CP-violating Fritzsch mass matrices

Lincoln Wolfenstein
Carnegie-Mellon University, Physics Department, Schenley Park, Pittsburgh, Pennsylvania 15213
(Received 9 April 1986)

The phase-invariant measure J of CP violation defined by Jarlskog is applied to the Fritzsch mass matrices. Given the hierarchy of masses it is shown that the phases in the matrices do not maximize J. Two recent suggestions for maximal CP violation using the Fritzsch form are compared.

A number of recent papers[1-4] have discussed maximum CP violation starting with the Fritzsch form of mass matrix.[5] Writing the normalized Fritzsch mass matrices in Hermitian form[6]

$$M_a=\begin{pmatrix} 0 & A_a e^{i\alpha_a} & 0 \\ A_a e^{-i\alpha_a} & 0 & B_a e^{i\beta_a} \\ 0 & B_a e^{-i\beta_a} & 1 \end{pmatrix} , \qquad (1)$$

where $a=u$ or d and the normalization consists of dividing by M_t or M_b. The factors A_a, B_b determined from the mass eigenvalues are given approximately by

$$B_d \approx \left(\frac{M_s}{M_b}\right)^{1/2}, \quad B_u \approx \left(\frac{M_c}{M_t}\right)^{1/2} ,$$
$$A_d \approx \left(\frac{M_d}{M_s}\right)^{1/2}\left(\frac{M_s}{M_b}\right), \quad A_w \approx \left(\frac{M_u}{M_c}\right)^{1/2}\left(\frac{M_c}{M_t}\right) . \qquad (2)$$

Since it is possible to change the relative phases of differently flavored quark doublets, the only significant phases are

$$\alpha=\alpha_u-\alpha_d, \quad \beta=\beta_u-\beta_d . \qquad (3)$$

A given choice of these phases determines the Kobayashi-Maskawa (KM) matrix elements V_{ij} as functions of the quark masses. A discussion of the phenomenology following from different choices of α and β has been given by Shin.[3]

Here we want to analyze the CP violation using the method of Jarlskog.[7] She considers the determinant of the commutator of the mass matrices

$$\det(M_u,M_d)=-2iF_uF_dJ , \qquad (4a)$$

$$F_u=(M_t-M_c)(M_t-M_u)(M_c-M_u)/M_t^3 \approx M_c/M_t , \qquad (4b)$$

$$F_d=(M_b-M_s)(M_b-M_d)(M_s-M_d)/M_b^3 \approx M_s/M_b , \qquad (4c)$$

$$J=\mathrm{Im}(V_{11}V_{22}V_{12}^*V_{21}^*) . \qquad (4d)$$

The quantity J is invariant with respect to any phase transformations allowed for the KM matrix; further, all CP-violating intensities contain this factor.[8]

From Eqs. (1), (2), and (4) we obtain

$$J \approx \sin\alpha \left(\frac{M_d}{M_s}\right)^{1/2}\left(\frac{M_u}{M_c}\right)^{1/2}\left|\left(\frac{M_s}{M_b}\right)^{1/2}-\left(\frac{M_c}{M_t}\right)^{1/2}e^{-i\beta}\right|^2 - \sin\beta\left(\frac{M_s}{M_b}\right)^{1/2}\left(\frac{M_c}{M_t}\right)^{1/2}\frac{M_d}{M_b}+\cdots , \qquad (5)$$

where the approximation corresponds to those indicated in Eqs. (2), (4b), and (4c), and the terms omitted in Eq. (5) are definitely smaller given the known quark masses. The order of magnitude of terms may be found using the mnemonic of Ref. 2:

$$M_u:M_c:M_t=\lambda^6:\lambda^2:1 ,$$
$$M_d:M_s:M_b=\lambda^4:\lambda^2:1 , \qquad (6)$$

where $\lambda \approx 0.22$ is the sine of the Cabibbo angle, and it is assumed that M_t is of order 40 GeV. It appears at first that

$$J \sim (\sin\alpha)\lambda^5+(\sin\beta)\lambda^6 . \qquad (7)$$

Indeed, if α and β are chosen to maximize J, we find $J \sim \lambda^5$. This would give a value of the CP-violating parameter $\varepsilon \sim \lambda^3$. If we accept the Fritzsch form and the empirical mass hierarchy, this choice may be called maximal CP violation. However, as discussed in the next paragraph, this maximal CP violation is ruled out by the empirical value of $|V_{cb}|$.

Experiments on the lifetime of B mesons can be summarized[9] by

$$|V_{cb}|=A\lambda^2 , \qquad (8)$$

with A approximately equal to unity. From the Fritzsch form,

$$|V_{cb}| \approx \left|\left(\frac{M_s}{M_b}\right)^{1/2}-\left(\frac{M_c}{M_t}\right)^{1/2}e^{-i\beta}\right| . \qquad (9)$$

As has been noted in many papers, it is necessary that the two terms in Eq. (9) approximately cancel in order to agree with Eq. (8). This has two consequences: (1) the first term in Eq. (5) is of order $\lambda^7\sin\alpha$; (2) $\beta \leq \lambda$. It follows that both terms in Eq. (5) are at most of order λ^7. As a consequence $\varepsilon \sim \lambda^5$.

In Refs. 2-4 the condition of maximal CP violation is chosen as

$$\alpha=\frac{\pi}{2},\ \beta=0. \tag{10}$$

Thus,

$$J=\left(\frac{M_d}{M_s}\right)^{1/2}\left(\frac{M_u}{M_c}\right)^{1/2}|V_{cb}|^2 . \tag{11}$$

In contrast, Gronau, Johnson, and Schechter[1] (GJS), following Stech,[10] write the Fritzsch matrices as

$$M_d=M_u+D\ , \tag{12}$$

with M_u symmetric and CP conserving, while D is antisymmetric and CP violating. It then follows using (2) and (6) that

$$M_u\sim\begin{pmatrix}0 & \lambda^4 & 0\\ \lambda^4 & 0 & \lambda\\ 0 & \lambda & 1\end{pmatrix},\quad D\sim\begin{pmatrix}0 & \pm i\lambda^3 & 0\\ \mp i\lambda^3 & 0 & \pm i\lambda^2\\ 0 & \mp i\lambda^2 & 0\end{pmatrix}. \tag{13}$$

The term in D proportional to λ^2 is required by Eq. (9) in order to agree with Eq. (8). From Eq. (13) we see that GJS have

$$\alpha\approx\frac{\pi}{2}\pm\lambda,\ \beta=\pm\lambda\ . \tag{14}$$

This is an equally good definition of CP violation since $D=M_d-M_u$ has maximum CP-violating phases. From Eqs. (5) and (9) we then find

$$J\simeq\left(\frac{M_d}{M_s}\right)^{1/2}\left(\frac{M_u}{M_c}\right)^{1/2}|V_{cb}|^2\mp|V_{cb}|\left(\frac{M_s}{M_b}\right)^{1/2}\left(\frac{M_d}{M_b}\right) . \tag{15}$$

Choosing the plus sign gives roughly twice the value of J given in Eq. (11).

It should be emphasized that the use of an expansion in powers of λ is very crude, both because Eq. (6) is very approximate and because λ is not very small. However, we believe it is useful in obtaining a qualitative picture of CP violation for the Fritzsch mass matrix. In conclusion, we have found that given the mass hierarchy of Eq. (6) the CP-violation measure J is at most of order $\lambda^5(\varepsilon\sim\lambda^3)$. When the empirical constraint on V_{cb} given by Eq. (8) is also considered it follows that J is at most of order $\lambda^7(\varepsilon\sim\lambda^5)$. Two definitions of maximal CP violation (Ref. 1 and Refs. 2–4) which are designed to meet this empirical constraint have been compared.

I wish to thank Ling-Fong Li and Jiang Liu for useful discussions. This research has been supported in part by the U.S. Department of Energy.

[1] M. Gronau, R. Johnson, and J. Schechter, Phys. Rev. Lett. **54**, 2176 (1985).

[2] T. P. Cheng and L. F. Li, Phys. Rev. Lett. **55**, 2249 (1985).

[3] M. Shin, Phys. Lett. **145B**, 285 (1984).

[4] A. Davidson *et al.*, Phys. Rev. D **29**, 1513 (1984).

[5] H. Fritzsch, Phys. Lett. **73B**, 317 (1978); Nucl. Phys. **B155**, 189 (1979).

[6] P. H. Frampton and C. Jarlskog, Phys. Lett. **154B**, 421 (1985).

[7] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985).

[8] Independent discussions of the quantity J have been given by O. W. Greenberg, Phys. Rev. D **32**, 1841 (1985); D. D. Wu, *ibid.* **33**, 860 (1986).

[9] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[10] B. Stech, Phys. Lett. **130B**, 189 (1983).