma_1}{}^{(2n)}(x)j_{\Gamma_2}{}^{(0)}(0))|0\rangle , \qquad (16)$$

where

$$j_\Gamma{}^{(2n)}(x)=\overline{\Psi}(x)\Gamma(ix\cdot\overleftrightarrow{D})^{2n}\Psi(x) , \qquad (17)$$

$\Gamma=\gamma_\mu\gamma_5$ for $j_{A\mu}{}^{(2n)}(x)$ in the case of π,η_c meson; and $\Gamma=\gamma_\mu$ for $j_{V\mu}{}^{(2n)}(x)$ in the case of ρ meson.

For the background field of a quark, one can expand it in a form

$$\Psi(x)=\Psi(0)+x^\alpha D_\alpha\Psi(0)+\tfrac{1}{2}x^\alpha x^\beta D_\alpha D_\beta(0)+\cdots . \qquad (18)$$

The Feynman diagrams, which contribute to the two-point correlation function are shown in Figs. 1(a)–1(k).[6]

For the light-quark case, one can neglect the quark mass in the propagator, and the contributions of Figs. 1(m) and 1(l) are approximately equal to zero. It is clear that the results for the vector-meson distribution amplitude depend on the helicity of the meson. For the longitudinal component of the ρ meson, the moments are apparently similar to those of the pion. However, the currents in Eq. (16) are different, so are the sign of the coefficients of Figs. 1(g), 1(h), and 1(e). Therefore the moment values are different for the π and ρ_L meson due to the different spin.

Using the background-field method, the correlation function has the general form

$$\Pi_{\mu\nu}{}^{2n,0}(q^2,z\cdot q)=(z\cdot q)^{2n}q_\mu q_\nu I^a{}_{2n,0}(q^2) -(z\cdot q)^{2n}g_{\mu\nu}I^b{}_{2n,0}(q^2) -(z\cdot q)^{2n}q_\mu z_\nu I^c{}_{2n,0}(q^2)\ , \tag{19}$$

where the functions $I^{a,b,c}{}_{2n,0}(q^2)$ are sums of nonvanishing expectation values:

$$I^{a,b,c}{}_{2n,0}(q^2)=\sum_N \frac{C_N{}^{a,b,c}}{(q^2)^{N/2}}\ . \tag{20}$$

Keeping only the contributions of lowest order in α_s and

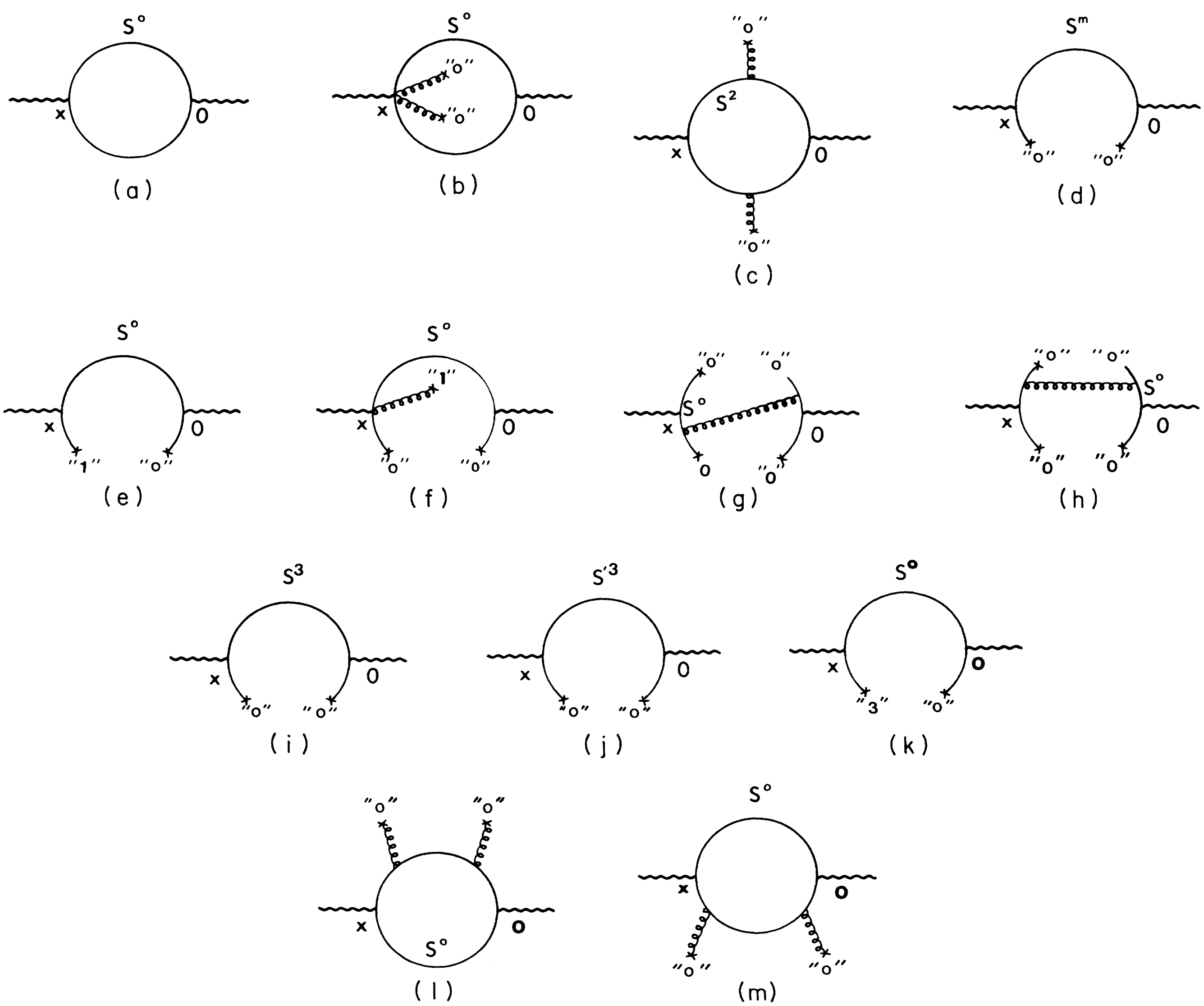

FIG. 1. Feynman diagrams for the two-point correlation function, keeping only the contributions of lowest order in α_s and the terms of the operators up to dimensions six including the linear term of the light-quark mass in the OPE of the two-point correlation function. In all diagrams, "×" stands for background-field operators, and "0","1", . . . , represent "zero", "first", . . . , derivatives of background-field operators, respectively. S^m is the mass term of the quark propagator in the background fields with $m\neq0$. The perturbative term corresponds to the contribution from (a). The gluon condensate terms come from (b) and (c). (d) and (e) give the quark condensate terms $\langle 0\,|\,m\bar{\Psi}\Psi\,|\,0\rangle$. (f)—(k) contribute to the four-quark condensates. (l) and (m) represent the nonvanishing contribution of the gluon condensate term in the heavy-quark case.

the terms of the operators up to dimension six (including the linear term in the quark mass in the OPE of the two-point correlation function), we obtain the detailed results listed in Ref. 6. Here we only give some terms from which we can see spin effects:

$$\Pi_{\mu\nu}^{2n,0}(q)\,|_{\text{Fig. 1(g)+Fig. 1(h)}} = \pm\frac{32\pi^2\langle 0|\sqrt{\alpha_s}\bar{\Psi}\Psi|0\rangle^2}{9q^6}(z\cdot q)^{2n}(q_\mu q_\nu - q_{\mu\nu}q^2)\,, \tag{21}$$

$$\Pi_{\mu\nu}^{2n,0}(q)\,|_{\text{Fig. 1(e)}} = \pm\left[\frac{-m_u\langle 0|\bar{u}u|0\rangle}{q^4}(z\cdot q)^{2n}g_{\mu\nu}q^2 + u\to d\right], \tag{22}$$

where the minus sign is for the pion and the other is for the ρ_L meson. In the case of light quarks, Fig. 1(e) is not important. In order to see the influence due to the different spin states, we compare Eqs. (21) and (22) with the perturbative contribution from Fig. 1(a):

$$\Pi_{\mu\nu}^{2n,0}(q,q\cdot z) = \frac{-3}{4\pi}\frac{(z\cdot q)^{2n}}{(2n+1)(2n+3)} \times (q_\mu q_\nu - g_{\mu\nu}q^2 - n g_{\mu\nu}q^2)\ln\frac{-q^2}{\mu^2}\,. \tag{23}$$

It may be seen that Eqs. (21) and (22) have the same sign as the perturbative contribution in the case of π meson, but the opposite for ρ_L meson. So the moment values of the pion's wave function are much larger than the asymptotic moment values (for $n\neq 0$), and the moments for the ρ_L meson wave function will be smaller than the pion case. A detailed calculation shows that the corresponding distribution amplitude of the ρ_L meson will be quite narrower than the pion's, but it is still wider than the asymptotic form.[3] Therefore every meson distribution amplitude will have a distinct form according to the different interactions with the nonperturbative condensates.

Now we come to see the case of η_c meson, in which the charm-quark mass cannot be neglected in the propagator. To simplify our calculations, we define the two-point correlation function:

$$\Pi^{2n,0}(q,z\cdot q) = z^\mu z^\nu \Pi_{\mu\nu}^{2n,0}(q,z\cdot q) = i\int d^4x e^{iq\cdot x}\langle 0|T(\bar{\Psi}(x)z^\mu\gamma_\mu\gamma_5(iz\cdot\overleftrightarrow{D})^{2n}\Psi(x)\bar{\Psi}(0)z^\nu\gamma_\nu\gamma_5\Psi(0))|0\rangle\,. \tag{24}$$

The calculation is a little bit lengthy, but the result is not too complicated. The perturbative contribution from Fig. 1(a) is

$$\Pi^{2n,0}(q^2,z\cdot q)\,|_{\text{Fig. 1(a)}} = \frac{8n_c}{(4\pi)^2}(z\cdot q)^{2n+2}\frac{1}{2(2n+1)(2n+3)} \times \Bigg\{\ln\frac{4\pi\mu^2}{m^2} - \gamma - [(2n+3)S_{n+1}(a^2-1) - (2n+1)S_{n+2}(a^2-1)] - (-1)^{n+1}[(2n+1)a^2+2](a^2-1)^{n+1}\Pi(a^2-1)\Bigg\}\,, \tag{25}$$

where γ is the Euler constant and $S_n(a^2-1)$, $\Pi(a^2-1)$ are functions of $a^2-1=4m^2/q^2-1$, which are defined by

$$\Pi(x) = \frac{\theta(x)}{\sqrt{x}}\arctan\frac{1}{\sqrt{x}} - \frac{\theta(-x)}{2\sqrt{-x}}\ln\left|\frac{1+\sqrt{-x}}{1+\sqrt{-x}}\right|\,, \tag{26}$$

$$S_n(x) = \sum_{\iota=0}^{n=1}(-1)^\iota\frac{x^\iota}{2n-2\iota-1}\quad (S_0=0)\,. \tag{27}$$

The nonvanishing condensate contributions from Figs. 1(b) and 1(c) are

$$\begin{aligned}\Pi^{2n,0}(q,z\cdot q)\,|_{\text{Fig. 1(b)}} = &-\frac{n}{6\pi(2n+1)}\frac{m^2}{q^6}\langle 0|\alpha_s GG|0\rangle(z\cdot q)^{2n+2}\\ &\times\Bigg\{2(2n-1)S_n(a^2-1) + 4[2n+(2n-1)(a^2-1)]S_n'(a^2-1)\\ &+[2n+1+(2n-1)(a^2-1)]a^2S_n''(a^2-1)\\ &-\frac{(-1)^n(a^2-1)^{n-2}}{4}(4n-3)[2n+1+(2n-1)(a^2-1)]\\ &-\frac{(-1)^n(a-1)^{n-1}}{2a^2}[6n-1+3(2n-1)(a^2-1)] + (-1)^n(a^2-1)^{n-2}(2n-1)\\ &\times\left[4n+\frac{(2n-1)(2n-3)}{4}\right](a^2-1)^2 + n(2n+1)(a^2-1) + \tfrac{1}{4}(2n-3)(2n+1)\Big]\Pi(a^2-1)\Bigg\}\,,\end{aligned} \tag{28}$$

$$\Pi^{2n,0}(q^2,z\cdot q)\,|_{\text{Fig. 1(c)}}=\frac{\langle 0|\alpha_s GG|0\rangle}{12\pi}\frac{(z\cdot q)^{2n+2}}{q^4}\times\left[\frac{1}{2n+1}+\frac{a^2}{2(a^2-1)}\left[2a^2+\frac{2n-1}{2(a^2-1)}a^4\right][S_n(a^2-1)+(-1)^n(a^2-1)^n\Pi(a^2-1)]\right], \tag{29}$$

where $S_n'(a^2-1)$ is defined by $S_n'(x)=dS_n(x)/dx$ ($x=a^2-1$). The contributions from Figs. 1(d)—1(k) are zero for the heavy-quark system. Now, let us discuss Figs. 1(l) and 1(m). Their contributions are

$$\Pi^{2n,0}(q^2,z\cdot q)\,|_{\text{Fig1(l)+Fig. 1(m)}}=-\frac{8m^2}{12\pi}\frac{\langle 0|\alpha_s GG|0\rangle}{q^6}(z\cdot q)^{2n+2}\times\left[\frac{1}{a^2(a^2-1)}+\frac{1}{8(a^2-1)^2}[3(2n+3)(a^2-1)-(2n+5)]+\frac{1}{8(a^2-1)^2}[S_n(a^2-1)+(-1)^n(a^2-1)^n\Pi(a^2-1)]\times[(2n-1)(2n-3)-2(4n^2-1)(a^2-1)-3(2n+3)(2n+1)(a^2-1)^2]\right]. \tag{30}$$

In principle, one can calculate moments for the heavy-quark system from the correlation function $\Pi^{2n,0}(q^2,z\cdot q)$. We do not give them here. Qualitatively, one can see that Eq. (28) becomes zero for $n=0$ and the signs of the contributions of Fig. 1(l) + Fig. 1(m) and Fig. 1(a) are opposite. Because the contribution from Fig. 1(l) + Fig. 1(m) does not vanish only when $m\neq 0$, the quark mass m certainly makes the distribution amplitude narrower than the asymptotic form. Therefore, when the quark mass becomes very large, it will probably make the heavy-quark system nonrelativistic through its interaction with background fields.

IV. CONCLUSIONS

We now come to the conclusion that (a) all light-meson distribution amplitudes have the same perturbative contribution (i.e., asymptotic form) without nonperturbative condensates; (b) the influence due to nonperturbative condensates are very important, and are quite distinct for different mesons; (c) in the case of the heavy-quark system, we have to consider the contributions from Figs. 1(l) and 1(m) (gluon condensation with the heavy quark), which will overwhelm the other contributions and make the distribution amplitude $\phi(x_i,Q^2)$ go over to the nonrelativistic case; and (d) even for the light-quark case, the effect of the nonperturbative condensates depends specifically on the helicity interaction and is most important for the pion at an intermediate Q^2 region.[7]

ACKNOWLEDGMENTS

We are grateful to S. J. Brodsky for suggestions and discussions. X. N. W. thanks N. G. Deshpande and K. Tesima for helpful discussions. The work was supported by the Science Fund of the Chinese Academy of Sciences.

*Present address: Institute of Theoretical Science and Department of Physics, University of Oregon, Eugene, OR 97403.

†Present address: Physics Department, University of Kentucky, Lexington, KE.

[1]A. J. Buras, Rev. Mod. Phys. **52**, 199 (1980); A. H. Mueller, Phys. Rep. **73**, 237 (1981); S. J. Brodsky, T. Huang, and G. P. Lepage, *Particles and Fields,* edited by A. Z. Capri and A. N. Kamal (Plenum, New York and London, 1983), Vol. 2, p. 143.

[2]M. A. Shifman *et al.*, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).

[3]T. Huang, X. N. Wang, and X. D. Xiang, Chinese Phys. Lett. 2, 67 (1985); X. D. Xiang, X. N. Wang, and T. Huang, BIHEP-TH-29, 1984 (unpublished); V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).

[4]X. N. Wang, X. D. Xiang and T. Huang, Commun. Theor. Phys. 2, (1986); V. L. Chernkyak and A. R. Zhitnitsky, Nucl. Phys. **B201**, 492 (1982); Phys. Rep. **112**, 173 (1984).

[5]J. Govaerts *et al.*, Phys. Lett. **128B**, 262 (1983).

[6]E. V. Shuryak and A. I. Vainshtein, Nucl. Phys. **B199**, 451 (1982); **B201**, 141 (1982); L. J. Reinders *et al.*, Phys. Rep. **127**, 1 (1985); S. Mallik, Nucl. Phys. **B234**, 45 (1984).

[7]See Xiang, Wang, and Huang, Ref. 3.