

Superconducting cosmic strings

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Superconducting loops of string formed in the early Universe, if they are relatively light, can be an important source of relativistic particles in the Galaxy. They can be observed as sources of synchrotron radiation at centimeter wavelengths. We propose a string model for two recently discovered radio sources, the "thread" in the galactic center and the source G357.7-0.1, and predict that the filaments in these sources should move at relativistic speeds. We also consider superheavy superconducting strings, and the possibility that they be observed as extragalactic radio sources.

I. INTRODUCTION

In recent years there has been considerable interest in cosmological effects of strings which could be produced at a phase transition in the early Universe. Superheavy strings can generate cosmologically interesting density fluctuations and can also produce a number of distinctive observational effects. For a review of strings and their cosmological implications see Ref. 1.

Until recently, it was thought that strings can be detected only through their gravitational interactions. However, Witten² has demonstrated that in some grand unified models strings can behave as superconducting wires and thus can have strong electromagnetic interactions. Moving through magnetized cosmic plasmas, such strings can develop large currents and can be observed as sources of synchrotron radiation. In this paper we shall discuss the astrophysical effects of superconducting strings. The next section reviews the physical properties of strings in general and of superconducting strings in particular. In Sec. III we analyze the interaction of current-carrying strings with cosmic plasmas and estimate the power of synchrotron radiation from a string. The prospects of observing a superconducting string in our Galaxy are discussed in Sec. IV. We suggest that the recently observed radio source G357.7-0.1 may be due to a string and propose a simple observational test of this hypothesis. (The test is so direct that our model may be ruled out by the time the paper appears in print.) Another string candidate is the "thread" recently discovered in the galactic center. Particle production by strings is estimated in Sec. V, and the possibility that superheavy superconducting cosmic strings be observed as extragalactic radio sources is considered in Sec. VI.

II. SUPERCONDUCTING STRINGS

In this section we shall briefly review the physical properties of strings with the main focus on the properties of

superconducting strings.

Let η be the symmetry-breaking mass scale of strings. We shall assume that $\eta \ll m_{\text{Pl}}$, where $m_{\text{Pl}} \sim 10^{19}$ GeV is the Planck mass. The mass per unit length of string is

$$\mu \sim c\eta^2/\hbar. \quad (2.1)$$

The gravitational interactions of strings are characterized by a dimensionless parameter,

$$\epsilon = G\mu/c^2 \sim (\eta/m_{\text{Pl}})^2. \quad (2.2)$$

The string scenario of galaxy formation¹ requires $\epsilon \sim 10^{-6}$, which corresponds to a grand-unification scale of symmetry breaking, $\eta \sim 10^{16}$ GeV. (Strings with $\epsilon \gg 10^{-6}$ have been ruled out by observation.) In this scenario, galaxies condense around oscillating loops of string while the loops gradually lose their energy by gravitational radiation and disappear. The rate of gravitational energy loss is

$$\dot{E}_g \sim 50\epsilon\mu c^3 \quad (2.3)$$

and the lifetime of the loops is

$$\tau_g \sim R/50\epsilon c, \quad (2.4)$$

where R is the length of the loop.

If dissipational effects are neglected, the motion of loops is strictly periodic with a period

$$T = M/2\mu c \sim R/2c, \quad (2.5)$$

where M is the mass of the loop. The rms velocity of a segment of string in the center-of-mass frame of the loop is

$$\bar{v} \sim c/\sqrt{2}. \quad (2.6)$$

It is possible that the strings responsible for galaxy formation are superconducting. It is also possible to have light superconducting strings ($\epsilon \ll 10^{-6}$). Such light strings are practically impossible to detect through their

gravitational interactions. However, as we will see, electromagnetic interactions of light superconducting strings can lead to observable effects.

If an electric field E is present on the axis of a superconducting string, the string builds up an electric current i at the rate

$$\frac{di}{dt} = \beta(ce^2/\hbar)E, \quad (2.7)$$

where e is the electron charge and $\beta \sim 1$ is a model-dependent coefficient. When the current reaches a critical value i_{\max} , its growth terminates, and the string starts producing particles and antiparticles at the rate (per unit time per unit length)

$$\frac{d^2N}{dt dl} \sim \frac{eE}{\hbar}. \quad (2.8)$$

The critical current i_{\max} can be written as

$$i_{\max} \sim emc^2/\hbar, \quad (2.9)$$

where $m < \eta$ is the characteristic mass of the charge carriers on the string.^{3,4} The actual value of m is model dependent.

The electric field E in Eqs. (2.7) and (2.8) differs from the applied field, because the time-varying magnetic field produced by the current (2.7) induces a field of opposite sign equal to $|L di/dt|$, where L is the inductance per unit length of the string. With this effect taken into account, Eq. (2.7) takes the form

$$\frac{di}{dt} = \frac{\beta ce^2}{\hbar} \left(E - L \frac{di}{dt} \right), \quad (2.10)$$

where

$$L = (2/c^2) \ln(R_c/\Delta) \quad (2.11)$$

and $\Delta \sim \hbar/\eta c$ is the thickness of the string. The cutoff radius R_c is given by the typical scale of variation of the electric field (the wavelength in the case of an electromagnetic wave) or by the size of the closed loop of string, whichever is smaller. From Eqs. (2.10) and (2.11) we obtain²

$$\frac{di}{dt} = \frac{\beta ce^2}{\hbar} \left(1 + 2\beta \frac{e^2}{\hbar c} \ln \frac{R_c}{\Delta} \right)^{-1} E. \quad (2.12)$$

Since the logarithm in Eq. (2.12) typically does not exceed 100, the correction factor in the parentheses does not change the order of magnitude of di/dt , and we shall use Eq. (2.7) for our order-of-magnitude estimates.

Consider a segment of string moving with velocity \mathbf{v} in an external magnetic field \mathbf{B}_0 . In the frame of the string there is an electric field $\mathbf{E} = c^{-1} \mathbf{v} \times \mathbf{B}_0$; and the current builds up at the rate $di/dt \sim (e^2/\hbar) v B_0$. A closed loop oscillating in external magnetic field will develop an alternating current. Using Eqs. (2.5) and (2.7), we can estimate the typical magnitude of the current in a loop of length R :

$$i \sim 0.2\beta e^2 B_0 R / \hbar. \quad (2.13)$$

The magnetic field produced by this current in the vicinity of the string is

$$B = 2i/cr, \quad (2.14)$$

where $r \ll R$ is the distance from the string.

III. INTERACTION OF STRINGS WITH COSMIC PLASMAS

So far we have discussed the physical properties of strings in the vacuum. Now we turn to the interaction of superconducting strings with cosmic plasmas. Consider a string with a current i moving through interstellar plasma. Charged particles cannot penetrate the region of strong magnetic field near the string, and since the motion of the string is highly supersonic, a shock front is formed at some distance r_s from the string. When viewed in the rest frame of the string, the physical situation here is similar to the interaction of the solar wind with the geomagnetic field. The latter problem has been intensively studied for several decades using analytic and numerical methods, as well as *in situ* observations and laboratory experiments. For a review see Ref. 5. Using the insights and results described there, we arrive at the picture of the shock shown in Fig. 1.

The plasma is approaching from the left (region I) with a supersonic velocity v_0 ; its density is n and its temperature is $T \ll mv_0^2/2$ (we set $k_B = 1$). The flow is reduced to subsonic speed at the bow shock. In the nonrelativistic limit ($v_0 \ll c$) the density, temperature, and velocity behind the shock are given by⁶

$$n' = 4n, \quad T' = \frac{3}{16} mv_0^2, \quad v' = \frac{1}{4} v_0, \quad (3.1)$$

where m is the mean mass per particle, about $0.6 m_H$ for fully ionized interstellar gas. For $v_0 \sim c$ these equations still give the right order of magnitude. The bow shock is followed by a tangential discontinuity (TD) where the plasma pressure in region II on the left of the discontinuity is balanced by the magnetic pressure in region III on the right. A thin current sheet on TD screens the magnetic field of the string, so that the field in region II is embedded in the incoming magnetized plasma; throughout

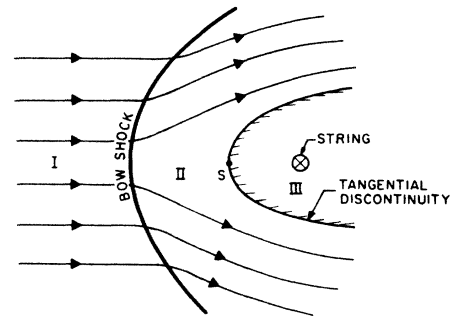


FIG. 1. A schematic diagram of the flow of interstellar plasma around a string. The fluid velocity is indicated by the arrows; it vanishes at the stagnation point S located at the tip of the tangential discontinuity.

most of region II the field is so weak that its dynamical effects may be neglected. The flow of plasma around TD is like a flow around a blunt body; the region III behind the discontinuity is essentially free of plasma. The distances from the string to the bow shock and to TD have comparable magnitudes $\sim r_s$.

To estimate r_s , we first note that the dynamical pressure of the incoming plasma is eventually balanced by the magnetic pressure in region III:

$$m n v_0^2 = B_s^2 / 8\pi . \quad (3.2)$$

Here B_s is the magnetic field at the stagnation point S in the middle of TD. With $v_0 \sim c/\sqrt{2}$, Eq. (3.2) gives

$$B_s \sim 0.1 n^{1/2} \text{ G} . \quad (3.3)$$

In the vicinity of the string, $r \ll r_s$, the magnetic field is given by Eq. (2.14). Although for $r \sim r_s$ this equation is quantitatively incorrect, it can still be used to obtain a rough order-of-magnitude estimate of r_s :

$$r_s \sim 7 \times 10^{-10} n^{-1/2} i \text{ cm} .$$

(Throughout the paper we use cgs units.) Another useful expression for r_s can be obtained with the aid of Eq. (2.13), which expresses the typical string current in terms of the external magnetic field B_0 :

$$r_s \sim 0.4 \beta \frac{e^2 v_A}{\hbar c} R , \quad (3.4)$$

where $v_A = B_0(4\pi n m)^{-1/2}$ is the Alfvén speed far from the string.

Of course the surface dividing the regions of hot plasma (II) and strong magnetic field (III) is not infinitely thin. Because of the diffusion of the magnetic field lines from region III, there is a thin layer of strongly magnetized plasma with $B \sim B_s$ at TD. As we shall see, a substantial amount of synchrotron radiation comes from this region, and so it is important to estimate its thickness. To do this, we will need a quantitative description of the flow near TD.

Concentrating on the neighborhood of the stagnation point S , we introduce a Cartesian coordinate system with origin at S , the x axis along the velocity of the incoming plasma, \mathbf{v}_0 , and the y axis perpendicular both to \mathbf{v}_0 and to the string.

We shall consider the part of region II near the stagnation point, where both $|x|$ and $|y| \ll r_s$, so that TD can be approximated by a plane, $x=0$. According to Ref. 5, in most of this region the gas pressure is much greater than the local ram pressure $m n v^2$ and also, the local magnetic pressure $B^2/8\pi$, and we can therefore use the approximation that the flow is incompressible:

$$\nabla \cdot \mathbf{v} = 0 . \quad (3.5)$$

The solution of this equation describing the flow near the stagnation point is

$$v_x = -ax, \quad v_y = ay , \quad (3.6)$$

where a is a constant which can be approximated by extrapolating this solution to $x \sim -r_s$. This gives $a \sim v'/r_s \sim v_0/4r_s$.

The plasma density near S is not much different from n' and the temperature can be found from the pressure balance condition: $n' T_s \sim B_s^2/8\pi \sim n m v_0^2$ or $T_s \sim m v_0^2/4$. If the electron and proton temperatures are the same, the electrons are accelerated to ultrarelativistic energies with a typical Lorentz factor

$$\gamma \sim T_s/m_e c^2 \sim 140 .$$

The diffusion of magnetic field lines is insignificant when the magnetic Reynolds number is large:

$$\mathcal{R}_m(x) \sim |x| v_x \sigma / c^2 \gg 1 , \quad (3.7)$$

where σ is the effective electrical conductivity. The thickness δ of the layer of strongly magnetized plasma can be determined from the condition

$$\mathcal{R}_m(\delta) \sim 1 . \quad (3.8)$$

In a dense relativistic plasma with a mean free path much smaller than all other length scales, the conductivity is given by⁷

$$\sigma \sim n e^2 \tau c^2 / 3 T_s , \quad (3.9)$$

where τ is the electron mean free time. Unfortunately, Eq. (3.9) does not apply to our problem. The cosmic plasma we are interested in is effectively collisionless, as the mean free paths of both ions and electrons are much greater than any other relevant scale. Depending on the physical situation, it is reasonable to assume that the role of the mean free path is played by the Larmor radius, the Debye length, or the wavelength associated with the fastest-growing plasma instability. We have not attempted to calculate the electrical conductivity of such a plasma, and so we cannot reliably estimate the value of δ . However, we note that for our particular values of the magnetic field, density, and temperature, the Larmor radii of protons and electrons are comparable to one another and to the Debye length for a relativistic plasma:

$$r_L \sim T_s / e B_s \sim (T_s / 4\pi n e^2)^{1/2} \sim \lambda_D . \quad (3.10)$$

This suggests that we can take, say, r_L as the relevant dissipation length and set $\tau \sim r_L/c$ and $\sigma \sim n e c / 3 B_s$. Then Eqs. (3.6)–(3.8) give

$$\delta \sim (96\sqrt{2}\pi r_s r_L)^{1/2} . \quad (3.11)$$

The magnetohydrodynamic description is justified because $r_L(x)$ is small compared to the characteristic scale, x ; therefore, $\delta/r_s \sim (r_L/r_s)^{1/2} \ll 1$.

For $|x| \gg \delta$, diffusion is unimportant and the magnetic field is that embedded in the incoming magnetized plasma.⁸ We shall assume for simplicity that the incoming plasma is uniformly magnetized in the direction of the y axis. The magnetic field lines are frozen in the plasma, and as the plasma flows around the string, the field lines pile up in region II (see Fig. 2). Eventually they diffuse through the layer of strongly magnetized plasma into region III (if \mathbf{B}_0 is parallel to the field of the string) or annihilate some of the field lines in the layer (in the antiparallel case). The condition that the magnetic field is frozen into the plasma is

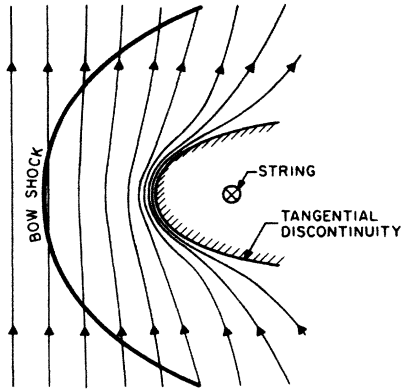


FIG. 2. The structure of the magnetic field corresponding to the flow depicted in Fig. 1. The field is shown outside the tangential discontinuity (TD); the intense field inside TD is not shown.

$$\nabla \times (\mathbf{B} \times \mathbf{v}) = 0. \quad (3.12)$$

With \mathbf{v} from Eq. (3.6) this gives⁹

$$B_y = b/x, \quad (3.13)$$

where b is a constant. Extrapolating this solution to $x \sim -r_s$, we find $b \sim B' r_s \sim 4B_0 r_s$, where B_0 and $B' = 4B_0$ are the values of the magnetic field at the bow shock in regions I and II, respectively. Equation (3.13) applies in the range¹⁰ $\delta \ll |x| \lesssim r_s$.

We can now estimate the synchrotron radiation from the string. The total power radiated by the strongly magnetized layer with $B \sim B_s$ is

$$\dot{E}_r \sim \frac{e^4}{m_e^2 c^3} \gamma^2 B_s^2 n R r_s \delta, \quad (3.14)$$

where R is the length of the string and γ is the electron Lorentz factor, which as we have shown is $\gg 1$. The characteristic wavelength of this radiation is¹¹

$$\lambda \sim \frac{20 m_e c^2}{e \gamma^2 B_s} \sim 20 n^{-1/2} \text{ cm}. \quad (3.15)$$

To estimate the contribution of the rest of region II, we note that according to Eq. (3.13) the magnetic field in a layer of thickness $x \gg \delta$ is $B \propto x^{-1}$ and the total radiated power is $\dot{E}_r \propto B^2 x \propto x^{-1}$. Hence, the main contribution to \dot{E}_r is given by a thin layer with $B \sim B_s$ near TD.

Particle acceleration can occur in the shock and magnetic reconnection regions, possibly resulting in a power-law, nonthermal distribution of high-energy particles,

$$dn \propto \xi^{-\alpha} d\xi, \quad (3.16)$$

where ξ is the electron energy. At wavelength λ the power radiated by these particles is proportional to the integral

$$\int_{T_s}^{\xi_{\max}} \xi^{2-\alpha} d\xi. \quad (3.17)$$

For $\alpha > 3$ the contribution of high-energy particles is unimportant while for $\alpha < 3$ the main contribution to (3.17) comes from the most energetic particles. The energies accessible to the particles are limited, in principle, only by the requirement that their Larmor radii should not exceed r_s (Ref. 12). This gives

$$\xi_{\max} \sim e B_s r_s = 2 \times 10^5 B_{-6} R_{20} \text{ GeV}.$$

The corresponding synchrotron wavelength is

$$\lambda_{\min} \sim \frac{20}{r_s^2} \left[\frac{m_e c^2}{e B_s} \right]^3. \quad (3.18)$$

We cannot estimate the radiated power in this case, since we do not know α and the factors which multiply Eq. (3.16). All we can say is that there may be a substantial power radiated at a wavelength $\gtrsim \lambda_{\min}$ in addition to that given by Eqs. (3.14) and (3.15).

IV. STRINGS IN THE GALAXY

The typical length of a loop we can expect to find in a galaxy is given by the galactic scale at the time of horizon crossing, $R_0 \sim 30 \text{ pc} \sim 10^{20} \text{ cm}$. Larger loops can occur with a probability $\sim (R_0/R)^{3/2}$. The number of loops with $R < R_0$ per galaxy is

$$N_R \sim (R_0/R)^{3/2}. \quad (4.1)$$

This distribution does not apply for R less than

$$R_{\min} \sim 50 \epsilon t_0 \sim 1.5 \times 10^{11} \epsilon \text{ pc}, \quad (4.2)$$

where $t_0 \sim 3 \times 10^{17} \text{ sec}$ is the age of the Universe. All smaller loops have already decayed by gravitational radiation. We see that there are no loops left in the Galaxy unless $R_{\min} \lesssim R_0$ or $\epsilon \lesssim 10^{-10}$. Strings with $\epsilon \lesssim 10^{-10}$, if they exist, are unimportant for galaxy formation, but we will show that they can still lead to observable effects.¹³

We estimated the lifetime of the loops using Eq. (2.4), which assumes that the dominant energy loss mechanism is the gravitational radiation. For superconducting strings, we have to compare the gravitational lifetime τ_g with

$$\tau_d \sim \mu R c^2 / \dot{E}_d, \quad (4.3)$$

where \dot{E}_d is the energy dissipated by shock heating of the plasma. A string moving through plasma with velocity v experiences a retarding force per unit length

$$f \sim n m v^2 r_s, \quad (4.4)$$

where r_s is the shock radius estimated in the previous section [see Eq. (3.4)]. The rate of energy loss for a loop of length R is

$$\dot{E}_d \sim f R v. \quad (4.5)$$

To get a numerical estimate of τ_d , we shall use typical values of the parameters for an interstellar plasma:

$$B_0 \sim 3 \times 10^{-6} \text{ G}, \quad n \sim 1 \text{ cm}^{-3}, \quad v_A \sim 10^6 \text{ cm/sec}. \quad (4.6)$$

We shall also use the notations B_{-6} and R_{20} , which stand for the values of B_0 and R in units of 10^{-6} G and 10^{20} cm, respectively. With these notations, Eqs. (3.4), (4.3), and (4.5) yield

$$r_s \sim 7 \times 10^{12} B_{-6} R_{20} n^{-0.5} \text{ cm}, \quad (4.7)$$

$$\dot{E}_d \sim 6 \times 10^{39} B_{-6} R_{20}^2 n^{0.5} \text{ erg/sec.}, \quad (4.8)$$

$$\tau_d \sim 3 \times 10^{29} \epsilon B_{-6}^{-1} R_{20}^{-1} n^{-0.5} \text{ sec.} \quad (4.9)$$

Note that τ_d decreases as we increase R , and so the first loops to be affected by shock dissipation are the largest ones with $R = R_0 \sim 30$ pc. They have already been affected if $\tau_d \leq t_0$, that is, if $\epsilon \leq 5 \times 10^{-14}$. For such values of ϵ , the largest loops surviving in a galaxy have size $R_0 \sim 6 \times 10^{14} \epsilon$ pc. Comparing Eqs. (2.4) and (4.9) we find that for $\epsilon \geq 10^{-11}$ the gravitational radiation is still the dominant energy loss mechanism for all galactic loops.

So far we have assumed implicitly that the interaction of loops with the magnetic field and with the plasma does not significantly disturb their motion. This is the case if the dissipational force (4.4) and the magnetic force, $f_B \sim iB/c$, are small compared to the force of string tension, $f_t \sim \mu c^2/R$. It is easily checked that these conditions are satisfied if $\epsilon \gg 10^{-22}$.

If there are any loops of string in our Galaxy, one can expect that they are orbiting around the galactic center with a typical virial velocity $u \sim 200$ km/sec. If the strings are superconducting, the force due to their shock dissipation in the ionized interstellar medium tends to decelerate the loops, so that the largest loops may end up at the galactic center. (Loops with $R \leq 30$ pc and $\epsilon \leq 10^{-10}$ have masses $M \leq 10^5 M_\odot$. The dynamical friction due to the gravitational interactions of such loops is not sufficient to bring them down to the galactic center.)

The force per unit length of string is given by Eq. (4.4). If the string moves through the plasma with a center-of-mass velocity u , then the forces acting on different parts of the string are not exactly balanced, and the net force is of the order

$$F \sim n m v u r_s R. \quad (4.10)$$

The deceleration time can be estimated by requiring that the work done by this force is comparable to the translational kinetic energy of the loop:

$$F u t \sim \frac{1}{2} \mu R u^2. \quad (4.11)$$

This gives the deceleration time $\sim \tau_d/4$, where τ_d is given by Eq. (4.9). For $\epsilon \leq 10^{-11}$ the deceleration time is less than the age of the Universe, and the largest loops are likely to be found at the galactic center.

To estimate the synchrotron radiation from a string, we use Eq. (3.14) with $\gamma \sim 140$ and δ from Eq. (3.11). This gives

$$\delta \sim 6 \times 10^{11} n^{1/2} B_{-6}^{1/2} R_{20}^{1/2}, \quad (4.12)$$

$$\dot{E}_r \sim 2 \times 10^{32} B_{-6}^3 R_{20}^5 n^2 \text{ erg/sec.} \quad (4.13)$$

The typical wavelength of the radiation is $\lambda \sim 20n^{-0.5}$ cm. [In addition, there may also be x-ray emission at $\lambda_{\text{max}} \sim 10^{-10}$ cm; see Eq. (3.18).]

Up to now we have been assuming that the string is moving into undisturbed plasma. This will be true if the width of the wake of the string, w , is smaller than the distance $d = Ru/2c \simeq 2 \times 10^{-2}$ pc, between successive passages of the string through space. We estimate that $w \simeq d$, so that in reality the plasma may be undisturbed, or, on the other hand, highly disturbed. We have analyzed only the first case in this paper.

We see from Eq. (4.13) that the radio emission of a superconducting string is not large. However, the radio sources associated with such strings should have a rather peculiar appearance: they should look more or less like closed lines with a thickness much smaller than their size. In fact, one such radio source has recently been observed.¹⁴ (The object is called G357.7-0.1.) It has a linear extent of about ~ 36 pc (which corresponds to the total length $R \sim 3 \times 10^{20}$ cm $\sim 3R_0$) and has a number of thin filaments with length/width ratios of up to 60. With $R_{20} \sim 3$ and $B_{-6} \sim 3$, Eqs. (4.13) and (3.15) give the radio luminosity $\dot{E}_r \sim 1 \times 10^{34} n^2$ erg/sec at $\lambda \sim 20n^{-0.5}$ cm, which is not inconsistent with the observed luminosity¹⁴ of the object for $n \simeq 1$ cm⁻³.

The string model of G357.7-0.1 makes a very simple and easily testable prediction: *the filaments should move at relativistic speeds*. For an object at a distance ~ 10 kpc this corresponds to ~ 4 arc sec/yr, and one should be able to observe a displacement of the filaments in a period of a year or so.

Morris and Yusef-Zadeh¹⁵ have recently discovered a remarkable radio source at the galactic center which they call a "thread." This object is much longer (> 30 pc) than it is wide (~ 0.3 pc), curves gently, and has a uniform brightness along its length. It is associated with complex filamentary structure in such a way as to suggest that the filaments interact with the thread.

We tentatively identify the thread as the immediate environs of a cosmic string. Its observed radio emission can be explained if $B_6 \sim 3$, $R_{20} \sim 3$, and $n \sim 0.2$ cm⁻³. The width of the "thread" is much greater than the shock radius, $r_s \sim 10^{14}$ cm. This is not necessarily a problem, since the magnetic reconnection region behind the string can be much wider than r_s .

If the thread is to be interpreted as a string, then its location at the galactic center indicates that it has been affected by frictional deceleration. Then our analysis implies $\tau_d/4 \leq t_0 \leq \tau_d$, which yields a narrow range for the possible values of ϵ : $4 \times 10^{-12} \leq \epsilon \leq 1.6 \times 10^{-11}$. The corresponding symmetry-breaking scale is $\eta \sim 3 \times 10^{13}$ GeV. The filamentary structure accompanying the thread could be part of the turbulent string wake. As in the case of G357.7-0.1, observations of the proper motion of the object could confirm or reject its identification as a string.

V. PARTICLE PRODUCTION

So far we have assumed that the current in the strings is less than the critical current, i_{max} . [Otherwise, Eq. (2.13) and all equations following from it do not apply.] When the current reaches the value i_{max} , the string starts

producing particles and antiparticles at the rate given by Eq. (2.8).

From Eq. (2.13), the current in a loop of length R is

$$i \sim 5 \times 10^{21} B_{-6} R_{20} \quad (5.1)$$

in cgs units. (As noted above, this equation applies as long as $i < i_{\max}$.) For loops in a galaxy $R_{20} < 1$ and the particle production can occur only if $i_{\max} < 10^{22}$ or

$$mc^2 < 10^7 \text{ GeV} . \quad (5.2)$$

If this condition is satisfied, then all loops with $R \geq R_* \sim 3 \times 10^{-6} m_{\text{GeV}} B_{-6}^{-1}$ pc are producing particles. (Here m_{GeV} is the mass m in units of 1 GeV.) Using Eqs. (2.8) and (4.1) we find the total rate of particle production in a galaxy:

$$\begin{aligned} \frac{dN}{dt} &\sim \left(\frac{R_0}{R_*} \right)^{3/2} \frac{eB_0}{\sqrt{2}\hbar} R_* \\ &\sim 8 \times 10^{34} B_{-6}^{3/2} (m_{\text{GeV}})^{-1/2} \text{ sec}^{-1} . \end{aligned} \quad (5.3)$$

The corresponding energy output is

$$\dot{E}_p \sim \frac{dN}{dt} mc^2 \sim 10^{32} B_{-6}^{3/2} m_{\text{GeV}}^{1/2} \text{ erg/sec} . \quad (5.4)$$

From Eq. (5.2) we see that \dot{E}_p does not exceed 2×10^{36} erg/sec. The main contribution to both dN/dt and \dot{E}_p comes from the smallest loops with $i \sim i_{\max}$ (because of their large number).

If we suppose that the energy density in galactic cosmic rays ($\sim 10^{-12}$ erg cm $^{-3}$) is uniformly distributed in a disk of radius 10 kpc and thickness 200 pc, the total energy in galactic cosmic rays is $\sim 2 \times 10^{54}$ erg. The typical lifetime of a cosmic ray in the disk is estimated to be $\sim 10^{14}$ sec, so that the required energy source is $\sim 2 \times 10^{40}$ erg sec $^{-1}$. We see from Eq. (5.4) that direct particle production by cosmic strings is negligible when compared to this.

On the other hand, according to Eq. (4.8) the total amount of energy dissipated by the source G357.7-0.1 if it is a string is 2×10^{41} erg sec $^{-1}$ if $B_{-6} = 3$, $R_{20} = 3$, and $n = 1$. If there is equipartition between electrons and ions behind the shock, the typical particle energy is $\sim \frac{1}{4} m v_0^2 = \frac{1}{8} m c^2 = 70$ MeV, so that the particles accelerated by the shock associated with a cosmic string could serve as a significant source of injected particles. The total power available is sufficient to provide the entire energy of cosmic rays in the galaxy.

Particles may be accelerated up to energies as high as

$$E_{\max} = eB_s r_s = 2 \times 10^5 B_{-6} R_{20} \text{ GeV}$$

by plasma instabilities in the wake of the string, as suggested in Sec. III. Moreover, it can be shown that in general two points on the string attain the velocity of light

during each period of oscillation. In the neighborhood of these points, the motion of a small segment of string of length Δl during a short interval of time Δt is ultrarelativistic, with a typical Lorentz factor $\gamma \sim R / (\Delta l + c\Delta t)$. This suggests that particles can be accelerated to large values of γ , and that the fraction of particles which acquire Lorentz factor γ or larger is proportional to γ^{-2} . This corresponds to a differential energy spectrum like that in Eq. (3.16) with $\alpha = 3$; this value is not far from the observed value for cosmic rays, $\alpha = 2.7$.

VI. SUPERHEAVY STRINGS

Suppose that strings are responsible for galaxy formation (which means that $\epsilon \sim 10^{-6}$), and further, that these strings are superconducting. Then all of the loops in our Galaxy have already decayed by gravitational radiation, and we have to look for such strings elsewhere.

The number density of loops of length $\sim R \geq ct_{\text{eq}} \sim 1$ kpc is given by

$$n_R \sim (c^2 t_0^2 R)^{-1}, \quad (6.1)$$

where $t_{\text{eq}} \sim 10^{11}$ sec is the time when matter and radiation densities are equal and t_0 is the present cosmic time. The typical distance between such loops is $d_R \sim n_R^{-1/3}$. The smallest surviving loops have length

$$R_{\min} \sim 50 \epsilon c t_0 \sim 0.15 \epsilon_{-6} \text{ Mpc} , \quad (6.2)$$

so that the "radius" of such loops is ~ 25 kpc. Here $\epsilon_{-6} = \epsilon / 10^{-6}$. The distance to the nearest loop is $\sim 100 \epsilon_{-6}^{1/3}$ Mpc, and its mass is $\sim 10^{13} \epsilon_{-6}^2 M_{\odot}$. Hence the loops have the size and mass of giant galaxies, and there should be about one in every supercluster.

Normal matter will accumulate in the gravitational potential well of the loop. If the normal matter contains even a small magnetic field, it will interact with the loop in the same manner that we have described for smaller loops, and the gas will be shock heated. If the interaction is strong enough, the gas is unlikely to cool sufficiently to form stars, so that the matter would not form a normal visible galaxy. On the other hand, the gas would emit radio waves which might be observable. Some objects which could conceivably be explained in this way have been observed.¹⁶

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³For strings with a bosonic type of superconductivity, the criti-

- cal current is $i_{\max} \sim e\eta c^2/\hbar$, corresponding to $m \sim \eta$. In this case the particle production process is more complicated, and Eq. (2.8) cannot be used. See Ref. 2 for details.
- ⁴Since the radius of the string is microscopic, in its vicinity the magnetic field may exceed the critical field of QED, 4×10^{13} G. If radiative corrections are ignored, the Dirac vacuum in an intense magnetic field, unlike one in an intense electric field, is stable [M. Soffel, B. Muller, and W. Greiner, *Phys. Rep.* **85**, 51 (1982)]. Whether radiative corrections alter this conclusion is an open question.
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- ⁸It is not difficult to see that $\mathcal{R}_m(x)$ grows with $|x|$ at least like x^2 , and so $\mathcal{R}_m(x) \gg 1$ for $|x| \gg \delta$.
- ⁹The solution (3.6), (3.13) has been discussed in a different context, in E. R. Priest, *Rep. Prog. Phys.* **48**, 955 (1985).
- ¹⁰In the general case, Eqs. (3.6) and (3.13) apply for $|x| \gg \max\{\delta, x_0\}$, where $x_0 \sim B_0 r_s / B_s$ is the point at which B_y in (3.13) becomes $\sim B_s$. The magnetic field in region II cannot become greater than B_s , for otherwise it would produce a pressure greater than that supporting TD on the other side. When B becomes $\sim B_s$, the magnetic pressure becomes comparable to the gas pressure, and the incompressible flow approximation can no longer be used. It is easily checked that $\delta \gg x_0$ for the cases of interest discussed in the following sections.
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- ¹³It is interesting to note that superconducting strings of intermediate energy scale naturally arise in superstring models of elementary particles. See E. Witten, *Phys. Lett.* **153B**, 243 (1985); G. Lazarides, C. Panagiotakopoulos, and Q. Shafi, *Phys. Rev. Lett.* **56**, 432 (1986).
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