Isospin violation in mesons and the constituent-quark masses

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Isospin-violating mass differences and mixing angles in mesons, which arise in part from an intrinsic mass difference between up and down quarks, are studied in a relativized quark potential model. Satisfactory results, comparable to those of nonrelativistic calculations, are obtained, but in addition we find that observed isospin breaking leads to tight constraints on the allowed values of constituent-quark masses: $\frac{1}{2}(m_u+m_d)$ must be between about 200 and 300 MeV.

I. INTRODUCTION

The idea that isospin violations in hadrons are at least partially due to the intrinsic mass difference of the up and down quarks is now fairly well accepted. One approach to the study of such isospin violations is via constituent-quark models based on QCD.[1,2] In this article we reconsider such calculations in the meson sector using a relativized version of the quark model[3] in order to study possible effects due to p/m being of order unity in light-quark systems. We place special emphasis, in fact, not on the quality of our results, which are comparable to those obtained before in the nonrelativistic quark model, but rather on a constraint that isospin splittings put on quark models. We find that the observed splittings require that the constituent masses of the u and d quarks lie in a narrow range around 250 MeV, thereby revealing some important facets of the structure of QCD in the confinement regime.

Consider, as an example of how such a constraint can arise, a relativized quark model in which the familiar Schrödinger Hamiltonian operator is altered by replacing the nonrelativistic kinetic energy term with a relativistic expression:

$$\sum_i \left[m_i + \frac{p_i^2}{2m_i} \right] \to \sum_i (p_i^2 + m_i^2)^{1/2} \; . \tag{1}$$

It has been noted[3] that in such relativized models the spectroscopy of mesons is essentially unchanged for up and down constituent-quark masses in the range of 0 to $\sim$250 MeV upon suitable adjustment of the other parameters of the model. However, including isospin-violating effects puts tight constraints on the allowed values of the constituent-quark masses. To see this, consider $K \equiv (p^2 + m^2)^{1/2}$. If m is changed from m_u to $m_d = m_u + \delta m$, then

$$\delta K = \left[\frac{m_d + m_u}{2K} \right] \delta m \; . \tag{2}$$

From current algebra we know that δm is in the range from 5 to 10 MeV, while experiment tells us that δK is of comparable magnitude. Since for a light quark K will be of the order of $\Lambda_{\rm QCD}$, we see from (2) that the light-quark constituent mass m must also be of the order of $\Lambda_{\rm QCD}$ if isospin splittings of the observed magnitude are to be generated.

This observation leads us to consider what we mean by the constituent-quark mass. The constituent-quark model is based on the approximate saturation of the Fock-space expansion of a hadron by valence-quark configurations. To achieve this saturation the underlying quark and gluon field theory must be cut off at some appropriate scale μ to produce an effective low-energy theory of dressed quarks with a finite size and with a mass (the constituent-quark mass) renormalized at the scale μ. Thus we should certainly not expect the quarks of the constituent-quark model to have current-quark masses.

In this paper we will substantiate the above comments by investigating the phenomenology of isospin violations in mesons using a relativized quark model which includes the main features expected from quantum chromodynamics. In what follows, we first examine isospin splittings in mesons and then look at isoscalar-isovector mixing angles.

II. ISOMULTIPLET MASS DIFFERENCES

The mass difference between two members of an isomultiplet results from several effects:[1,2] (1) the change in K from the shift δm in the quark masses; (2) the change induced by δm in the spin-dependent parts of the potential: for example, in the nonrelativistic limit the Fermi contact term from one-gluon exchange is proportional to

$$\frac{\mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2} \delta^3(\mathbf{r}_{12}) \; ; \tag{3}$$

(3) the change induced by δm in the annihilation amplitudes $A_{qq'}$ for the self-conjugate mesons; (4) electromagnetic interactions.

The calculation of the strong-interaction effects (1)–(3) has been described in detail in Ref. 3; here we simply extend those calculations to the case $m_u \neq m_d$. To calculate the electromagnetic contribution (4) to isospin splittings, we proceed in parallel to the Ref. 3 calculation of chromoelectric and chromomagnetic effects in a relativistic setting. (For example, the effective quark mass in these electromagnetic hyperfine interactions is the same as that appearing in strong hyperfine interactions and in meson magnetic dipole decays in Ref. 3.) The only change we need to make is to introduce a quark-charge smearing function

$$\rho_{\rm em}(\mathbf{r}-\mathbf{r}')\equiv\frac{\gamma_{\rm em}^{3}}{\pi^{3/2}}e^{-\gamma_{\rm em}^{2}(\mathbf{r}-\mathbf{r}')^2} . \tag{4}$$

(Thus $\gamma_{\rm em}$ here is analogous to σ in Ref. 3; it need not, of course, be equal to it.) Physically, this smearing arises from many sources, including relativistic nonlocalities, cutoff dependence, and strong-interaction corrections to the quark current operator. With this one parameter in addition to δm [since we consider here only systems with at least one of the three light quarks, we can ignore the variation of $\gamma_{\rm em}$ with the reduced mass of the system (see, e.g., the variation of σ under these conditions in Ref. 3)] we can then proceed with the calculation of the splittings shown in Table I. The best fit is obtained using reasonable values of δm and $\gamma_{\rm em}$.

Our results are in qualitative agreement with experiment. One quantitative discrepancy on which we would like to comment is the $K^{*+}-K^{*0}$ mass difference where our prediction is considerably smaller than the measured splitting. This theoretical result is a very general one; since

$$\delta m_K\equiv m_{K^+}-m_{K^0}=-\delta m\frac{\partial m_{K^+}}{\partial m_u}+\delta E_{\rm em} \tag{5}$$

and

$$\delta m_{K^*}\equiv m_{K^{*+}}-m_{K^{*0}}=-\delta m\frac{\partial m_{K^{*+}}}{\partial m_u}+\delta E^*_{\rm em} , \tag{6}$$

it follows that

$$\Delta\equiv\delta m_{K^*}-\delta m_K\simeq-\delta m\frac{\partial}{\partial m_u}(m_{K^{*+}}-m_{K^+}) \tag{7}$$

since $\delta E_{\rm em}$ and $\delta E^*_{\rm em}$ are mainly Coulombic. From (7) we see that $\Delta>0$ so that the predicted K^+-K^0 mass difference will be more negative than the K^{*+}-K^{*0} mass difference. This discrepancy with experiment may be due to the neglect of $K\pi$ final-state interactions; in any event, it is interesting because it is such a general property of constituent models.

TABLE I. Meson isomultiplet mass splitting in MeV. $\delta m=+8$ MeV, $\gamma_{\rm em}=0.47$ GeV.

	Strong	Electromagnetic	Other	Total	Experiment
π^+-π^0	+0.1	+3.9		+4.0	+4.6
ρ^+-ρ^0	~0	+2.1	$-1.4^{\rm a}$	+0.7	-0.3 ± 2.2
A_2^+-A_2^0	~0	+1.4		+1.4	
K^+-K^0	−6.5	+2.3		−4.2	-4.0 ± 0.1
K^{*+}-K^{*0}	−3.0	+1.4		−1.6	-4.1 ± 0.6
K_2^+-K_2^0	−3.3	+1.0		−2.3	
D^+-D^0	+3.8	+3.2		+7.0	$+4.7\pm0.3$
D^{*+}-D^{*0}	+2.5	+2.5		+5.0	$+2.9\pm1.0$
D_2^+-D_2^0	+2.6	+1.8		+4.4	
B^+-B^0	−2.7	+1.2		−1.5	
B^{*+}-B^{*0}	−2.3	+1.1		−1.2	

[a]From $\rho^0\to\gamma\to\rho^0$.

III. ISOVECTOR-ISOSCALAR MIXING

In addition to isomultiplet mass splittings, δm will also cause isovector-isoscalar mixing[1,2] in the π-η-$\eta'(P)$, ρ-ω-$\phi(V)$, A_2-f-$f'(T)$, g-ω_3-$\phi_3(3)$, and δ-h-$\phi_4(4)$ systems. To calculate these mixings, we treat the change in the relevant mass matrix due to δm as a perturbation with the unperturbed mass matrix approximated by the observed masses and widths, and with the unperturbed eigenvectors being those of the strong interaction with $m_u=m_d$.[3] There are two contributions to the perturbation in the mass matrix. The first is given simply by the potential model mass difference between unmixed $d\bar d$ and $u\bar u$ mesons of a given type:

$$(m_{d\bar d}-m_{u\bar u})_{\substack{\rm potential\\ \rm model}}=(m_{d\bar d}-m_{u\bar u})_{\substack{\rm unmixed\\ \rm strong}}+(m_{d\bar d}-m_{u\bar u})_{\rm em} ; \tag{8}$$

our calculations give for these differences the values listed in Table II. The second contribution is due to the perturbation of the annihilation matrix which we denote by $\delta A_{qq'}$. The "diagonal" contributions of $\delta A_{qq'}$ are also shown in Table II; the off-diagonal effects of annihilation are negligible ($\delta A_{qq'}<0.1$ MeV) except for the pseudoscalars for which (doing our perturbations around the mean quark mass)

$$\begin{aligned}&\delta A_{ud}\simeq0,\ \delta A_{us}=+2.2,\ \delta A_{ds}=-2.2 ,\\ &\delta A_{uu'}=-1.4,\ \delta A_{ud'}=-1.1,\ \delta A_{us'}=-1.0 ,\\ &\delta A_{du'}=+1.2,\ \delta A_{dd'}=+1.5,\ \delta A_{ds'}=+1.0 ,\end{aligned} \tag{9}$$

where u, d, and s denote the $u\bar u$, $d\bar d$, and $s\bar s$ ground states, u', d', and s' denote the first radial excitations, and all the δA's are in MeV. The mixing problem can then be expressed in terms of the physical masses and the isovector-isoscalar matrix elements:

$$\begin{pmatrix} m_1-i\Gamma_1/2 & m_{10} & m_{10'}\\ m_{10} & m_0-i\Gamma_0/2 & 0\\ m_{10'} & 0 & m_{0'}-i\Gamma_{0'}/2\end{pmatrix}\begin{matrix}|\bar M_1\rangle\\ |\bar M_0\rangle ,\\ |\bar M_{0'}\rangle\end{matrix} \tag{10}$$

where

$$|\bar M_1\rangle=\frac{1}{\sqrt2}(u\bar u-d\bar d)_{1s} , \tag{11a}$$

$$|\bar M_0\rangle=\frac{\alpha}{\sqrt2}(u\bar u+d\bar d)_{1s}+\beta(s\bar s)_{1s}+\frac{\gamma}{\sqrt2}(u\bar u+d\bar d)_{2s}+\varepsilon(s\bar s)_{2s} , \tag{11b}$$

$$|\bar M_{0'}\rangle=\frac{\alpha'}{\sqrt2}(u\bar u+d\bar d)_{1s}+\beta'(s\bar s)_{1s}+\frac{\gamma'}{\sqrt2}(u\bar u+d\bar d)_{2s}+\varepsilon'(s\bar s)_{2s} . \tag{11c}$$

Here $1s$ refers to the ground states and $2s$ to the first radial excitations. The coefficients, taken from Ref. 3, are reproduced for convenience in Table III. As an example of the calculation of the mixing matrix elements in this

TABLE II. Contributions to $m_{d\bar{d}}-m_{u\bar{u}}$.

	$\delta m_{\text{unmixed strong}}$	δm_{em}	$A_{d\bar{d}}-A_{u\bar{u}}$	Total
$P\equiv 1^1S_0$	26.3	2.6	-15.7	13.2
$V\equiv 1^3S_1$	7.5	1.4	~ 0	8.9
$T\equiv 1^3P_2$	7.5	0.9	$+0.4$	8.8
$3\equiv 1^3D_3$	6.6	0.8	~ 0	7.4
$4\equiv 1^3F_4$	5.8	0.7	~ 0	6.5

method, in the pseudoscalars we have

$$\begin{aligned} m_{10} &= \langle \overline{M}_1 | \delta H | \overline{M}_0 \rangle \\ &= \frac{\alpha}{2}(m_{u\bar{u}}-m_{d\bar{d}}) + \frac{\beta}{\sqrt{2}}(\delta A_{us}-\delta A_{ds}) \\ &\quad + \frac{\gamma}{2}(\delta A_{uu'}+\delta A_{ud'}-\delta A_{du'}-\delta A_{dd'}) \\ &\quad + \frac{\varepsilon}{\sqrt{2}}(\delta A_{us'}-\delta A_{ds'}) \ . \end{aligned}$$

In this way one finds

$$\begin{aligned} &P{:}m_{10}=-7.0,\ m_{10'}=-3.3\ , \\ &V{:}m_{10}=-4.4,\ m_{10'}=-0.09\ , \\ &T{:}m_{10}=-4.4,\ m_{10'}=+0.3\ , \\ &3{:}m_{10}=-3.7\ , \\ &4{:}m_{10}=-3.2\ . \end{aligned} \tag{12}$$

These results can be reexpressed in terms of mixing angles defined by

$$\begin{aligned} |M_1\rangle &= |\overline{M}_1\rangle - \chi|\overline{M}_0\rangle - \chi'|\overline{M}_0'\rangle\ , \\ |M_0\rangle &= |\overline{M}_0\rangle + \chi|\overline{M}_1\rangle\ , \\ |M_{0'}\rangle &= |\overline{M}_0\rangle + \chi'|\overline{M}_1\rangle\ , \end{aligned} \tag{13}$$

TABLE III. The approximate composition of some mixed isoscalar mesons according to model $P1$ of Ref. 3. These are amplitude decompositions in terms of the eigenstates in the absence of annihilation. We have denoted $(\frac{1}{2})^{1/2}(u\bar{u}+d\bar{d})$ by ns; the label n refers to the radial quantum number.

	$n=1$		$n=2$	
	ns	$s\bar{s}$	ns	$s\bar{s}$
η	$+0.67$	-0.73	$+0.11$	$+0.042$
η'	$+0.58$	$+0.62$	$+0.47$	$+0.13$
ω	$+1.0$	-0.02		
ϕ	$+0.02$	1.0		
f	$+0.997$	$+0.06$		
f'	-0.07	$+0.997$		

which have the values

$$\begin{aligned} &\chi_P=-0.017,\ \chi_P'=-0.004\ , \\ &\chi_V=-0.06e^{-i80^\circ},\ \chi_V'=-0.0003e^{-i16^\circ}\ , \\ &\chi_T=+0.08e^{-i38^\circ},\ \chi_T'=+0.0015e^{-i5^\circ}\ . \end{aligned} \tag{14}$$

Note that in the 3^{--} and 4^{++} mesons, we obtain large mixing angles but with large errors.

IV. CONCLUSIONS

In the preceding we have presented a description of isospin violations induced by m_d-m_u and electromagnetic effects in a relativized quark model. The resulting picture is reasonably satisfactory. It demonstrates that a relativistic version of the quark potential model can provide a description of isospin-violating effects comparable to that of the usual nonrelativistic model. Equally significant, however, is the observation that relativistic effects on isospin breaking place strong constraints on the values of the light constituent-quark masses.

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